$Y(2175)$ state in the QCD sum rule

Hua-Xing Chen,[1,2,*] Xiang Liu,[1,+] Atsushi Hosaka,[2,‡] and Shi-Lin Zhu[1,§]

[1]*Department of Physics, Peking University, Beijing 100871, China*
[2]*Research Center for Nuclear Physics, Osaka University, Ibaraki 567–0047, Japan*
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We study the mass of the state $Y(2175)$ of $J^{PC} = 1^{--}$ in the QCD sum rule. We construct both the diquark-antidiquark currents $(ss)(\bar{s}\,\bar{s})$ and the meson-meson currents $(\bar{s}s)(\bar{s}s)$. We find that there are two independent currents for both cases and derive the relations between them. The operator product expansion convergence of these two currents is sufficiently fast, which enables us to perform good sum rule analysis. Both the SVZ (Shifman-Vainshtein-Zakharov) sum rule and the finite energy sum rule lead to a mass around 2.3 ± 0.4 GeV, which is consistent with the observed mass within the uncertainties of the present QCD sum rule. The coupling of the four-quark currents to lower lying states such as $\phi(1020)$ turns out to be rather small. We also discuss possible decay properties of $Y(2175)$ if it is a tetraquark state.

I. INTRODUCTION

The theory of the strong interactions, quantum chromodynamics (QCD), originated from the systematics of hadron spectroscopy. The spectroscopy contains meson and baryon states, many of which are well classified by the quark model with the quark content $q\bar{q}$ and qqq. Besides the quark model, QCD allows much richer hadron spectrum such as multiquark states, hadron molecules, hybrid states, glueballs, etc. However, the spectrum of QCD seems to saturate at $q\bar{q}$ and qqq. Since 2003, there have been important developments in hadron spectroscopy, which is triggered by the observation of the pentaquark Θ^+. After three years of intense study, the status of Θ^+ is still controversial. However, we have the charm-strange mesons $D_{sJ}(2317)$, $D_{sJ}(2460)$ [1,2]; the charmonium state $X(3872)$ [3], $Y(4260)$ [4], and many Xs and Ys, whose properties seem difficult to be explained by the conventional picture of $q\bar{q}$.

Recently *BABAR* Collaboration observed a resonance $Y(2175)$ near the threshold in the process $e^+e^- \rightarrow \phi f_0(980)$ via initial-state radiation [5–7]. It has the quantum numbers $J^{PC} = 1^{--}$. The Breit-Wigner mass is $M = 2.175 \pm 0.010 \pm 0.015$ GeV, and width is $\Gamma = 0.058 \pm 0.016 \pm 0.020$ GeV. It has been also confirmed by BES Collaboration in the process $J/\psi \rightarrow \eta\phi f_0(980)$. A fit with a Breit-Wigner function gives the peak mass and width of $M = 2.186 \pm 0.010 \pm 0.006$ GeV and $\Gamma = 0.065 \pm 0.023 \pm 0.017$ GeV [8].

There are many suggestions to interpret this resonance. Ding and Yan interpreted it as a strangeonium hybrid and studied its decay properties in the flux-tube model and the constituent gluon model. Furthermore, for testing

$s\bar{s}g$ scenario, they suggested searching decay modes such as $Y(2175) \rightarrow K_1(1400)K \rightarrow \pi K^*(892)K$, $Y(2175) \rightarrow K_1(1270)K \rightarrow \rho KK$, and $Y(2175) \rightarrow K_1(1270)K \rightarrow \pi K_0^*(1430)$ [9]. In Ref. [10], the authors explored $Y(2175)$ as a 2^3D_1 $s\bar{s}$ meson, and calculated its decay modes by using both the $^3P_0$ model and the flux-tube model. They suggested experimental search of the decay modes KK, K^*K^*, $K(1460)K$, and $h_1(1380)\eta$. The characteristic decay modes of $Y(2175)$ as either a hybrid state or an $s\bar{s}$ state are quite different, which may be used to distinguish the hybrid and $s\bar{s}$ schemes. Wang studied $Y(2175)$ as a tetraquark state $ss\bar{s}\,\bar{s}$ by using QCD sum rule and suggested that there may be some tetraquark components in the state $Y(2175)$ [11]. In a recent article [12], Zhu reviewed $Y(2175)$ and indicated that the possibility of $Y(2175)$ arising from S-wave threshold effects can not be excluded. Napsuciale, Oset, Sasaki, and Vaquera-Araujo studied the reaction $e^+e^- \rightarrow \phi\pi\pi$ for pions in an isoscalar S-wave channel which is dominated by the loop mechanism. By selecting the $\phi f_0(980)$ contribution as a function of the e^+e^- energy, they also reproduced the experimental data except for the narrow peak [13]. Bystritskiy, Volkov, Kuraev, Bartos, and Secansky calculated the total probability and the differential cross section of the process $e^+e^- \rightarrow \phi f_0(980)$ by using the local NJL model [14]. Anikin, Pire, and Teryaev studied the reaction $\gamma^*\gamma \rightarrow \rho\rho$, and calculated the mass of the isotensor exotic meson [15]. In Ref. [16], the authors performed a QCD sum rule study for 1^{--} hybrid meson, and the mass is predicted to be 2.3–2.4, 2.3–2.5, and 2.5–2.6 GeV for $q\bar{q}g$, $q\bar{s}g$, and $s\bar{s}g$, respectively.

In this work, we revisit the possibility of $Y(2175)$ as a tetraquark state $ss\bar{s}\,\bar{s}$. With the approach developed in our previous work [17], we construct the general tetraquark interpolating currents with the quantum numbers $J^{PC} = 1^{--}$. We find that there are two independent currents. They can have a structure of diquark-antidiquark $(ss)(\bar{s}\,\bar{s})$, or

*hxchen@rcnp.osaka-u.ac.jp
+xiangliu@pku.edu.cn
‡hosaka@rcnp.osaka-u.ac.jp
§zhusl@phy.pku.edu.cn

have a structure of meson-meson $(\bar{s}s)(\bar{s}s)$. We show that they are equivalent, and derive the relations between them. Then by using these two independent currents, we also perform a QCD sum rule analysis. We calculate the OPE up to the dimension 12, which contains the $\langle \bar{q}q \rangle^4$ condensates. In these two respects, our study differs from the previous one of Ref. [11].

This paper is organized as follows. In Sec. II, we construct the tetraquark currents using both diquark (qq) and antidiquark $(\bar{q}\bar{q})$ fields, as well as quark-antiquark $(\bar{q}q)$ pairs. In Sec. III, we perform a QCD sum rule analysis by using these currents. In Sec. IV, the numerical result is obtained for the mass of $Y(2175)$. In Sec. V, we use finite energy sum rule to calculate its mass again. Section VI is a summary.

II. INTERPOLATING CURRENTS

In this section, we construct currents for the state $Y(2175)$ of $J^{PC} = 1^{--}$. From the decay pattern $Y(2175) \rightarrow \phi(1020)f_0(980)$, we expect that there is a large $ss\bar{s}\bar{s}$ component in $Y(2175)$. We may add further quark and antiquark pairs, but the simplest choice would be $ss\bar{s}\bar{s}$. We will discuss later how this simplest quark content will be compatible with the above decay pattern when considering the possible structure of $\phi(1020)$ and $f_0(980)$.

Let us now briefly see the flavor structure of the current. In the diquark-antidiquark construction $(ss)(\bar{s}\bar{s})$ where ss and $\bar{s}\bar{s}$ pairs have a symmetric flavor structure, the flavor decomposition goes as

$$\mathbf{6_f} \otimes \mathbf{\bar{6}_f} = \mathbf{1_f} \oplus \mathbf{8_f} \oplus \mathbf{27_f}. \quad (1)$$

Therefore, the $(ss)(\bar{s}\bar{s})$ state is a mixing of 1_f, 8_f, and 27_f multiplets in the ideal mixing scheme.

Now we find that there are two nonvanishing currents for each state with the quantum number $J^{PC} = 1^{--}$. For the state $ss\bar{s}\bar{s}$:

$$\eta_{1\mu} = (s_a^T C \gamma_5 s_b)(\bar{s}_a \gamma_\mu \gamma_5 C \bar{s}_b^T) - (s_a^T C \gamma_\mu \gamma_5 s_b)$$
$$\times (\bar{s}_a \gamma_5 C \bar{s}_b^T), \quad (2)$$

$$\eta_{2\mu} = (s_a^T C \gamma^\nu s_b)(\bar{s}_a \sigma_{\mu\nu} C \bar{s}_b^T) - (s_a^T C \sigma_{\mu\nu} s_b)(\bar{s}_a \gamma^\nu C \bar{s}_b^T), \quad (3)$$

where the sum over repeated indices (μ for Dirac spinor indices, and a, b for color indices) is taken. $C = i\gamma_2\gamma_0$ is the Dirac field charge conjugation operator, and the superscript T represents the transpose of the Dirac indices only.

Besides the diquark-antidiquark currents, we can also construct the tetraquark currents by using quark-antiquark $(\bar{s}s)$ pairs. We find that there are four nonvanishing currents:

$$\eta_{3\mu} = (\bar{s}_a s_a)(\bar{s}_b \gamma_\mu s_b),$$

$$\eta_{4\mu} = (\bar{s}_a \gamma^\nu \gamma_5 s_a)(\bar{s}_b \sigma_{\mu\nu} \gamma_5 s_b),$$

$$\eta_{5\mu} = \lambda_{ab}\lambda_{cd}(\bar{s}_a s_b)(\bar{s}_c \gamma_\mu s_d),$$

$$\eta_{6\mu} = \lambda_{ab}\lambda_{cd}(\bar{s}_a \gamma^\nu \gamma_5 s_b)(\bar{s}_c \sigma_{\mu\nu} \gamma_5 s_d).$$

In Ref. [11], the author used $\eta_{5\mu}$ to perform QCD sum rule analysis, which is a mixing of $\eta_{1\mu}$ and $\eta_{2\mu}$. We can verify the following relations by using the Fierz transformation:

$$\eta_{5\mu} = -\tfrac{5}{3}\eta_{3\mu} - i\eta_{4\mu}, \qquad \eta_{6\mu} = 3i\eta_{3\mu} + \tfrac{1}{3}\eta_{4\mu}. \quad (4)$$

Therefore, among the four $(\bar{q}q)(\bar{q}q)$ currents, two are independent. We can also verify the relations between $(ss)(\bar{s}\bar{s})$ currents and $(\bar{s}s)(\bar{s}s)$ currents, by using the Fierz transformation:

$$\eta_{1\mu} = -\eta_{3\mu} + i\eta_{4\mu}, \qquad \eta_{2\mu} = 3i\eta_{3\mu} - \eta_{4\mu}. \quad (5)$$

Therefore, these two constructions are equivalent, and we will use $\eta_{1\mu}$ and $\eta_{2\mu}$ for QCD sum rule analysis.

III. QCD SUM RULE ANALYSIS

For the past decades QCD sum rule has proven to be a very powerful and successful nonperturbative method [18,19], and it has been applied to study tetraquark states in many references [17,20–25]. In sum rule analyses, we consider two-point correlation functions:

$$\Pi_{\mu\nu}(q^2) \equiv i \int d^4x e^{iqx} \langle 0|T\eta_\mu(x)\eta_\nu^\dagger(0)|0\rangle, \quad (6)$$

where η_μ is an interpolating current for the tetraquark. The Lorentz structure can be simplified to be

$$\Pi_{\mu\nu}(q^2) = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi^{(1)}(q^2) + \frac{q_\mu q_\nu}{q^2}\Pi^{(0)}(q^2). \quad (7)$$

We compute $\Pi(q^2)$ in the operator product expansion (OPE) of QCD up to certain order in the expansion, which is then matched with a hadronic parametrization to extract information of hadron properties. At the hadron level, we express the correlation function in the form of the dispersion relation with a spectral function:

$$\Pi^{(1)}(q^2) = \int_{16m_s^2}^{\infty} \frac{\rho(s)}{s - q^2 - i\varepsilon}ds, \quad (8)$$

where the subscript is $(4m_s)^2 = 16m_s^2$, and

$$\rho(s) \equiv \sum_n \delta(s - M_n^2)\langle 0|\eta|n\rangle\langle n|\eta^\dagger|0\rangle$$
$$= f_Y^2 \delta(s - M_Y^2) + \text{higher states}. \quad (9)$$

For the second equation, as usual, we adopt a parametrization of one pole dominance for the ground state Y and a continuum contribution. The sum rule analysis is then performed after the Borel transformation of the two expressions of the correlation function, (6) and (8)

$$\Pi^{(\text{all})}(M_B^2) \equiv \mathcal{B}_{M_B^2}\Pi^{(1)}(p^2) = \int_{16m_s^2}^{\infty} e^{-s/M_B^2}\rho(s)ds. \quad (10)$$

Assuming the contribution from the continuum states can be approximated well by the spectral density of OPE above a threshold value s_0 (duality), we arrive at the sum rule equation

$$\Pi(M_B^2) \equiv f_Y^2 e^{-M_Y^2/M_B^2} = \int_{16m_s^2}^{s_0} e^{-s/M_B^2}\rho(s)ds. \quad (11)$$

Differentiating Eq. (11) with respect to $1/M_B^2$ and dividing it by Eq. (11), finally we obtain

$$M_Y^2 = \frac{\frac{\partial}{\partial(-1/M_B^2)}\Pi(M_B^2)}{\Pi(M_B^2)} = \frac{\int_{16m_s^2}^{s_0} e^{-s/M_B^2}s\rho(s)ds}{\int_{16m_s^2}^{s_0} e^{-s/M_B^2}\rho(s)ds}. \quad (12)$$

In the following, we study both Eqs. (11) and (12) as functions of the parameters such as the Borel mass M_B and the threshold value s_0 for various combinations of the tetraquark currents.

For the currents $\eta_{1\mu}$ and $\eta_{2\mu}$, we have calculated the OPE up to dimension 12, which contains the $\langle\bar{q}q\rangle^4$ condensate:

$$\begin{aligned}
\Pi_1(M_B^2) = \int_{16m_s^2}^{s_0}&\left[\frac{s^4}{18432\pi^6} - \frac{m_s^2 s^3}{256\pi^6} + \left(-\frac{\langle g^2 GG\rangle}{18432\pi^6} + \frac{m_s\langle\bar{s}s\rangle}{48\pi^4}\right)s^2 + \left(\frac{\langle\bar{s}s\rangle^2}{18\pi^2} - \frac{m_s\langle g\bar{s}\sigma Gs\rangle}{48\pi^4} + \frac{17m_s^2\langle g^2 GG\rangle}{9216\pi^6}\right)s\right.\\
&\left.+ \left(\frac{\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{12\pi^2} - \frac{m_s\langle g^2 GG\rangle\langle\bar{s}s\rangle}{128\pi^4} - \frac{29m_s^2\langle\bar{s}s\rangle^2}{12\pi^2}\right)\right]e^{-s/M_B^2}ds + \left(\frac{5\langle g^2 GG\rangle\langle\bar{s}s\rangle^2}{864\pi^2} + \frac{\langle g\bar{s}\sigma Gs\rangle^2}{48\pi^2} + \frac{20m_s\langle\bar{s}s\rangle^3}{9}\right.\\
&\left.- \frac{5m_s\langle g^2 GG\rangle\langle g\bar{s}\sigma Gs\rangle}{2304\pi^4} - \frac{3m_s^2\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{2\pi^2}\right) + \frac{1}{M_B^2}\left(-\frac{32g^2\langle\bar{s}s\rangle^4}{81} - \frac{\langle g^2 GG\rangle\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{576\pi^2}\right.\\
&\left.- \frac{10m_s\langle\bar{s}s\rangle^2\langle g\bar{s}\sigma Gs\rangle}{9} + \frac{m_s^2\langle g^2 GG\rangle\langle\bar{s}s\rangle^2}{576\pi^2} + \frac{m_s^2\langle g\bar{s}\sigma Gs\rangle^2}{12\pi^2}\right),
\end{aligned} \quad (13)$$

$$\begin{aligned}
\Pi_2(M_B^2) = \int_{16m_s^2}^{s_0}&\left[\frac{s^4}{12288\pi^6} - \frac{3m_s^2 s^3}{512\pi^6} + \left(\frac{\langle g^2 GG\rangle}{18432\pi^6} + \frac{m_s\langle\bar{s}s\rangle}{32\pi^4}\right)s^2 + \left(\frac{\langle\bar{s}s\rangle^2}{12\pi^2} - \frac{m_s\langle g\bar{s}\sigma Gs\rangle}{32\pi^4} + \frac{35m_s^2\langle g^2 GG\rangle}{9216\pi^6}\right)s\right.\\
&\left.+ \left(\frac{\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{8\pi^2} - \frac{3m_s\langle g^2 GG\rangle\langle\bar{s}s\rangle}{128\pi^4} - \frac{29m_s^2\langle\bar{s}s\rangle^2}{8\pi^2}\right)\right]e^{-s/M_B^2}ds + \left(\frac{5\langle g^2 GG\rangle\langle\bar{s}s\rangle^2}{288\pi^2} + \frac{\langle g\bar{s}\sigma Gs\rangle^2}{32\pi^2} + \frac{10m_s\langle\bar{s}s\rangle^3}{3}\right.\\
&\left.- \frac{5m_s\langle g^2 GG\rangle\langle g\bar{s}\sigma Gs\rangle}{768\pi^4} - \frac{9m_s^2\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{4\pi^2}\right) + \frac{1}{M_B^2}\left(-\frac{16g^2\langle\bar{s}s\rangle^4}{27} - \frac{\langle g^2 GG\rangle\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{192\pi^2}\right.\\
&\left.- \frac{5m_s\langle\bar{s}s\rangle^2\langle g\bar{s}\sigma Gs\rangle}{3} - \frac{m_s^2\langle g^2 GG\rangle\langle\bar{s}s\rangle^2}{576\pi^2} + \frac{m_s^2\langle g\bar{s}\sigma Gs\rangle^2}{8\pi^2}\right).
\end{aligned} \quad (14)$$

In the above equations, $\langle\bar{s}s\rangle$ is the dimension $D = 3$ strange quark condensate; $\langle g^2 GG\rangle$ is a $D = 4$ gluon condensate; $\langle g\bar{s}\sigma Gs\rangle$ is $D = 5$ mixed condensate. There are many terms which give minor contributions, such as $\langle g^3 G^3\rangle$, and we omit them. As usual, we assume the vacuum saturation for higher dimensional condensates such as $\langle 0|\bar{q}q\bar{q}q|0\rangle \sim \langle 0|\bar{q}q|0\rangle\langle 0|\bar{q}q|0\rangle$. To obtain these results, we keep the terms of order $O(m_q^2)$ in the propagators of a massive quark in the presence of quark and gluon condensates:

$$\begin{aligned}
iS^{ab} &\equiv \langle 0|T[q^a(x)q^b(0)]|0\rangle\\
&= \frac{i\delta^{ab}}{2\pi^2 x^4}\hat{x} + \frac{i}{32\pi^2}\frac{\lambda_{ab}^n}{2}g_c G_{\mu\nu}^n\frac{1}{x^2}(\sigma^{\mu\nu}\hat{x} + \hat{x}\sigma^{\mu\nu})\\
&\quad- \frac{\delta^{ab}}{12}\langle\bar{q}q\rangle + \frac{\delta^{ab}x^2}{192}\langle g_c\bar{q}\sigma Gq\rangle - \frac{m_q\delta^{ab}}{4\pi^2 x^2}\\
&\quad+ \frac{i\delta^{ab}m_q\langle\bar{q}q\rangle}{48}\hat{x} + \frac{i\delta^{ab}m_q^2}{8\pi^2 x^2}\hat{x}.
\end{aligned} \quad (15)$$

We find that there is an approximate relation between the correlation functions of $\eta_{1\mu}$ and $\eta_{2\mu}$:

$$3\Pi_1(M_B^2) \sim 2\Pi_2(M_B^2), \quad (16)$$

which is valid for the continuum, $\langle\bar{s}s\rangle$, and $\langle g_c\bar{q}\sigma Gq\rangle$ terms, etc. So the numerical results by using them are also very similar.

IV. NUMERICAL ANALYSIS

In our numerical analysis, we use the following values for various condensates and m_s at 1 GeV and α_s at 1.7 GeV [26–33]:

$$\begin{aligned}
\langle\bar{q}q\rangle &= -(0.240 \text{ GeV})^3,\\
\langle\bar{s}s\rangle &= -(0.8 \pm 0.1) \times (0.240 \text{ GeV})^3,\\
\langle g_s^2 GG\rangle &= (0.48 \pm 0.14) \text{ GeV}^4,\\
\langle g_s\bar{q}\sigma Gq\rangle &= -M_0^2 \times \langle\bar{q}q\rangle,\\
M_0^2 &= (0.8 \pm 0.2) \text{ GeV}^2,\\
m_s(1 \text{ GeV}) &= 125 \pm 20 \text{ MeV},\\
\alpha_s(1.7 \text{ GeV}) &= 0.328 \pm 0.03 \pm 0.025.
\end{aligned} \quad (17)$$

There is a minus sign in the definition of the mixed condensate $\langle g_s\bar{q}\sigma Gq\rangle$, which is different from that used in some other QCD sum rule studies. This difference just comes from the definition of coupling constant g_s [27,34].

First we want to study the convergence of the operator product expansion, which is the cornerstone of the reliable QCD sum rule analysis. By taking s_0 to be ∞ and the integral subscript $16m_s^2$ to be zero, we obtain the numerical series of the OPE as a function of M_B:

$$\Pi_1(M_B^2) = 1.4 \times 10^{-6}M_B^{10} - 3.8 \times 10^{-7}M_B^8 - 6.2$$
$$\times 10^{-7}M_B^6 + 4.2 \times 10^{-7}M_B^4 - 1.2$$
$$\times 10^{-6}M_B^2 + 4.7 \times 10^{-8} - 1.5 \times 10^{-7}M_B^{-2},$$
(18)

$$\Pi_2(M_B^2) = 2.0 \times 10^{-6}M_B^{10} - 5.7 \times 10^{-7}M_B^8 - 8.0$$
$$\times 10^{-7}M_B^6 + 6.4 \times 10^{-7}M_B^4 - 1.7$$
$$\times 10^{-6}M_B^2 + 1.0 \times 10^{-7} - 2.2 \times 10^{-7}M_B^{-2}.$$
(19)

After careful testing of the free parameter Borel mass M_B, we find for $M_B^2 > 2$ GeV^2, which is the region suitable for the study of $Y(2175)$, the Borel mass dependence is weak. Moreover, the convergence of the OPE is satisfied in this region. The correlation function of the current $\eta_{1\mu}$ is shown in Fig. 1, when we take $s_0 = 5.7$ GeV^2 (the integral subscript is still $16m_s^2$). We find that in the region of 2 GeV$^2 < M_B^2 < 5$ GeV^2, the perturbative term (the solid line in Fig. 1) gives the most important contribution, and the convergence is quite good.

It is important to note that the $Y(2175)$ state is not the lowest state in the 1^{--} channel containing $s\bar{s}$ and that the interpolating currents see only the quantum number of the states. It is possible that the low-lying states $\phi(1020)$ and
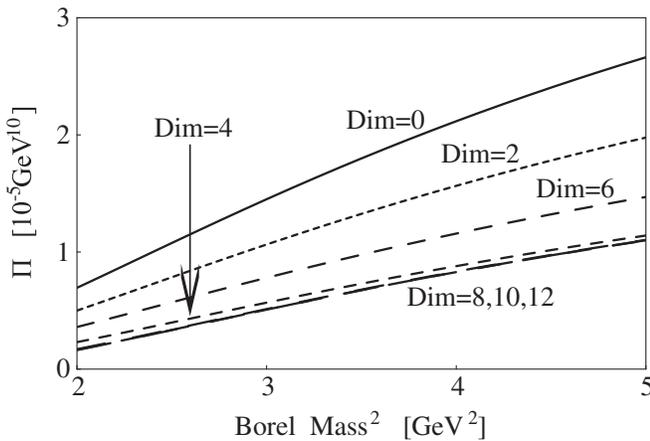
$\phi(1680)$ also couple to the tetraquark currents $\eta_{1\mu}$ and $\eta_{2\mu}$. If so, their contribution to the spectral density and the resulting correlation function should be positive definite.

However, we find that (1) the spectral densities $\rho(s)$ of Eq. (9) for both currents $\eta_{1\mu}$ and $\eta_{2\mu}$ are negative when $s < 2$ GeV^2; (2) the Borel transformed correlation function $\Pi(M_B^2)$ in Eq. (11) is also negative in the region $s_0 < 4.3$ GeV^2 and 1 GeV$^2 < M_B^2 < 4$ GeV^2. As an illustration, we show the correlation function as a function of s_0 in Fig. 2. This fact indicates that the $ss\bar{s}\bar{s}$ tetraquark currents couple weakly to the lower states $\phi(1020)$ and $\phi(1680)$ in the present QCD sum rule analysis.

The pole contribution is not large enough for both currents due to the $D = 10$ perturbative term $\int_0^{s_0} e^{-s/M_B^2}s^4 ds$, which is a common feature for any multiquark interpolating currents with high dimensions. The mixing of the currents $\eta_{1\mu}$ and $\eta_{2\mu}$ does not improve the rate of the pole contribution. The small pole contribution suggests that the continuum contribution to the spectral density is dominant, which demands a very careful choice of the parameters of the QCD sum rule. In our numerical analysis, we require the extracted mass have a dual minimum dependence on both the Borel parameter M_B and the threshold parameter s_0. In this way, we can find a good working region of M_B and s_0 (Borel window), where the mass of $Y(2175)$ can be determined reliably.

Now the mass is shown as functions of the Borel mass M_B and the threshold value s_0 in Figs. 3 and 4. The threshold value is taken to be around $5 \sim 7$ GeV^2, where its square root is around $2.2 \sim 2.7$ GeV. We find that there is a mass minimum around 2.4 GeV for the current $\eta_{1\mu}$, when we take $M_B^2 \sim 4$ GeV^2 and $s_0 \sim 5.7$ GeV^2, while this minimum is around 2.3 GeV for the current $\eta_{2\mu}$, when we take $M_B^2 \sim 4$ GeV^2 and $s_0 \sim 5.4$ GeV^2.

In short summary, we have performed the QCD sum rule analysis for both $\eta_{1\mu}$ and $\eta_{2\mu}$. The obtained results are

FIG. 1.   Various contribution to the correlation function for the current $\eta_{1\mu}$ as functions of the Borel mass M_B in units of GeV^{10} at $s_0 = 5.7$ GeV^2. The labels indicate the dimension up to which the OPE terms are included.
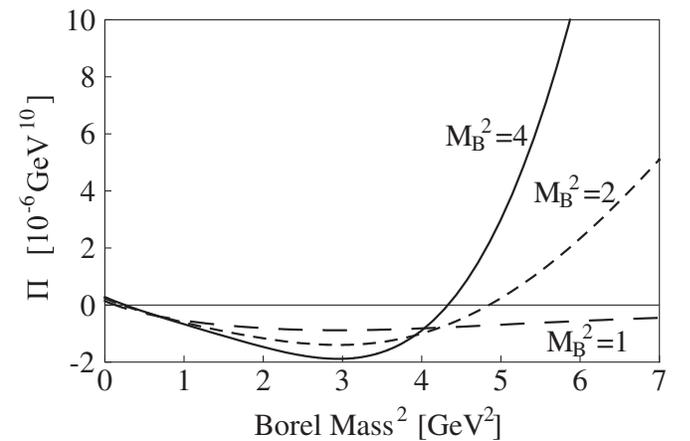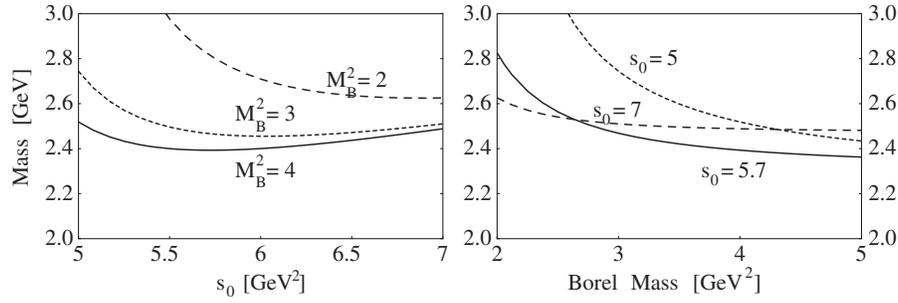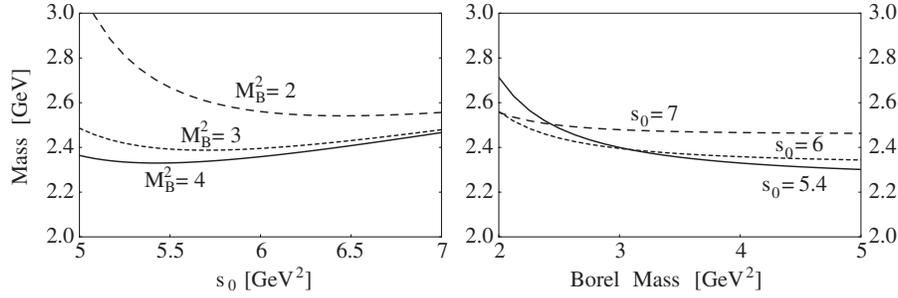

FIG. 2.   The correlation function for the current $\eta_{1\mu}$ as a function of s_0 in units of GeV^{10}. The curves are obtained by setting $M_B^2 = 1$ GeV^2 (long-dashed line), 2 GeV^2 (short-dashed line), and 4 GeV^2 (solid line).

FIG. 3.   The mass of $Y(2175)$ as a function of M_B (left) and s_0 (right) in units of GeV for the current $\eta_{1\mu}$.

FIG. 4.   The mass of $Y(2175)$ as a function of M_B (left) and s_0 (right) in units of GeV for $\eta_{2\mu}$.

quite similar. This is due to the similarity of the two correlation functions as shown in Eq. (16). We have also considered their mixing, which also give a similar result. The mass is predicted to be around $2.3 \sim 2.4$ GeV in the QCD sum rule.

V. FINITE ENERGY SUM RULE

In this section, we use the method of finite energy sum rule. In order to calculate the mass in the finite energy sum rule, we first define the nth moment by using the spectral function $\rho(s)$ in Eq. (9)

$$W(n, s_0) = \int_0^{s_0} \rho(s) s^n ds. \qquad (20)$$

This integral is used for the phenomenological side, while the integral along the circular contour of radius s_0 on the q^2 complex plain should be performed for the theoretical side. The lower integral bound $s = 0$ is taken in order to include the *delta*-function contribution in the OPE [Eqs. (23) and (24)].

With the assumption of quark-hadron duality, we obtain

$$W(n, s_0)|_{\text{Hadron}} = W(n, s_0)|_{\text{OPE}}. \qquad (21)$$

The mass of the ground state can be obtained as

$$M_Y^2(n, s_0) = \frac{W(n + 1, s_0)}{W(n, s_0)}. \qquad (22)$$

For the currents $\eta_{1\mu}$ and $\eta_{2\mu}$, the spectral functions $\rho_1(s)$ and $\rho_2(s)$ can be drawn from Eqs. (13) and (14). The $d = 12$ terms which are proportional to $1/(q^2)^2$ do not contribute to the function $W(n, s_0)$ of Eq. (20) for $n = 0$, or they have a very small contribution for $n = 1$, when the theo-

retical side is computed by the integral over the circle of radius s_0 on the complex q^2 plain. Therefore, the spectral densities for $\eta_{1\mu}$ and $\eta_{2\mu}$ take the following form up to dimension 10,

$$\begin{aligned}
\rho_1(s) = {}& \frac{s^4}{18432\pi^6} - \frac{m_s^2 s^3}{256\pi^6} + \left(-\frac{\langle g^2 GG\rangle}{18432\pi^6} + \frac{m_s\langle\bar{s}s\rangle}{48\pi^4}\right)s^2 \\
& + \left(\frac{\langle\bar{s}s\rangle^2}{18\pi^2} - \frac{m_s\langle g\bar{s}\sigma Gs\rangle}{48\pi^4} + \frac{17 m_s^2\langle g^2 GG\rangle}{9216\pi^6}\right)s \\
& + \left(\frac{\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{12\pi^2} - \frac{m_s\langle g^2 GG\rangle\langle\bar{s}s\rangle}{128\pi^4} - \frac{29 m_s^2\langle\bar{s}s\rangle^2}{12\pi^2}\right) \\
& + \left(\frac{5\langle g^2 GG\rangle\langle\bar{s}s\rangle^2}{864\pi^2} + \frac{\langle g\bar{s}\sigma Gs\rangle^2}{48\pi^2} + \frac{20 m_s\langle\bar{s}s\rangle^3}{9}\right. \\
& \left. - \frac{5 m_s\langle g^2 GG\rangle\langle g\bar{s}\sigma Gs\rangle}{2304\pi^4} - \frac{3 m_s^2\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{2\pi^2}\right)\delta(s),
\end{aligned}$$
$$(23)$$

$$\begin{aligned}
\rho_2(s) = {}& \frac{s^4}{12288\pi^6} - \frac{3 m_s^2 s^3}{512\pi^6} + \left(\frac{\langle g^2 GG\rangle}{18432\pi^6} + \frac{m_s\langle\bar{s}s\rangle}{32\pi^4}\right)s^2 \\
& + \left(\frac{\langle\bar{s}s\rangle^2}{12\pi^2} - \frac{m_s\langle g\bar{s}\sigma Gs\rangle}{32\pi^4} + \frac{35 m_s^2\langle g^2 GG\rangle}{9216\pi^6}\right)s \\
& + \left(\frac{\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{8\pi^2} - \frac{3 m_s\langle g^2 GG\rangle\langle\bar{s}s\rangle}{128\pi^4} - \frac{29 m_s^2\langle\bar{s}s\rangle^2}{8\pi^2}\right) \\
& + \left(\frac{5\langle g^2 GG\rangle\langle\bar{s}s\rangle^2}{288\pi^2} + \frac{\langle g\bar{s}\sigma Gs\rangle^2}{32\pi^2} + \frac{10 m_s\langle\bar{s}s\rangle^3}{3}\right. \\
& \left. - \frac{5 m_s\langle g^2 GG\rangle\langle g\bar{s}\sigma Gs\rangle}{768\pi^4} - \frac{9 m_s^2\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{4\pi^2}\right)\delta(s).
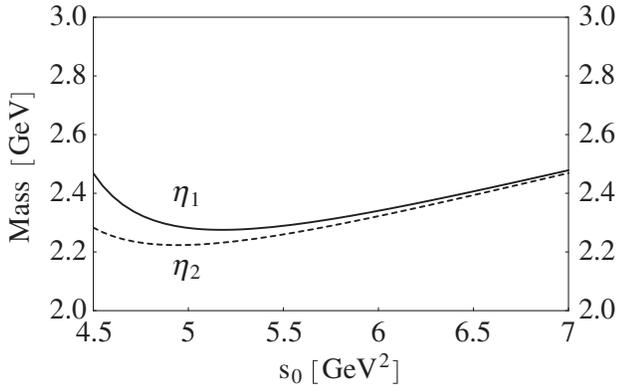\end{aligned}$$
$$(24)$$

FIG. 5. The mass of $Y(2175)$ by using the current $\eta_{1\mu}$ (solid line) and $\eta_{2\mu}$ (dashed line) as a function of s_0 in units of GeV.

The mass is shown as a function of the threshold value s_0 in Fig. 5, where n is chosen to be 1. We find that there is a mass minimum. It is around 2.3 GeV for the current $\eta_{1\mu}$ when we take $s_0 \sim 5.2$ GeV^2, while it is around 2.2 GeV for the current $\eta_{2\mu}$ when we take $s_0 \sim 4.8$ GeV^2. For the current $\eta_{1\mu}$, the minimum point occurs at $\sqrt{s_0} = 2.28$ GeV where the mass takes 2.3 GeV, and the threshold value is slightly smaller than the mass, unlike the ordinary expectation that $\sqrt{s_0}$ is larger than the obtained mass. However, the minimum point is on the very shallow minimum curve and the resulting mass is rather insensitive to the change in the $\sqrt{s_0}$ value. Therefore, we can increase $\sqrt{s_0}$ slightly more, for example, 2.45 GeV, but the mass still remains at around 2.35 GeV, which is smaller than $\sqrt{s_0}$ now. This fact is due to the uncertainty of our sum rule analysis as well as the negative part of the spectral densities. For example, if we take the lower limit of integrations in Eq. (22) to be 1 GeV^2 instead of $16m_s^2$ (the positive part starts at around 3 GeV^2), the mass minimum will be
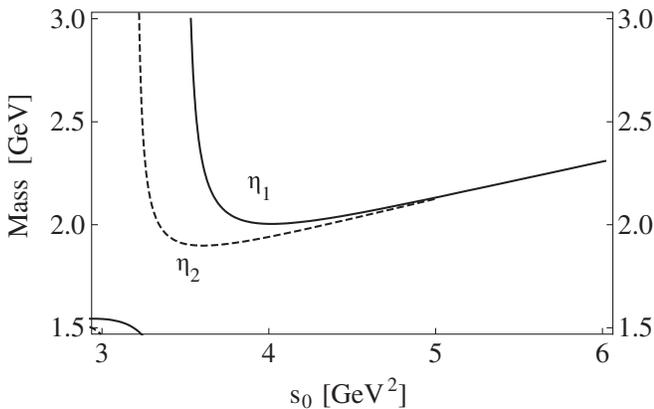

FIG. 6. The mass of $Y(2175)$ by using the current $\eta_{1\mu}$ (solid line) and $\eta_{2\mu}$ (dashed line) as a function of s_0 in units of GeV, when the lower limit of integrations in Eq. (22) is 2 GeV^2 instead of $16m_s^2$.

around 2.1 GeV, when s_0 is around 4.5 GeV^2; if we take the lower limit to be 2 GeV^2, the mass minimum would be around 2.0 GeV, when s_0 is around 4 GeV^2. We show the second case in Fig. 6. The region $5 < s_0 < 6$ GeV^2 is suitable for the QCD sum rule analysis, and the mass obtained is around 2.2 GeV. Therefore, considering the uncertainty of the QCD sum rule, we obtain the same result as the previous one.

VI. SUMMARY AND DISCUSSIONS

In this work we have studied the mass of the state $Y(2175)$ with the quantum numbers $J^{PC} = 1^{--}$ in the QCD sum rule. We have constructed both the diquark-antidiquark currents $(ss)(\bar{s}\,\bar{s})$ and the meson-meson currents $(\bar{s}s)(\bar{s}s)$. We find that there are two independent currents for both cases and verify the relations between them. Then using the two $(ss)(\bar{s}\,\bar{s})$ currents, we calculate the OPE up to dimension 12, which contains the $\langle\bar{s}s\rangle^4$ condensates. The convergence of the OPE turns our to be very good. We find that the OPE's of the two currents are similar, and therefore, the obtained results are also similar. By using both the SVZ (Shifman-Vainshtein-Zakharov) sum rule and the finite energy sum rule, we find that there is a mass minimum. For SVZ sum rule, the minimum is in the region $5 < s_0 < 7$ GeV^2 and $2 < M_B^2 < 4$ GeV^2. For finite energy sum rule, the minimum is in the region $4.5 < s_0 < 5.5$ GeV^2. Considering the uncertainty, the mass obtained is around 2.3 ± 0.4 GeV. The state $Y(2175)$ can be accommodated in the QCD sum rule formalism although the central value of the mass is about 100 MeV higher than the experimental value. We calculate the OPE up to dimension 12 and include many terms, but still the accuracy is around 20%. This is the usual accuracy of the QCD sum rule. In our analysis it is partly due to the many omitted condensates such as $\langle GGG\rangle$ etc.

We have investigated the coupling of the currents to the lower lying states including $\phi(1020)$ and found that the relevant spectral density becomes negative, implying that the present four-quark currents cannot describe those states properly. This fact indicates that the four-quark interpolating currents couple rather weakly to $\phi(1020)$, which is a pure $s\bar{s}$ state.

We can test the tetraquark structure of $Y(2175)$ by considering its decay properties. Naively, the $ss\bar{s}\,\bar{s}$ tetraquark would fall apart via S-wave into the $\phi(1020)f_0(980)$ pair, and would have a very large width. The experimental width of $Y(2175)$ is only about 60 MeV, which seems too narrow to be a pure tetraquark state. We can discuss the decay of the $Y(2175)$ by borrowing an argument based on a valence quark picture. The $(\bar{s}s)(\bar{s}s)$ configuration for $Y(2175)$ can be a combination of $^3S_1$ and $^3P_0$, which may fall apart into two mesons of 1^- and 0^+ in the s-wave. In the QCD sum rule the 1^- $\bar{s}s$ meson is well identified with $\phi(1020)$, while the 0^+ $\bar{s}s$ meson has a mass around 1.5 GeV and is hard to be identified with the observed $f_0(980)$. Therefore, such a

fall-apart decay would simply be suppressed due to the kinematical reason. The physical $f_0(980)$ state may be a tetraquark state as discussed in the previous QCD sum rule study [17]. Then the transition $Y(2175) \rightarrow \phi(1020) + f_0(\text{tetraquark})$ should be accompanied by a $\bar{q}q$ creation violating the OZI (Okuba-Zweig-Iizuka) rule, as well as by an annihilation of one quanta of orbital angular momentum. These facts may once again suppress the decay of $Y(2175) \rightarrow \phi(1020) + f_0(980)$. This fact was studied in the recent paper by Torres, Khemchandani, Geng, Napsuciale, and Oset [35]. They studied the $\phi K\bar{K}$ system with the Faddeev equations where the contained $K\bar{K}$ form the $f_0(980)$ resonance. The decay width they calculated is around 18 MeV, not far from the experimental value. The above evidences would imply that the $Y(2175)$ is a possible candidate of a tetraquark state.

$Y(2175)$ could be a threshold effect, a hybrid state $s\bar{s}G$, a tetraquark, an excited $s\bar{s}$ state, or a mixture of all of the above possibilities. Because of its nonexotic quantum number, it is not easy to establish its underlying structure. Clearly more experimental and theoretical investigations are required.

One by-product of the present work is the interesting observation that some type of four-quark interpolating currents may couple weakly to the conventional $q\bar{q}$ ground states. If future work confirms this point, we may have a novel framework to study the excited $q\bar{q}$ mesons using the four-quark interpolating currents, which is not feasible for the traditional $q\bar{q}$ interpolating currents.

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