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Estimate of the branching fraction $\tau^- \to \eta\pi^-\nu_\tau$, the $a_0^-(980)$, and nonstandard weak interactions

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We consider the "second-class current" decay $\tau^- \to \pi^-\eta\nu_\tau$ from several points of view. We first focus on the decay rate as expected within standard weak interaction and QCD due to isospin violation. The decay contributions divide into P- and S-wave parts. The former can be reliably estimated using the $\rho\eta\pi$ coupling inferred from the rates and Dalitz-plot distributions of $\eta \to 3\pi$ decays. The somewhat larger S-wave part, which was previously computed using chiral perturbation theory, is estimated from a simple $\bar{q}q$ model. Both estimates of the S-wave part depend on whether the $a_0(980)$ scalar particle is a $\bar{q}q$ or some other (4-quark) state. Finally, we discuss genuinely new, non-$V-A$ scalar weak interactions. The $\tau^- \to \pi^-\eta\nu_\tau$ decay provides information on this question, which nicely complements that from precision β decay experiments. In summary, we discuss the possible implications of putative values of the branching fraction $\mathcal{B}(\tau^- \to \pi^-\eta\nu_\tau)$. In the case of larger values, in particular, of the S-wave part, not only will detection of the decay be more likely and more reliable, its implications will be more far-reaching and interesting.

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I. INTRODUCTION

The weak decay $\tau^- \to \pi^-\eta\nu_\tau$, an example of "second class current" decays introduced by Weinberg [1], may soon be observed or tightly bounded by the B factories. This isospin- and G parity-violating decay is suppressed by the small value of $(m_d - m_u)/\Lambda_{\rm QCD}$ or $\alpha_{\rm EM}$. Various estimates [2] using chiral perturbation theory or other methods have predicted this decay's branching fraction to be

$$\mathcal{B} \equiv \mathcal{B}(\tau^- \to \pi^-\eta\nu_\tau) = (1.3 \pm 0.2) \times 10^{-5}, \qquad (1)$$

far below the present CLEO upper bound of 1.4×10^{-4} [3]. In view of the possibility of new measurements, we point out interesting consequences of various $\mathcal{B}$ values.

The plan of this paper is as follows: In Sec. II, we present the kinematics and some other general aspects of the $\tau^- \to \pi^-\eta\nu_\tau$. The contribution of the vector ($L = 1$) $\pi^-\eta$ final state to $\mathcal{B}$ is discussed in Sec. III, assuming that the $L = 1$ and $I = 1$, $\pi^-\eta$ final state is dominated by the ρ^- meson. Sec. IV addresses the contribution of the $J^P = 0^+$ $\pi^-\eta$ state to $\mathcal{B}$. The analog of the ρ^- here is the $I = 1$, $a_0^-(980)$ state, whose coupling to the vector current relates to a longstanding question on whether the $a_0^-(980)$ is a $\bar{u}d$ state or a $\bar{u}d\bar{s}s/\bar{K}K$-threshold state. The $\bar{u}d$ assumption was implicitly made in the chiral-Langrangian calculations predicting Eq. (1), where $a_0^-(980)$ dominance was used to analytically continue the calculation of low-energy decays to the τ^- decay of interest. We briefly discuss another naive quark-model-based estimate. Sec. V addresses the possible relation between $\mathcal{B}$ and precise measurements of β-decay spectra from trapped radioactive ions. Such measurements can be used to search for scalar interactions, in addition to the standard electroweak $(V - A)\cdot(V - A)$ interaction. In the concluding Sec. VI we present putative $\mathcal{B}$ values and/or bounds on $\mathcal{B}$ with implications for the discussions in the former sections.

II. KINEMATICS OF THE $\tau^- \to \pi^-\eta\nu_\tau$ DECAY

Only the vector weak current $V_\mu(x) = \bar{u}(x)\gamma_\mu d(x)$ contributes to the hadronic part $\langle 0|J_\mu^W|\eta\pi^-\rangle = H_\mu$ of the current-current interaction, since the 1^+ and 0^- parts of the axial current cannot create natural-parity states of two pseudoscalars. The matrix element H_μ can be decomposed into a $J^P = 0^+$ part and a 1^- part in the rest frame of the $\eta\pi^-$ system as follows:

$$\langle 0|V_\mu|\pi\eta\rangle = f_1(s)q_\mu + f_0(s)Q_\mu, \qquad (2)$$

where f_L is the coefficient of the state with angular-momentum L,

$$Q \equiv q_\pi + q_\eta, \qquad q \equiv a(s)q_\pi - q_\eta, \qquad s \equiv Q^2, \quad (3)$$

q_x is the four momentum of particle x, and

$$a(s) \equiv \frac{m_\eta^2 + q_1 \cdot q_2}{m_\pi^2 + q_1 \cdot q_2} \qquad (4)$$

is chosen so that $Q \cdot q = 0$. In the rest frame of the $\eta\pi^-$ system, q is a spacelike vector

$$q = (0, |q|\cos\theta, |q|\sin\theta, 0), \qquad (5)$$

where θ is the angle in this frame between $\vec{q}$ and the recoiling neutrino momentum. The $L = 0$ and $L = 1$ amplitudes interfere in the angular dependence $d\Gamma/d(\cos(\theta)$, but not in the total decay rate obtained by integrating over $d(\cos(\theta)$, namely,

$$\frac{d\Gamma}{ds} = K_1|f_1(s)|^2 + K_0|f_0(s)|^2, \tag{6}$$

with the K_L being kinematic factors. Thus, either the S- or P-wave contribution yields a lower bound on the total rate. We proceed with an estimation of the magnitudes of these contributions.

III. ESTIMATING THE $L = 1$ CONTRIBUTION OF THE $\pi^-\eta$ STATE

The decay $\tau^- \to \pi^-\pi^0\nu_\tau$ comprises 25.5% of all τ^- decays, and is completely dominated by ρ^- exchange. Similarly, our estimate of the $L = 1$ contribution to the decay $\tau^- \to \pi^-\eta\nu_\tau$ assumes ρ^- dominance, taking place via $\tau^- \to \rho^-\nu_\tau$ followed by $\rho^- \to \eta\pi^-$. We thus expect the $L = 1$ component of $\mathcal{B}$ to be

$$\mathcal{B}_{L=1} = \left(\frac{g_{\eta\rho\pi}}{g_{\rho\pi\pi}}\right)^2\left(\frac{p_{\rho\to\eta\pi}}{p_{\rho\to\pi\pi}}\right)^3\mathcal{B}(\tau^- \to \rho^-\nu_\tau), \tag{7}$$

where $g_{\eta\rho\pi}$ and $g_{\rho\pi\pi}$ are the $\rho \to \eta\pi$ and $\rho \to \pi\pi$ coupling constants, respectively, and the cubed ratio between the daughter momenta in the two decays is $(p_{\rho\to\eta\pi}/p_{\rho\to\pi\pi})^3 = 0.07$.

Since the decay $\rho^- \to \eta\pi^-$ has not been observed, we obtain the coupling constant $g_{\eta\rho\pi}$ from the Dalitz-plot distribution of the decay $\eta \to \pi^+\pi^-\pi^0$ and the branching fraction $\mathcal{B}(\eta \to \pi^0\pi^0\pi^0)$. The three-pion Dalitz plot is customarily described with the variables

$$X \equiv \frac{\sqrt{3}}{Q}(T_+ - T_-), \qquad Y \equiv \frac{3}{Q}T_0 - 1, \tag{8}$$

where T_c is the kinetic energy of the pion with charge c, and

$$Q \equiv m_\eta - 2m_{\pi^+} - m_{\pi^0} \approx m_\eta - 3m_\pi. \tag{9}$$

Henceforth, we ignore the difference between the charged and neutral pion masses. The matrix element for $\eta \to \pi^+\pi^-\pi^0$ is taken to be the sum of a scalar and a vector exchange contribution, the latter dominated by the $\rho(770)$

$$\mathcal{M}_{+-0} = \mathcal{M}_S + \mathcal{M}_{\rho^+} + \mathcal{M}_{\rho^-}. \tag{10}$$

A ρ^0 contribution is forbidden due to charge conjugation conservation. Properly accounting for the number of diagrams and identical particles, the $\eta \to \pi^0\pi^0\pi^0$ matrix element is

$$\mathcal{M}_{000} = \frac{3}{\sqrt{3!}}\mathcal{M}_S. \tag{11}$$

The branching fraction of this decay gives the absolute value of the scalar matrix element

$$\begin{aligned}|\mathcal{M}_S|^2 &= 8(2\pi)^3 m_\eta\Gamma_\eta\mathcal{B}(\eta \to \pi^0\pi^0\pi^0)\frac{6\sqrt{3}}{Q^2S_1}\frac{3!}{9} \\ &= 0.065,\end{aligned} \tag{12}$$

where we used the measured values of the η mass, width, and $\pi^0\pi^0\pi^0$ branching fraction [4], the phase-space differential is $dE_1dE_2 = (Q^2/6\sqrt{3})dXdY$, and $S_1 = 2.75$ is the area of the Dalitz plot. The scalar particle exchanged is assumed to be very broad, so that the distribution of events over the relatively small Dalitz plot is essentially uniform.

We take the vector matrix element to be

$$\begin{aligned}\mathcal{M}_{\rho^\mp} &= -g_{\eta\rho\pi}g_{\rho\pi\pi}\frac{(P_\eta + P_\pm)\cdot(P_\mp - P_0)}{(P_\mp + P_0)^2 - m_\rho^2 - i\Gamma_\rho m_\rho} \\ &= -g_{\eta\rho\pi}g_{\rho\pi\pi}\frac{2m_\eta(E_0 - E_\mp)}{2m_\eta E_\pm + M_0^2 - \frac{2}{3}m_\eta^2},\end{aligned} \tag{13}$$

where E_+, E_-, and E_0 are the η-rest-frame energies of the π^+, π^-, and π^0, respectively, and

$$M_0^2 \equiv m_\rho^2 - \frac{1}{3}m_\eta^2 - m_\pi^2 + i\Gamma_\rho m_\rho. \tag{14}$$

Replacing the energies with the Dalitz-plot quantities of Eqs. (8) and (9), the sum of the ρ^+ and ρ^- contributions is

$$\begin{aligned}\mathcal{M}_{\rho^-} + \mathcal{M}_{\rho^+} &= -2g_{\eta\rho\pi}g_{\rho\pi\pi}\frac{rY - \frac{1}{3}r^2(Y^2 + X^2)}{1 - \frac{2}{3}rY + \frac{1}{3}r^2(\frac{1}{3}Y^2 - X^2)} \\ &\approx -g_{\eta\rho\pi}g_{\rho\pi\pi}2\bigg[rY + \frac{r^2}{3}(Y^2 - X^2) \\ &\quad + \frac{r^3}{9}(X^2Y - Y^3)\bigg],\end{aligned} \tag{15}$$

where

$$r \equiv \frac{m_\eta Q}{M_0^2} = 0.14 + 0.03i. \tag{16}$$

and the last line of Eq. (15) is obtained from a Taylor expansion to order r^3.

Squaring the sum of the scalar and vector terms, again keeping terms to order r^3, we obtain

$$\frac{|\mathcal{M}_{+-0}|^2}{|\mathcal{M}_S|^2} \approx 1 + \alpha Y + \beta Y^2 + \gamma X^2 + \delta Y^3 - \delta YX^2, \tag{17}$$

where

$$\begin{aligned}\alpha &= -4g_{\eta\rho\pi}g_{\rho\pi\pi}\Re\{\mathcal{M}_S^* r\}\frac{1}{|\mathcal{M}_S|^2}, \\ \beta &= \left[-\frac{4}{3}g_{\eta\rho\pi}g_{\rho\pi\pi}\Re\{\mathcal{M}_S^* r^2\} + 4(g_{\eta\rho\pi}g_{\rho\pi\pi})^2|r|^2\right]\frac{1}{|\mathcal{M}_S|^2}, \\ \gamma &= \frac{4}{3}g_{\eta\rho\pi}g_{\rho\pi\pi}\Re\{\mathcal{M}_S^* r^2\}\frac{1}{|\mathcal{M}_S|^2}, \\ \delta &= \left[\frac{4}{9}g_{\eta\rho\pi}g_{\rho\pi\pi}\Re\{\mathcal{M}_S^* r^3\} + \frac{8}{3}(g_{\eta\rho\pi}g_{\rho\pi\pi})^2\Re\{r(r^2)^*\}\right] \\ &\quad \times\frac{1}{|\mathcal{M}_S|^2}.\end{aligned} \tag{18}$$

The product of coupling constants $g_{\eta\rho\pi}g_{\rho\pi\pi}$ is obtained by comparing the coefficients of Eq. (17) with the Dalitz-plot distribution of the decay $\eta \to \pi^+\pi^-\pi^0$. A high-statistics study of this distribution has been recently performed by the KLOE collaboration [5], yielding the parameterization

$$|\mathcal{M}_{+-0}|^2 \propto 1 - 1.09Y + 0.124Y^2 + 0.057X^2 + 0.14Y^3. \tag{19}$$

We ignore the measured coefficient errors, as they are much smaller than the theoretical errors associated with our model. From the coefficient of the Y term in Eq. (19) and the first of Eqs. (18), one obtains the product of coupling constants

$$g_{\eta\rho\pi}g_{\rho\pi\pi} = \frac{1.09}{4}\frac{\mathcal{M}_S}{\Re(r)} = 0.51, \tag{20}$$

where $\mathcal{M}_S$ was taken to be real. The accuracy of the model may be judged from the values it obtains for the other coefficients

$$|\mathcal{M}_{+-0}|^2 \propto 1 - 1.09Y + 0.27Y^2 + 0.05X^2 + 0.03Y^3 - 0.03YX^2. \tag{21}$$

Allowing $\mathcal{M}_S$ to have a complex phase does not improve the agreement between Eqs. (19) and (21) significantly. A related cross-check is provided by the ratio of branching fractions $\mathcal{B}(\eta \to \pi^+\pi^-\pi^0)/\mathcal{B}(\eta \to \pi^0\pi^0\pi^0) = 0.70$. The value predicted by Eqs. (12) and (17) is 0.71 when using the experimental coefficients of Eq. (19), and 0.76 using those of Eq. (21).

Taking the matrix element for the decay $\rho \to \pi\pi$ to be

$$\mathcal{M}_\rho = g_{\rho\pi\pi}\varepsilon_\mu^{(\xi)}(P_+ - P_-)^\mu, \tag{22}$$

the coupling constant $g_{\rho\pi\pi}$ is determined to be

$$g_{\rho\pi\pi} = \sqrt{\frac{6\pi m_\rho^2\Gamma_\rho}{p^3_{\rho\to\pi\pi}}} = 6.0. \tag{23}$$

Equations (20) and (23) then give

$$g_{\eta\rho\pi} \approx 0.085. \tag{24}$$

A similar calculation by Ametller and Bramon [6] yielded the ratio $g_{\eta\rho\pi}/g_{\rho\pi\pi} = 0.011 \pm 0.002$, consistent with our results.

From Eqs. (7), (23), and (24), we calculate the $L = 1$ component of the $\tau^- \to \pi^-\eta\nu_\tau$ branching fraction,

$$\mathcal{B}_{L=1} \approx 3.6 \times 10^{-6}. \tag{25}$$

We also obtain

$$\mathcal{B}(\rho \to \eta\pi) = \frac{g^2_{\eta\rho\pi}p^3_{\rho\to\eta\pi}}{6\pi m_\rho^2\Gamma_\rho} \approx 1.4 \times 10^{-5}, \tag{26}$$

far below the current experimental limit of 6×10^{-3} [4].

IV. THE $L = 0$ CONTRIBUTION

The contribution of the $(L = 0)$ $\pi^-\eta$ state to $\mathcal{B}$ is not as readily accessible to a phenomenological estimate as that of the $L = 1$ state. The observed ρ^- dominance in the $\pi^-\pi^0$ final state of the τ^- decay is expected, since the ρ^- has the quantum numbers of the hadronic vector current $\bar{u}\gamma_\mu d$. It is therefore natural to assume that it also dominates the $(L = 1)$ $\pi^-\eta$ final state, although this decay is suppressed by isospin violation. This is not so for the superficially analog case of $a_0^-(980)$ and the scalar contribution to $\mathcal{B}$. In Ref. [2], the $a_0^-(980)$ dominance of the $(L = 0)$ $\pi^-\eta$ channel in weak decays was used to extrapolate the low-energy amplitude for $\eta \to \pi^- e^+\nu_e$ (computed via chiral perturbation theory) to the decay $\tau^- \to \pi^-\eta\nu_\tau$ and obtain the estimate of Eq. (1). The resulting scalar contribution to $\mathcal{B}$ is then ~3 times larger than the vector contribution. This extrapolation is questionable not only because of the large change in Q^2 from ~0.15 GeV^2 to ~1 GeV^2. The key point is that $a_0^-(980)$ [just like its $I = 0$ counterpart $f_0(980)$] may well be a four-quark $\bar{u}d\bar{s}s$ state, a view suggested early on [7] and adopted recently by the Particle Data Group [4]. In this case, the $a_0(980)$ coupling to the $\bar{u}d$ scalar current is "Zweig-rule" suppressed, and the four-quark state will not dominate the decay in question.

Several considerations suggest that the $a_0(980)$ and $f_0(980)$ states have significant four-quark contributions:

(1) The widths $\Gamma(f_0(980) \to \pi\pi) \sim \Gamma(a_0(980) \to \pi\eta) \sim 50$ MeV are anomalously small for an S-wave $\bar{q}q$ state. Since the lighter, 770-MeV ρ has a P-wave decay width of 150 MeV, the $a_0(980)$ $f_0(980)$ and widths should have been vastly larger. This is the case for the so-called $\sigma(600)$ scalar, often used in nuclear potentials, which has a width of about 600 MeV.

(2) The fact that $a_0(980)$ and $f_0(980)$ decay also into $K\bar{K}$ despite the highly reduced phase space (the decay is kinematically forbidden over most of the widths) is an argument against their being $\bar{q}q$ states. Indeed, four-quark states would much more readily fall apart to $q\bar{s}\bar{q}s = \bar{K}K$ than would $\bar{q}q$ scalars. In principle, the a_0 and f_0 could be "molecular," lightly bound $\bar{K}K$ threshold states, in analogy with the $X(3872)$, which may be a $D^*\bar{D}$ threshold state [8]. For states of similar size, the kinetic energy in the $D^*\bar{D}$ system is four times smaller than that of the $\bar{K}K$ system. On the other hand, roughly the same meson-meson potentials are generated by couplings of the light quarks. Therefore, binding $\bar{K}K$ to form $a_0(980)$ and $f_0(980)$ seems unlikely. The features 1 and 2 above, which are particularly puzzling in a $\bar{q}q$ picture, can conceivably be resolved if one notes the special role of t'Hooft's anomaly induced $\bar{u}u\bar{d}d\bar{s}s$ six-quark coupling [9].

(3) Further indirect support for the four-quark picture comes from the suggestion [10] that in collision or decay processes with few initial quarks, $\bar{q}q$ meson production should exceed considerably that of more complex baryonic and exotic four-quark states. Comparison of $a_0(980)$ and $f_0(980)$ with bonafide $\bar{q}q$ states such as $\rho(770)$ mesons in e^+e^- or $p\pi$ collisions and in B decays suggests that the former are significantly suppressed, again supporting the four-quark hypothesis. If the initial state has many quarks and, in particular, many $\bar{s}s$ pairs, as is the case at the Relativistic Heavy Ion Collider, then the suppression of $\bar{q}q\bar{s}s$ production is expected to be weaker. This may be easier to test for $f_0(980)$ than for $a_0(980)$, whose identification requires good photon reconstruction. As further example, we note that 11% of the decay $D_s^+ \to K^+K^-\pi$ is due to $f_0 \to K^+K^-$ [4].

If $a_0(980)$ is indeed a four-quark state, then $\mathcal{B}$ will be smaller than the value predicted utilizing $a_0(980)$ dominance and assuming it is a $\bar{q}q$ state, Eq. (1). If a search for $\tau^- \to \pi^-\eta\nu_\tau$ that is sensitive to a branching fraction of order 10^{-5} fails to detect a $\sim$50 MeV-wide peak around 980 MeV in the $\eta\pi^-$ invariant mass spectrum, this would constitute a fourth argument in support of the four-quark view. Conversely, observation of a clear peak would strongly suggest that $a_0(980)$ is in fact a regular $\bar{u}d$ state, as early arguments by Bramon and Masso have suggested [11].

Next, we present some general arguments regarding the expected scalar ($L = 0$) contribution $\mathcal{B}_S$ to the branching fraction $\mathcal{B}$, assuming that it is dominated by the exchange of the $a_0(980)$, which is taken to be a $\bar{u}d$ state. Key to its small magnitude is the operator equation expressing the fact that the weak vector current is conserved up to small electromagnetic and $m_d - m_u$ mass difference effects

$$\nabla^\mu V_\mu(x) = (m_d - m_u)\bar{u}(x)d(x) + eA^\mu_{em}(x)V_\mu(x). \quad (27)$$

The contribution of the electromagnetic interaction term to $\tau^- \to \eta\pi^-\nu_\tau$ is related to $\tau^- \to \eta\pi^-\nu_\tau\gamma$, but given the difficulty in observing $\tau^- \to \eta\pi^-\nu_\tau$, there is little hope that the decay involving an additional photon in the final state can be studied in the near future. The corresponding one-loop electromagnetic corrections are suppressed by $\alpha/\pi \sim 1/500$. The first term of Eq. (27) is $\sim(m_d - m_u)/m_h \sim 1/200$ for $(m_d - m_u) \sim 4$ MeV and a typical hadronic mass of $m_h \sim 0.8$ GeV, hence we focus on this term in what follows. The matrix element $\langle 0|\nabla^\mu V_\mu|h\rangle$ [with $h = \eta\pi$ or $h = a_0^-(980)$, if $a_0^-(980)$ dominance holds] of the operator Eq. (27) then yields

$$Q^\mu\langle 0|V_\mu|h\rangle = Q^2 f_0(s) = (m_d - m_u)\langle 0|S^-|h\rangle, \quad (28)$$

where S^- is the scalar current $\bar{u}(x)d(x)$, and $Q^2 = s = m_h^2$ is the squared mass of the hadronic system. The left-hand side of Eq. (28) yields the middle expression by using Eqs. (2) and (3). Thus, computing $\mathcal{B}_{L=0}$, the $L = 0$ contribution to $\mathcal{B}$, reduces to estimating the low-energy hadronic parameter $\langle 0|S^-|h\rangle$. A first-principles, unquenched lattice QCD calculation is lacking at present, but recent progress in dealing with light quarks/pseudoscalars may soon make it feasible [12]. The calculation is circumvented in the chiral perturbation theory approach, which uses effective Lagrangians (including isospin violation) and couplings fitted together to known low-energy processes and extrapolated to the τ decay of interest. The fact that as many as three calculations of this type yielded the same result [Eq. (1)] indicates that this is a well-defined framework, but does not test its reliability.

Here, we present a simpler quark model-motivated estimate. Unlike the $A \sim V$ and $S \sim P$ chiral symmetry-motivated relation, we relate the axial and scalar matrix elements, since both pertain to P-wave ($a_1(1260)$ and $a_0(980)$) rather than S-wave (ρ and π) $\bar{q}q$ states. We assume that $a_0^-(980)$ dominates the $\tau^- \to \pi^-\eta\nu_\tau$ decay and that the decay $\tau \to \pi^-\pi^+\pi^-\nu_\tau$ is dominated by the $a_1^-(1260)$. Defining the matrix elements

$$v \equiv \langle 0|S^-|a_0^-(980)\rangle, \qquad a \equiv \langle 0|A_i|a_1^-(1260)_i\rangle, \quad (29)$$

where i is a helicity state index, we expect

$$\frac{\mathcal{B}_{L=0}}{\mathcal{B}(\tau^- \to a_1^-(1260)\nu_\tau)} \sim 1.3\frac{v^2}{a^2}\left(\frac{m_d - m_u}{m_{a_1(1260)}}\right)^2, \quad (30)$$

where the 1.3 enhancement is due to the larger phase space for the decay into the lighter $a_0^-(980)$. The couplings of the local scalar and axial currents to the two ${}^3P_0$ and ${}^3P_1$ $\bar{u}d$ states of similar mass are expected to be roughly equal, namely, $a \sim v$. Indeed, these couplings are fixed by quark-model wave functions which, apart from relatively small $L \cdot S$ effects, are the P-wave ground states of the same Hamiltonian. From Eq. (30) we find

$$\mathcal{B}_{L=0} \sim 1 \times 10^{-5}, \quad (31)$$

similar to the contribution of the ρ^- and, within our crude approximations, consistent with the chiral-perturbation-theory estimates. We note that Eq. (31) may require an additional suppression factor of up to $\sim$3, due to the three helicity states available to the $a_1^-(1260)$).

V. TEST FOR NEW WEAK INTERACTIONS

The general Lorentz-invariant "current × current" weak interactions could include, in addition to $(V - A) \cdot (V - A)$, products of scalar (S), pseudoscalar (P), and tensor (T) "currents." Exchanging new, heavy elementary particles cannot generate the nonminimal T part (however, see Ref. [13] regarding the possibility of generating tensor interactions via a Fierz transformation of a scalar leptoquark contribution in the S channel), hence we focus on the S and P parts. Experimentally, the amplitudes of the $V \cdot V$, $V \cdot A$, and $A \cdot A$ current products can be compared with those of $S \cdot S$, $P \cdot P$, or $S \cdot P$ terms in nuclear beta decays involving both $u \rightarrow d$ and $e \rightarrow \nu_e$ weak transitions [14]. It is convenient to parameterize the corrections to the standard-model currents using the same weak coupling g_W^2, attributing the smallness of the $S \cdot S$, $S \cdot P$, and $P \cdot P$ terms to heavy (pseudo) scalar mesons with masses m_P, $m_S \gg m_W$. A positive result implying m_S, m_P masses smaller than $O(\mathrm{TeV})$ would motivate searching for such particles at the upcoming LHC.

A stringent limit on the pseudoscalar mass M_P comes from its contribution of g_W^2/M_P^2 to the amplitude $A(\pi^- \rightarrow e^- \nu_e)$. The branching fraction for this decay, $(1.230 \pm 0.004) \times 10^{-4}$, is in agreement with the expectation of the standard electroweak model, where its small value is due to the $m_e/m_\mu \sim 1/200$ helicity suppression of the $V - A$ amplitude. We therefore use the error of this result to obtain an approximate limit on the pseudoscalar contribution

$$\left(\frac{M_W}{M_P}\right)^2 < 0.004 \times 10^{-4} \frac{1}{200} \sim 3 \times 10^{-6}. \tag{32}$$

In order for measurements using unsuppressed nuclear beta decays to compete with this limit, a precision of about 3×10^{-6} is needed. Similarly, the decays $K^- \rightarrow e^- \bar{\nu}_e$ and $B^- \rightarrow e^- \bar{\nu}_e$ yield stringent bounds on pseudoscalar couplings involving second- and third-generation quarks [15]. We note that direct production of a pseudoscalar with mass $M_P > 10^3 M_W$ is far beyond the reach of the LHC.

The case of the scalar part is different. Current limits from high-precision nuclear beta-decay experiments will continue to be unchallenged by accelerator-based experiments, until an eventual B-factory limit on or observation of the decay $\tau^- \rightarrow \pi^- \eta \nu_\tau$, whose small standard-model branching fraction makes it sensitive to new scalar interactions. In a nuclear beta decay, the distribution of the angle between the neutrino and the lepton is

$$W(\theta) = 1 + b\frac{m_e}{E_e} + a\beta_e \cos(\theta), \tag{33}$$

where m_e, E_e, and β_e are, respectively, the electron mass, energy, and velocity. The beautiful new experiments using traps to also measure with high precision the recoil velocity of the daughter nucleus have observed $b = -0.0027 \pm 0.0029$ [16], $a = 0.9981\,^{+0.0044}_{-0.0048}$ [14]. The deviation of a from the $V - A$ prediction $a = 1$ leads to the (so far relatively weak) bound on the scalar mass

$$\frac{M_S}{M_W} \sim (0.004)^{-1/4} \sim 4. \tag{34}$$

A tighter bound of $(M_S/M_W) > 6$–7 is expected from improved measurements of a. Once the lower part of the beta spectrum is more precisely measured, the overall normalization of the rate will yield a more sensitive bound of $M_S > 15 M_W$ by utilizing interference of the S and $V - A$ amplitudes [17].

In passing, we note that standard beta decay experiments such as KATERIN [18], which will measure the electron-neutrino mass (or rather m_{ν_1}) down to 0.4 eV, will have very high statistics of $\sim 10^{11}$ events. Still, beta spectra with or without recoiling atoms are also affected by radiative and hadronic effects, and precise calculations of the latter will be required if the experimental precision is to yield strong limits on nonstandard couplings.

A scalar $\bar{u}d$ weak current contributes to G-parity-violating second-class-current transitions, such as $\tau^- \rightarrow \pi^- \eta \nu_\tau$, provided that it couples to τ^- and ν_τ. As discussed above, the present experimental upper bound on the branching ratio for this mode is an order of magnitude greater than the estimated standard-model contribution $\sim 10^{-5}$, which is at the level that may be detected by the BABAR and Belle experiments. Since interference between a nonstandard contribution and the small $V - A$ amplitude will not contribute much, a limit of the branching fraction at the level of 3×10^{-5} would imply

$$\frac{M_S}{M_W} > (3 \times 10^{-5})^{-1/4} \sim 12, \tag{35}$$

comparable to the expected future bounds from beta decay experiments.

Unlike the universal gauged weak interactions, the scalar couplings could discriminate between different lepton generations. Thus, the S particle could be "first-generation oriented," coupling to the u and d quarks and the e and ν_e leptons but not to τ or ν_τ. In such a case, it will affect the beta decays but not the $\tau^- \rightarrow \pi^- \eta \nu_\tau$ decays. Conversely, S particles may couple more strongly to the third-generation $\tau \nu_\tau$ vertex than to $e \nu_e$. Thus, *a priori*, the limit from nuclear beta decays and the one from the $\tau^- \rightarrow \pi^- \eta \nu_\tau$ decay are complementary and, furthermore, observation of S-coupling effects in one mode and not the other would indicate nonuniversality.

On the particle theory side, many lines of argument [19] suggest that new physics, particularly novel weak couplings different from standard $V - A$, will most strongly manifest in higher generations. This would enhance S effects in the τ decays relative to the first-generation beta decays. More generally, M_S is unlikely to be much smaller than M_P, for which the very strict bound above applies, unless M_S is protected by $SU(2)_L$, namely, S couples to the Z^0. In that case, the S^+, S^-, and S^0 form an $SU(2)_L$ triplet, helping produce S particles at the LHC via an intermediate Z^0 or $W^\pm$. Otherwise, production of $S^+ S^-$ pairs is smaller by $(\alpha_{\mathrm{EM}}/\alpha_{\mathrm{Weak}})^2 \sim 10^{-2}$. In general, if we have left-right symmetry at relatively low scales [20] the stringent limits on M_P push M_S to very high values.

VI. CONCLUSIONS

We have considered the S- and P-wave contributions to the branching fraction of the decay $\tau^- \rightarrow \pi^- \eta \nu_\tau$. We find the P-wave contribution, which is more robustly calculated, to be 3.6×10^{-6}, and the S-wave one to be around 1×10^{-5}, both in agreement with previous calculations. Given the capability of experiments at the B factories to measure or set a limit on the branching fraction $\mathcal{B}(\tau^- \rightarrow \pi^- \eta \nu_\tau)$ at the 10^{-5} level, it is interesting to note the implications of the possible experimental results:

(i) A "minimal" result of $\mathcal{B} \sim (0.2\text{–}0.4) \times 10^{-5}$ with the $\pi^- \eta$ invariant mass around the ρ^- peak, which may be hard to extract experimentally, involves no new surprises.

(ii) A larger value of $\mathcal{B}$, in the range $(1\text{–}1.5) \times 10^{-5}$, consistent with the chiral perturbation theory calculations and with our quark-model estimate, would strongly suggest that $a_0^-(980)$ dominates the S-wave part of the decay. In this case, a narrow invariant-mass peak around 980 MeV should be seen. This would strongly suggest that the $a_0^-(980)$ is a $\bar{u}d$ scalar meson after all.

(iii) A somewhat larger value, $\mathcal{B} > (2 - 3) \times 10^{-5}$ with scalar-meson dominance, may indicate novel scalar components in the weak interactions.

We note that an upper limit on the branching fraction of the related decay $\tau^- \rightarrow \eta' \pi^- \nu_\tau$ has been set at 7.210^{-6} [21]. Discussion of the implications of this limit will be the subject of a follow-up publication.

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