Exclusive $B \to \rho l^+ l^-$ decay in the standard model with fourth-generation quarks

K. Zeynali[1,*] and V. Bashiry[2,+]

[1]Faculty of Medicine, Ardabil University of Medical Sciences (ArUMS), Daneshgah Street, Ardabil, Iran

[2]Engineering Faculty, Cyprus International University, Haspolat-Lefkosa, North Cyprus, via Mersin 10, Turkey

(Received 16 April 2008; published 5 August 2008)

We investigate the influence of the fourth generation of quarks on the branching ratio, the *CP* asymmetry, and the polarization asymmetries in $B \to \rho \ell^+ \ell^-$ decay. Taking $|V_{t'd}V_{t'b}| \sim 0.001$ with phase about 10°, which is consistent with the $\sin 2\phi_1$ of the Cabibbo-Kobayashi-Maskawa matrix and the B_d mixing parameter Δm_{B_d}, we obtain that for both (μ, τ) channels the branching ratio is increased and the magnitude of *CP* asymmetry and polarization asymmetries decreased by the mass and mixing parameters of the 4th generation of quarks. These results can serve as a good tool to search for new physics effects, precisely, to search for the fourth generation of quarks (t', b') via its indirect manifestations in loop diagrams.

DOI: 10.1103/PhysRevD.78.033001 PACS numbers: 12.15.Ji, 13.25.Hw

I. INTRODUCTION

Flavor changing natural current (FCNC) and lepton flavor violation (LFV) are at the forefront of our study both for precision test of the standard model (SM) and for new physics effects. FCNC, forbidden in the tree level, is induced by the quantum loop level. The new physics (NP) can either contribute to the effective Hamiltonian by the new operators which are absent in the SM or alter the Wilson coefficients of the Hamiltonian. A consequential extension of the SM with a new generation of fermions belongs to the classes of the new physics where the Wilson coefficients change comparing to the corresponding three-generation standard model (SM3).

The existence of the 4th generation of fermions, if their mass is less than the half of the mass of the Z boson, is excluded by the CERN LEP II experiment [1]. In this sense, the status of the fourth generation is more subtle [2] from the experimental point of view. However, a consequential extension of the SM3 can address some of the puzzles and fundamental questions from the theoretical point of view. In this respect, the consequential 4th generation of quarks and leptons are interesting in different ways, i.e., [3–9]. The 4th generation of quarks can include new weak phases and mixings in the Cabibbo-Kobayashi-Maskawa matrix (CKM). Thus, the four-generation standard model (SM4) can demonstrate a better solution to the baryogenesis than the SM3.

Two type of studies can be conducted to discover the 4th generation of fermions. The first type is the direct search of the 4th generation of quarks and leptons which can be accessed by increasing the center of mass energy of colliders with high luminosity. Here the cross section of production will increase and such fermions can be created as real states; i.e., the 4th generation of quarks can be created by gluon-gluon fusion at CERN LHC [10]. The second type is the indirect search dealing with the effects of the 4th generation of fermions in the FCNC decays [3–9] and LFV [11]. In these classes of studies, one studies the contribution of the 4th generation of fermions at the quantum loop level; Ref. [11] studied the effects of the 4th generation of heavy neutrinos (heavier than the half of the Z boson mass) in the $\mu \to e\gamma$ decay and anomalous magnetic moment of the μ. The result was an upper limit for the mass of ν_4 which is up to $\sim$100 GeV. Considering these constraints, one can study the branching ratio of the $\mu^- \to 2e^- e^+$ decay.

The $b \to s(d)$ transition is at the forefront for searching for the 4th generation of quarks. This transition is forbidden at tree level in the standard model. A consequential extension of the three-generation standard model to the four-generation standard model (SM4) maintains the same property at tree level, but at the quantum loop level the 4th generation of heavy quark (t') can contribute to the quantum loop. This contribution can affect physical observables, i.e. branching ratio, *CP* asymmetry, polarization asymmetries, and forward-backward (FB) asymmetries. The study of these physical observables is a good tool to look for the 4th generation of up-type quarks [3–9].

There are some constraints on a fourth family [12]. From the strong constraint on the number of light neutrinos, we know that the fourth family of neutrinos is heavy. The S and ρ parameter are sensitive to a fourth family, but the experimental limits on these parameters have been evolved through the years in such a way that the constraint on a fourth family has lessened. In addition, the masses of the fourth family of leptons may produce negative S and T. As discussed in [13] and the reference therein, the constraints from S and T do not prohibit the fourth family, but instead serve only to constrain the mass spectrum of the fourth family of quarks and leptons.

FCNC and *CP* violation (CPV) are indeed the most sensitive probes of NP contributions to penguin operators.

*k.zeinali@arums.ac.ir

+bashiry@ipm.ir

Rare decays, induced by FCNC $b \to s(d)$ transitions, are at the forefront of our quest to understand flavor and the origins of CPV, offering one of the best probes for new physics beyond the SM [14–16]. In addition, there are some important QCD corrections, which have recently been calculated in the next-to-next-to-leading-order [17]. Moreover, $b \to s(d)\ell^+\ell^-$ decay is also very sensitive to the new physics beyond the SM. New physics effects manifest themselves in rare decays in two different ways, either through new combinations to the new Wilson coefficients or through the new operator structure in the effective Hamiltonian, which is absent in the SM. A crucial problem in the new physics search within flavor physics is the optimal separation of new physics effects from uncertainties. It is well known that inclusive decay modes are dominated by partonic contributions; nonperturbative corrections are in general smaller [18]. Also, ratios of exclusive decay modes such as asymmetries for $B \to K(K^*, \rho, \gamma)\ell^+\ell^-$ decay [19–28] are well studied for new physics search. Here large parts of the hadronic uncertainties partially cancel out.

In this paper, we investigate the possibility of searching for new physics in the $B \to \rho\ell^+\ell^-$ decay using the SM with the fourth generation of quarks (b', t'). The fourth quark (t'), like u, c, t quarks, contributes in the $b \to s(d)$ transition at loop level. Clearly, it would change the branching ratio, CP asymmetry, and polarization asymmetries. Note that fourth-generation effects on the branching ratio have been widely studied in baryonic and semileptonic $b \to s$ transition [7,8,29–36]. But few studies related to the $b \to d$ transitions [3] exist.

The sensitivity of the branching ratio and CP asymmetry to the existence of the fourth generation of quarks in the $B \to \pi\ell^+\ell^-$ decay is investigated in [3] and it was observed that branching ratio, CP asymmetry, and lepton polarization asymmetries are very sensitive to the fourth-generation parameters $(m_{t'}, V_{t'b}V^*_{t'd})$. In this regard, it is interesting to ask whether the branching ratio, CP asymmetry, and lepton polarization asymmetries in $B \to \rho\ell^+\ell^-$ decay are sensitive to the fourth-generation parameters in the same way. In the work presented here we try to answer these questions.

The paper is organized as follows: In Sec. II, using the effective Hamiltonian, the general expressions for the matrix element and CP asymmetry of $B \to \rho\ell^+\ell^-$ decay is derived. Section III is devoted to calculations of lepton polarization. In Sec. IV, we investigate the sensitivity of these functions to the fourth-generation parameters $(m_{t'}, V_{t'b}V^*_{t'd})$.

II. MATRIX ELEMENT DIFFERENTIAL DECAY RATE AND *CP* ASYMMETRY

The QCD corrected effective Lagrangian for the decays $b \to d\ell^+\ell^-$ can be achieved by integrating out the heavy quarks and the heavy electroweak bosons in the SM4 as follows:

$$\begin{aligned} M = \frac{G_F\alpha_{em}\lambda_t}{\sqrt{2}\pi}[&C_9^{\text{tot}}(\bar{d}\gamma_\mu P_L b)\bar{\ell}\gamma_\mu\ell \\ &+ C_{10}^{\text{tot}}(\bar{d}\gamma_\mu P_L b)\bar{\ell}\gamma_\mu\gamma^5\ell \\ &- 2C_7^{\text{tot}}\bar{d}i\sigma_{\mu\nu}\frac{q^\nu}{q^2}(m_bP_R + m_dP_L)b\bar{\ell}\gamma_\mu\ell]. \end{aligned} \tag{1}$$

In this formula, unitarity of the CKM matrix has been used. Here the $\lambda_t = V^*_{tb}V_{td}$ is factored out and q denotes the four momentum of the lepton pair. The Wilson coefficients C_i^{tot}'s are as follows:

$$\lambda_t C_i^{\text{tot}} = \lambda_t C_i^{\text{SM}} + \lambda_{t'} C_i^{\text{new}}, \tag{2}$$

hereby, $\lambda_f = V^*_{fb}V_{fd}$ and the last term in this expression describes the contributions of the t' quark to the Wilson coefficients. The explicit forms of the C_i^{new} can be obtained from the corresponding expression of the Wilson coefficients in the SM by substituting $m_t \to m_{t'}$.

A general 4×4 CKM matrix can be written as follows:

$$\hat{V}^4_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{pmatrix}. \tag{3}$$

Using the Wolfenstein parametrization, the values of 3×3 CKM matrix elements, keeping $\mathcal{O}(\lambda^5)$, is obtained in [37]. On the other hand, one can estimate the elements appearing in the fourth column and row of the 4×4 CKM matrix by studying the experimental results of the $B_{s,d}$ mixing [29] and $b \to s(d)$ transitions [5,30]. The former sharply constrains the phases of elements and the latter generally constrains the magnitudes. If we summarize all these experimental constraints with the unitarity condition of the 4×4 CKM matrix, then the following values for the elements of $\hat{V}^4_{\text{CKM}}$ can be obtained:

$$\hat{V}^4_{\text{CKM}} \approx \begin{pmatrix} 0.9745 & 0.224 & 0.0038e^{-i60^\circ} & 0.0281e^{i61^\circ} \\ -0.224 & 0.9667 & 0.0415 & 0.1164e^{i66^\circ} \\ 0.0073e^{-i25^\circ} & 0.0555e^{-i25^\circ} & 0.9746 & 0.2168e^{-i1^\circ} \\ -0.0044e^{-i10^\circ} & -0.1136e^{-i70^\circ} & -0.2200 & 0.9688 \end{pmatrix}. \tag{4}$$

The unitarity of the 4×4 CKM matrix leads to

$$\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0. \tag{5}$$

Then $\lambda_t = -\lambda_u - \lambda_c - \lambda_{t'}$. Now we can rewrite Eq. (2) as

$$\lambda_t C_{7,10}^{\rm SM} + \lambda_{t'} C_{7,10}^{\rm new} = (-\lambda_c - \lambda_u) C_{7,10}^{\rm SM} + \lambda_{t'} (C_{7,10}^{\rm new} - C_{7,10}^{\rm SM}). \quad (6)$$

It is clear that for the $m_{t'} \to m_t$ or $\lambda_{t'} \to 0$ the $\lambda_{t'}(C_{7,10}^{\rm new} - C_{7,10}^{\rm SM})$ term vanishes and the SM3 results are recovered. If we parameterize $\lambda_{t'}$ as

$$\lambda_{t'} = V_{t'b}^* V_{t'b} = r_{db} e^{i\phi_{db}} \quad (7)$$

it is obvious from Eq. (4) that $\phi_{db} \sim 10^0$ and $r_{db} \sim \mathcal{O}(10^{-3})$.

Neglecting the terms of $O(m_q^2/m_W^2)$, $q = u, d, c$, the analytic expressions for all Wilson coefficients, except $C_9^{\rm eff}$, can be found in [38]. Note that just $C_9^{\rm tot}$ has weak and strong phases, i.e.

$$C_9^{\rm tot} = \xi_1 + \lambda_{tu}\xi_2 + \lambda_{tt'} C_9^{\rm new}, \quad (8)$$

where the CP violating parameter $\lambda_{tt'} = \frac{\lambda_{t'}}{\lambda_t}$ and $\lambda_{tu} = \frac{\lambda_u}{\lambda_t}$.

The explicit expressions of functions ξ_1 and ξ_2 in $\mu = m_b$ scale are, respectively, [38–42]

$$\begin{aligned}\xi_1 = {} & C_9(x_i, m_b) + 0.138\omega(\hat{s}) + g(\hat{m}_c, \hat{s})(3C_1 + C_2 \\ & + 3C_3 + C_4 + 3C_5 + C_6) - \tfrac{1}{2} g(\hat{m}_d, \hat{s})(C_3 + C_4) \\ & - \tfrac{1}{2} g(\hat{m}_b, \hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) \\ & + \tfrac{2}{9}(3C_3 + C_4 + 3C_5 + C_6), \end{aligned} \quad (9)$$

$$\xi_2 = [g(\hat{m}_c, \hat{s}) - g(\hat{m}_u, \hat{s})](3C_1 + C_2), \quad (10)$$

where $\hat{m}_q = m_q/m_b$, $\hat{s} = \frac{q^2}{m_b^2}$, and $x_i = \frac{m_i^2}{m_W^2}$; here, $i = t(t')$ for $C_9^{\rm eff}(C_9^{\rm new})$.

The function $g(\hat{m}_q, \hat{s})$, which includes the strong phase, represents the one loop corrections to the four-quark operators $O_1 - O_6$ [42] and is defined as

$$\begin{aligned} g(\hat{m}_q, \hat{s}) = {} & -\frac{8}{9}\ln(\hat{m}_q) + \frac{8}{27} + \frac{4}{9} y_q - \frac{2}{9}(2 + y_q) \\ & \times \sqrt{|1 - y_q|}\left\{\Theta(1 - y_q)\left[\ln\left(\frac{1 + \sqrt{1 - y_q}}{1 - \sqrt{1 - y_q}}\right) - i\pi\right]\right. \\ & \left. + \Theta(y_q - 1) 2\arctan\frac{1}{\sqrt{y_q - 1}}\right\}. \end{aligned} \quad (11)$$

Although long-distance effects of $c\bar{c}$ bound states could contribute to $C_9^{\rm eff}$, for simplicity, they are not included in the present study. On the other hand, the bound states could be excluded experimentally by cutting the phase space at the resonant regions. In the case of the J/ψ family, this is usually accomplished by introducing a Breit-Wigner distribution for the resonances through the replacement [43]

$$g(\hat{m}_c, \hat{s}) \to g(\hat{m}_c, \hat{s}) - \frac{3\pi}{\alpha^2} \sum_{V = J/\psi, \psi', \ldots} \frac{\hat{m}_V Br(V \to l^+ l^-)\hat{\Gamma}_{\rm total}^V}{\hat{s} - \hat{m}_V^2 + i\hat{m}_V \hat{\Gamma}_{\rm total}^V}. \quad (12)$$

One has to sandwich the inclusive effective Hamiltonian between the initial hadron state $B(p_B)$ and the final hadron state $\rho(p_\rho)$ to obtain the matrix element for the exclusive decay $B(p_B) \to \rho(p_\rho)\ell^+(p_+)\ell^-(p_-)$. It follows from Eq. (1) that in order to calculate the decay width and other physical observables of the exclusive $B \to \rho\ell^+\ell^-$ decay, we need the following matrix elements, defined in terms of form factors [44]:

$$\begin{aligned} & \langle \rho(p_\rho, \varepsilon)|\bar{d}\gamma_\mu(1 - \gamma^5)b|B(p_B)\rangle \\ & = -\epsilon_{\mu\nu}\lambda_\sigma \varepsilon^{\nu*} p_\rho^\lambda p_B^\sigma \frac{2V(q^2)}{m_B + m_\rho} \\ & \quad - i\varepsilon_\mu^*(m_B + m_\rho)A_1(q^2) + i(p_B + p_\rho)(\varepsilon^* q) \\ & \quad \times \frac{A_2(q^2)}{m_B + m_\rho} + iq_\mu(\varepsilon^* q)\frac{2m_\rho}{q^2}[A_3(q^2) - A_0(q^2)], \end{aligned} \quad (13)$$

$$\begin{aligned} & \langle \rho(p_\rho, \varepsilon)|\bar{d}i\sigma_{\mu\nu}q^\nu(1 \pm \gamma^5)b|B(p_B)\rangle \\ & = 4\epsilon_{\mu\nu}\lambda_\sigma \varepsilon^{\nu*} p_\rho^\lambda q^\sigma T_1(q^2) \pm 2i[\varepsilon_\mu^*(m_B^2 - m_\rho^2) \\ & \quad - (p_B + p_\rho)\mu(\varepsilon^* q)]T_2(q^2) \\ & \quad \pm 2i(\varepsilon^* q)\left(q_\mu - (p_B + p_\rho)_\mu \frac{q^2}{m_B^2 - m_\rho^2}\right)T_3(q^2), \end{aligned} \quad (14)$$

$$\langle \rho(p_\rho, \varepsilon)|\bar{d}(1 + \gamma^5)b|B(p_B)\rangle = -\frac{2im_\rho}{m_b}(\varepsilon^* q)A_0(q^2), \quad (15)$$

where p_ρ and ε denote the four momentum and polarization vector of the ρ meson, respectively.

From Eqs. (13)–(15) we get the following expression for the matrix element of the $B \to \rho\ell^+\ell^-$ decay:

$$\begin{aligned} M^{B\to\rho} = {} & \frac{G_F \alpha_{em}\lambda_t}{\sqrt{2}\pi}[i\epsilon_{\mu\nu\alpha\beta}\epsilon^{\nu*} p_B^\beta q^\beta A + \epsilon_\mu^* B + (\epsilon^*.q) \\ & \times (p_B)C](\bar{\ell}\gamma^\mu\ell) + [i\epsilon_{\mu\nu\alpha\beta}\epsilon^{\nu*} p_B^\beta q^\beta D + \epsilon_\mu^* E \\ & + (\epsilon^*.q)(p_B)F](\bar{\ell}\gamma^\mu\ell) + G(\epsilon^*.q)(\bar{\ell}\gamma_5\ell), \end{aligned} \quad (16)$$

where

$$A = \frac{4(m_b + m_d)T_1(q^2)}{m_B^2 s} C_7^{\text{tot}} + \frac{V(q^2)}{m_B + m_\rho} C_9^{\text{tot}}, \qquad B = -\frac{2(m_b - m_d)(1 - r_\rho)T_2(q^2)}{s} C_7^{\text{tot}} - \frac{(m_B + m_\rho)A_1(q^2)}{2} C_9^{\text{tot}},$$

$$C = \frac{4(m_b - m_d)}{m_B^2 s}\left(T_2(q^2) + \frac{s}{1 - r_\rho} T_3(q^2)\right) C_7^{\text{tot}} + \frac{A_2(q^2)}{m_B + m_\rho} C_9^{\text{tot}}, \qquad D = \frac{V(q^2)}{m_B + m_\rho} C_{10}^{\text{tot}}, \tag{17}$$

$$E = -\frac{(m_B + m_\rho)A_1(q^2)}{2} C_{10}^{\text{tot}}, \qquad F = \frac{A_2(q^2)}{m_B + m_\rho} C_{10}^{\text{tot}}, \qquad G = \left(-\frac{m_\ell}{m_B + m_\rho} A_2(q^2) + \frac{2 m_\rho m_\ell}{m_B^2 s}(A_3(q^2) - A_0(q^2))\right) C_{10}^{\text{tot}}.$$

From this expression of the matrix element, for the differential decay width, we get the following result:

$$\left(\frac{d\Gamma^\rho}{ds}\right)_0 = \frac{G_F^2 \alpha^2}{3 \times 2^{10} \pi^5} |\lambda_t|^2 m_B^5 \upsilon \sqrt{\lambda_\rho} \Delta_\rho, \tag{18}$$

$$\begin{aligned}
\Delta_\rho = {} & \left(1 + \frac{2t^2}{s}\right)\lambda_\rho \left[4 m_B^2 s |A|^2 + \frac{2}{m_B^2 r_\rho}\left(1 + 12 \frac{s r_\rho}{\lambda_\rho}\right)|B|^2 + \frac{m_B^2}{2 r_\rho}\lambda_\rho |C|^2 + \frac{2}{r_\rho}(1 - r_\rho + s)\,\mathrm{Re}(B^* C)\right] + 4 m_B^2 \lambda_\rho (s - 4t^2)|D|^2 \\
& + \frac{4(2t^2 + s) - 4(2t^2 + s)(r_\rho + s) + 4t^2(r_\rho^2 - 26 r_\rho + s^2) + 2s(r_\rho^2 + 10 s r_\rho + s^2)}{m_B^2 s r_\rho}|E|^2 + \frac{m_B^2}{2 s r_\rho}\lambda_\rho[(2t^2 + s) \\
& \times (\lambda_\rho + 2s + 2r_\rho) - 2\{2t^2(r_\rho + 5s) + s(r_\rho + s)\}]|F|^2 + 3\frac{s}{r_\rho}\lambda_\rho |G|^2 + \frac{2\lambda_\rho}{s r_\rho}[-2t^2(r_\rho - 5s) + (2t^2 + s) \\
& - s(r_\rho + s)]\,\mathrm{Re}(E^* F) + \frac{12 t}{m_B r_\rho}\lambda_\rho\,\mathrm{Re}(G^* E) + \frac{2 m_B t}{r_\rho}\lambda_\rho (1 - r_\rho + s)\,\mathrm{Re}(G^* F),
\end{aligned} \tag{19}$$

with $r_\rho = m_\rho^2/m_B^2$, $\lambda_\rho = r_\rho^2 + (s - 1)^2 - 2 r_\rho (s + 1)$, $\upsilon = \sqrt{1 - \frac{4t^2}{s}}$, and $t = m_\ell/m_B$.

Another physical observable is the CP-violating asymmetry which can be defined for both polarized and unpolarized leptons. We aim to obtain the normalized CP-violating asymmetry for the unpolarized leptons. The standard definition is given as

$$A_{CP}^\rho(\hat{s}) = \frac{(\frac{d\Gamma^\rho}{d\hat{s}})_0 - (\frac{d\bar{\Gamma}^\rho}{d\hat{s}})_0}{(\frac{d\Gamma^\rho}{d\hat{s}})_0 + (\frac{d\bar{\Gamma}^\rho}{d\hat{s}})_0} = \frac{\Delta_\rho - \bar{\Delta}_\rho}{\Delta_\rho + \bar{\Delta}_\rho}, \tag{20}$$

where

$$\frac{d\Gamma^\rho}{d\hat{s}} = \frac{d\Gamma^\rho(b \to d\ell^+\ell^-)}{d\hat{s}}, \qquad \frac{d\bar{\Gamma}^\rho}{d\hat{s}} = \frac{d\bar{\Gamma}^\rho(b \to d\ell^+\ell^-)}{d\hat{s}},$$

and $(d\bar{\Gamma}^\rho/d\hat{s})_0$ can be obtained from $(d\Gamma^\rho/d\hat{s})_0$ by making the replacement

$$C_9^{\text{tot}} = \xi_1 + \lambda_{tu}\xi_2 + \lambda_{tt'} C_9^{\text{new}} \to \bar{C}_9^{\text{tot}} = \xi_1 + \lambda_{tu}^*\xi_2 + \lambda_{tt'}^* C_9^{\text{new}}. \tag{21}$$

Using this definition and the expression for $\Delta^\rho(\hat{s})$, the CP-violating asymmetry contributed from the SM3 and new contributions from the SM4 are

$$A_{CP}^\rho(\hat{s}) = \frac{-\Sigma^{\text{SM}} - \Sigma^{\text{new}}}{\Delta_1^\rho + \Sigma^{\text{SM}} + \Sigma^{\text{new}}} \tag{22}$$

where

$$\Sigma^{\text{SM}}(s) = 4\,\mathrm{Im}(\lambda_{tu})\left(B_4\,\mathrm{Im}(\xi_1^*\xi_2) + \frac{B_2 + B_3}{2}\,\mathrm{Im}(C_7^{\text{eff}*}\xi_2)\right), \tag{23}$$

$$\begin{aligned}
\Sigma^{\text{new}}(s) = {} & 4\,\mathrm{Im}(\lambda_{tt'})(B_1\,\mathrm{Im}(C_7^{\text{new}} C_7^{\text{eff}*}) + B_2\,\mathrm{Im}(C_9^{\text{new}} C_7^{\text{eff}*}) + B_3\,\mathrm{Im}(C_7^{\text{new}}\xi_1^*) + B_4\,\mathrm{Im}(C_9^{\text{new}}\xi_1^*)) + 4\,\mathrm{Im}(\lambda_{tt'}\lambda_{tu}) \\
& \times \left(\frac{B_2 + B_3}{2}\,\mathrm{Im}(C_7^{\text{eff}*}\xi_2) + B_4\,\mathrm{Im}(\xi_1^*\xi_2)\right) + 4\,\mathrm{Im}(\lambda_{tt'}^*\lambda_{tu})\left(\frac{B_2 + B_3}{2}\,\mathrm{Im}(C_7^{\text{new}*}\xi_2) + B_4\,\mathrm{Im}(C_9^{\text{new}*}\xi_2)\right) \\
& + 4\,\mathrm{Im}(\lambda_{tu})|\lambda_{tt'}|^2\left(\frac{B_2 + B_3}{2}\,\mathrm{Im}(C_7^{\text{new}*}\xi_2) + B_4\,\mathrm{Im}(C_9^{\text{new}*}\xi_2)\right),
\end{aligned} \tag{24}$$

with

$$B_1 = \frac{8m_b^2}{m_B^2 s r_\rho \lambda_\rho}(2r_\rho s(\lambda\rho - 12r_\rho + 4r_\rho^2 + 6) + \lambda_\rho(2r_\rho - 1 - r_\rho^2 - 2s - \lambda_\rho))T_2^2 + \frac{16m_b^2}{m_B^2(1-r_\rho)r_\rho s}(r_\rho(s - r_\rho + 2) + (\lambda_\rho - s - 1))T_2T_3 + \frac{64m_b^2}{m_B^2 s}T_1^2 + \frac{8m_b^2\lambda_\rho}{m_B^2(1-r_\rho)^2 r_\rho}T_3^2, \tag{25}$$

$$B_2 = \frac{2m_b}{(m_B + m_\rho)r_\rho s}(2r - \rho(s + r_\rho + 2) + (2s + \lambda_\rho + 2))A_2T_2 + \frac{2m_b}{m_B^2 s r_\rho \lambda_\rho}(\lambda_\rho(m_B + m_\rho)(1 - r_\rho) + 12sr_\rho(m_\rho - r_\rho(m_N + m_\rho) + \lambda_\rho))A_1T_2 + \frac{16m_b}{(m_B + m_\rho)}T_1V + \frac{2m_b\lambda_\rho}{(m_B + m_\rho)(1 - r_\rho)r_\rho}A_2T_3, \tag{26}$$

$$B_3 = B_2 + \frac{4m_b(1 - r_\rho + s)}{m_B^2(m_B + m_\rho)(1 - r_\rho)sr_\rho}(m_B^2(1 - r_\rho)^2A_2T_2 - ((m_B + m_\rho)^2(1 - r_\rho)T_2 + sT_3)A_1), \tag{27}$$

$$B_4 = \frac{1}{2m_B^2(m_B + m_\rho)^2 r_\rho \lambda_\rho}(2m_B^2(m_B + m_\rho)^2(r_\rho - s - 1)\lambda_\rho A_1A_2 + (m_B + m_\rho)^4(12sr_\rho + \lambda_\rho)A_1^2 + m_B^4\lambda_\rho(8sr_\rho V^2 + \lambda_\rho A_2^2)), \tag{28}$$

and

$$\Delta_1^\rho = \frac{2s\Delta_\rho}{(s + 2t^2)\lambda_\rho}. \tag{29}$$

From this expression it is easy to see that in the $\lambda_{t'} \to 0$ the SM3 result can be obtained. Secondly, when $m_{t'} \to m_t$ the result of the SM4 coincides with the SM3, as it has to be, even if it is not obviously visible from the expressions (see figures).

III. LEPTON POLARIZATION

In order to calculate the polarization asymmetries of the lepton defined in the effective four fermion interaction of Eq. (16), we must first define the orthogonal vectors (components of S) in the rest frame of ℓ^-. Note that we should use the subscripts L, N, and T to correspond to the lepton being polarized along the longitudinal, normal, and transverse directions, respectively.

$$\begin{aligned} S_L^\mu &\equiv (0, \mathbf{e_L}) = \left(\mathbf{0}, \frac{\mathbf{p}-}{|\mathbf{p}-|}\right),\\ S_N^\mu &\equiv (0, \mathbf{e_N}) = \left(\mathbf{0}, \frac{\mathbf{p}_\rho \times \mathbf{p}-}{|\mathbf{p}_\rho \times \mathbf{p}-|}\right),\\ S_T^\mu &\equiv (0, \mathbf{e_T}) = (\mathbf{0}, \mathbf{e_N} \times \mathbf{e_L}), \end{aligned} \tag{30}$$

where $\mathbf{p}_-$ and $\mathbf{p}_\rho$ are the three momenta of the ℓ^- and ρ particles, respectively. The longitudinal unit vector is boosted to the CM frame of $\ell^-\ell^+$ by Lorenz transformation:

$$S_L^\mu = \left(\frac{|\mathbf{p}_-|}{m_\ell}, \frac{E_\ell \mathbf{p}_-}{m_\ell|\mathbf{p}_-|}\right);$$

here, the other two vectors remain unchanged. The polarization asymmetries can now be calculated using the spin projector $\frac{1}{2}(1 + \gamma_5 \not{S})$ for ℓ^-.

Regarding the expressions above, now we can define the single lepton polarization. The definition of the polarized and normalized differential decay rate is

$$\frac{d\Gamma^\rho(s, \vec{n})}{ds} = \frac{1}{2}\left(\frac{d\Gamma^\rho}{ds}\right)_0[1 + P_i^\rho \vec{e}.\vec{n}], \tag{31}$$

where a summation over $i = L, T, N$ is implied. Polarized components P_i^ρ in Eq. (31) are as follows:

$$P_i^\rho = \frac{d\Gamma^\rho(\vec{n} = \vec{e}_i)d\hat{s} - d\Gamma^\rho(\vec{n} = -\vec{e}i)/d\hat{s}}{d\Gamma^\rho(\vec{n} = \vec{e}_i)/d\hat{s} + d\Gamma^\rho(\vec{n} = -\vec{e}_i)/d\hat{s}}. \tag{32}$$

As a result the different components of the P_i^ρ are given as

$$P_L = \bigg\{24\,\mathrm{Re}(A * B)(1 - r_\rho - s)s(-1 + v) + 4m_B^2 s\lambda_\rho v\,\mathrm{Re}(A^*D) + \frac{1}{r\rho}(3 + v)[2\,\mathrm{Re}(B^*E)(1 + r_\rho^2 + 2sr_\rho + s^2 - 2(r_\rho + s)) + m_B^2\,\mathrm{Re}(C^*E)\{1 - 3(r_\rho + s) - (r_\rho - s)^2(r_\rho + s) + (3r_\rho^2 + 2sr_\rho + 3s^2)\}] + \frac{1}{r_\rho}\{\mathrm{Re}(B^*F) \times (1 - r_\rho - s) + \mathrm{Re}(C^*F)m_B^2\lambda_\rho\}[(3 + v)(1 + r_\rho(r\rho - s) - 2r_\rho) + (3 - 7v)s(r_\rho - s) - 8sv]\bigg\}/\Delta_\rho; \tag{33}$$

$$P_T = \frac{\pi t\sqrt{s\lambda_\rho}}{\Delta_\rho}\Big[-4\,\mathrm{Re}(A^*B) + \frac{1}{4sr_\rho}\{2(2(1-s-r_\rho) \times \mathrm{Re}(B^*E)m_B^2\lambda_\rho\,\mathrm{Re}(C^*E))\}\Big]; \quad (34)$$

$$P_N = \frac{\pi\sqrt{\lambda_\rho(s-4t^2)}}{\Delta_\rho}\Big[\frac{2(1+r_\rho-s)}{r_\rho}\,\mathrm{Im}(E^*F) + 2\,\mathrm{Im}(A^*E+B^*D)\Big]. \quad (35)$$

These results for P_L, P_T, and P_N agree with those given in [45] when $\lambda_{t'} = 0$. It also can be seen from the explicit expression of P_i^ρ involving various combination of the Wilson coefficients that they are quite sensitive to the fourth-generation effects. Furthermore, P_N^ρ is proportional to the imaginary parts of the product of the Wilson coefficients. The existence of the new weak phase as a result of the fourth generation contributes constructively to the magnitude of the P_N^ρ.

IV. NUMERICAL ANALYSIS

In this section, we will study the dependence of the total branching ratio, averaged CP asymmetry and lepton polarizations to the mass of fourth quark ($m_{t'}$) and the product of quark mixing matrix elements ($V_{t'b}^* V_{t'd} = r_{sb}e^{i\phi_{db}}$). The main input parameters in the calculations are the form factors. The definitions of the form factors are (see [45])

$$\begin{aligned}
V(q^2) &= \frac{V(0)}{1-q^2/5^2},\\
A_1(q^2) &= A_1(0)(1-0.023q^2),\\
A_2(q^2) &= A_2(0)(1+0.034q^2),\\
A_0(q^2) &= \frac{A_3(0)}{1-q^2/4.8^2},\\
A_3(q^2) &= \frac{m_B+m_\rho}{2m_\rho}A_1(q^2) - \frac{m_B-m_\rho}{2m_\rho}A_2(q^2),\\
T_1(q^2) &= \frac{T_1(0)}{1-q^2/5.3^2},\\
T_2(q^2) &= T_2(0)(1-0.02q^2),\\
T_3(q^2) &= T_3(0)(1+0.005q^2),
\end{aligned} \quad (36)$$

with $V(0) = 0.47$, $A_1(0) = 0.37$, $A_2(0) = 0.4$, $T_1(0) = 0.19$, $T_2(0) = 0.19$, $T_3(0) = -0.7$ We also use the SM parameters shown in Table I.

In order to perform quantitative analysis of the total branching ratio, CP asymmetry, and the lepton polarizations, the values of the new parameters ($m_{t'}$, r_{db}, ϕ_{db}) are needed. In the foregoing numerical analysis, we vary $m_{t'}$ in the range $175 \le m_{t'} \le 600$ GeV. The former is lower range because of the fact that the fourth-generation up quark is expected to be heavier than the third-generation ones ($m_t \le m_{t'}$) [12]. The upper range comes from the experimental bounds on the ρ and S parameters of the SM; furthermore, a mass greater than 600 GeV also contradicts with partial wave unitarity [12]. As for mixing, we use the result of Ref. [30] where it was found that $|V_{t'd}V_{t'b}| \sim 0.001$ with the phase about 10°, which is consistent with the $\sin 2\phi_1$ of the CKM and the B_d mixing parameter Δm_{B_d} [30].

TABLE I. The values of the input parameters used in the numerical calculations.

Parameter	Value
α_{em}	1/129
m_u	2.3 (MeV)
m_d	4.6 (MeV)
m_c	1.25 (GeV)
m_b	4.8 (GeV)
m_μ	0.106 (GeV)
m_τ	1.780 (GeV)

We can still move one more step further. From explicit expressions of the physical observables, one can easily see that they depend on both $\hat{s}$ and the new parameters ($m_{t'}$, r_{db}). One may eliminate the dependence of the lepton polarization on one of the variables. We eliminate the variable $\hat{s}$ by performing integration over $\hat{s}$ in the allowed kinematical region. The total branching ratio, CP asymmetry, and the averaged lepton polarizations are defined as

$$\begin{aligned}
\mathcal{B}_r &= \int_{4m_\ell^2/m_B^2}^{(1-\sqrt{\hat{r}_\rho})^2}\frac{d\mathcal{B}}{d\hat{s}}d\hat{s},\\
\langle Pi(A_{CP})\rangle &= \frac{\int_{4m_\ell^2/m_B^2}^{(1-\sqrt{\hat{r}_\rho})^2}P_i(A_{CP})\frac{d\mathcal{B}}{d\hat{s}}d\hat{s}}{\mathcal{B}_r}.
\end{aligned} \quad (37)$$

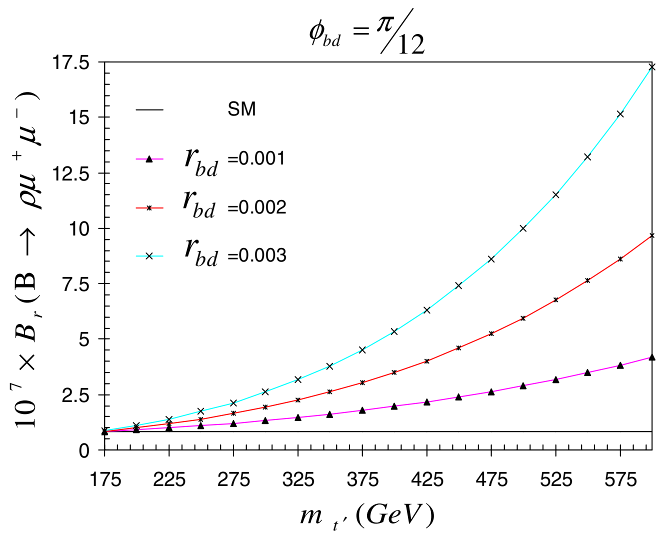

FIG. 1 (color online). The dependence of the branching ratio of $B \to \rho\mu^+\mu^-$ on $m_{t'}$ for $r_{db} = 0.001, 0.002, 0.003$.

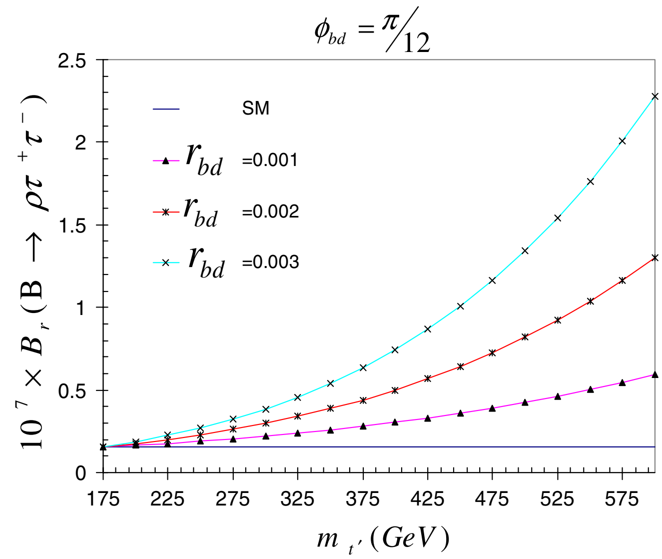

FIG. 2 (color online). The same as in Fig. 1, but for the τ lepton.

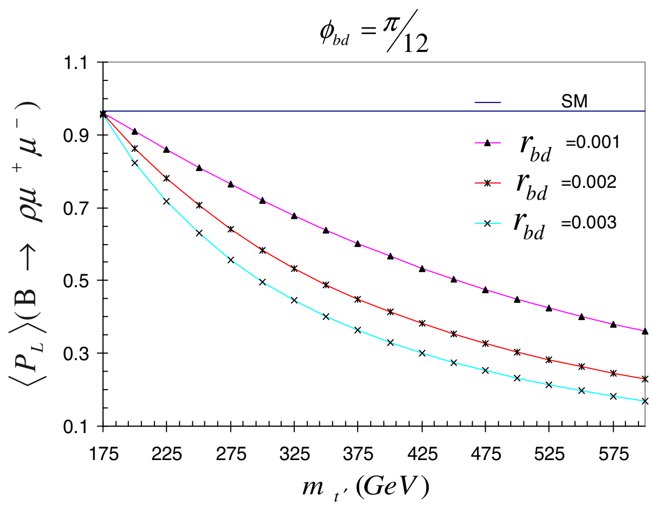

FIG. 4 (color online). The dependence of the $\langle P_L \rangle$ for μ lepton on $m_{t'}$ for $r_{db} = 0.001, 0.002, 0.003$.

Figures 1–6 depict the dependence of the total branching ratio, unpolarized CP asymmetry, and averaged lepton polarization for various r_{db} in terms of $m_{t'}$. We should note here that as the dependency for various $\phi_{db} \sim \{0°–30°\}$ is too weak we just show the result only for $\phi_{db} = 15°$. Also, we do not present the deviation of some observables for which the corresponding three-generation standard model values are less than 1%, i.e. $\langle P_N^\rho \rangle$ for muon and tau leptons and $\langle A_{CP}^\rho \rangle$, $\langle P_N^\rho \rangle$ for tau lepton.

From the figures it can be concluded that

(i) $\mathcal{B}_r$ strongly depends on the mass of the fourth quark ($m_{t'}$) and quark mixing matrix product (r_{db}) for both μ and τ channels. Furthermore, for both channels, $\mathcal{B}_r$ is enhanced sizably in terms of both $m_{t'}$ and r_{db}.

(ii) P_L^ρ and A_{CP}^ρ are independent from the lepton mass [see Eq. (20) and (33)]. For this reason, considering a fixed value of $\hat{s}$, they are the same for e, μ, and τ channels. The situation for the $\langle P_L^\rho \rangle$ and $\langle A_{CP}^\rho \rangle$ is different; those values for the τ channel are less than for the μ channel. This is because of the fact that the phase integral space is suppressed by increasing the lepton mass (m_ℓ). The SM3 value of $\langle P_L^\rho \rangle$ and $\langle A_{CP}^\rho \rangle$ almost vanishes for the τ channel. The SM4 suppresses those values even more. On the other hand, $\langle P_L^\rho \rangle$ and $\langle A_{CP}^\rho \rangle$ for the μ channel strongly depend on the SM4 parameters. The mag-

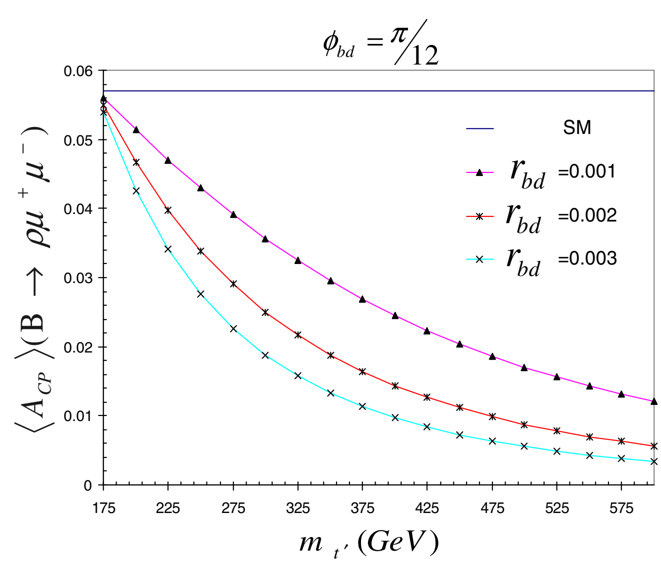

FIG. 3 (color online). The dependence of the $\langle A_{CP} \rangle$ of $B \to \rho\mu^+\mu^-$ on $m_{t'}$ for $r_{db} = 0.001, 0.002, 0.003$.

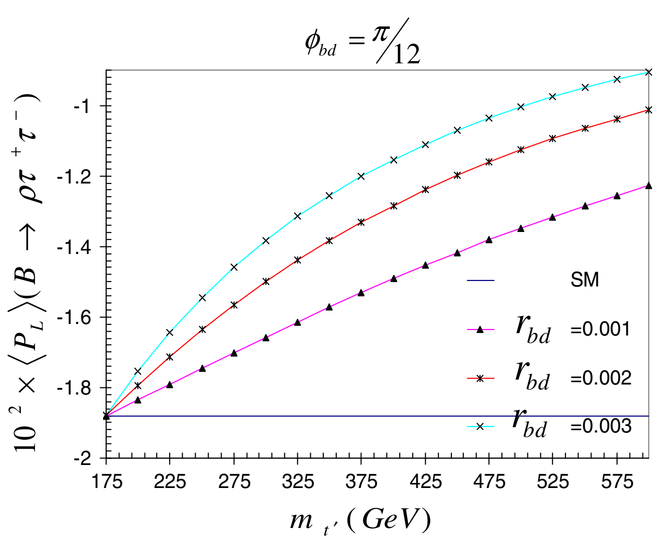

FIG. 5 (color online). The same as in Fig. 5, but for the τ lepton.

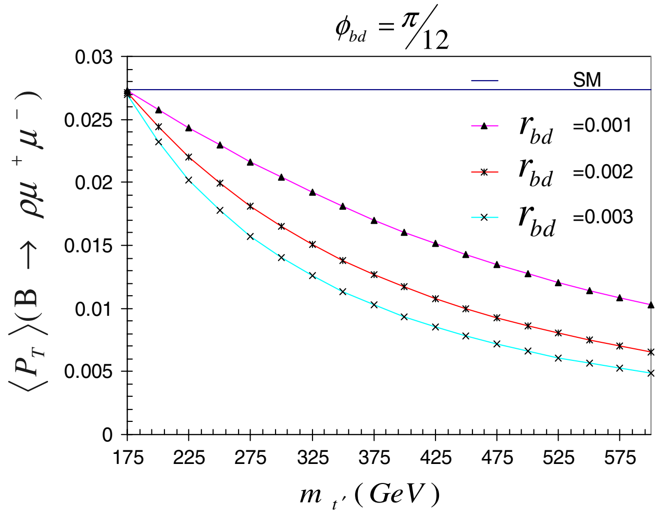

FIG. 6 (color online). The dependence of the $\langle P_T \rangle$ for μ lepton on $m_{t'}$ for $r_{db} = 0.001, 0.002, 0.003$.

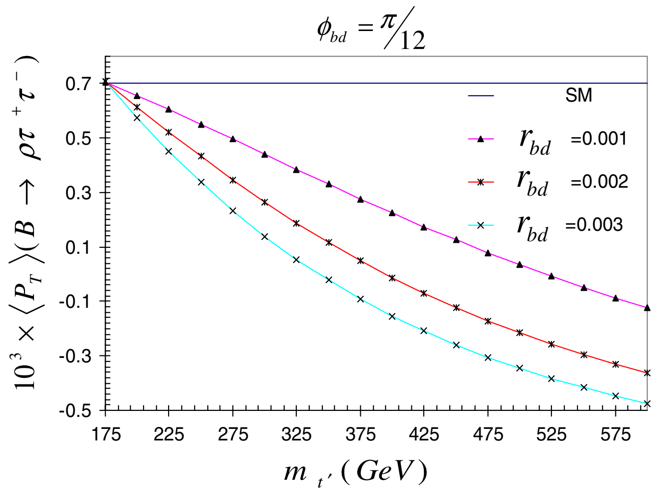

FIG. 7 (color online). The same as in Fig. 6, but for the τ lepton.

nitude of both is a decreasing function of the r_{db} and $m_{t'}$.

(iii) $\langle P_T^\rho \rangle$ strongly depends on the fourth quark mass ($m_{t'}$) and quark mixing matrix product (r_{sb}) for both the μ and τ channels. Its magnitude is a decreasing function of both $m_{t'}$ and r_{sb}. The measurement of the magnitude and sign of this observable can be used as a good tool to search for fourth-generation effects (see Figs. 6 and 7).

To sum up, we presented the systematic analysis of the $B \to \rho \ell^- \ell^+$ decay by using the SM with the fourth generation. The sensitivity of the total branching ratio, *CP* asymmetry, and lepton polarization on the new parameters, which come out of the fourth generation, were studied. We found out that the above-mentioned physical observables depict a strong dependence on the fourth quark ($m_{t'}$) and on the product of quark mixing matrix elements ($V_{t'b}^* V_{t'd} = r_{db} e^{i\phi_{db}}$). We found that while the branching ratio and $\langle P_N^\rho \rangle$ are enhanced, *CP* asymmetry, $\langle P_L^\rho \rangle$, and $\langle P_T^\rho \rangle$ are suppressed by fourth-generation effects. The measurement of the magnitude and sign of these readily measurable observables, in particular, for the μ case, can serve as a good tool to search for physics beyond the SM. In particular, the results can be used for an indirect search to look for the fourth generation of quarks.

[1] S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).
[2] P. H. Frampton, P. Q. Hung, and M. Sher, Phys. Rep. **330**, 263 (2000).
[3] V. Bashiry and K. Zeynali, J. High Energy Phys. 12 (2007) 055.
[4] V. Bashiry and K. Azizi, J. High Energy Phys. 07 (2007) 064.
[5] F. Zolfagharpour and V. Bashiry, Nucl. Phys. **B796**, 294 (2008).
[6] V. Bashiry and F. Falahati, Phys. Rev. D **77**, 015001 (2008).
[7] W. j. Huo, C. D. Lu, and Z. j. Xiao, arXiv:hep-ph/0302177.
[8] T. M. Aliev, A. Ozpineci, and M. Savci, Eur. Phys. J. C **29**, 265 (2003).
[9] A. Arhrib and W. S. Hou, Eur. Phys. J. C **27**, 555 (2003).
[10] E. Arik, S. Atağ, Z. Z. Aydin, A. Çelikel, Z. Çiçek, A. K. Çiftçi, A. Mailov, S. Sultansoy, and G. Ünel, Phys. Rev. D **58**, 117701 (1998).
[11] W. J. Hou and T. F. Feng, arXiv:hep-ph/0203207.
[12] S. Sultansoy, arXiv:hep-ph/0004271.
[13] B. Holdom, J. High Energy Phys. 08 (2006) 076.
[14] A. J. Buras and M. Münz, Phys. Rev. D **52**, 186 (1995).
[15] W. S. Hou, R. S. Willey, and A. Soni, Phys. Rev. Lett. **58**, 1608 (1987); **60**, 2337(E) (1988).
[16] N. G. Deshpande and J. Trampetic, Phys. Rev. Lett. **60**, 2583 (1988).
[17] T. Huber, E. Lunghi, M. Misiak, and D. Wyler, Nucl. Phys. **B740**, 105 (2006).
[18] T. Hurth, Rev. Mod. Phys. **75**, 1159 (2003).
[19] J. L. Hewett, Phys. Rev. D **53**, 4964 (1996).
[20] T. M. Aliev, V. Bashiry, and M. Savci, Eur. Phys. J. C **35**, 197 (2004).
[21] T. M. Aliev, V. Bashiry, and M. Savci, Phys. Rev. D **72**, 034031 (2005).
[22] T. M. Aliev, V. Bashiry, and M. Savci, J. High Energy Phys. 05 (2004) 037.
[23] T. M. Aliev, V. Bashiry, and M. Savci, Phys. Rev. D **73**, 034013 (2006).

[24] T. M. Aliev, V. Bashiry, and M. Savci, Eur. Phys. J. C **40**, 505 (2005).
[25] F. Krüger and L. M. Sehgal, Phys. Lett. B **380**, 199 (1996).
[26] Y. G. Kim, P. Ko, and J. S. Lee, Nucl. Phys. **B544**, 64 (1999).
[27] C. H. Chen and C. Q. Geng, Phys. Lett. B **516**, 327 (2001).
[28] V. Bashiry, Chin. Phys. Lett. **22**, 2201 (2005).
[29] W. S. Hou, H. n. Li, S. Mishima, and M. Nagashima, Phys. Rev. Lett. **98**, 131801 (2007).
[30] W. S. Hou, M. Nagashima, and A. Soddu, Phys. Rev. Lett. **95**, 141601 (2005).
[31] W. J. Huo, arXiv:hep-ph/0203226.
[32] T. M. Aliev, A. Ozpineci, and M. Savci, Eur. Phys. J. C **27**, 405 (2003).
[33] T. M. Aliev, A. Ozpineci, and M. Savci, Nucl. Phys. **B585**, 275 (2000).
[34] C. S. Huang, W. J. Huo, and Y. L. Wu, Mod. Phys. Lett. A **14**, 2453 (1999).
[35] S. Abachi *et al.* (D0 Collaboration), Phys. Rev. Lett. **78**, 3818 (1997).
[36] D. London, Phys. Lett. B **234**, 354 (1990).
[37] A. J. Buras, arXiv:hep-ph/0505175.
[38] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
[39] B. Grinstein, M. J. Savage, and M. B. Wise, Nucl. Phys. **B319**, 271 (1989).
[40] G. Cella, G. Ricciardi, and A. Viceré, Phys. Lett. B **258**, 212 (1991).
[41] M. Misiak, Nucl. Phys. **B393**, 23 (1993).
[42] M. Misiak, Nucl. Phys. **B439**, 461(E) (1995).
[43] C. S. Lim, T. Morozumi, and A. I. Sanda, Phys. Lett. B **218**, 343 (1989); N. G. Deshpande, J. Trampetić, and K. Panose, Phys. Rev. D **39**, 1461 (1989); P. J. O'Donnell and H. K. K. Tung, *ibid.* **43**, R2067 (1991); P. J. O'Donnell, M. Sutherland, and H. K. K. Tung, *ibid.* **46**, 4091 (1992).
[44] P. Coleangelo, F. De Fazio, P. Santorelli, and E. Scrimieri, Phys. Rev. D **53**, 3672 (1996).
[45] S. R. Choudhury and N. Gaur, Phys. Rev. D **66**, 094015 (2002).