# Correlations among new *CP* violating effects in $\Delta F = 2$ observables

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We point out that the observed *CP* violation in $B_d - \bar{B}_d$ mixing, taking into account the measured ratio $\Delta M_d / \Delta M_s$, the recently decreased lattice value of the nonperturbative parameter $\hat{B}_K$ and an additional effective suppression factor $\kappa_\epsilon \simeq 0.92$ in $\epsilon_K$ neglected so far in most analyses, may be insufficient to describe the measured value of $\epsilon_K$ within the standard model (SM), thus hinting at new *CP* violating contributions to the $K - \bar{K}$ and/or $B_d - \bar{B}_d$ systems. Furthermore, assuming that $\Delta M_d / \Delta M_s$ is SM-like, the signs and the magnitudes of new-physics effects in $\epsilon_K$ and in the *CP* asymmetries $S_{\psi K_s}$ and $S_{\psi \phi}$ may turn out to be correlated. For example, in a scenario with new *CP* phases in $B_d$ and $B_s$ mixings being approximately equal and negative, a common new phase $\approx -5°$ could remove the tension between $\epsilon_K$ and $S_{\psi K_s}$ present in the SM and simultaneously accommodate, at least partly, the recent claim of $S_{\psi \phi}$ being much larger than the SM expectation. We emphasize the importance of precise determinations of $V_{cb}$, $\hat{B}_K$, $F_K$, and $\xi_s$, to which the parameter $\epsilon_K$ and its correlation with the *CP* violation in the $B_d - \bar{B}_d$ system are very sensitive.

## 1. INTRODUCTION

The major task achieved in quark flavor physics up to the present is a sound test of the standard model (SM) mechanism of flavor and *CP* violation. This mechanism has proven to be able to accommodate dozens of measured processes, to a degree of accuracy sometimes unexpected. These processes have consequently allowed a redundant determination of the CKM matrix parameters, in particular $\bar{\rho}$, $\bar{\eta}$. Indeed, the $(\bar{\rho}, \bar{\eta})$ plots by the UTfit and CKMfitter collaborations have become somewhat an icon of the SM performance in flavor physics. To the present level of accuracy, the "big picture" in flavor and *CP* violation looks therefore quite solid.

Nonetheless, hints of discrepancies with respect to the SM expectations do exist in some flavor observables. The most recent is the claim of a $B_s$ mixing phase much larger than the SM prediction. This conclusion—first signalled in 2006 by Lenz and Nierste [1]—has been recently reported as an evidence by the UTfit collaboration [2] on the basis of a combined fit to the time-dependent tagged angular analyses of $B_s \rightarrow \psi \phi$ decays by the CDF [3] and D0 [4] collaborations. The result of [2] urges higher-statistics data from Tevatron, but, if confirmed, would be the first evidence of physics beyond the SM from collider data.

Another emblematic example, also emphasized in [2,5], is that of the penguin-dominated nonleptonic $b \rightarrow s$ decays. The mixing-induced *CP* asymmetries measured in these decays allow one to access $\sin 2\beta$, where $\beta$ is one of the angles of the unitarity triangle (UT), defined below in Eq. (3). The $\sin 2\beta$ determinations obtained from these decay modes are systematically lower than the value measured in the tree-level decay $B_d \rightarrow \psi K_s$. The latter direct determination has in turn been found to be lower than the one extracted indirectly from tree-level measurements, in

particular $|V_{ub}/V_{cb}|$ [6–8]. Conclusions in this respect depend mostly on the $|V_{ub}|$ estimate, which is a not yet a settled issue. Independently of this, the problem has been recently revived in [5] as a consequence of a new lattice estimate of the $\hat{B}_K$ parameter [9], which reads $\hat{B}_K = 0.720(13)(37)$.[1] The parameter $\hat{B}_K$ enters the *CP*-violating observable $\epsilon_K$ and, in the context of the SM, the decrease of $\hat{B}_K$ found in [9,10] with respect to previous determinations favors $\sin 2\beta$ again substantially higher than the one extracted from $B_d \rightarrow \psi K_s$.

Here we would like to gather these pieces of information and try to address the question of whether existing data on the $B_d$ and $K$ systems do already signal the presence of inconsistencies in the SM picture of *CP* violation from a somewhat different point of view than the analysis in [5]. More concretely, the most updated theoretical input in $K$ physics—in particular the quite low central value from the aforementioned new lattice determination of $\hat{B}_K$ and an additional effective suppression factor $\kappa_\epsilon \simeq 0.92$ in the SM $\epsilon_K$ formula neglected in most analyses to date—tend both to lower the SM prediction for $|\epsilon_K|$ beneath its measured value if the amount of *CP* violation in the $B_d$ system, quantified by $\sin 2\beta$ from $B_d \rightarrow \psi K_s$, is used as input.

In order to cure this potential inconsistency, one should then introduce either a new *CP* phase in the $B_d$ or, respectively, in the $K$ system, or alternatively two smaller phases in both systems. The case of a single additional $B_d$ mixing phase is especially interesting. In this instance, the SM

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[1]Similar results have been obtained in [10], while $\hat{B}_K = 0.83(18)$ has been reported in [11]. It may also be interesting to note that some nonlattice estimates of $\hat{B}_K$, e.g. those in the large $N_c$ approach, feature $\hat{B}_K \lesssim 0.70$. See, in particular, Refs. [12–14].

formula for the mixing-induced *CP* asymmetry $S_{\psi K_s}$ generalizes to

$$S_{\psi K_s} = \sin(2\beta + 2\phi_d) = 0.681 \pm 0.025, \qquad (1)$$

where $\phi_d$ is the new phase. The information mentioned above points toward a small *negative* value of $\phi_d$. On the other hand, the mixing-induced *CP* asymmetry $S_{\psi\phi}$ is given by [7]

$$S_{\psi\phi} = \sin(2|\beta_s| - 2\phi_s), \qquad (2)$$

where the SM phases $\beta$, $\beta_s$ are defined from the CKM matrix entries $V_{td}$, $V_{ts}$ through

$$V_{td} = |V_{td}|e^{-i\beta}, \qquad V_{ts} = -|V_{ts}|e^{-i\beta_s}, \qquad (3)$$

with $\beta_s \approx -1°$. From Eq. (2) one finds that a negative $\phi_s$ is also required to explain the claim of [2]. It is then tempting to investigate whether, at least to first approximation, the same new phase $\phi_d \approx \phi_s \approx \phi_B$ could fit in both $B_d$ and $B_s$ systems, being a small correction in the former case—where the SM phase is large—and the bulk of the effect in the latter.[2]

The rest of this paper is an attempt to explore the above possibilities in more detail. For the sake of clarity, we introduce here some notation details. The amplitudes for $B_q$ $(q = d, s)$ meson mixings are parameterized as follows:

$$\langle B_q|\mathcal{H}^{\text{full}}_{\Delta F=2}|\bar{B}_q\rangle \equiv A^{\text{full}}_q e^{2i\beta^{\text{full}}_q}, \qquad (4)$$

where, to make contact with the conventions on the SM phases $\beta$, $\beta_s$, one has

$$\beta^{\text{full}}_d = \beta + \phi_d, \qquad \beta^{\text{full}}_s = \beta_s + \phi_s. \qquad (5)$$

The magnitudes $A^{\text{full}}_q$ can be written as

$$\begin{aligned} A^{\text{full}}_q &= A^{\text{SM}}_q C_q, \quad \text{with} \\ A^{\text{SM}}_q &\equiv |\langle B_q|\mathcal{H}^{\text{SM}}_{\Delta F=2}|\bar{B}_q\rangle| = \Delta M^{\text{SM}}_q/2. \end{aligned} \qquad (6)$$

Concerning $C_q$, with present theoretical errors on the $B_q$ system mass differences $\Delta M_q$, it is impossible to draw conclusions on the presence of new physics (NP). Therefore, one typically considers the ratio $\Delta M_d/\Delta M_s$, where the theoretical error is smaller, and is dominated by the uncertainty in the lattice parameter $\xi_s$, defined as

$$\xi_s \equiv \frac{F_{B_s}\sqrt{\hat{B}_s}}{F_{B_d}\sqrt{\hat{B}_d}}. \qquad (7)$$

The resulting SM prediction for $\Delta M_d/\Delta M_s$ is in good agreement with the experimentally measured ratio.[3]

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[2]This simple correlation is unrelated to more involved correlations that invoked $\Delta F = 1$ transitions, as in [15] and references therein.

[3]Variations of the SM formula due to different CKM input are much smaller than the relative theoretical error, which is roughly $2 \times \sigma_{\xi_s}$.

Hence it is plausible, at least to first approximation, to assume $\Delta M_d/\Delta M_s$ as unaffected by NP, i.e., recalling Eq. (6), that

$$C_d = C_s = C_B. \qquad (8)$$

We will comment on this assumption later on in the analysis.

## II. $\epsilon_K$ AND sin2β

We start our discussion by looking more closely at the $\epsilon_K$ parameter. For the latter, we use the following theoretical formula [16]:

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon\left(\frac{\text{Im}(M^K_{12})}{\Delta M_K} + \xi\right), \qquad \xi = \frac{\text{Im}A_0}{\text{Re}A_0}, \qquad (9)$$

with $A_0$ the 0-isospin amplitude in $K \to \pi\pi$ decays, $M^K_{12} = \langle K|\mathcal{H}^{\text{full}}_{\Delta F=2}|\bar{K}\rangle$ and $\Delta M_K$ the $K - \bar{K}$ system mass difference. The phase $\phi_\epsilon$ is measured to be [17]

$$\phi_\epsilon = (43.51 \pm 0.05)°. \qquad (10)$$

Formula (9) can for instance be derived from any general discussion of the $K - \bar{K}$ system formalism, like [18,19], and can be shown to be equivalent to Eq. (1.171) of [20], where all the residual uncertainties are explicitly indicated and found to be well below 1%. In contrast with the $\epsilon_K$ formula used in basically all phenomenological applications, Eq. (9) takes into account $\phi_\epsilon \neq \pi/4$ and $\xi \neq 0$. Specifically, the second term in the parenthesis of Eq. (9) constitutes an O(5%) correction to $\epsilon_K$ and in view of other uncertainties was neglected until now in the standard analyses of the UT, with the notable exception of [21,22]. Most interestingly for the discussion to follow, both $\xi \neq 0$ and $\phi_\epsilon < \pi/4$ imply suppression effects in $\epsilon_K$ relative to the approximate formula. In order to make the impact of these two corrections transparent, we will parameterize them through an overall factor $\kappa_\epsilon$ in $\epsilon_K$:

$$\kappa_\epsilon = \sqrt{2} \sin\phi_\epsilon \bar{\kappa}_\epsilon, \qquad (11)$$

with $\bar{\kappa}_\epsilon$ parameterizing the effect of $\xi \neq 0$. The calculation by Nierste in [20] (page 58), the analyses in [21,22], and our very rough estimate at the end of the paper show that $\bar{\kappa}_\epsilon \lesssim 0.96$, with $0.94 \pm 0.02$ being a plausible figure. Consequently, we find

$$\kappa_\epsilon = 0.92 \pm 0.02. \qquad (12)$$

In view of the improvements in the input parameters entering $\epsilon_K$, the correction (12) may start having a non-negligible impact in UT analyses. Therefore, a better evaluation of this factor would certainly be welcome.

One can now identify the main parametric dependencies of $\epsilon_K$ within the SM through the formula

$$\begin{aligned} |\epsilon^{\text{SM}}_K| = \kappa_\epsilon C_\epsilon \hat{B}_K |V_{cb}|^2 \lambda^2 \bar{\eta}(&|V_{cb}|^2(1 - \bar{\rho})\eta_{tt}S_0(x_t) \\ &+ \eta_{ct}S_0(x_c, x_t) - \eta_{cc}x_c), \end{aligned}$$

$$\text{with} \quad C_\epsilon = \frac{G^2_F F^2_K m_{K^0} M^2_W}{6\sqrt{2}\pi^2 \Delta M_K}, \qquad (13)$$

TABLE I.    Input parameters. Quantities lacking a reference are taken from [17].

| | |
|---|---|
| $G_F = 1.16637 \cdot 10^{-5}$ GeV$^{-2}$ | $\lambda = 0.2255(7)$ [23] |
| $M_W = 80.403(29)$ GeV | $\lvert V_{cb} \rvert = 41.2(1.1) \cdot 10^{-3}$ [24] |
| $M_Z = 91.1876(21)$ GeV | $\eta_{cc} = 1.43(23)$ [25] |
| $\alpha_s(M_Z) = 0.1176(20)$ | $\eta_{ct} = 0.47(4)$ [25] |
| $m_c(m_c) = 1.25(9)$ GeV | $\eta_{tt} = 0.5765(65)$ [26] |
| $M_t = 172.6(1.4)$ GeV$^a$ [27] | $F_K = 0.1561(8)$ GeV [23] |
| $M_{B_d} = 5.2795(5)$ GeV | $M_{K^0} = 0.49765$ GeV |
| $M_{B_s} = 5.3661(6)$ GeV | $\Delta M_K = 0.5292(9) \cdot 10^{-2}$/ps |
| $\Delta M_d = 0.507(5)$/ps | $\lvert \epsilon_K \rvert = 2.232(7) \cdot 10^{-3}$ |
| $\Delta M_s = 17.77(12)$/ps [28] | $\kappa_\epsilon = 0.92(2)$ |
| $\xi_s = 1.21(6)$ [29–32] | $\phi_\epsilon = 43.51(5)°$ |

$^a$The $\overline{\text{MS}}$ mass value $m_t(m_t) = 162.7(1.3)$ is derived using [33].

and where notation largely follows Ref. [18], in particular $x_i = m_i^2(m_i)/M_W^2$, $i = c, t$. As far as CKM parameters are concerned, Eq. (13) reproduces the "exact" SM result, where no expansion in $\lambda$ is performed, to 0.5% accuracy. Now, $1 - \bar{\rho} = R_t \cos\beta$ and $\bar{\eta} = R_t \sin\beta$, where the UT side $R_t$ is given by

$$R_t \approx \frac{1}{\lambda} \frac{\lvert V_{td} \rvert}{\lvert V_{ts} \rvert} = \frac{\xi_s}{\lambda} \sqrt{\frac{M_{B_s}}{M_{B_d}}} \sqrt{\frac{\Delta M_d}{\Delta M_s}} \sqrt{\frac{C_s}{C_d}} \qquad (14)$$

with $C_d = C_s$ assumed here [see Eq. (8)] and $\xi_s$ introduced in Eq. (7). Therefore, for the leading contribution to $\epsilon_K$, due to top exchange, one can write

$$\lvert \epsilon_K \rvert \propto \kappa_\epsilon F_K^2 \hat{B}_K \lvert V_{cb} \rvert^4 \xi_s^2 \frac{C_s}{C_d} \sin 2\beta, \qquad (15)$$

showing that the prediction for $\epsilon_K$ is very sensitive to the value of $\lvert V_{cb} \rvert$ but also to $\xi_s$ and $F_K$. All the input needed in Eqs. (13)–(15) and in the rest of our paper is reported in Table I.

## III. THREE NEW-PHYSICS SCENARIOS

Next we note that the most updated values for all the parameters on the right-hand side of Eq. (15), with exception of $\sin 2\beta$, are lower with respect to previous determinations. Notably, the central value of the most recent estimate of $\hat{B}_K$ [9] is lower by roughly 9%, with a similar effect due to the $\kappa_\epsilon$ factor [see Eq. (12)]. One can then investigate whether the value of $\sin 2\beta$ required to accommodate $\lvert \epsilon_K \rvert$ within the SM may be too high with respect to the $\sin 2\beta$ determination from $B_d$ physics, as already investigated in [5] for $\kappa_\epsilon = 1$. Here we would like to emphasize that, more generally, this could entail the presence of a new phase either dominantly in the $B_d$ system or, respectively, in the $K$ system, or, alternatively, of two smaller phases in both systems, defining in turn three NP scenarios. Addressing the significance of either scenario crucially depends on the errors associated with the theoretical input entering the $\epsilon_K^{\text{SM}}$ formula. We will come back to this point quantitatively in the discussion to follow, where all the present uncertainties are taken into account.

However, since these uncertainties in the input do not yet allow clear-cut conclusions, we would like to first illustrate the three just mentioned NP scenarios by setting all input parameters except $\hat{B}_K$ at their central values. This would correspond to the hypothetical situation in which all the input, including the CKM parameters, were controlled with higher accuracy than $\hat{B}_K$, for which we assume a 3% uncertainty. In Fig. 1 (left panel) we then show $\lvert \epsilon_K^{\text{SM}} \rvert$ as a function of $\sin 2\beta$ for $\hat{B}_K \in \{0.65, 0.70, 0.75, 0.80\} \pm 3\%$. The vertical ranges centered at $\sin 2\beta \in \{0.681, 0.75, 0.88\}$, with a relative error chosen at 3.7% as in the $\sin 2\beta_{\psi K_s}$ case, define the scenarios in question. The horizontal range, representing the experimental result for $\epsilon_K$, shows that $\sin 2\beta \approx \sin 2\beta_{\psi K_s}$ would require NP in $\epsilon_K$ in
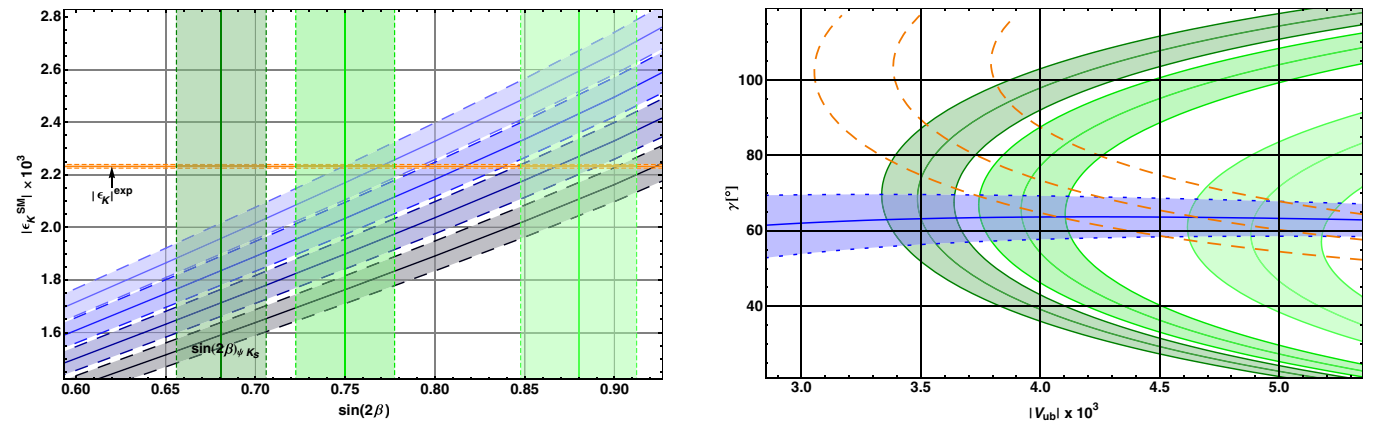


FIG. 1 (color online).    Left panel: $\lvert \epsilon_K^{\text{SM}} \rvert$ vs $\sin 2\beta$ with only $\hat{B}_K$ errors included. $\hat{B}_K \in \{0.65, 0.70, 0.75, 0.80\} \pm 3\%$ are shown as areas delimited by long-dashed lines, in the order darker to lighter (blue). Vertical (green) areas display $\sin 2\beta \in \{0.681, 0.75, 0.88\} \pm 3.7\%$ (see text). Right panel: $\gamma$ vs $\lvert V_{ub} \rvert$ for $\sin 2\beta \in \{0.681, 0.75, 0.88\} \pm 3.7\%$, corresponding to the areas delimited by solid lines, in the order darker to lighter (green). The (blue) area delimited by dotted lines corresponds to $R_t = R_t^{\text{SM}}$, while the long-dashed (orange) lines represent the contours of $\epsilon_K^{\text{exp}}$.

TABLE II. Indicative values for various quantities of interest in the scenarios represented in the left panel of Fig. 1 (see also text).

| | | $\sin 2\beta$ | |
|---|---|---|---|
| | 0.681 | 0.75 | 0.88 |
| $10^3 \cdot |\epsilon_K^{\rm SM}| \begin{cases} \hat{B}_K = 0.7 \\ \hat{B}_K = 0.8 \end{cases}$ | 1.71 | 1.90 | 2.27 |
| | 1.96 | 2.17 | 2.59 |
| $\phi_d[°]$ | 0 | $-2.8$ | $-9.4$ |
| $S_{\psi\phi}$ | 0.04 | 0.14 | 0.36 |
| $10^3 \cdot |V_{ub}|$ | 3.50 | 3.92 | 4.90 |
| $\gamma[°]$ | 63.5 | 64.0 | 63.9 |



FIG. 3 (color online). $\hat{B}_K$ ranges compatible with the experimental $\epsilon_K$ result as a function of $\sin 2\beta$. $\hat{B}_K$ and/or $\xi_s$ are taken with present or 2.5% uncertainties. Comparing the areas corresponding to the different assumed uncertainties (see legend), one can note the role of a decrease in the $\xi_s$ error.

order to fit the data, unless $\hat{B}_K \gtrsim 0.85$. Conversely, in the last scenario, as considered in [5], no NP is required to fit the data on $\epsilon_K$, even for $\hat{B}_K \approx 0.65$. In this case, however, the discrepancy with respect to the $\sin 2\beta_{\psi K_s}$ determination reveals the need for a NP phase in the $B_d$ system around $-9°$. In Table II we report indicative values for various quantities of interest obtained from the scenarios shown in Fig. 1 (left panel). In particular, values for $|\epsilon_K^{\rm SM}|$ are shown for $\hat{B}_K = \{0.7, 0.8\}$. In giving the result for $S_{\psi\phi}$ we set $\phi_d = \phi_s$ (see discussion below). We observe that values of $\hat{B}_K$ in the ballpark of 0.7 would imply a NP correction to $|\epsilon_K^{\rm SM}|$ exceeding $+20\%$, which should be visible if the input parameters could be controlled with, say, 2% accuracy.

The above discussion, and the scenarios in Table II, assume that the UT side $R_t$ be equal to its SM value [see Eq. (8)] and imply $\gamma$ not larger than around 65°. Figure 1 (right panel) shows the correlation existing for fixed $\sin 2\beta$ between $\gamma$ and $|V_{ub}|$ (or, equivalently, the side $R_b$ [34]). From the figure one can note that, if $\gamma$ from tree-level decays turns out to be larger than the values in Table II, consistency of $\sin 2\beta$ with Eq. (1) can be recovered by increasing the side $R_t$ with respect to the SM value (thus shifting upwards the area delimited by dotted lines in the figure). As one can see from the same figure, this would
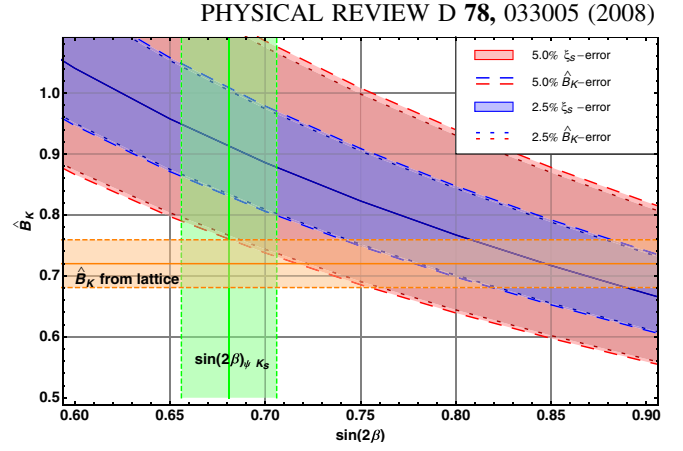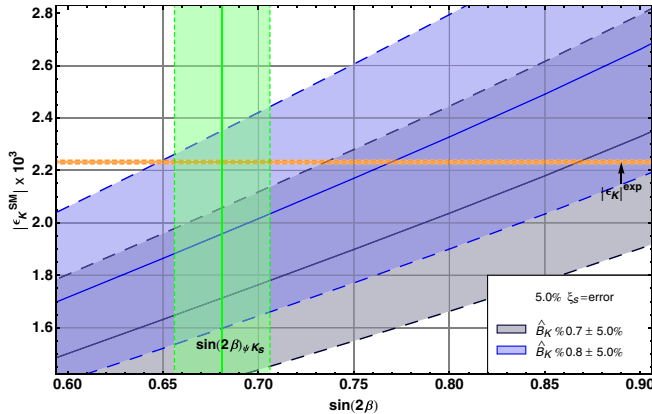
also accommodate $\epsilon_K$, since an upward shift in $R_t$ from NP corresponds to $C_s > C_d$ [cf. Eqs. (14) and (15)], and could come, in particular, from $C_d < 1$, as $\Delta M_d$, in contrast to $\Delta M_s$, is directly sensitive to $R_t$.

Plots analogous to that of Fig. 1 (left panel), but with all present uncertainties on the input taken into account, are shown in Figs. 2 and 3. These plots are obtained by the following procedure. The $\Delta M_d/\Delta M_s$ constraint is used to solve for $\bar{\rho}, \bar{\eta}$ depending on the $\sin 2\beta$ value. The range of solutions implied by the $\Delta M_d/\Delta M_s$ error (with $\bar{\rho}, \bar{\eta}$ highly correlated) can be translated into a range of values for $|\epsilon_K^{\rm SM}|$. The rest of the contributions to the $\epsilon_K$ error, mostly due to $m_c$, $m_t$, to the CKM entry $|V_{cb}|$ and to the assumed ranges for $\hat{B}_K$ and $\kappa_\epsilon$, can be treated as uncorrelated, and plugged in an error-propagation formula. As one can see, this procedure only assumes that $\Delta M_d/\Delta M_s$ be SM-like.

Figure 2 confirms that the combined information of $\sin 2\beta_{\psi K_s}$ and $|\epsilon_K^{\rm exp}|$ tends to prefer "high" values of $\hat{B}_K \gtrsim 0.85$ (cf. estimate in [22]). However, use of present errors
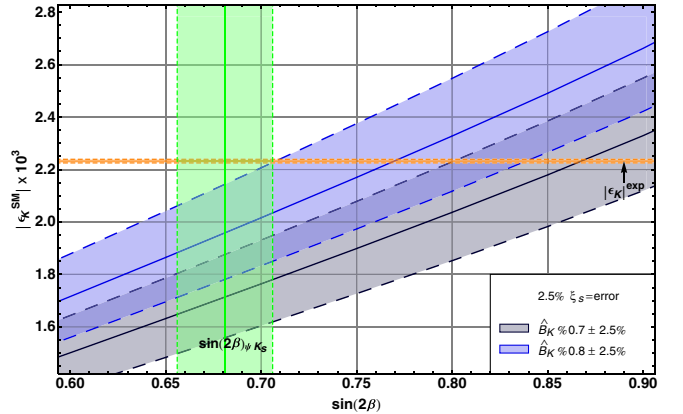


FIG. 2 (color online). $|\epsilon_K^{\rm SM}|$ vs $\sin 2\beta$ with inclusion of all input uncertainties. Left panel assumes present $\hat{B}_K$ and $\xi_s$ errors, whereas right panel shows the situation with errors on both quantities shrunk to 2.5%.

on $\hat{B}_K$ and $\xi_s$ (both $\approx 5\%$), as in the left panel of Fig. 2, impairs any clear-cut conclusion. The situation in the case of $\hat{B}_K$ and $\xi_s$ errors hypothetically halved can be appreciated from the right panel of the same figure, where actually a large part of the improvement is driven by the shrinking in the $\xi_s$ error, allowing a better determination of $\bar{\rho}$, $\bar{\eta}$. Therefore, an alternative or complementary strategy to an improvement in $\xi_s$ would be a major advance in the angle $\gamma$ through tree-level decays.

Finally, as an alternative viewpoint on the above facts (in particular on the role of the $\hat{B}_K$ and $\xi_s$ errors), Fig. 3 displays, as a function of $\sin 2\beta$, the $\hat{B}_K$ range compatible with the experimental $\epsilon_K$ result. For $\sin 2\beta = \sin 2\beta_{\psi K_s}$ the required $\hat{B}_K$ agrees well with the one found in [22].

## IV. $S_{\psi\phi}$ AND $\sin 2\beta$

As a last case, we would like to focus on the possibility that NP contributions to $\epsilon_K$ be negligible, as assumed in [5] and in scenario 3 discussed in the previous section. As one can infer from the above considerations, this would favor values of $\sin 2\beta \gtrsim 0.80$, implying the presence of a sizable new phase in $B_d$ mixing with a possible correlation with the $B_s$ system, which we discuss next.

Let us start with the $B_s$ mixing phase $\beta_s^{\text{full}}$, Eq. (5), using the information from [2]. In the notation of our Eqs. (4) and (5), the range for the NP phase $\phi_s$ at 95% probability is found to be

$$\phi_s \in [-30.45, -9.29]°$$
$$\cup [-78.45, -58.2]°,$$

corresponding to $S_{\psi\phi} \in [0.35, 0.89]$.      (16)

Assuming generic NP, the SM contribution to the phase amounts instead to [2] $\beta_s = -1.17(11)°$, where to estimate the error we have simply propagated that on $\sin 2\beta_s$.

Let us now compare these findings with the $B_d$ case. If a NP phase contributes to the mixing amplitude, the *CP* asymmetry in $B_d \rightarrow \psi K_s$ measures the quantity $\beta_d^{\text{full}}$ [see Eq. (5)]. Then, one can extract information on the NP phase $\phi_d$, provided the SM phase $\beta$ is estimated in some other way. An example is the determination of Ref. [5], where the main assumptions are the absence of NP in the ratio $\Delta M_d / \Delta M_s$ and in $\epsilon_K$ (as we are supposing in the present scenario). Using the CKMfitter package [35], we find

$$\sin(2\beta) = 0.88^{+0.11}_{-0.12}, \quad\quad (17)$$

where we have used the $\hat{B}_K$ result from Ref. [9] and the $\kappa_\epsilon$ factor in Table I and, similarly to Ref. [5], we have treated all the input errors as Gaussian. The result in Eq. (17) is compatible with that of [5], in particular, the inclusion of the $\kappa_\epsilon$ correction pushes the $\sin 2\beta$ determination further upwards, even if its associated error introduces an additional uncertainty in the $\epsilon_K$ evaluation.

If the high value implied by Eq. (17) for $\beta$ were indeed correct, this would indicate the presence of a *negative* NP phase in the $B_d$ system, with absolute value of O(10°). Quite interestingly, the solutions found in [2] for the NP phase in the $B_s$ system [see Eq. (16)] go in the same (negative) direction and the lowest solution is also compatible with $\approx -10°$.

One is then tempted to envisage a scenario characterized by a significant NP phase roughly equal in both $B_d$ and $B_s$ systems, i.e.

$$\phi_B = \phi_d \approx \phi_s \approx -9° \Rightarrow \begin{cases} \beta_{\psi K_s} < \beta \approx 30° \\ S_{\psi\phi} \approx 0.4 \end{cases} \quad (18)$$

with no NP in the $K$ system. The interesting aspect of this scenario is the correlation between new *CP* violation in the $B_d$ and $B_s$ systems. In the limiting case of exact equality between the NP phases in the two sectors, we show in Fig. 4 the predicted $S_{\psi\phi}$ as a function of $S_{\psi K_s}$ [see Eqs. (1) and (2) for the definitions]. If improvements on the $\sin 2\beta$ determination should indicate a large figure like Eq. (17) and $S_{\psi\phi}$ were measured as large as 0.4, this could be a hint in favor of this scenario. On the other hand, the scenario in Eq. (18) seems to be problematic with regards to the implied $|V_{ub}|$ value. As seen already in the right panel of Fig. 1, the value of $|V_{ub}|$ is generically larger than the present exclusive result. To address this issue, we plot in Fig. 5 the $|V_{ub}|$ range implied by a given NP phase $\phi_d$. We note that, since $|V_{ub}|$ is determined from the side $R_b$, its error depends mostly on the $|V_{cb}|$ uncertainty and is estimated through the propagation formula. On the other hand, for fixed $\sin 2\beta$, $|V_{ub}|$ depends only very weakly on the error due to the $\bar{\rho}$, $\bar{\eta}$ determination, as expected.

From Fig. 5 and Table II it is evident that $\phi_d \approx -9°$ would imply $|V_{ub}| \approx 4.9 \times 10^{-3}$, which is even higher than the inclusive averages in [36]. For comparison, the most recent combination of the inclusive and exclusive
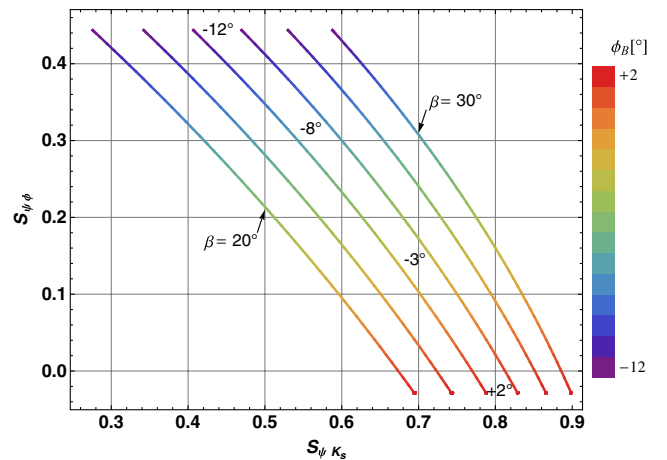


FIG. 4 (color online). *CP* asymmetry $S_{\psi\phi}$ as a function of $S_{\psi K_s}$ for a common NP phase $\phi_B \in [-12, +2]°$. Sample values within the chosen $\phi_B$ range are indicated also numerically.
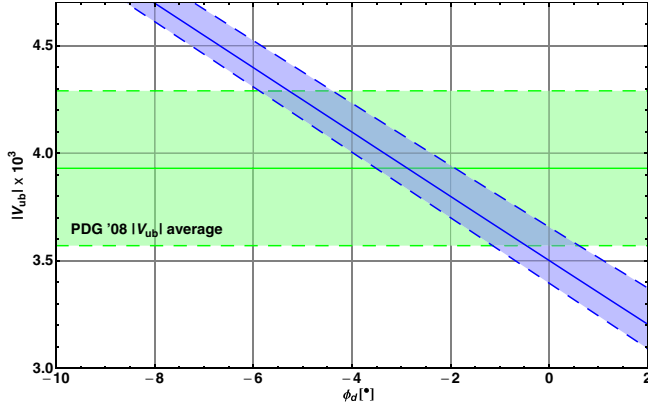
FIG. 5 (color online). $|V_{ub}|$ ranges implied by a given NP phase in the $B_d$ system, $\phi_d$. The (green) horizontal band displays the most recent $|V_{ub}|$ average quoted by the PDG [24].

$|V_{ub}|$ determinations quoted by the Particle Data Group (PDG) [24], namely,

$$|V_{ub}| = (3.93 \pm 0.36) \times 10^{-3}, \qquad (19)$$

is reported in Fig. 5 as a horizontal band, and can be seen to be compatible with no phase. Results similar to Eq. (19) can be found in [37].

Therefore, assuming that $|V_{ub}| \lesssim 4 \times 10^{-3}$ and that $S_{\psi\phi}$ should be confirmed as large as implied by Eq. (16), the middle scenario presented in the previous section, characterized by smaller NP effects in both the $B_d$ and $K$ sectors, would be a more plausible possibility. In this case, the NP phases in the $B_d$ and $B_s$ systems would be (mostly) uncorrelated with each other. In fact, in the case of exact correlation (see Fig. 4), $\beta \approx 26°$, corresponding to scenario 2, would imply $S_{\psi\phi} \lesssim 0.2$. As for the $K$ system, ascertaining the presence of NP would require a leap forward in the errors on the input parameters, $\hat{B}_K$ and $|V_{cb}|$ in the first place.

## V. CONCLUSIONS

In the present paper we have pointed out a possible inconsistency between the size of $CP$ violation in $K - \bar{K}$ and $B_d - \bar{B}_d$ mixing within the SM. The recent decrease in $\hat{B}_K$ from lattice [9,10] and the inclusion of the suppression factor $\kappa_\epsilon$ in the formula for $\epsilon_K$ are mostly responsible for this finding. Such an inconsistency has been already noted in [5], but we differ from that paper as we do not assume the absence of NP in $\epsilon_K$. Moreover, in [5] $\kappa_\epsilon \simeq 0.92$ has not been taken into account.

Under the single assumption that $\Delta M_d/\Delta M_s$ be unaffected by NP, the general pattern of correlations between $CP$ violation in the $K - \bar{K}$ and $B_d - \bar{B}_d$ systems is as follows:

(i) In the absence of new $CP$ violation in the $B_d$ system, the measured size of $S_{\psi K_s}$ implies $\epsilon_K$ with a central

value as much as 20% below the data, hinting at NP in $K - \bar{K}$ mixing.

(ii) In the absence of new $CP$ violation in $K - \bar{K}$ mixing, the size of the measured value of $\epsilon_K$ implies $\sin 2\beta$ by 10%–20% larger [5] than $S_{\psi K_s}$, so that a negative new phase $\phi_d$ is required in order to fit the experimental value of $S_{\psi K_s}$.

(iii) Since $\phi_d$ can reach O($-10°$), the limiting case of a new phase roughly equal in both $B_d$ and $B_s$ systems allows an enhancement of the asymmetry $S_{\psi\phi}$ by roughly an order of magnitude with respect to its SM value. This could then explain, at least to a first approximation, the effect found in [2].

If, on the other hand, one allows for contributions of NP to $\Delta M_d/\Delta M_s$, so that $R_t$ is increased with respect to its SM value, one can remove the discrepancy between the two systems, provided $R_t$ is increased by, say, 10%–15%. This would require, for instance, a destructive interference between SM and NP contributions to $\Delta M_d$—i.e., recalling Eq. (14), $C_d < 1$—and would automatically increase also $\gamma$.

Finally, we would like to emphasize that our results are very sensitive to the used value of $V_{cb}$, as can be anticipated from Eq. (15). Therefore, in addition to an accurate calculation of $\hat{B}_K$ and $\xi_s$, a very precise determination of $V_{cb}$ is required in order to fully exploit the power of the $\epsilon_K$ constraint on NP.

We hope that the results and the plots in our paper will help to monitor the developments in the field of $\Delta F = 2$ transitions in the coming years, when various input parameters and the data on $CP$ violation in $b \rightarrow s$ transitions will steadily improve.

## APPENDIX: ESTIMATE OF THE PARAMETER $\bar{\kappa}_\epsilon$

A rough estimate of the factor $\bar{\kappa}_\epsilon$, discussed at the beginning of Sec. II, can be obtained as follows. Starting from the general formula for $\epsilon_K$ in Eq. (9), one finds

$$\bar{\kappa}_\epsilon \simeq 1 + \frac{\xi}{\sqrt{2}|\epsilon_K|} \equiv 1 + \Delta_\epsilon, \qquad (A1)$$

where terms of O($\xi^2$) on the right-hand side have been neglected. Then $\Delta_\epsilon$ can in principle be extracted from the

analyses of $\epsilon'/\epsilon$. One has [18]

$$\frac{\epsilon'}{\epsilon} = -\omega\Delta_\epsilon(1 - \Omega), \qquad (A2)$$

where $\omega = \mathrm{Re}A_2/\mathrm{Re}A_0 = 0.045$ and $\Omega$ summarizes the isospin-breaking corrections, that are dominated by electroweak penguin contributions. It is well known that $\Omega > 0$ in the SM and in most known SM extensions. Therefore, setting $\Omega = 0$ and using the experimental value for $\epsilon'/\epsilon = 1.66(26) \times 10^{-3}$ [17], one finds

$$\Delta_\epsilon = -\frac{1}{\omega}\frac{\epsilon'}{\epsilon} = (-3.7 \pm 0.6) \times 10^{-2}, \qquad (A3)$$

which is compatible with [21,22]. This value can be considered as a plausible lower bound on $|\Delta_\epsilon|$.

However, it is well known that $\Omega$ cannot be neglected, but the evaluation of this quantity is subject to significant hadronic uncertainties, although, as discussed in Ref. [38], these uncertainties appear to be smaller than in $\xi$ itself. We recall that $\xi$ and $\Omega$ are dominated by QCD penguin and electroweak penguin operators, respectively, and the evaluation of $\xi$ and $\Omega$ requires the knowledge of their hadronic matrix elements.

One method [20] is to evaluate $\Omega$ and extract $\Delta_\epsilon$ from $\epsilon'/\epsilon$. From the analysis of [38], that combined various nonperturbative approaches, we find $\Omega = 0.4 \pm 0.1$ in

the SM. Yet, one has to remember that $\Omega$ is sensitive to NP contributions, in contrast with $\Delta_\epsilon$, whose NP sensitivity turns out to be much smaller. For this reason we have also calculated $\Delta_\epsilon$ directly in the large $N_c$ approach [39]. Both routes give

$$\Delta_\epsilon \simeq -6 \times 10^{-2}. \qquad (A4)$$

Calculations (A3) and (A4) and the fact that the SM estimate of $\epsilon'/\epsilon$ in the large $N_c$ approach agrees well with the data [40] drive us to the estimate

$$\bar{\kappa}_\epsilon \approx 0.94 \pm 0.02. \qquad (A5)$$

This agrees well with the 6% effect estimated in [20]. The error quoted in (A5) is no more than a guesstimate, but we believe it to be realistic. Clearly a better calculation of $\bar{\kappa}_\epsilon$ should be attempted, using e.g. lattice methods. The result obtained in [22] through a direct calculation of $\xi$ corresponds to $\bar{\kappa}_\epsilon \simeq 0.90(3)$ and implies $\epsilon'/\epsilon \approx 4.5 \times 10^{-3}$ from QCD penguins alone, roughly by a factor 3 larger than the data. Such result requires a very large negative electroweak penguin component for the predicted $\epsilon'/\epsilon$ to agree with experiment and a certain fine-tuning between the two contributions. Consequently we believe that Eq. (A5) represents a very plausible estimate of $\bar{\kappa}_\epsilon$.

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