$\mu \to e\gamma$ and $\tau \to l\gamma$ decays in the fermion triplet seesaw model

A. Abada,[1] C. Biggio,[2] F. Bonnet,[1] M. B. Gavela,[3] and T. Hambye[4]

[1]*Laboratoire de Physique Théorique UMR 8627, Université de Paris-Sud 11, Bâtiment 210, 91405 Orsay Cedex, France*

[2]*Max-Planck-Institut für Physik, 80805 München, Germany*

[3]*Departamento de Física Teórica and Instituto de Física Teórica UAM/CSIC, Universidad Autónoma de Madrid, 28049 Cantoblanco, Madrid, Spain*

[4]*Service de Physique Théorique, Université Libre de Bruxelles, 1050 Brussels, Belgium*

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In the framework of the seesaw models with triplets of fermions, we evaluate the decay rates of $\mu \to e\gamma$ and $\tau \to l\gamma$ transitions. We show that although, due to neutrino mass constraints, those rates are in general expected to be well under the present experimental limits, this is not necessarily always the case. Interestingly enough, the observation of one of those decays in planned experiments would nevertheless contradict bounds stemming from present experimental limits on the $\mu \to eee$ and $\tau \to 3l$ decay rates, as well as from μ to e conversion in atomic nuclei. Such detection of radiative decays would therefore imply that there exist sources of lepton flavor violation not associated to triplet fermions.

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I. INTRODUCTION

The search for flavor changing rare leptonic decays, in particular, for $\mu \to e\gamma$, $\tau \to \mu\gamma$, and $\tau \to e\gamma$ decays, has been the object of intense experimental investigations for decades [1]. With respect to the present experimental upper limit, $\mathrm{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [1], $\mathrm{Br}(\tau \to \mu\gamma) < 4.5 \times 10^{-8}$ [2], $\mathrm{Br}(\tau \to e\gamma) < 1.1 \times 10^{-7}$ [1], new experiments are expected to improve in the near future their branching ratios by as much as three orders of magnitudes for the first decay mode [3] and by one or two for the two others [4].

The recent experimental evidence for neutrino masses has shown that lepton flavor is violated in the neutrino sector and that, consequently, in a model independent way, these decay rates are predicted to be different from zero. The actual predicted rate, however, turns out to be highly model dependent. There are three basic models which can explain the neutrino masses at tree level, from the exchange of heavy states, through the seesaw mechanism. The above rare decays have been studied at length in the framework of two of these seesaw models, with right-handed neutrinos [5] (type I seesaw [6]) and with one or several Higgs triplets [7] (type II seesaw [8]). In this paper we perform the calculation of these decay rates in the framework of the third seesaw model, with heavy triplets of fermions (type III seesaw [9]). This model has been studied in detail, both from the theoretical and phenomenological point of view, in Ref. [10], where the result on these rare decays has already been presented without the detailed calculation. This paper also contains a determination of the constraint that μ to e conversion in atomic nuclei implies on the type III seesaw model.

II. THE TYPE III SEESAW LAGRANGIAN

The type III seesaw model consists in the addition to the standard model of SU(2) triplets of fermions with zero hypercharge, Σ. In this model at least two such triplets are necessary in order to have two nonvanishing neutrino masses. A nonvanishing $l_1 \to l_2\gamma$ rate can nevertheless be induced already with only one fermionic triplet. In the following, we will not specify the number of triplets so that our calculation is valid for any number of them. Being in the adjoint representation of the electroweak group, the Majorana mass term of such triplets is gauge invariant. In terms of the usual and compact two-by-two notation for triplets, the beyond the standard model (SM) interactions are described by the Lagrangian (with implicit flavor summation)

$$\begin{aligned}\mathcal{L} &= \mathrm{Tr}[\bar{\Sigma}_i \not{D} \Sigma] - \tfrac{1}{2}\mathrm{Tr}[\bar{\Sigma} M_\Sigma \Sigma^c + \bar{\Sigma}^c M_\Sigma^* \Sigma] \\ &\quad - \tilde{\phi}^\dagger \bar{\Sigma} \sqrt{2} Y_\Sigma L - \bar{L} \sqrt{2} Y_\Sigma^\dagger \Sigma \tilde{\phi},\end{aligned} \tag{1}$$

with $L \equiv (l, \nu)^T$, $\phi \equiv (\phi^+, \phi^0)^T \equiv (\phi^+, (v + H + i\eta)/\sqrt{2})^T$, $\tilde{\phi} = i\tau_2\phi^*$, $\Sigma^c \equiv C\bar{\Sigma}^T$ and with, for each fermionic triplet,

$$\begin{aligned}\Sigma &= \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}, \\ \Sigma^c &= \begin{pmatrix} \Sigma^{0c}/\sqrt{2} & \Sigma^{-c} \\ \Sigma^{+c} & -\Sigma^{0c}/\sqrt{2} \end{pmatrix}, \\ D_\mu &= \not{\partial}_\mu - i\sqrt{2}g \begin{pmatrix} W_\mu^3/\sqrt{2} & W_\mu^+ \\ W_\mu^- & -W_\mu^3/\sqrt{2} \end{pmatrix}.\end{aligned} \tag{2}$$

Without loss of generality, in the following we will assume that we start from the basis where M_Σ is real and diagonal. In order to consider the mixing of the triplets with the charged leptons, it is convenient to express the 4 degrees of freedom of each charged triplet in terms of a single Dirac spinor

$$\Psi \equiv \Sigma_R^{+c} + \Sigma_R^-. \tag{3}$$

The neutral fermionic triplet components on the other hand can be left in two-component notation, since they have only 2 degrees of freedom and mix with neutrinos, which are also described by two-component fields. This leads to the Lagrangian

$$\begin{aligned}\mathcal{L} = {}& \bar{\Psi} i\not{\partial}\Psi + \bar{\Sigma}_R^0 i\not{\partial}\Sigma_R^0 - \bar{\Psi}M_\Sigma\Psi - \left(\bar{\Sigma}_R^0\frac{M_\Sigma}{2}\Sigma_R^{0c} + \text{H.c.}\right)\\ & + g(W_\mu^+\bar{\Sigma}_R^0\gamma_\mu P_R\Psi + W_\mu^+\bar{\Sigma}_R^{0c}\gamma_\mu P_L\Psi + \text{H.c.})\\ & - gW_\mu^3\bar{\Psi}\gamma_\mu\Psi - (\phi^0\bar{\Sigma}_R^0 Y_\Sigma\nu_L + \sqrt{2}\phi^0\bar{\Psi}Y_\Sigma l_L\\ & + \phi^+\bar{\Sigma}_R^0 Y_\Sigma l_L - \sqrt{2}\phi^+\bar{\nu}_L^c Y_\Sigma^T\Psi + \text{H.c.}). \end{aligned} \tag{4}$$

The mass term of the charged sector shows then the usual aspect for Dirac particles

$$\begin{aligned}\mathcal{L} - {}&(\bar{l}_R\bar{\Psi}_R)\begin{pmatrix} m_l & 0 \\ Y_\Sigma v & M_\Sigma\end{pmatrix}\begin{pmatrix} l_L \\ \Psi_L\end{pmatrix}\\ &- (\bar{l}_L\bar{\Psi}_L)\begin{pmatrix} m_l & Y_\Sigma^\dagger v \\ 0 & M_\Sigma\end{pmatrix}\begin{pmatrix} l_R \\ \Psi_R\end{pmatrix},\end{aligned} \tag{5}$$

with $v \equiv \sqrt{2}\langle\phi^0\rangle = 246$ GeV. The symmetric mass matrix for the neutral states is on the other hand given by

$$\begin{aligned}\mathcal{L} \ni {}& -(\bar{\nu}_L\bar{\Sigma}^{0c})\begin{pmatrix} 0 & Y_\Sigma^\dagger v/2\sqrt{2} \\ Y_\Sigma^* v/2\sqrt{2} & M_\Sigma/2\end{pmatrix}\begin{pmatrix}\nu_L^c \\ \Sigma^0\end{pmatrix}\\ & - (\bar{\nu}_L^c\bar{\Sigma}^0)\begin{pmatrix} 0 & Y_\Sigma^T v/2\sqrt{2} \\ Y_\Sigma v/2\sqrt{2} & M_\Sigma/2\end{pmatrix}\begin{pmatrix}\nu_L \\ \Sigma^{0c}\end{pmatrix}.\end{aligned} \tag{6}$$

A. Diagonalization of the mass matrices

To calculate the $l_1 \to l_2\gamma$ decay rates, we will work in the mass eigenstates basis. As it happens with any Dirac mass, the charged lepton mass matrix can be diagonalized by a bi-unitary transformation

$$\begin{pmatrix} l_{L,R} \\ \Psi_{L,R}\end{pmatrix} = U_{L,R}\begin{pmatrix} l'_{L,R} \\ \Psi'_{L,R}\end{pmatrix}, \tag{7}$$

where $U_{L,R}$ are $(3+n)$-by-$(3+n)$ matrices, if n triplets are present. On the contrary, the symmetric neutral lepton mass matrix can be diagonalized by a single unitary matrix

$$\begin{pmatrix} \nu_L \\ \Sigma^{0c}\end{pmatrix} = U_0\begin{pmatrix} \nu'_L \\ \Sigma'^{0c}\end{pmatrix}. \tag{8}$$

It is convenient to write the mixing matrices in terms of three-leptons-plus-n-triplets sub-blocks

$$U_L \equiv \begin{pmatrix} U_{Lll} & U_{Ll\Psi} \\ U_{L\Psi l} & U_{L\Psi\Psi}\end{pmatrix}, \qquad U_R \equiv \begin{pmatrix} U_{Rll} & U_{Rl\Psi} \\ U_{R\Psi l} & U_{R\Psi\Psi}\end{pmatrix},$$
$$U_0 \equiv \begin{pmatrix} U_{0\nu\nu} & U_{0\nu\Sigma} \\ U_{0\Sigma\nu} & U_{0\Sigma\Sigma}\end{pmatrix}. \tag{9}$$

In the following we will calculate the decay rates at $\mathcal{O}((Y_\Sigma v/M_\Sigma)^2)$, which is a good approximation as long as M_Σ is sufficiently big compared to $Y_\Sigma v$. In order to do so it can be checked that it is enough to calculate all the mixing matrix elements at order $\mathcal{O}([(Y_\Sigma v, m_l)/M_\Sigma]^2)$. We obtain

$$\begin{aligned} U_{Lll} &= 1 - \epsilon & U_{Ll\Psi} &= Y_\Sigma^\dagger M_\Sigma^{-1} v\\ U_{L\Psi l} &= -M_\Sigma^{-1}Y_\Sigma v & U_{L\Psi\Psi} &= 1 - \epsilon'\\ U_{Rll} &= 1 & U_{Rl\Psi} &= m_l Y_\Sigma^\dagger M_\Sigma^{-2} v\\ U_{R\Psi l} &= -M_\Sigma^{-2}Y_\Sigma m_l v & U_{R\Psi\Psi} &= 1\\ U_{0\nu\nu} &= \left(1 - \frac{\epsilon}{2}\right)U_{\text{PMNS}} & U_{0\nu\Sigma} &= Y_\Sigma^\dagger M_\Sigma^{-1}\frac{v}{\sqrt{2}}\\ U_{0\Sigma\nu} &= -M_\Sigma^{-1}Y_\Sigma\frac{v}{\sqrt{2}}U_{0\nu\nu} & U_{0\Sigma\Sigma} &= \left(1 - \frac{\epsilon'}{2}\right)\end{aligned} \tag{10}$$

where $\epsilon = \frac{v^2}{2}Y_\Sigma^\dagger M_\Sigma^{-2}Y_\Sigma$, $\epsilon' = \frac{v^2}{2}M_\Sigma^{-1}Y_\Sigma Y_\Sigma^\dagger M_\Sigma^{-1}$, and U_{PMNS} is the lowest order Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix which is unitary. Note that ϵ is nothing but the coefficient of the unique low energy dimension-six operator induced by the triplets, once they have been integrated out [10].[1] Equation (10) shows as expected that the $(3+n)$-by-$(3+n)$ mixing matrices $U_{L,R,0}$ are unitary but the various submatrices are not. The neutrino mass matrix in this model is given by[2]:

$$m_\nu = -\frac{v^2}{2}Y_\Sigma^T\frac{1}{M_\Sigma}Y_\Sigma. \tag{11}$$

B. Lagrangian in the mass basis

After the diagonalization of the mass matrices, we obtain the following Lagrangian in the mass basis (omitting from now on the primes on the mass eigenstate fields):

$$\mathcal{L} = \mathcal{L}_{Kin} + \mathcal{L}_{CC} + \mathcal{L}_{NC} + \mathcal{L}_{H,\eta} + \mathcal{L}_{\phi^-}, \tag{12}$$

where

$$\begin{aligned}\mathcal{L}_{CC} = {}& \frac{g}{\sqrt{2}}(\bar{l} \quad \bar{\Psi})\gamma^\mu W_\mu^-(P_L g_L^{CC} + P_R g_R^{CC}\sqrt{2})\begin{pmatrix}\nu \\ \Sigma\end{pmatrix}\\ & + \text{H.c.}\end{aligned} \tag{13}$$

$$\mathcal{L}_{NC} = \frac{g}{\cos\theta_W}(\bar{l} \quad \bar{\Psi})\gamma^\mu Z_\mu(P_L g_L^{NC} + P_R g_R^{NC})\begin{pmatrix} l \\ \Psi\end{pmatrix} \tag{14}$$

[1]The ϵ' contribution does not appear in the low energy effective theory as it involves external Σ's.

[2]As for the masses of the charged leptons, they are essentially unaffected by the presence of the Σ's as the difference between the physical masses of the l' and the ones of the l's, m_l, is of order $m_l Y_\Sigma^2 v^2/M_\Sigma^2$.

$$\mathcal{L}_{H,\eta} = \frac{g}{2M_W}(\bar{l} \quad \bar{\Psi})H(P_L g_L^H + P_R g_R^H)\begin{pmatrix} l \\ \Psi \end{pmatrix} + i\frac{g}{2M_W}(\bar{l} \quad \bar{\Psi})\eta(P_L g_L^\eta + P_R g_R^\eta)\begin{pmatrix} l \\ \Psi \end{pmatrix} \tag{15}$$

$$\mathcal{L}_{\phi^-} = -\phi^- \bar{l}\frac{g}{\sqrt{2}M_W}\{(P_L g_{L_\nu}^{\phi^-} + P_R g_{R_\nu}^{\phi^-})\nu + (P_L g_{L_\Sigma}^{\phi^-} + P_R g_{R_\Sigma}^{\phi^-})\Sigma\} + \text{H.c.} \tag{16}$$

with

$$g_L^{CC} = \begin{pmatrix} g_{L_{l\nu}}^{CC} & g_{L_{l\Sigma}}^{CC} \\ g_{L_{\Psi\nu}}^{CC} & g_{L_{\Psi\Sigma}}^{CC} \end{pmatrix} = \begin{pmatrix} (1+\epsilon)U_{0_{\nu\nu}} & -Y_\Sigma^\dagger M_\Sigma^{-1}\frac{v}{\sqrt{2}} \\ 0 & \sqrt{2}(1-\frac{\epsilon'}{2}) \end{pmatrix} \tag{17}$$

$$g_R^{CC} = \begin{pmatrix} g_{R_{l\nu}}^{CC} & g_{R_{l\Sigma}}^{CC} \\ g_{R_{\Psi\nu}}^{CC} & g_{R_{\Psi\Sigma}}^{CC} \end{pmatrix} = \begin{pmatrix} 0 & -m_l Y_\Sigma^\dagger M_\Sigma^{-2} v \\ -M_\Sigma^{-1} Y_\Sigma^* U_{0_{\nu\nu}}^* \frac{v}{\sqrt{2}} & 1-\frac{\epsilon'^*}{2} \end{pmatrix} \tag{18}$$

$$g_L^{NC} = \begin{pmatrix} g_{L_{ll}}^{NC} & g_{L_{l\Psi}}^{NC} \\ g_{L_{\Psi l}}^{NC} & g_{L_{\Psi\Psi}}^{NC} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}-\cos^2\theta_W-\epsilon & \frac{1}{2}Y_\Sigma^\dagger M_\Sigma^{-1} v \\ \frac{1}{2}M_\Sigma^{-1}Y_\Sigma v & \epsilon' - \cos^2\theta_W \end{pmatrix} \tag{19}$$

$$g_R^{NC} = \begin{pmatrix} g_{R_{ll}}^{NC} & g_{R_{l\Psi}}^{NC} \\ g_{R_{\Psi l}}^{NC} & g_{R_{\Psi\Psi}}^{NC} \end{pmatrix} = \begin{pmatrix} 1-\cos^2\theta_W & m_l Y_\Sigma^\dagger M_\Sigma^{-2} v \\ M_\Sigma^{-2} Y_\Sigma m_l v & -\cos^2\theta_W \end{pmatrix} \tag{20}$$

$$g_L^H = \begin{pmatrix} g_{L_{ll}}^H & g_{L_{l\Psi}}^H \\ g_{L_{\Psi l}}^H & g_{L_{\Psi\Psi}}^H \end{pmatrix} = \begin{pmatrix} m_l(3\epsilon-1) & -m_l Y_\Sigma^\dagger M_\Sigma^{-1} v \\ -Y_\Sigma v(1-\epsilon) - M_\Sigma^{-2}Y_\Sigma m_l^2 v & \dots \end{pmatrix} \tag{21}$$

$$g_R^H = \begin{pmatrix} g_{R_{ll}}^H & g_{R_{l\Psi}}^H \\ g_{R_{\Psi l}}^H & g_{R_{\Psi\Psi}}^H \end{pmatrix} = \begin{pmatrix} (3\epsilon-1)m_l & -(1-\epsilon)Y_\Sigma^\dagger v - m_l^2 Y_\Sigma^\dagger M_\Sigma^{-2} v \\ -M_\Sigma^{-1}Y_\Sigma m_l v & \dots \end{pmatrix} \tag{22}$$

$$g_R^\eta = \begin{pmatrix} g_{R_{ll}}^\eta & g_{R_{l\Psi}}^\eta \\ g_{R_{\Psi l}}^\eta & g_{R_{\Psi\Psi}}^\eta \end{pmatrix} = \begin{pmatrix} -(\epsilon+1)m_l & (1-\epsilon)Y_\Sigma^\dagger v - m_l^2 Y_\Sigma^\dagger M_\Sigma^{-2} v \\ -M_\Sigma^{-1}Y_\Sigma m_l v & \dots \end{pmatrix} \tag{23}$$

$$g_L^\eta = \begin{pmatrix} g_{L_{ll}}^\eta & g_{L_{l\Psi}}^\eta \\ g_{L_{\Psi l}}^\eta & g_{L_{\Psi\Psi}}^\eta \end{pmatrix} = \begin{pmatrix} m_l(\epsilon+1) & m_l Y_\Sigma^\dagger M_\Sigma^{-1} v \\ -Y_\Sigma v(1-\epsilon) + M_\Sigma^{-2}Y_\Sigma m_l^2 v & \dots \end{pmatrix} \tag{24}$$

and

$$\begin{cases} g_{L_\nu}^{\phi^-} = m_l U_{0_{\nu\nu}} \\ g_{R_\nu}^{\phi^-} = -(1-\epsilon)m_\nu^* U_{0_{\nu\nu}}^* \end{cases} \quad \begin{cases} g_{L_\Sigma}^{\phi^-} = m_l Y_\Sigma^\dagger M_\Sigma^{-1}\frac{v}{\sqrt{2}} \\ g_{R_\Sigma}^{\phi^-} = (1-\epsilon)Y_\Sigma^\dagger\frac{v}{\sqrt{2}}(1-\frac{\epsilon'^*}{2}) - \sqrt{2}m_\nu^* Y_\Sigma^T M_\Sigma^{-1} v \end{cases}. \tag{25}$$

The dots in Eqs. (21)–(24) refer to Ψ-Ψ interactions which we omit here since they do not contribute to the one-loop $l_1 \to l_2\gamma$ rates.

III. $\mu \to e\gamma$ AND $\tau \to l\gamma$ DECAYS

In the following we perform the calculation of the $\mu \to e\gamma$ rate. The τ decay rates will be obtained straightforwardly from it later on. As it is well-known, the on shell transition $\mu \to e\gamma$ is a magnetic transition so that its amplitude can be written, in the $m_e \to 0$ limit, as:

$$T(\mu \to e\gamma) = A \times \bar{u}_e(p-q)[iq^\nu \varepsilon^\lambda \sigma_{\lambda\nu}(1+\gamma_5)]u_\mu(p), \tag{26}$$

with ε the polarization of the photon, p_μ the momentum of the incoming muon, q_μ the momentum of the outgoing photon, and $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. Using the Gordon decomposition we can rewrite it as

$$T(\mu \to e\gamma) = A \times \bar{u}_e(p-q)(1+\gamma_5)(2p\cdot\varepsilon - m_\mu \not\varepsilon) \times u_\mu(p). \tag{27}$$

In the following we will calculate only the $p\cdot\varepsilon$ terms. The terms proportional to $\not\varepsilon$ can be recovered from the $p\cdot\varepsilon$ terms through Eq. (27). All in all, this gives

$$\Gamma(\mu \to e\gamma) = \frac{m_\mu^3}{4\pi}|A|^2. \tag{28}$$

In the mass eigenstate basis, from the Lagrangian of Eqs. (13)–(16), there are 14 diagrams contributing to

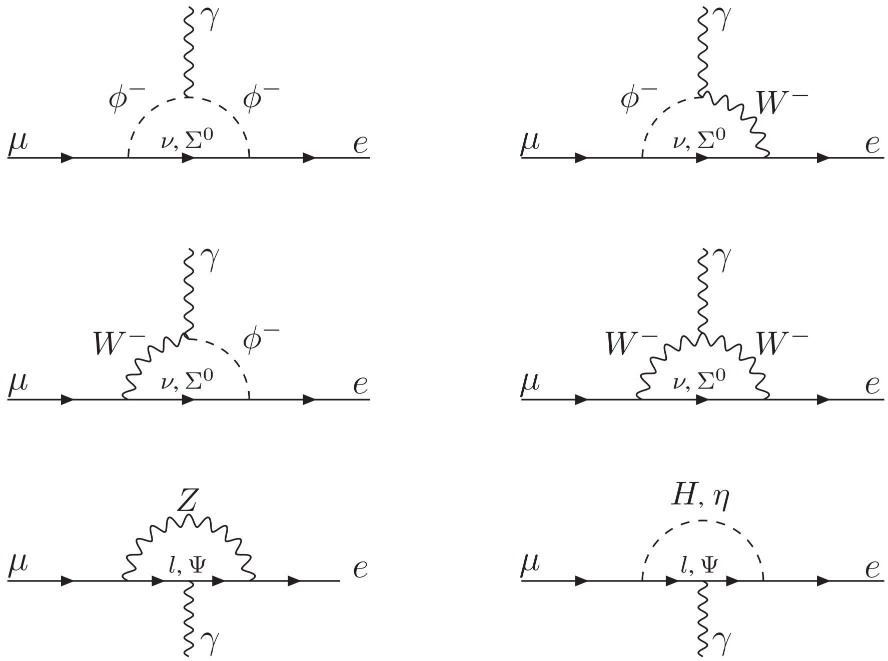

FIG. 1. Diagrams contributing to $\mu \to e\gamma$. $\phi^{\pm}$, η are the three Goldstone boson associated with the W^- and Z bosons. H stands for the physical Higgs boson.

$\mu \to e\gamma$, as shown in Fig. 1. The detailed calculation is presented in the Appendix.[3] In the limit in which $M_\Sigma \gg M_W$, at $\mathcal{O}((\frac{Y_\Sigma v}{M_\Sigma})^2)$, the total amplitude is given by

$$T(\mu \to e\gamma) = i\frac{G_F^{\mathrm{SM}}}{\sqrt{2}}\frac{e}{32\pi^2} m_\mu \bar{u}_e(p-q)(1+\gamma_5) \times i\sigma_{\lambda\nu}\varepsilon^\lambda q^\nu u_\mu(p)\bigg\{\bigg(\frac{13}{3}+C\bigg)\epsilon_{e\mu} - \sum_i x_{\nu_i}(U_{\mathrm{PMNS}})_{ei}(U^\dagger_{\mathrm{PMNS}})_{i\mu}\bigg\}, \quad (29)$$

where $C = -6, 56$, and $x_{\nu_i} \equiv \frac{m^2_{\nu_i}}{M_W^2}$. Note that the second term is the usual contribution from neutrino mixing [12], while the first one is the explicit contribution of the fermion triplet(s). As is well-known, a Glashow-Iliopoulos-Maiani cancellation operates in the second term. The total decay rate is then given by

$$\Gamma(\mu \to e\gamma) = \frac{G_F^{\mathrm{SM}^2} e^2 m_\mu^5}{8192\pi^5}\bigg|\bigg(\frac{13}{3}+C\bigg)\epsilon_{e\mu} - \sum_i x_{\nu_i}(U_{\mathrm{PMNS}})_{ei}(U^\dagger_{\mathrm{PMNS}})_{i\mu}\bigg|^2 \quad (30)$$

[3]General formulas for radiative fermion decays have been derived in detail in Ref. [11], although restricted to the case in which all fermion masses arise from the standard Higgs mechanism. In consequence, isospin invariant mass terms as those essential in seesaw models were not taken into account.

and the branching ratio reads

$$\mathrm{Br}(\mu \to e\gamma) = \frac{3}{32}\frac{\alpha}{\pi}\bigg|\bigg(\frac{13}{3}+C\bigg)\epsilon_{e\mu} - \sum_i x_{\nu_i}(U_{\mathrm{PMNS}})_{ei}(U^\dagger_{\mathrm{PMNS}})_{i\mu}\bigg|^2. \quad (31)$$

$\tau \to l\gamma$ decays can be obtained from Eq. (31) by replacing μ by τ, e by l, and by multiplying the obtained result by $\mathrm{Br}(\tau \to e\nu_\tau\bar{\nu}_e) = (17.84 \pm 0.05) \times 10^{-2}$ [1].

IV. PHENOMENOLOGY

A. Bounds on $\epsilon_{e\mu}$, $\epsilon_{\mu\tau}$, and $\epsilon_{e\tau}$

From the result above, it is not surprising that in general we expect a very tiny $\mu \to e\gamma$ rate. For instance, omitting flavor indices, for a given value of M_Σ we would expect in general from the seesaw formula that $Y_\Sigma^2 \simeq m_\nu M_\Sigma/v^2 \sim \sqrt{\delta m^2_{\mathrm{atm}}} M_\Sigma/v^2 \sim M_\Sigma/(10^{15}\ \mathrm{GeV})$. This gives $\epsilon \sim m_\nu/M_\Sigma \sim 10^{-25}(10^{15}\ \mathrm{GeV}/M_\Sigma)$ and $x_\nu \sim \delta m^2_{\mathrm{atm}}/M_W^2 \sim 10^{-24}$ which leads to $\mathrm{Br}(\mu \to e\gamma) \sim 10^{-52} \cdot (10^{15}\ \mathrm{GeV}/M_\Sigma)^2$, far below the present upper limit 1.2×10^{-11}. In this case, even for M_Σ as low as 100 GeV, we get $\mathrm{Br}(\mu \to e\gamma) \sim 10^{-26}$. Similarly, for $\tau \to \mu\gamma$ and $\tau \to e\gamma$, we get both rates of order $10^{-53}(10^{15}\ \mathrm{GeV}/M_\Sigma)^2$, far below the present upper limit 4.5×10^{-8} and 1.1×10^{-7}), respectively.

There are cases, however, in which the branching ratio can be much larger without any fine-tuning of the Yukawa couplings and mass parameters. This is the case if neutrino masses are generated through "direct lepton violation" (see Ref. [10]), i.e., if neutrino masses are directly proportional to a small lepton number violating scale rather than inversely proportional to a high scale. Direct lepton violation appears naturally in the type II seesaw model, since two scales are present there: the mass of the heavy scalar triplet M_Δ and the dimension-full trilinear coupling μ between the scalar triplet and two Higgs doublets. In this case $m_\nu \sim Y_\Delta \mu v^2/M_\Delta^2$, where Y_Δ is the Yukawa coupling, but $\mathrm{Br}(\mu \to e\gamma) \sim Y_\Delta^4 M_W^4/M_\Delta^4$. If the scale μ is sufficiently small to suppress neutrino masses, Y_Δ/M_Δ can be large enough to generate visible effects in rare lepton decays. A similar pattern can be realized also in the type III seesaw, if besides a high scale M_Σ, a low scale μ, responsible for lepton number violation, is present. This has indeed been studied in the context of type I seesaw [10,13], but it can be applied here as well. In this case the $\epsilon_{e\mu}$ term in Eqs. (29)–(31) is enhanced to much larger values and the x_{ν_i} term can be neglected.

With such a pattern the $\mu \to e\gamma$ branching ratio could be as large as $\sim 10^{-4}$ for the extreme case where the Yukawa couplings would be as large as unity with triplets as light as a few hundreds GeV. This shows that the present experimental bound is already relevant to exclude too large values of the Yukawas associated to too small values of the triplet mass. The present experimental bounds on the branching ratios give the following constraints on the $\epsilon_{\alpha\beta}$ coefficients[4]:

$$|\epsilon_{e\mu}| = \frac{v^2}{2}\left| Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger}\frac{1}{M_\Sigma} Y_\Sigma \right|_{\mu e} \lesssim 1.1 \times 10^{-4} \tag{32}$$

$$|\epsilon_{\mu\tau}| = \frac{v^2}{2}\left| Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger}\frac{1}{M_\Sigma} Y_\Sigma \right|_{\tau\mu} \lesssim 1.5 \times 10^{-2} \tag{33}$$

$$|\epsilon_{e\tau}| = \frac{v^2}{2}\left| Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger}\frac{1}{M_\Sigma} Y_\Sigma \right|_{\tau e} \lesssim 2.4 \times 10^{-2}. \tag{34}$$

B. Comparison of $l \to l'\gamma$ and $l \to 3l'$ decays

The bounds of Eqs. (32)–(34) from $l \to l'\gamma$ decays turn out to be on the same parameters ϵ as the ones obtained from $\mu \to 3e$ or $\tau \to 3l$ decays, derived in Ref. [10]. This can be understood from the fact that, at order $1/M_\Sigma^2$, for example, for $\mu \to e\gamma$ and $\mu \to 3e$, there is only one way to combine two Yukawa couplings and two inverse M_Σ mass matrices to induce a μ-e transition along a same fermionic line: through the combination $\epsilon_{e\mu}$ (i.e., the flavor structure of the μ to e fermionic line is the same for both processes, it corresponds to a μ which mixes with a fermion triplet which mixes with an electron). This can also be understood from the related fact that the number of independent parameters contained in the coefficients of the dimension five operators (proportional to the neutrino mass matrix) and dimension-six operators (encoded in the $\epsilon_{\alpha\beta}$ [10]) of the low energy theory (obtained in the limit of large fermion triplet mass) equals the number of independent parameters of the original theory. This implies that any physical transition studied at order $1/M_\Sigma^2$, necessarily has to be proportional to the dimension-six operator coefficients, and there is only one which gives a μ to e transition, $\epsilon_{e\mu}$.

As a result we obtain the following fixed ratios for these branching ratios:

$$\mathrm{Br}(\mu \to e\gamma) = 1.3 \times 10^{-3} \cdot \mathrm{Br}(\mu \to eee), \tag{35}$$

$$\begin{aligned}\mathrm{Br}(\tau \to \mu\gamma) &= 1.3 \times 10^{-3} \cdot \mathrm{Br}(\tau \to \mu\mu\mu) \\ &= 2.1 \times 10^{-3} \cdot \mathrm{Br}(\tau^- \to e^- e^+ \mu^-),\end{aligned} \tag{36}$$

$$\begin{aligned}\mathrm{Br}(\tau \to e\gamma) &= 1.3 \times 10^{-3} \cdot \mathrm{Br}(\tau \to eee) \\ &= 2.1 \times 10^{-3} \cdot \mathrm{Br}(\tau^- \to \mu^- \mu^+ e^-).\end{aligned} \tag{37}$$

The ratios are much smaller than unity because $l \to 3l'$ is induced at tree level through mixing of the charged leptons with the charged components of the fermion triplets [10], while $l \to l'\gamma$ is a one-loop process. The results of Eqs. (35)–(37) hold in the limit where $M_\Sigma \gg M_{W,Z,H}$, as they are based on Eq. (31). Not taking this limit, i.e., using Eq. (A26) of the Appendix, for values of M_Σ as low as $\sim$100 GeV, these ratios can vary around these values by up to 1 order of magnitude. Numerically it turns out that the bounds in Eqs. (32)–(34) are thus not as good as the ones coming from $\mu \to eee$, $\tau \to eee$, and $\tau \to \mu\mu\mu$ decays, which give $|\epsilon_{e\mu}| < 1.1 \times 10^{-6}$, $|\epsilon_{\mu\tau}| < 4.9 \times 10^{-4}$, $|\epsilon_{e\tau}| < 5.1 \times 10^{-4}$, respectively, [using the experimental bounds: $\mathrm{Br}(\mu \to eee) < 1 \times 10^{-12}$ [1], $\mathrm{Br}(\tau \to eee) < 3.6 \times 10^{-8}$ [14], and $\mathrm{Br}(\tau \to \mu\mu\mu) < 3.2 \times 10^{-8}$ [14]].[5] This shows that even if the upper limits on $\mu \to e\gamma$ and $\tau \to l\gamma$ are improved in the future by 3 or 2 orders of magnitude, respectively, the $\mu \to 3e$ and $\tau \to 3l$ will still provide the most competitive bounds on the $\epsilon_{\alpha\beta}$ ($\alpha \neq \beta$). This can be clearly seen from the bounds, $\mathrm{Br}(\mu \to e\gamma) < 10^{-15}$, $\mathrm{Br}(\tau \to \mu\gamma) < 4 \times 10^{-11}$, and

[4]Note that these bounds show that the approximation we made in the above to work only at first order in $Y^2 v^2/M_\Sigma^2$ is justified.

[5]Note that these bounds from τ decays are better than the ones quoted in Table 8 of Ref. [10], as we have used the new experimental limits on $\tau \to 3l$ decays of Ref. [14]. This also leads to the new following bounds: $|\epsilon_{\mu\tau}| < 5.6 \times 10^{-4}$ [from $\mathrm{Br}(\tau \to e^+ e^- \mu^-) < 2.7 \times 10^{-8}$] and $|\epsilon_{e\tau}| < 7.2 \times 10^{-4}$ [from $\mathrm{Br}(\tau \to \mu^+ \mu^- e^-) < 4.1 \times 10^{-8}$]. We thank M. Nemevšek for pointing to us the existence of Ref. [14].

$\mathrm{Br}(\tau \to e\gamma) < 5 \cdot 10^{-11}$, that one obtains from Eqs. (35)–(37) using the experimental bounds on the $l \to 3l'$ decays.

This leads to the conclusion that the observation of one leptonic radiative decay by upcoming experiments would basically rule out the seesaw mechanism with only triplets of fermions, i.e., with no other source of lepton flavor changing new physics. To our knowledge this is a unique result.

This is different from other seesaw models. For instance, in type I seesaw, for the same reasons as for the type III model, the ratios of Eqs. (35)–(37) are also fixed at order $1/M_N^2$, but unlike for this type III model, both processes are instead realized at one loop. As a result, generically, $l \to l'\gamma$ dominates over $l \to 3l'$ because the latter suffers an extra α suppression. On the other hand, in type II seesaw, no definite predictions for these ratios can be done, because both types of decays depend on different combinations of the parameters [10]. This stems from the fact that in the type II model the Yukawa coupling Y_Δ couples a scalar triplet to two light fermions, so it carries two light lepton flavor indices, instead of one in the type I and type III models. As a result there are several combinations of the Yukawa couplings which can lead to a μ to e transition in this model.[6]

V. μ-TO-e CONVERSION IN ATOMIC NUCLEI

Beside $l \to l'\gamma$ and $l \to 3l$ decays, fermion triplets can also induce μ-to-e conversion in atomic nuclei. The relevant diagram turns out to be a tree level one, as for $l \to 3l$ decays, where μ goes to $e + Z$ with the Z connected to a u or d quark fermion line. For the reasons given above, or simply from the fact that this diagram involves exactly the same μ-e-Z vertex as the $\mu \to eee$ decay, μ-to-e conversion gives a constraint on the same $\varepsilon_{e\mu}$ parameter than from $\mu \to eee$ decay (or than from $\mu \to e\gamma$ decay). Using the experimental upper bound for the μ-to-e conversion rate to total nucleon muon capture rate ratio for ${}^{48}_{22}Ti$ nuclei, $R^{\mu\to e} < 4.3 \times 10^{-12}$ [15], the bound one obtains actually turns out to be even more stringent than from $\mu \to eee$

$$|\varepsilon_{e\mu}| < 1.7 \times 10^{-7}. \tag{38}$$

This bound can be straightforwardly obtained by determining the quark-lepton effective interaction induced by the Z exchange

$$\mathcal{L}_{\mathrm{eff}} = -\sqrt{2}G_F(\bar{l}_i\gamma^\alpha P_L g^{NC}_{Lij} l_j)(\bar{u}\gamma_\alpha[(1 - \tfrac{8}{3}\sin^2\theta_W) - \gamma_5]u + \bar{d}\gamma_\alpha[(-1 + \tfrac{4}{3}\sin^2\theta_W) + \gamma_5]d) \tag{39}$$

which using standard formula, for example, Eq. (2.16) of Ref. [16], gives

$$R^{\mu\to e} = 1.4 \times 10^1 \cdot |\varepsilon_{e\mu}|^2. \tag{40}$$

This leads to the following fixed ratio predictions for ${}^{48}_{22}Ti$

$$\mathrm{Br}(\mu \to eee) = 2.4 \times 10^{-1} R^{\mu\to e} \tag{41}$$

$$\mathrm{Br}(\mu \to e\gamma) = 3.1 \times 10^{-4} R^{\mu\to e} \tag{42}$$

which allows further possibilities to test and/or exclude the model. Results from the gold nuclei, which experimentally gives $R^{\mu\to e} < 7 \times 10^{-13}$ [17], are of same order of magnitude. Note that the PRISM collaboration [18] is expected to improve the experimental bound on $R^{\mu\to e}$ for the ${}^{48}_{22}Ti$ nuclei by several orders of magnitude in the long term.

VI. SUMMARY

We have calculated the $\mu \to e\gamma$ and $\tau \to l\gamma$ decay rates in the presence of one or more triplets of fermions. As with right-handed neutrinos, the obtained rate is in general extremely suppressed but in special cases (not necessarily tuned) it can exceed the present experimental bounds. Unlike for other seesaw models, the observation of a leptonic radiative decay rate close to the present bounds, would nevertheless be incompatible with bounds which arise in this model from $l \to 3l'$ decays. Similarly it would be incompatible with the bound from μ to e conversion we have determined. This provides an interesting possibility to exclude this model as the unique low energy source of lepton flavor changing new physics.

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APPENDIX

The 14 diagrams of Fig. 1 can be grouped according to the fermion circulating in the loop. Performing the calculation in the 't Hooft-Feynman gauge, after loop integration, the various amplitudes, at $\mathcal{O}((\frac{Y_\Sigma v}{M_\Sigma})^2)$, are

[6] For instance the $\mu \to 3e$ transition involves the combination $Y_{\Delta\mu e}Y^\dagger_{\Delta ee}$ while the $\mu \to e\gamma$ involve the combination $Y_{\Delta\mu l}Y^\dagger_{\Delta le}$ with $l = e, \mu, \tau$ see, e.g., [10].

$$T_{\nu_i}^{\phi^-,W^-} = T_{\nu_i}^{\phi^-} + T_{\nu_i}^{\phi^-,W^-} + T_{\nu_i}^{W^-,\phi^-} + T_{\nu_i}^{W^-}$$
$$= i\frac{G_F^{\rm SM}}{\sqrt{2}}\frac{e}{32\pi^2}m_\mu \bar{u}_e(p-q)(1+\gamma_5)(2p\cdot\varepsilon)u_\mu(p)[(U_{0_{\nu\nu}})_{ei}(U^\dagger_{0_{\nu\nu}})_{i\mu}F_1(x_{\nu_i}) + (\epsilon U_{0_{\nu\nu}})_{ei}(U^\dagger_{0_{\nu\nu}})_{i\mu}F_2(x_{\nu_i})$$
$$+ (U_{0_{\nu\nu}})_{ei}(U^\dagger_{0_{\nu\nu}}\epsilon)_{i\mu}F_3(x_{\nu_i})] \tag{A1}$$

$$T_{\Sigma_i}^{\phi^-,W^-} = T_{\Sigma_i}^{\phi^-} + T_{\Sigma_i}^{\phi^-,W^-} + T_{\Sigma_i}^{W^-,\phi^-} + T_{\Sigma_i}^{W^-}$$
$$= i\frac{G_F^{\rm SM}}{\sqrt{2}}\frac{e}{32\pi^2}m_\mu \bar{u}_e(p-q)(1+\gamma_5)(2p\cdot\varepsilon)u_\mu(p)\bigg\{(Y_\Sigma^\dagger M_\Sigma^{-1})_{ei}(M_\Sigma^{-1}Y_\Sigma)_{i\mu}\frac{v^2}{2}F_4(x_{\Sigma_i})$$
$$+ (Y_\Sigma^\dagger M_\Sigma^{-1})_{ei}(M_\Sigma^{-1}Y_\Sigma\epsilon)_{i\mu}\frac{v^2}{2}x_{\Sigma_i}F_5(x_{\Sigma_i}) + \frac{1}{M_W^2}\bigg[(Y_\Sigma^\dagger)_{ei}(\epsilon'^T Y_\Sigma)_{i\mu}\frac{v^2}{4} + (Y_\Sigma^\dagger)_{ei}(M_\Sigma^{-1}Y_\Sigma^* m_\nu^T)_{i\mu}v^2\bigg]F_5(x_{\Sigma_i})$$
$$+ \frac{1}{M_W^2}\bigg[(Y_\Sigma^\dagger\epsilon'^*)_{ei}(Y_\Sigma)_{i\mu}\frac{v^2}{4} + (m_\nu^* Y_\Sigma^T M_\Sigma^{-1})_{ei}(Y_\Sigma)_{i\mu}v^2\bigg]F_6(x_{\Sigma_i}) + (\epsilon Y_\Sigma^\dagger M_\Sigma^{-1})_{ei}(M_\Sigma^{-1}Y_\Sigma)_{i\mu}\frac{v^2}{2}x_{\Sigma_i}F_6(x_{\Sigma_i})\bigg\} \tag{A2}$$

$$T_{\Psi_i}^{Z,H,\eta} = T_{\Psi_i}^{Z} + T_{\Psi_i}^{H} + T_{\Psi_i}^{\eta}$$
$$= i\frac{G_F^{\rm SM}}{\sqrt{2}}\frac{e}{32\pi^2}m_\mu \bar{\mu}_e(p-q)(1+\gamma_5)(2p\cdot\varepsilon)u_\mu(p)\bigg[(Y_\Sigma^\dagger M_\Sigma^{-1})_{ei}(M_\Sigma^{-1}Y_\Sigma)_{i\mu}\frac{v^2}{2}(F_7(y_{\Sigma_i}) + F_8(z_{\Sigma_i}))$$
$$- (\epsilon Y_\Sigma^\dagger M_\Sigma^{-1})_{ei}(M_\Sigma^{-1}Y_\Sigma)_{i\mu}\frac{v^2}{2}(F_8(y_{\Sigma_i}) + F_8(z_{\Sigma_i})) - (Y_\Sigma^\dagger M_\Sigma^{-1})_{ei}(M_\Sigma^{-1}Y_\Sigma\epsilon)_{i\mu}\frac{v^2}{2}(F_9(y_{\Sigma_i}) + F_9(z_{\Sigma_i}))\bigg] \tag{A3}$$

$$T_{l_i}^{Z,H,\eta} = T_{l_i}^{Z} + T_{l_i}^{H} + T_{l_i}^{\eta} = i\frac{G_F^{\rm SM}}{\sqrt{2}}\frac{e}{32\pi^2}m_\mu \bar{u}_e(p-q)(1+\gamma_5)(2p\cdot\varepsilon)u_\mu(p)\epsilon_{e\mu}G(y_{li}, z_{l_i}), \tag{A4}$$

where $x_{\nu_i} \equiv \frac{m_{\nu_i}^2}{M_W^2}$, $x_{\Sigma_i} \equiv \frac{m_{\Sigma_i}^2}{M_W^2}$, $y_{l_i} = \frac{m_{l_i}^2}{M_Z^2}$, $z_{l_i} = \frac{m_{l_i}^2}{M_H^2}$, $y_{\Sigma_i} = \frac{m_{\Sigma_i}^2}{M_Z^2}$, $z_{\Sigma_i} = \frac{m_{\Sigma_i}^2}{M_H^2}$, and $F_i(x)$ and $G(x)$ are the following functions:

$$F_1(x) = \frac{10 - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3\log(x)}{3(-1+x)^4} \tag{A5}$$

$$F_2(x) = \frac{2(5 - 24x + 39x^2 - 20x^3 + 6x^2(-1+2x)\log(x))}{3(-1+x)^4} \tag{A6}$$

$$F_3(x) = \frac{7 - 33x + 57x^2 - 31x^3 + 6x^2(-1+3x)\log(x)}{3(-1+x)^4} \tag{A7}$$

$$F_4(x) = \frac{-38 + 185x - 246x^2 + 107x^3 - 8x^4 + 18(4-3x)x^2\log(x)}{3(-1+x)^4} \tag{A8}$$

$$F_5(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2\log(x)}{3(-1+x)^4} \tag{A9}$$

$$F_6(x) = \frac{7 - 12x - 3x^2 + 8x^3 - 6x(-2+3x)\log(x)}{3(-1+x)^4} \tag{A10}$$

$$F_7(x) = \frac{40 - 46x - 3x^2 + 2x^3 + 7x^4 + 18x(4 - 3x)\log(x)}{3(-1 + x)^4} \tag{A11}$$

$$F_8(x) = \frac{x(-16 + 45x - 36x^2 + 7x^3 + 6(-2 + 3x)\log(x))}{3(-1 + x)^4} \tag{A12}$$

$$F_9(x) = \frac{x(2 + 3x - 6x^2 + x^3 + 6x\log(x))}{3(-1 + x)^4} \tag{A13}$$

$$\begin{aligned} G(y_{l_i}, z_{l_i}) = {} & \delta_{ie}\left[8\left(\frac{1}{2} - \cos^2\theta_W\right)\frac{4 - 9y_{l_i} + 5y_{l_i}^3 + 6(1 - 2y_{l_i})y_{l_i}\log(y_{l_i})}{6(-1 + y_{l_i})^4}\right] \\ & + \delta_{i\mu}\left[z_{l_i}\frac{16 - 45z_{l_i} + 36z_{l_i}^2 - 7z_{l_i}^3 - 6(-2 + 3z_{l_i})\log(z_{l_i})}{2(-1 + z_{l_i})^4}\right. \\ & + 8\left(\frac{1}{2} - \cos^2\theta_W\right)\frac{4 - 9y_{l_i} + 5y_{l_i}^3 + 6(1 - 2y_{l_i})y_{l_i}\log(y_{l_i})}{6(-1 + y_{l_i})^4} \\ & - 8(1 - \cos^2\theta_W)\frac{2(-1 + y_{l_i}^2 - 2y_{l_i}\log(y_{l_i}))}{(-1 + y_{l_i})^3} \\ & \left. - y_{l_i}\frac{-20 + 39y_{l_i} - 24y_{l_i}^2 + 5y_{l_i}^3 + 6(-2 + y_{l_i})\log(y_{l_i})}{6(-1 + y_{l_i})^4}\right]. \end{aligned} \tag{A14}$$

Since $y_{l_i}, z_{l_i}, x_{\nu_i} \ll 1$ it is a good approximation to take the lepton flavor conserving quantities y_{l_i} and z_{l_i} to zero and to keep only the linear term in the flavor changing quantities x_{ν_i}

$$F_1(x_{\nu_i}) \simeq \tfrac{10}{3} - x_{\nu_i} \tag{A15}$$

$$F_2(x_{\nu_i}) \simeq \tfrac{10}{3} - \tfrac{8}{3}x_{\nu_i} \tag{A16}$$

$$F_3(x_{\nu_i}) \simeq \tfrac{7}{3} - \tfrac{5}{3}x_{\nu_i} \tag{A17}$$

$$G(y_i, z_i) = C = -6,56. \tag{A18}$$

Summing over i and neglecting terms of $\mathcal{O}((Y_\Sigma v/M_\Sigma)^n)$ with $n > 2$, we obtain

$$T_\nu^{\phi^-,W^-} = \sum_i T_{\nu_i}^{\phi^-,W^-} = i\frac{G_F^{\rm SM}}{\sqrt{2}}\frac{e}{32\pi^2}m_\mu\bar{u}_e(p - q)(1 + \gamma_5)(2p\cdot\varepsilon)u_\mu(p)\left\{\frac{7}{3}\epsilon_{e\mu} - \sum_i x_{\nu_i}(U_{\rm PMNS})_{ei}(U_{\rm PMNS}^\dagger)_{i\mu}\right\} \tag{A19}$$

$$T_\Sigma^{\phi^-,W^-} = \sum_i T_{\Sigma_i}^{\phi^-,W^-} = i\frac{G_F^{\rm SM}}{\sqrt{2}}\frac{e}{32\pi^2}m_\mu\bar{u}_e(p - q)(1 + \gamma_5)(2p\cdot\varepsilon)u_\mu(p)\left\{-\frac{8}{3}\epsilon_{e\mu} + \sum_i\frac{v^2}{2}(Y_\Sigma^\dagger M_\Sigma^{-1})_{ei}(M_\Sigma^{-1}Y_\Sigma)_{i\mu}A(x_{\Sigma_i})\right\} \tag{A20}$$

$$T_l^{Z,H,\eta} = \sum_i T_{l_i}^{Z,H,\eta} = i\frac{G_F^{\rm SM}}{\sqrt{2}}\frac{e}{32\pi^2}m_\mu\bar{u}_e(p - q)(1 + \gamma_5)(2p\cdot\varepsilon)u_\mu(p)\epsilon_{e\mu}\times C \tag{A21}$$

$$T_{\Psi}^{Z,H,\eta} = \sum_i T_{\Psi_i}^{Z,H,\eta}$$
$$= i\frac{G_F^{\rm SM}}{\sqrt{2}}\frac{e}{32\pi^2}m_\mu \bar{u}_e(p-q)(1+\gamma_5)(2p\cdot\varepsilon)u_\mu(p)\left\{\frac{14}{3}\epsilon_{e\mu} + \sum_i \frac{v^2}{2}(Y_\Sigma^\dagger M_\Sigma^{-1})_{ei}(M_\Sigma^{-1}Y_\Sigma)_{i\mu}(B(y_{\Sigma_i}) + C(z_{\Sigma_i}))\right\}, \tag{A22}$$

where

$$A(x_{\Sigma_i}) = \frac{-30 + 153x_{\Sigma_i} - 198x_{\Sigma_i}^2 + 75x_{\Sigma_i}^3 + 18(4-3x_{\Sigma_i})x_{\Sigma_i}^2 \log x_{\Sigma_i}}{3(x_{\Sigma_i}-1)^4} \tag{A23}$$

$$B(y_{\Sigma_i}) = \frac{33 - 18y_{\Sigma_i} - 45y_{\Sigma_i}^2 + 30y_{\Sigma_i}^3 + 18(4-3y_{\Sigma_i})y_{\Sigma_i} \log y_{\Sigma_i}}{3(y_{\Sigma_i}-1)^4} \tag{A24}$$

$$C(z_{\Sigma_i}) = \frac{-7 + 12z_{\Sigma_i} + 3z_{\Sigma_i}^2 - 8z_{\Sigma_i}^3 + 6(3z_{\Sigma_i}-2)z_{\Sigma_i} \log z_{\Sigma_i}}{3(z_{\Sigma_i}-1)^4}. \tag{A25}$$

The total amplitude is then

$$T(\mu\to e\gamma) = i\frac{G_F^{\rm SM}}{\sqrt{2}}\frac{e}{32\pi^2}m_\mu \bar{u}_e(p-q)(1+\gamma_5)(2p\cdot\varepsilon)u_\mu(p)\left\{\left(\frac{13}{3}+C\right)\epsilon_{e\mu} - \sum_i x_{\nu_i}(U_{\rm PMNS})_{ei}(U_{\rm PMNS}^\dagger)_{i\mu} + \sum_i \frac{v^2}{2}(Y_\Sigma^\dagger M_\Sigma^{-1})_{ei}(M_\Sigma^{-1}Y_\Sigma)_{i\mu}(A(x_{\Sigma_i}) + B(y_{\Sigma_i}) + C(z_{\Sigma_i}))\right\}. \tag{A26}$$

This result is valid at $\mathcal{O}((\frac{Y_\Sigma v}{M_\Sigma})^2)$. For $x_{\Sigma_i}, y_{\Sigma_i}, z_{\Sigma_i} \gg 1$, the additional limit $x_{\Sigma_i}, y_{\Sigma_i}, z_{\Sigma_i} \to \infty$ can be taken, which leads to the result displayed in the text, Eq. (29).

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