Comment on "Can infrared gravitons screen Λ?"

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We reply to the recent criticism by Garriga and Tanaka of our proposal that quantum gravitational loop corrections may lead to a secular screening of the effective cosmological constant. Their argument rests upon a renormalization scheme in which the composite operator $(R\sqrt{-g} - 4\Lambda\sqrt{-g})_{\rm ren}$ is defined to be the trace of the renormalized field equations. Although this is a peculiar prescription, we show that it *does not preclude secular screening*. Moreover, we show that a constant Ricci scalar *does not even classically* imply a constant expansion rate. Other important points are: (1) the quantity $R_{\rm ren}$ of Garriga and Tanaka is neither a properly defined composite operator, nor is it constant; (2) gauge dependence does not render a Green's function devoid of physical content; (3) scalar models on a nondynamical de Sitter background (for which there is no gauge issue) can induce arbitrarily large secular contributions to the stress tensor; (4) the same secular corrections appear in observable quantities in quantum gravity; and (5) the prospects seem good for deriving a simple stochastic formulation of quantum gravity in which the leading secular effects can be summed and for which the expectation values of even complicated, gauge invariant operators can be computed at leading order.

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I. INTRODUCTION

Some years ago we proposed that the continual production of infrared gravitons induces a sort of quantum friction which gradually slows inflation [1]. When inflation rips a pair of infrared gravitons from the vacuum, which is a 1-loop effect, they induce a gravitational potential that contributes to the energy density at next order. This potential remains imprinted on the spacetime even after the pair has been pulled out of causal contact. The number density of pairs remains constant because the vast expansion of the spatial volume cancels the continual creation of pairs. However, the induced gravitational potential grows without bound. Taking account of the number of infrared gravitons inside the past light cone of a local observer gives $\Phi \sim -GH^2 \cdot Ht$ [2]. The effect of this potential must be to slow inflation because gravity is attractive. Hence, the induced energy density of interaction is negative, $\rho \sim -GH^6 \cdot Ht$. Because gravity is weak, this energy density grows very slowly, in other words, $|\dot{\rho}| \ll H|\rho|$ for $Ht \gg 1$. Combining that fact with stress-energy conservation, $\dot{\rho} = -3H(\rho + p)$, implies the induced pressure is $p \sim -\rho$. Hence, one gets a growing, negative vacuum energy.

It is tempting to speculate that this mechanism might simultaneously explain why the observed cosmological constant is so much smaller than the natural scales of particle physics, and also provide a model of inflation which has no fundamental scalar. The idea is that primordial inflation would start because the bare cosmological constant is positive and of GUT scale. This avoids the problem of needing a scalar inflaton to be unnaturally homogeneous over a super-Hubble volume [3]. Inflation would persist for a long time because gravity is a weak interaction, thereby dispensing with the need for a shallow potential. Inflation would be brought to an end by the gradual accumulation of negative vacuum energy.

Explicit computations in this scheme are challenging because the first effect should be at two loops and because the putative effect would be nonperturbatively strong during the current epoch. Ford was early able to show that there is no secular deviation from de Sitter background at one loop [4], in agreement with the mechanism. This 1-loop result has been confirmed by two other groups [5,6]. A year-long computation of the graviton 1-point function revealed secular slowing at the expected 2-loop order [7].[1] Because backreaction should be small until the last few e-foldings, it was possible to use the 2-loop result to estimate the scalar and tensor power spectra [8]. Some thought has also been given to how the mechanism might operate after it becomes nonperturbatively strong at the end of inflation [9].

Our proposal has recently been criticized by Garriga and Tanaka [10]. They raise two objections:

(1) The graviton 1-point function cannot show slowing, or anything else, because it is gauge dependent.

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[1] This work has not been confirmed, nor redone using dimensional regularization [6].

(2) There can be no slowing of inflation in pure quantum gravity because the field equations set the renormalized Ricci scalar to a constant.

In Sec. II we point out that physical information can reside in gauge-dependent quantities, that the secular growth we found was not injected through our gauge fixing functional, that the only invariant quantities so far checked in quantum gravity show the same type of secular dependence, that this secular dependence also affects the vacuum energy of scalar models for which there is no gauge issue, and that it could only drop out of the quantum gravitational vacuum energy through an infinite series of cancellations. Section III contains two important points:

(1) Garriga and Tanaka did not show that the renormalized Ricci scalar is constant and the relation they actually derived is completely consistent with secular slowing.
(2) They did not define the Ricci scalar as a properly renormalized composite operator but rather the Ricci scalar times the volume element.

We also comment on some peculiarities of their renormalization scheme. Section IV makes the short but crucial point that a constant Ricci scalar does not even classically imply expansion. In Sec. V we review recent progress in developing a computational formalism which is powerful enough to evolve into the nonperturbative regime.

II. GAUGE DEPENDENCE

The fact that the expectation value of the metric cannot be formed into a gauge invariant quantity in the same way as a classical metric was originally pointed out by Unruh [11]. We agree with him, and two years ago we advocated that the quantum gravitational backreaction on Λ-driven inflation should be quantified with an invariant operator which measures the geodesic deviation [12]. The expectation value of this operator has not yet been computed at 2-loop order owing to its complexity. It is worth noting that the difficulty of working with gauge invariant observables has led Losic and Unruh as well to the consideration of gauge-dependent quantities in their own interesting study of backreaction [13].

A. Gauge dependence does not automatically imply lack of physical content

A good example is provided by the 1PI functions of the standard model. These 1PI functions are certainly gauge dependent, yet they can be assembled to give the gauge-independent S-matrix, which has of course been subject to impressive experimental verification. So the gauge-dependent 1PI functions of the standard model must contain valid, physical, and gauge-independent information along with some unphysical and gauge-dependent behavior.

An especially noteworthy example is the gauge dependence of effective potentials [14]. In the days when people were studying the possibility for loop corrections to stabilize Kaluza-Klein compactifications [15], this gauge dependence was regarded with great suspicion and even despair. The question was asked, "Which result should be trusted if the effective potential shows a nontrivial minimum in one gauge but not in another?" It was even believed that a "field-space metric" could be identified that would allow construction of a gauge-independent (and also field redefinition independent) effective action [16,17].

It was eventually realized that gauge dependence is actually a blessing for this problem. What one really wants is to find a stationary point of the full effective action; it is only because the effective action is too complicated to evaluate for an arbitrary background field that one is reduced to studying the effective potential. Because different gauges are related by field-dependent gauge transformations, the same finite-parameter family of field configurations in one gauge can, in another gauge, probe a different direction in the full space of fields. Hence, the correct conclusion is not that both effective potentials are rubbish on account of their gauge dependence, or that one is right and the other wrong, but rather that *each one represents valid physical information about the theory.* For a putative solution to be correct it must make each of the effective potentials stationary—and even then it might still be spurious if there is a nonstationary direction that neither of the restricted probes of field space chances to access.

The expectation value of the graviton 1-point function in a fixed gauge is a perfectly valid measure of quantum corrections to the background *in that gauge*. It might be that the secular screening we found is an artifact of the gauge (or even an error in what was a very long and difficult computation) but several possibilities can be ruled out. First, the secular dependence we found was not introduced through the gauge fixing functional. We found "infrared logarithms;" these are logarithms of the ratio of the scale factor at the observation time to its value at the beginning of inflation. These logarithms break the part of the 10-parameter de Sitter group known as dilatations; the latter generate the transformation $x^\mu \longrightarrow k \cdot x^\mu$ in conformal coordinates. However, our gauge condition preserves this symmetry.

B. Infrared logarithms do not always drop out of physical and gauge invariant quantities

The only invariant so far checked is the inflationary power spectrum and Weinberg has shown that infrared logarithms *do correct this* [18,19]. Because the logarithms in that case are of the ratio of the current scale factor to the scale factor at horizon crossing, their enhancement is at most about $\ln(a/a_{\rm hc}) \sim 100$ for any perturbation whose spatial variation we can resolve. This is not enough to overcome the small loop counting parameter of $GH^2 \lesssim 10^{-12}$. However, there can be arbitrarily large infrared logarithms correcting things we perceive as spatially constant *such as the vacuum energy.*

It might still be that the vacuum energy is somehow protected from the secular effects which contaminate 1PI functions [7,20], the quantum-corrected mode functions [21] and the power spectrum [18,19]. But that is certainly not true for scalar models on nondynamical, de Sitter background. In that case the calculations are vastly simpler, and there is no gauge issue to frustrate drawing obvious physical conclusions. It suffices to compute the expectation value of the stress tensor. For example, a fully renormalized, 2-loop computation of the stress tensor of a massless, minimally coupled scalar with a φ^4 self-interaction which is released in Bunch-Davies vacuum at $t = 0$ [with scale factor $a(t) = e^{Ht}$] reveals the following secular growth for the induced energy density and pressure [22]:

$$\rho(t) = \frac{\lambda H^4}{(2\pi)^4}\left\{\frac{1}{8}\ln^2(a)\right\} + O(\lambda^2), \tag{1}$$

$$p(t) = \frac{\lambda H^4}{(2\pi)^4}\left\{-\frac{1}{8}\ln^2(a) - \frac{1}{12}\ln(a)\right\} + O(\lambda^2). \tag{2}$$

At leading logarithm order this result is in perfect agreement with the nonperturbative analysis of Yokoyama and Starobinskiĭ [23,24]. It also has the transparent physical interpretation that the inflationary production of scalars increases the scalar field strength, which drives the scalar up its φ^4 potential and thereby induces a growing vacuum energy.

One may dismiss the φ^4 result on the grounds that it shows an *increase* in the vacuum energy, whereas we claim a decrease for quantum gravity. But it is simple to find scalar models whose induced vacuum energy is negative. Consider scalar quantum electrodynamics, for which the analogous 2-loop results are [25,26]

$$\rho(t) = \frac{e^2H^4}{(2\pi)^4}\times\left[-\frac{3}{4}\ln(a)\right] + O(e^4), \tag{3}$$

$$p(t) = \frac{e^2H^4}{(2\pi)^4}\times\left[\frac{3}{4}\ln(a)\right] + O(e^4). \tag{4}$$

The physical interpretation seems to be that the inflationary production of charged scalars polarizes the vacuum [27,28], which lowers the scalar energy.

Again one may dismiss the scalar QED results on the grounds that they represent a transient effect that approaches a constant after a nonperturbatively large time $\Delta t \sim 1/(e^2H)$ [26]. The specious nature of this argument can be seen from a massless fermion which is Yukawa coupled to a massless, minimally coupled scalar on nondynamical de Sitter background. For that model the induced vacuum energy *falls without bound* [29]. This also has a transparent physical interpretation: the inflationary production of scalars increases the scalar field strength, which induces a fermion mass, whose effect is to decrease the vacuum energy. Had gravity been dynamical in this model, the resulting secular backreaction would *end inflation.* That the universe subsequently decays to a big rip singularity does not alter the model's demonstration that *secular backreaction can accumulate to dominate late time cosmology.*

Finally, it should be noted that the absence of secular quantum gravitational backreaction would not only require that a single infrared logarithm drops out at two loops. The number of infrared logarithms grows with each loop in a way that can be predicted from the number of undifferentiated massless, minimally coupled scalars or graviton fields in the basic interaction vertex. The number of extra infrared logarithms for each extra coupling constant in $\lambda\varphi^4$, Yukawa theory, and scalar QED are [26]

$$\lambda\varphi^4\sqrt{-g} \Rightarrow \ln^2(a) \quad \text{for every } \lambda, \tag{5}$$

$$-f\varphi\bar{\psi}\psi\sqrt{-g} \Rightarrow \ln(a) \quad \text{for every } f^2, \tag{6}$$

$$ie(\varphi^*\partial_\mu\varphi - \partial_\mu\varphi^*\varphi)A_\nu g^{\mu\nu}\sqrt{-g} \Rightarrow \ln(a) \quad \text{for every } e^2. \tag{7}$$

These expectations are verified in a large number of explicit 1-loop, 2-loop, and even 3-loop computations [30]. They follow as well from the stochastic analysis of Starobinskiĭ and Yokoyama [23,24,26].[2]

The basic interaction of quantum gravity takes the form $\kappa^n h^n \partial h \partial h a^{D-2}$ [32], where $\kappa^2 \equiv 16\pi G$ is the loop counting parameter. The number of extra infrared logarithms for each extra factor of GH^2 is [26]

$$\kappa^n h^n \partial h \partial h a^{D-2} \Rightarrow \ln(a) \quad \text{for every } GH^2. \tag{8}$$

So the absence of secular backreaction in quantum gravity would require that the escalating series of infrared logarithms *which do appear in 1PI functions and physical quantities* somehow contrives to cancel out of the expansion rate. We do not regard it as reasonable to suppose that this happens. Of course Garriga and Tanaka claim to have proven that it does happen, and to all orders. We turn now to an explanation of why their proof is not correct.

III. RENORMALIZING THE RICCI SCALAR

The basic argument of Garriga and Tanaka consists of a peculiar scheme for renormalizing the Ricci scalar so that the following relation applies:

$$\langle R_{\text{ren}}\sqrt{-g}\rangle = 4\Lambda\langle(1 + \delta_{\text{vol}})\sqrt{-g}\rangle. \tag{9}$$

[2]Certain 1PI functions have counterterms that can be used to absorb infrared logarithms at low loop orders [31] but factors of $\ln(a)$ always show up at higher orders.

We shall presently describe some technical problems with this scheme but the most significant point of this section is that (9) *does not imply a constant Ricci scalar*. We shall demonstrate that (9) is completely consistent with the relaxation mechanism of Sec. I. Moreover, $R_{\rm ren}$ as defined by Garriga and Tanaka (GT) is not a properly renormalized composite operator because it fails to produce finite results when inserted in 1PI functions.

A. Preliminaries

Many people have concluded that screening is impossible in pure quantum gravity because the trace of the classical field equations implies $R = 4\Lambda$:[3]

$$\mathcal{L} = \frac{1}{16\pi G}(R - 2\Lambda)\sqrt{-g} \Rightarrow 16\pi G g^{\mu\nu}\frac{\delta S}{\delta g^{\mu\nu}} = (-R + 4\Lambda)\sqrt{-g} = 0. \qquad (10)$$

The conclusion is premature because quantizing general relativity requires adding to the Lagrangian an infinite sequence of ever-higher dimensional, Bogoliubov-Parasuik-Hepp-Zimmerman (BPHZ) counterterms:

$$\Delta\mathcal{L} = \alpha_1 R^2\sqrt{-g} + \alpha_2 C^{\rho\sigma\mu\nu}C_{\rho\sigma\mu\nu}\sqrt{-g} + \cdots. \qquad (11)$$

The divergent parts of the α_i's are fixed by the need to cancel primitive divergences, but no physical principle seems to fix their finite parts, and what they are affects physical predictions. That is one way of expressing the problem of quantum gravity.

Of particular interest to us is the fact that even the lowest order counterterms alter the relation between R and Λ implied by the trace of the renormalized field equations:[4]

$$0 = 16\pi G g^{\mu\nu}\frac{\delta(S + \Delta S)}{\delta g^{\mu\nu}(x)} = (-R + 4\Lambda + 16\pi G \times 6\alpha_1 \Box R + \cdots)\sqrt{-g}. \qquad (12)$$

One would typically break the α_i's up into divergent and finite parts, and render this equation in terms of suitably renormalized composite operators:

$$0 = (-R\sqrt{-g} + 4\Lambda\sqrt{-g})_{\rm ren} + 16\pi G \times 6\alpha_{1,\rm finite}(\Box R\sqrt{-g})_{\rm ren} + O(G^2). \qquad (13)$$

The Eddington counterterm—the one proportional to α_1—was exploited by Starobinskiĭ in constructing an early model of what would later be called inflation [34]. It should therefore be obvious that the equations of quantum gravity cannot imply the Ricci scalar is constant unless the finite parts of a countably infinite number of α_i's are set to zero.

[3] We first heard the argument from Susskind in the mid-1980's [33].

[4] Computing the trace in dimensional regularization would result as well in finite contributions from the α_1 and α_2 counterterms whose inclusion would only strengthen the argument we shall make.

B. The Garriga and Tanaka argument

They propose to circumvent the problem of counterterms by absorbing them into the renormalized Ricci scalar $R_{\rm ren}$ and the renormalized volume element $(1 + \delta_{\rm vol})\sqrt{-g}$. Their Eq. (36) defines the operator whose vanishing in expectation values is claimed to preclude secular back-reaction:

$$\mathcal{O}_{\rm GT} \equiv \int d^4x W(x)[-R_{\rm ren} + 4\Lambda(1 + \delta_{\rm vol})]\sqrt{-g}. \qquad (14)$$

Here $W(x)$ is their "window function" which is supposed to be a scalar that does not depend upon the metric.[5] Combining Eqs. (38) and (41) of Garriga and Tanaka results in their definition for $R_{\rm ren}$:

$$R_{\rm ren} \equiv R - 16\pi G \times \frac{g^{\mu\nu}}{\sqrt{-g}}\frac{\delta\Delta S}{\delta g^{\mu\nu}} + 4\Lambda\delta_{\rm vol}. \qquad (15)$$

Substituting (15) in (14) reveals the key operator of Garriga and Tanaka as the smeared trace of the renormalized field equations:

$$\mathcal{O}_{\rm GT} = 16\pi G \times \int d^4x W(x) g^{\mu\nu}(x)\frac{\delta(S + \Delta S)}{\delta g^{\mu\nu}(x)}. \qquad (16)$$

Of course this does vanish, *as do all components of the renormalized field equations*, in all expectation values, with or without the smearing.

C. Consistency with screening

Garriga and Tanaka have not shown their definition of the renormalized Ricci scalar is constant but rather that it, times the volume element, equals another operator:

$$\langle R_{\rm ren}\sqrt{-g}\rangle = 4\Lambda\langle(1 + \delta_{\rm vol})\sqrt{-g}\rangle. \qquad (17)$$

This is completely consistent with the secular screening mechanism described in Sec. I. It is incorrect to think of R and $\sqrt{-g}$ as possessing distinct time dependences in (17) but one can do so at leading logarithm order. In that case, working in conformal coordinates, the mechanism of Sec. I implies the following relaxations for the Ricci scalar and volume element:

$$R = 4\Lambda + 8\pi G(\rho - 3p) \sim 4\Lambda[1 - \#G^2H^4\ln(a) + O(G^3)], \qquad (18)$$

$$\sqrt{-g} \sim a^4[1 - \#G^2H^4\ln^2(a) + O(G^3)]. \qquad (19)$$

[5] We remark that $W(x)$ must involve the metric if it is not constant. Garriga and Tanaka seem to feel that metric dependence in $W(x)$ would prevent the equations of motion from holding, but any obstruction of this sort can be absorbed into an operator ordering [35].

At leading logarithm order, the change in R is insignificant compared with that of the volume element and both sides of Eq. (17) are equal:

$$\langle R_{\rm ren}\sqrt{-g}\rangle = 4\Lambda\langle(1+\delta_{\rm vol})\sqrt{-g}\rangle, \sim 4\Lambda a^4\{1 - \#[GH^2\ln(a)]^2 + O(G^3)\}. \tag{20}$$

This continues to be true at all orders in the loop expansion, as per relation (8), because R involves differentiated metrics whereas $\sqrt{-g}$ does not. Of course Garriga and Tanaka want Eq. (17) to be exact and not just valid at leading logarithm order. However, the necessary subleading logarithms can easily be supplied by cross terms between R and $\sqrt{-g}$, and by the operator $\delta_{\rm vol}$.

D. Peculiar renormalization

Although the renormalization scheme (15) is consistent with secular slowing, we still find it dubious. As Garriga and Tanaka point out, using the equations of motion inside a functional integral requires the integrand to be an invariant [35]. Hence, one is not considering "R" and "1" but rather "$R\sqrt{-g}$" and "$\sqrt{-g}$." Renormalizing such composite operators requires that their insertion in a 1PI diagram be accompanied by the insertion of counteroperators:

$$(R\sqrt{-g})_{\rm ren} \equiv R\sqrt{-g} + \delta R\sqrt{-g}, \tag{21}$$

$$(\sqrt{-g})_{\rm ren} \equiv \sqrt{-g} + \delta_{\rm vol}\sqrt{-g}. \tag{22}$$

We know of no previous study of these operators with a nonzero cosmological constant but, on general grounds of symmetry and the degree of 1-loop divergences one expects [36,37]:

$$\delta_{\rm vol} = 16\pi G \times \gamma_1 R + O(G^2), \tag{23}$$

$$\delta R = 16\pi G \times (\beta_1 \Box R + \beta_2 R^2 + \beta_3 R^{\mu\nu}R_{\mu\nu} + \beta_4 C^{\rho\sigma\mu\nu}C_{\rho\sigma\mu\nu} + 4\gamma_1\Lambda R) + O(G^2). \tag{24}$$

The trace of the renormalized field equations (12) requires that the divergent part of β_1 should agree with the divergent part of $-6\alpha_1$, and that β_{2-4} should vanish. That is no problem. However, the renormalization scheme (15) of Garriga and Tanaka also requires that the finite part of β_1 should agree with the finite part of $-6\alpha_1$. An escalating series of similar relations must hold as one moves up the loop expansion.

There are two ways of viewing this. Either Garriga and Tanaka have solved the problem of quantum gravity by uniquely specifying the finite parts of every α_i except α_2,[6] or else they are defining the Ricci scalar so as to absorb known classical effects. In the latter case, one could also absorb the crucial R^2 term of Starobinskiĭ's model [34] to conclude that the "renormalized Ricci scalar" is constant. Indeed, there seems no reason to restrict the procedure to pure gravity. Were it applied to scalar-driven inflation—by including the scalar action in the "ΔS" they use in (15)—then one would conclude that scalar-driven inflation never ends.

It might be objected that a key distinction between the counterterms of quantum gravity and the inflaton potential is that the former contain factors of $\hbar$ whereas the latter does not. If so, then suppose the inflaton potential derives from a 1-loop correction, such as the Coleman-Weinberg potential of new inflation [38]. The inflaton potential now carries a factor of $\hbar$, so it can be absorbed into the "renormalized Ricci scalar" and we have just shown that new inflation is pure de Sitter.

E. The composite operator $R_{\rm ren}$ of (15) is neither properly defined nor is it constant

Correctly defined composite operators have the property that inserting them in 1PI functions gives finite results. With the conventions of Garriga and Tanaka, the composite operators $R_{\rm ren}\sqrt{-g}$ and $(1+\delta_{\rm ren})\sqrt{-g}$ have this property but $R_{\rm ren}$ does not. Simply divide out the volume element from both sides of their key relation (9):

$$R_{\rm ren}\sqrt{-g} = 4\Lambda(1+\delta_{\rm vol})\sqrt{-g} \Rightarrow R_{\rm ren} = 4\Lambda(1+\delta_{\rm vol}). \tag{25}$$

Now note two crucial features of the volume counterterm $\delta_{\rm vol}$:

(1) It is not finite.

(2) It is an operator rather than a constant.

The first feature can be seen by expanding the volume element operator in powers of the graviton field:

$$\sqrt{-g} = a^D\Big(1 + \frac{\kappa}{2}h^\rho{}_\rho + \frac{\kappa^2}{8}(h^\rho{}_\rho)^2 - \frac{\kappa^2}{4}h^{\rho\sigma}h_{\rho\sigma} + \cdots\Big). \tag{26}$$

Inserting $\sqrt{-g}$ in the vacuum amplitude gives a divergence at order $\kappa^2 = 16\pi G$ from the coincident propagator.[7] From expression (23) we see that this divergence is canceled by the 1-loop contribution to $\delta_{\rm vol}$, which is $\kappa^2\gamma_1 R = \gamma_1 D(D-1)\kappa^2H^2 + O(\kappa^4)$. It follows that γ_1 is divergent, so inserting just $\delta_{\rm vol}$ into a 1PI function, *without the factor of* $\sqrt{-g}$, does not produce finite results. We therefore conclude that Garriga and Tanaka cannot claim to have shown that the Ricci scalar is constant—or anything else—because they have not correctly defined it as a composite operator. Further, the quantity $R_{\rm ren}$, is not constant but rather an infinite series of time-dependent operators.

[6]The single parameter α_2 would not prevent us from making sense of quantum gravity.

[7]The divergence would be $\frac{\kappa^2H^2}{4\pi^2}\frac{1}{D-4}$ in our gauge, implying $\gamma_1 = -\frac{1}{48\pi^2}\frac{1}{D-4}$ + finite.

IV. RICCI SCALAR CONSTANCY AND EXPANSION

It is time to critically examine the unstated assumption of Garriga and Tanaka that proving the Ricci scalar is constant implies de Sitter expansion. This is quite incorrect. Neither the classical field equation, $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0$, nor its trace, $-R + 4\Lambda = 0$, precludes the presence of gravitational radiation. In fact, all gravitational wave solutions with $\Lambda \neq 0$ must obey these equations. Because gravity is a nonlinear theory, gravitational radiation interacts with itself. Because the gravitational force is attractive, this self-interaction opposes the expansion of spacetime. If enough gravitational radiation is present, the result is not expansion but rather collapse into one or more black holes. This is an unavoidable consequence of the singularity theorems. Therefore, one cannot conclude that constant Ricci scalar implies a constant expansion rate because this is not even true classically.

It might be objected that a positive cosmological constant opposes the tendency towards collapse of too much gravitational radiation. So the correct result is that a fixed, initial distribution of gravitational radiation may well collapse to one or more black holes, but the late time geometry away from these black holes will approach the local de Sitter form for which $R^{\rho}{}_{\sigma\mu\nu} = H^2(\delta^{\rho}_{\mu} g_{\sigma\nu} - \delta^{\rho}_{\nu} g_{\sigma\mu})$. That would be true enough classically, where no new gravitational radiation can be generated beyond what was present initially, but it is not correct when quantum effects are included. The simplest way to understand quantum effects is as the classical response to the source provided by uncertainty principle. In these terms, our screening mechanism is driven by the *steady injection* of gravitational radiation, throughout space, as more and more infrared virtual gravitons are ripped out of the vacuum. These gravitons must slow the expansion of spacetime because that is what a classical distribution of gravitational radiation would do. However, such a classical distribution would obey $R = 4\Lambda$.

V. EPILOGUE

It seems obvious that gravitons contribute to vacuum energy. At 1-loop order, where each mode contributes separately, infrared gravitons participate on an equal footing with ultraviolet ones; this can be shown by explicit computation [4–6]. Higher loop contributions involve nonlinear combinations of gravitons which are best viewed as integrals over position space. These integrals reach back from the point of observation, along the past light cone to the fixed initial value surface. Because the past light cone grows as the time of observation evolves, these higher loop contributions can grow as well. That has to be expected in view of the infrared singularities of quantum gravity on a locally de Sitter background [39].

Nothing ought to seem dubious about this. Although the effect has not yet been demonstrated for an invariant measure of the quantum gravitational backreaction on inflation, the same phenomenon injects secular dependence into other invariant quantities [18,19]. It also engenders secular contributions to the vacuum energies in scalar models [22,26,29] for which there is no gauge issue to prevent one from drawing obvious conclusions.

We have shown that the renormalization scheme employed by Garriga and Tanaka is highly dubious and that, even if accepted, it does not preclude secular slowing. Nor would proving any plausibly defined "Ricci scalar" to be constant preclude secular slowing. Of course the burden of proving that secular slowing does occur rests quite properly with us. Even an explicit perturbative demonstration of this at 2-loop order would not establish quantum gravitational backreaction either as an explanation for why the observed cosmological constant is so small or as the basis of a viable model of inflation. For that, one would need a reliable way of computing in the late time regime, after backreaction has become nonperturbatively strong.

We do not think such computations are beyond reach. Starobinskiĭ has proposed a simple stochastic formalism [40] that reproduces the leading infrared logarithms of scalar potential models to all orders [24] and which can be summed to give nonperturbative results [23]. We have recently extended Starobinskiĭ's formalism to massless, minimally coupled scalars that interact with other kinds of fields. This has led to explicit, nonperturbative results for Yukawa theory [29] and for scalar QED [26]. We do not yet know how to treat derivative interactions of the sort quantum gravity possesses but the problem does not seem insolvable.

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[1] N. C. Tsamis and R. P. Woodard, Phys. Lett. B **301**, 351 (1993); Nucl. Phys. **B474**, 235 (1996).
[2] N. C. Tsamis and R. P. Woodard, Ann. Phys. (N.Y.) **267**, 145 (1998); R. P. Woodard, Nucl. Phys. B, Proc. Suppl. **104**, 173 (2002).
[3] T. Vachaspati and M. Trodden, Phys. Rev. D **61**, 023502 (1999).
[4] L. H. Ford, Phys. Rev. D **31**, 710 (1985).
[5] F. Finelli, G. Marozzi, G. P. Vacca, and G. Venturi, Phys. Rev. D **71**, 023522 (2005).
[6] N. C. Tsamis and R. P. Woodard, Ann. Phys. (N.Y.) **321**, 875 (2006).
[7] N. C. Tsamis and R. P. Woodard, Ann. Phys. (N.Y.) **253**, 1 (1997); Phys. Rev. D **54**, 2621 (1996).
[8] L. R. W. Abramo, R. P. Woodard, and N. C. Tsamis, Fortschr. Phys. **47**, 389 (1999).
[9] N. C. Tsamis and R. P. Woodard, J. Cosmol. Astropart. Phys. 05 (2004) 006.
[10] J. Garriga and T. Tanaka, Phys. Rev. D **77**, 024021 (2008).
[11] W. G. Unruh, arXiv:astro-ph/9802323.
[12] N. C. Tsamis and R. P. Woodard, Classical Quantum Gravity **22**, 4171 (2005).
[13] B. Losic and W. G. Unruh, Phys. Rev. D **72**, 123510 (2005); Phys. Rev. D **74**, 023511 (2006).
[14] R. Jackiw, Phys. Rev. D **9**, 1686 (1974).
[15] P. Candelas and S. Weinberg, Nucl. Phys. **B237**, 397 (1984).
[16] I. Buchbinder, S. D. Odintsov, and P. M. Lavrov, Nucl. Phys. **B308**, 191 (1988).
[17] S. R. Huggins, G. Kunstatter, H. P. Leivo, and D. J. Toms, Nucl. Phys. **B301**, 627 (1988).
[18] S. Weinberg, Phys. Rev. D **72**, 043514 (2005); Phys. Rev. D **74**, 023508 (2006).
[19] K. Chaicherdsakul, Phys. Rev. D **75**, 063522 (2007).
[20] S. P. Miao and R. P. Woodard, Classical Quantum Gravity **23**, 1721 (2006).
[21] S. P. Miao and R. P. Woodard, Phys. Rev. D **74**, 024021 (2006).
[22] V. K. Onemli and R. P. Woodard, Classical Quantum Gravity **19**, 4607 (2002); Phys. Rev. D **70**, 107301 (2004).
[23] A. A. Starobinskiĭ and J. Yokoyama, Phys. Rev. D **50**, 6357 (1994).
[24] N. C. Tsamis and R. P. Woodard, Nucl. Phys. **B724**, 295 (2005).
[25] R. P. Woodard, J. Phys. Conf. Ser. **68**, 012032 (2007).
[26] T. Prokopec, N. C. Tsamis, and R. P. Woodard, Ann. Phys. (N.Y.) **323**, 1324 (2008).
[27] T. Prokopec, O. Tornkvist, and R. P. Woodard, Phys. Rev. Lett. **89**, 101301 (2002); Ann. Phys. (N.Y.) **303**, 251 (2003).
[28] T. Prokopec and R. P. Woodard, Am. J. Phys. **72**, 60 (2004).
[29] S. P. Miao and R. P. Woodard, Phys. Rev. D **74**, 044019 (2006).
[30] N. C. Tsamis and R. P. Woodard, Phys. Lett. B **426**, 21 (1998); T. Prokopec and R. P. Woodard, J. High Energy Phys. 10 (2003) 059; Ann. Phys. (N.Y.) **312**, 1 (2004); T. Brunier, V. K. Onemli, and R. P. Woodard, Classical Quantum Gravity **22**, 59 (2005); T. Prokopec, N. C. Tsamis, and R. P. Woodard, Classical Quantum Gravity **24**, 201 (2007); E. O Kahya and V. K. Onemli, Phys. Rev. D **76**, 043512 (2007).
[31] L. D. Duffy and R. P. Woodard, Phys. Rev. D **72**, 024023 (2005); E. O. Kahya and R. P. Woodard, Phys. Rev. D **72**, 104001 (2005); Phys. Rev. D **74**, 084012 (2006).
[32] N. C. Tsamis and R. P. Woodard, Commun. Math. Phys. **162**, 217 (1994).
[33] L. Susskind (private communication).
[34] A. A. Starobinskiĭ, Phys. Lett. **91B**, 99 (1980).
[35] N. C. Tsamis and R. P. Woodard, Classical Quantum Gravity **2**, 841 (1985); Ann. Phys. (N.Y.) **215**, 96 (1992).
[36] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, England, 1996), Vol. II, pp. 115–118.
[37] C. Itzykson and J. B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980), pp. 399–402, 684.
[38] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[39] N. C. Tsamis and R. P. Woodard, Classical Quantum Gravity **11**, 2969 (1994); Ann. Phys. (N.Y.) **238**, 1 (1995).
[40] A. A. Starobinskiĭ, in *Field Theory, Quantum Gravity and Strings*, edited by H. J. de Vega and N. Sanchez (Springer-Verlag, Berlin, 1986), pp. 107–126.