PHYSICAL REVIEW D **78,** 026005 (2008)

# Holographic duals of long open strings

Ofer Aharony[1] and David Kutasov[2]

[1]*Department of Particle Physics, The Weizmann Institute of Science, Rehovot 76100, Israel*
[2]*EFI and Department of Physics, University of Chicago, 5640 S. Ellis Avenue, Chicago, Illinois 60637, USA*
(Received 18 May 2008; published 16 July 2008)

We study the holographic map between long open strings, which stretch between $D$-branes separated in the bulk space-time, and operators in the dual boundary theory. We focus on a generalization of the Sakai-Sugimoto holographic model of QCD, where the simplest chiral condensate involves an operator of this type. Its expectation value is dominated by a semiclassical string world sheet, as for Wilson loops. We also discuss the deformation of the model by this operator, and, in particular, its effect on the meson spectrum. This deformation can be thought of as a generalization of a quark mass term to strong coupling. It leads to the first top-down holographic model of QCD with a non-Abelian chiral symmetry which is both spontaneously and explicitly broken, as in QCD. Other examples we study include half-supersymmetric open Wilson lines, and systems of $D$-branes ending on $NS5$-branes, which can be analyzed using world sheet methods.

DOI: 10.1103/PhysRevD.78.026005 PACS numbers: 11.25.Tq, 11.30.Rd

## I. INTRODUCTION AND SUMMARY

Holographic dualities [generalizing the anti-de Sitter/conformal field theory (AdS/CFT) correspondence [1]] have proven to be very useful, both for studying quantum gravity in backgrounds with appropriate boundaries, and for studying the dual theories living on these boundaries. However, the dictionary between the boundary theories and the corresponding quantum gravity duals is not yet complete.

There are two types of objects which we know how to translate between the bulk and boundary theories (at least in the limit in which the bulk geometry is weakly curved, and thus well described by supergravity). Local fields in the bulk map to local operators in the dual boundary theory; sources for these fields map to sources for the corresponding operators [2,3]. Extended $p$-dimensional branes in the bulk can end on closed $(p-1)$-dimensional surfaces on the boundary. They correspond to operators in the boundary theory that are associated with these surfaces.

For example, when the boundary theory is a large $N$ gauge theory, a Euclidean closed fundamental string world sheet in the bulk, which ends on a closed loop $C$ on the boundary, maps to (a locally supersymmetric version of) a Wilson loop in the dual field theory [4–6]. The latter can be thought of as associated with external (infinitely massive) $W$ bosons in the gauge theory.

In this note we add another entry to this dictionary. When the bulk background includes branes extending to the boundary, it is possible for other branes to end on these branes, and give additional observables in the theory. We will focus on the case where the bulk contains $D$-branes, and the branes ending on them are fundamental strings, but the discussion can be generalized to other systems.

String theory in the bulk contains in this case operators corresponding to open strings stretched between $D$-branes near the boundary. There are two qualitatively different classes of such operators. One corresponds to strings which can shrink to zero size (“short strings”). These are very similar to the closed string operators mentioned above; their duals in the boundary theory are local operators, which contain the degrees of freedom associated with the $D$-branes. The second class corresponds to “long strings,” that are stretched between $D$-branes which are separated by a finite amount near the boundary. Such operators depend on the choice of an open contour $\tilde{C}$, which ends on the two $D$-branes on the boundary. We propose that their duals in the boundary theory are certain “line operators.”

In the case of large $N$ gauge theories, $D$-branes ending on the boundary are associated with fields in the fundamental representation of $SU(N)$, and the line operators are open Wilson lines starting from a field in the fundamental representation associated with the first $D$-brane, and ending on a field in the antifundamental representation associated with the second one. We propose that an insertion of such an open Wilson line in the field theory corresponds in the bulk to an insertion of an open string ending on the corresponding contour on the boundary, as in the Wilson loop case. As there, some correlation functions of these operators are dominated by semiclassical string world sheets with the appropriate boundary conditions, and can thus be computed in the supergravity limit.

A case in which long open string operators play an important role is the Sakai-Sugimoto holographic model of QCD [7] and its generalizations studied in [8,9]. This model shares with QCD the phenomena of confinement and non-Abelian chiral symmetry breaking. As we discuss below, open Wilson lines play an important role in under-

*Ofer.Aharony@weizmann.ac.il
+dkutasov@uchicago.edu

 

standing the latter. Previous attempts to study them in this model appeared in [10,11], but our methods are different.

The Sakai-Sugimoto model describes a $4+1$ dimensional $SU(N_c)$ maximally supersymmetric Yang-Mills (SYM) theory, with 't Hooft coupling $\lambda_5$, compactified on a circle of radius $R(x^4 \equiv x^4 + 2\pi R)$ with antiperiodic boundary conditions for the fermions, and coupled to $N_f$ left- and right-handed fermions in the fundamental representation of $SU(N_c)$ localized at $x^4 = -L/2$ and $x^4 = L/2$, respectively.

The three parameters with dimensions of length, $\lambda_5$, $R$, and $L$, can be thought of as providing an overall scale and two dimensionless couplings on which the dynamics depends [8,9]. In the region of parameter space $\lambda_5 \ll L \sim R$, the $4+1$ dimensional gauge theory is weakly coupled at the scale $L \sim R$, and the model is equivalent at long distances (much larger than $L$, $R$, which can be viewed from this perspective as a UV cutoff), to massless $3+1$ dimensional QCD.

For large $\lambda_5$, the $4+1$ dimensional gauge theory is strongly coupled and needs to be UV completed. In string theory this is achieved by realizing the gauge theory as a low-energy theory on a stack of $N_c$ $D4$-branes wrapped around the twisted $x^4$ circle, intersecting $N_f$, $D8$, and $\bar{D}8$-branes along an $\mathbb{R}^{3,1}$, at $x^4 = \pm L/2$.

At strong coupling (and large $N_c$), one can replace the $D4$-branes by their near-horizon geometry, and study the dynamics of the eight-branes in this geometry. One finds that in the vacuum the $U(N_f)_L \times U(N_f)_R$ global chiral symmetry associated with the $D8$ and $\bar{D}8$-branes is spontaneously broken to the diagonal $U(N_f)$, due to the fact that the eight-branes connect in the bulk. To study this breaking in more detail, one would like to identify an operator in the field theory that transforms nontrivially under $U(N_f)_L \times U(N_f)_R$, and has a nonzero vacuum expectation value (VEV) that preserves the diagonal $U(N_f)$, i.e., an order parameter for the symmetry breaking.

Since the left- and right-handed fermions are separated in $x^4$, there are no local gauge-invariant operators in the $D4$-brane theory that are charged under both $U(N_f)$ groups. The simplest operators with the desired flavor quantum numbers are open Wilson lines of the type discussed above, such as (for a specific choice of the contour $\tilde{C}$)

$$OW_i^j(x^\mu) = \psi_L^{\dagger j}\left(x^\mu, x^4 = -\frac{L}{2}\right)\mathcal{P}\exp\left[\int_{-L/2}^{L/2}(iA_4 + \Phi)dx^4\right] \times \psi_{Ri}\left(x^\mu, x^4 = \frac{L}{2}\right), \tag{1.1}$$

where $\Phi$ is one of the scalar fields of the SYM theory, and $\mathcal{P}$ denotes path ordering.

In the weak coupling regime, the gauge field $A_4$ and scalar $\Phi$ are weakly coupled at the scale $L$, and the Wilson line in (1.1) can be neglected. Thus, the operator $OW_i^j$ reduces in this case to the local operator $\psi_L^{\dagger j}\psi_{Ri}$, which is the familiar order parameter of chiral symmetry breaking in field theory. In the QCD regime, its VEV is expected to be of order $\Lambda_{\rm QCD}^3$, where the QCD scale $\Lambda_{\rm QCD}$ also sets the scale of masses of mesons and glueballs in the theory.

At strong coupling, the Wilson line cannot be neglected, since the $4+1$ dimensional gauge theory degrees of freedom are strongly interacting at the scale $L$. We will compute the expectation value of $OW_i^j$ (1.1) below and find that it is exponentially large. For example, in the original model of [7] (in which $L = \pi R$), it scales like $\exp(\lambda_5/18\pi R)$. We interpret this exponentially large value as associated with the Wilson line contribution to (1.1), rather than with the fermions, since such exponentially large values do not appear in the effective action of the Nambu-Goldstone bosons (the "pions") and of the other mesons. Moreover, we will see that the expectation value depends strongly on the choice of contour $\tilde{C}$ connecting the two intersections.

Another interesting question in the Sakai-Sugimoto model is how to give a mass to the quarks.[1] As explained above, local operators which couple the left- and right-handed fermions are not gauge invariant in this model. The best we can do is to add to the Lagrangian the nonlocal operator (1.1). This breaks the chiral symmetry explicitly, and in the region in parameter space in which the model reduces to QCD, becomes equivalent to the quark mass deformation.

On the other hand, at strong coupling where we can use supergravity, this deformation is highly nonlocal and irrelevant (i.e., it grows in the UV). At low energies it leads to a change in the masses of the mesons, and, in particular, to a nonzero mass for the Nambu-Goldstone bosons associated with the symmetry breaking. We will study this deformation to leading order in the deformation parameter, and comment briefly on higher order effects.

In addition to this main example, we present two other examples of long open string operators. One involves a system of $k$ $NS5$-branes, with $N_f$ $Dp$ and $\bar{D}p$-branes a distance $L$ apart ending on them. For a critical value of the distance, the branes and antibranes can connect, and form a single curved $D$-brane, the hairpin brane of [12–14]. In the process, the $U(N_f)_L \times U(N_f)_R$ symmetry acting on the $D$-branes breaks to the diagonal subgroup, as before. The long open string stretched between the branes and antibranes near the boundary can again be viewed as an order parameter for the breaking. It has a nonzero VEV that can be computed in the same way as for the generalized Sakai-Sugimoto model, and one can again study the deformation that breaks the symmetry explicitly.

The main advantage of this example compared to the previous one is its tractability. The near-horizon limit of the

[1]So far there are no top-down holographic examples of quark masses in theories with a non-Abelian chiral symmetry.

$NS5$-branes is described by a solvable world sheet theory [the linear dilaton conformal field theory (CFT)], and the hairpin boundary state gives rise to a solvable boundary CFT, due to the fact that it preserves $\mathcal{N} = 2$ superconformal symmetry on the world sheet. One can write down explicitly the open string vertex operator corresponding to the long string, and compare the results of our semiclassical analysis to those obtained from the effective action of the stretched open strings, and to the exact solution of the world sheet CFT.

A second example which we present briefly is of a long open string operator in type IIB string theory on $\mathrm{AdS}_5 \times S^5$ with $D$-brane defects, which preserves half of the supersymmetry, and is analogous to the circular closed Wilson loop in the $d = 4$ $\mathcal{N} = 4$ SYM theory. It is easy to construct many other examples of supersymmetric open string operators, and it would be interesting to study them in more detail, generalizing the studies of supersymmetric closed string operators. It would also be interesting to understand if there is any relation (along the lines of [15–17]) between open Wilson lines of the type studied here and scattering amplitudes of quarks and gluons.

The organization of this paper is as follows. We begin in Sec. II with a general discussion of open Wilson line operators and their holographic description. In Sec. III we discuss some holographic computations of their correlation functions in the $D4 - D8 - \bar{D}8$ system. In Sec. IV we study the deformation of the Lagrangian of this system by the operator (1.1), which explicitly breaks the chiral symmetry, to leading order in the deformation. In Sec. V we describe the system of $D$-branes ending on $NS5$-branes. Finally, in Sec. VI we present a simple example of a supersymmetric long open string operator, and discuss cusplike divergences which occur in the computation of correlation functions of generic long open string operators (both at weak and at strong coupling).

## II. HOLOGRAPHIC OPEN WILSON LINES

As mentioned in the introduction, in this paper we will discuss certain nonlocal observables in the context of the AdS/CFT correspondence and its generalizations [1–3]. A class of such observables that has been widely studied is Wilson loops in large $N$ gauge theories with only adjoint fields. Locally supersymmetric Wilson loops in the fundamental representation dressed with scalar fields $\Phi_i$,

$$W[C] = \mathrm{tr}\left\{\mathcal{P}\exp\left[\oint_C ds(iA_\mu(x^\nu(s))\dot{x}^\mu(s) + n^i(s)\Phi_i(x^\nu(s))|\dot{x}|(s))\right]\right\}, \tag{2.1}$$

have been found [4–6] to correspond to strings ending on the closed boundary contour $C$ (parametrized by $x^\nu(s)$ in the noncompact space-time, and by the unit vector $\vec{n}(s)$ in the compact space). Thus, an insertion of the operator $W[C]$ on the boundary corresponds in the bulk path integral to summing over configurations which include a string world sheet[2] ending on the loop $C$ on the boundary. Wilson loops with generic (or no) couplings to scalar fields are more subtle; in particular, their correlation functions have perimeter divergences [unlike (2.1)] that need to be regularized. Nevertheless, the operators (2.1) already give a large amount of information about the theory. For instance, they can be used as a diagnostic for confinement.

When the boundary theory is a large $N$ gauge theory with a finite number of fields in the fundamental representation, the corresponding bulk description involves adding $D$-branes to the gravity background created by the adjoint fields.[3] The gauge symmetry on the $D$-branes corresponds to a global flavor symmetry in the dual field theory (which may or may not be a symmetry of the vacuum). From the (bosonic or fermionic) fundamental and antifundamental fields $\psi_i(x)$, $\bar{\psi}^j(x)$, one can form local gauge-invariant operators such as $\bar{\psi}^j(x)\psi_i(x)$. Such operators typically map under holography to local fields in the bulk, arising from short open strings stretching from the $i$th to the $j$th $D$-brane [22].

The situation is different when the $D$-branes are localized in some of the dimensions in which the gauge theory lives, and thus give rise to defects. Examples include the $D4 - D8 - \bar{D}8$ (generalized Sakai-Sugimoto) model [7–9] mentioned in the previous section, the closely related intersecting brane systems described in [23,24], and the $D3 - D5$ system that corresponds to adding $2 + 1$-dimensional hypermultiplets to $\mathcal{N} = 4$ SYM [25,26].

In these cases there are no local gauge-invariant operators that involve fundamental fields from different brane intersections (separated in space-time). The best one can do is to consider generalizations of (1.1),

$$OW_i^j[\tilde{C}] = \bar{\psi}^j(x_j)\mathcal{P}\exp\left[\int_{\tilde{C}} ds(iA_\mu(x^\nu(s))\dot{x}^\mu(s) + n^k(s)\Phi_k(x^\nu(s))|\dot{x}|(s))\right]\psi_i(x_i), \tag{2.2}$$

where $\tilde{C}$ is a contour between the point $x_j$ in the intersection at which the field $\bar{\psi}^j$ lives, and the point $x_i$ in the intersection at which the field $\psi_i$ lives. This contour is topologically a line segment; thus, we will refer to operators of the form (2.2) as open Wilson Lines, or OWLs.

Since locally along the contour $\tilde{C}$ the operator (2.2) looks just like (2.1), when $\vec{n}$ is a unit vector this operator is locally supersymmetric and its correlation functions do not exhibit divergences proportional to the length of the

[2]For Wilson loops in higher dimensional representations of the gauge group, the dominant configurations do not look like strings but rather like other branes carrying the same charges [18–20].

[3]This follows from 't Hooft's [21] mapping of Feynman diagrams to string world sheets, in which loops of fields in the fundamental representation correspond to holes in the world sheet.

path $\tilde{C}$. The holographic dual of (2.2) must involve a string world sheet ending on the open contour $\tilde{C}$ on the boundary. Thus, we propose that an insertion of the operator (2.2) into the path integral of the boundary gauge theory corresponds in the bulk to summing over configurations which include an open string world sheet which approaches the contour $\tilde{C}$ at the boundary of the bulk space-time, and near the boundary looks like a strip whose ends lie on the $i$th and $j$th $D$-branes.

As we will see, in some cases the computation of correlation functions of these operators is dominated by a saddle point corresponding to a semiclassical string world sheet, just like for many holographic closed Wilson loop computations. In particular, the one-point function $\langle OW_i^j\rangle$ is given to leading order in the semiclassical expansion by $\exp(-A/2\pi\alpha')$, where $A$ is the minimal area of the world sheet of such a string. If a finite area string world sheet does not exist, the one-point function of the OWL vanishes.

A few comments about the preceding discussion are in order:

(1) Just like for other holographic operators, in order to obtain finite correlation functions one needs to introduce a UV cutoff, and renormalize the OWL operators described above. In particular, the string world sheet that enters the calculation of the one-point function must only have finite area for finite UV cutoff.

(2) When performing the bulk path integral in the presence of the open string world sheet, one has to include all the couplings of the string to the background fields, such as the Neveu-Schwarz (NS)-NS $B_{\mu\nu}$ field, and the gauge fields that live on the $D$-branes.

(3) When the $i$th and/or $j$th $D$-branes give rise to more than one fundamental field in the gauge theory, the distinction between the corresponding bulk operators in the semiclassical calculation described above arises from quantization of zero modes on the world sheet of the string.

The example that motivated this investigation is the Sakai-Sugimoto model of holographic QCD. In this model, the large $N$ gauge theory lives on $D4$-branes in type IIA string theory, and the fundamental fields are left- and right-handed fermions, $\psi_L$ and $\psi_R$, which are localized at $3+1$ dimensional defects—the intersections of the $D4$-branes with $N_f$ $D8$ and $\bar{D}8$-branes, respectively. The $D8$ and $\bar{D}8$-branes are separated by a distance $L$ in the direction $x^4$ along the $D4$-branes. The model has a $U(N_f)_L \times U(N_f)_R$ global symmetry corresponding to the gauge symmetry on the $D8$ and $\bar{D}8$-branes.

In the strongly coupled regime $\lambda_5 \gg L$, $R$, the vacuum of this model corresponds to a brane configuration in which the $D8$ and $\bar{D}8$-branes are connected, and the chiral $U(N_f)_L \times U(N_f)_R$ symmetry is dynamically broken to the diagonal $U(N_f)$. Most of the work on the model involved light open 8–8 strings, such as the translational modes of the eight-branes and their world volume gauge fields. The latter correspond in the boundary theory to the $U(N_f)_L \times U(N_f)_R$ chiral symmetry currents $\psi_L^\dagger(x)\sigma^\mu\psi_L(x)$, $\psi_R^\dagger(x)\bar{\sigma}^\mu\psi_R(x)$.

These "short string" operators are useful for analyzing the low-lying spectrum of mesons, but in order to study chiral symmetry breaking it is better to consider operators such as (1.1), which transform as $(\mathbf{N_f}, \bar{\mathbf{N}}_\mathbf{f})$ under the chiral symmetry. In the next section we will use holography to show that the expectation value of these operators is nonzero at strong coupling; thus, they are natural order parameters for chiral symmetry breaking.

In QCD one can break the chiral symmetry explicitly by adding a mass term for the quarks. The closest analog of this at strong coupling is to add (1.1) to the Lagrangian. We will describe some results about this deformation in Sec. IV below.

OWL operators of the form (2.2) can in principle be also defined for theories in which the fundamental fields are not localized at defects, but they seem to be less useful in such cases. Consider, for example, the $D3-D7$ system, which corresponds to adding to $\mathcal{N}=4$ SYM a massless hypermultiplet in the fundamental representation of the gauge group. In the dual description this corresponds [22,27] to adding a $D7$-brane wrapping $\mathrm{AdS}_5 \times S^3$ to type IIB string theory on $\mathrm{AdS}_5 \times S^5$, where the $S^3$ is a maximal three-sphere inside $S^5$.

The one-point function of an open Wilson line operator (2.2) involves in this case a configuration with a string ending on the contour $\tilde{C}$ connecting the points $x_i$ and $x_j$ in $\mathbb{R}^{3,1}$. The world sheet of such a string can always reduce its area by contracting towards the boundary. Therefore, the corresponding one-point function depends on the UV regulator, and does not appear to be well-behaved.[4] This is reasonable from the general perspective described in the introduction. The open string in question is really a short $D7-D7$ string that does not shrink only because its two ends are held fixed at two different points in $\mathbb{R}^{3,1}$. A more natural basis for describing such strings is in terms of excited perturbative $D7-D7$ strings, rather than the OWL basis (2.2).

The last remark is also applicable to long open strings connecting widely separated $D$-branes. For a given pair of branes there is a preferred contour $\tilde{C}$ that has minimal length, and it is natural to study the OWL operator associated with it. For the $D4-D8-\bar{D}8$ system this is the operator (1.1). One can consider other contours that connect different points in $\mathbb{R}^{3,1}$ and/or vary nontrivially in the interior, as in (2.2), but these are less natural. They can be alternatively described by adding string oscillators to the operator corresponding to the minimal contour (1.1).

[4]This is also true at weak coupling, due to divergences associated with the screening by the fundamental representation fields.

## III. OPEN WILSON LINES IN THE $D4-D8-\bar{D}8$ SYSTEM

To demonstrate the general discussion of the previous section, we will consider here the following intersecting brane system in type IIA string theory. We start with $N_c$ $D4$-branes stretched in the $\mathbb{R}^{4,1}$ labeled by $(x^0, x^1, x^2, x^3, x^4)$, and add to them $N_f$ $D8$-branes localized at $x^4 = -L/2$, as well as $N_f$ $\bar{D}8$-branes localized at $x^4 = +L/2$.

This leads [8] to a nonconfining theory of massless left- and right-handed fermions, $\psi_L$, $\psi_R$, which are localized at the 4–8 and 4–$\bar{8}$ intersections, respectively, and interact via exchange of modes living on the $D4$-branes.[5] The strength of the interaction is determined by the 't Hooft coupling $\lambda_5 = (2\pi)^2 g_s N_c l_s$. When the interaction at the scale $L$ is strong ($\lambda_5 \gg L$), one can replace the $D4$-branes by their near-horizon geometry [28,29]. The metric is given by

$$ds^2 = \left(\frac{u}{R_{D4}}\right)^{3/2}\left[-(dx^0)^2 + \sum_{i=1}^{4}(dx^i)^2\right] + \left(\frac{R_{D4}}{u}\right)^{3/2}[du^2 + u^2 d\Omega_4^2], \tag{3.1}$$

where $R_{D4}^3 \equiv \pi g_s N_c l_s^3$. The Ramond-Ramond four-form and dilaton are

$$F_{(4)} = \frac{2\pi N_c}{\mathrm{Vol}(S^4)}\epsilon_4, \qquad e^{\Phi} = g_s\left(\frac{u}{R_{D4}}\right)^{3/4}. \tag{3.2}$$

The dynamics of the fermions are determined by the shape of the eight-branes in this background. It was found in [8] that in the lowest energy configuration the $D8$ and $\bar{D}8$-branes are connected by a tube and form a single stack of $N_f$ connected eight-branes. They are extended in the $\mathbb{R}^{3,1}$ labeled by $(x^0, x^1, x^2, x^3)$, wrap the four-sphere labeled by $\Omega_4$, and form a curve $u(x^4)$ in the $(u, x^4)$ plane, which is a solution of the first order differential equation

$$\frac{u^4}{\sqrt{1 + (\frac{R_{D4}}{u})^3 u'^2}} = u_0^4. \tag{3.3}$$

The solution of (3.3) is a $U$-shaped brane, with the distance between the two arms approaching $L$ at large $u$. The minimal value of $u$ to which the $D8$-branes extend, $u_0$, is determined by $L$,

$$L = \tfrac{1}{4}R_{D4}^{3/2}u_0^{-1/2}B\left(\tfrac{9}{16}, \tfrac{1}{2}\right). \tag{3.4}$$

For strong coupling, the curvature of the metric (3.1) near the $D8$-branes is small. The string coupling (3.2) diverges as $u \to \infty$, but in the 't Hooft large $N_c$ limit there is a parametrically wide region in $u$ in which it is small, and we can restrict attention to that region by placing a UV cutoff on $u$, $u \le u_{\max}$.

Since the $D8$ and $\bar{D}8$-branes are connected in the vacuum, the chiral $U(N_f)_L \times U(N_f)_R$ symmetry acting on them is spontaneously broken to its diagonal subgroup. The fermions $\psi_L$, $\psi_R$, which correspond in the brane picture to strings stretching from the bottom of the curved $D8$-branes towards $u = 0$, obtain a dynamically generated "constituent mass" $m = u_0/2\pi\alpha' \sim \lambda_5/L^2$.

One can use the effective action for the $D8$-branes (which includes the Dirac-Born-Infeld (DBI), Wess-Zumino, and fermionic terms) to study the low-lying excitations of the model, and, in particular, to verify the existence of $N_f^2$ massless Nambu-Goldstone mesons corresponding to the breaking of the chiral symmetry. The rest of the spectrum is massive; the masses of the lowest lying mesons are of order $1/L$. They are much lighter than the fermions, and can be thought of as tightly bound states of two fermions.

As explained above, the simplest operator which can serve as an order parameter for chiral symmetry breaking in this theory is the OWL operator (1.1). We next turn to a calculation of its one-point function at strong coupling, the chiral condensate, and comment on more general operators of the form (2.2).

### A. One-point functions of open Wilson lines

In order to compute the expectation value of the OWL operator (1.1) at strong coupling, we need to perform the gravitational path integral in the closed string background (3.1) and (3.2), in the presence of the curved $D8$-branes described around (3.3), and of a Euclidean fundamental string world sheet which near the (regularized) boundary at $u = u_{\max}$ stretches along a straight line in the $x^4$ direction between the two arms of the curved $D8$-branes.

This path integral is dominated by a semiclassical contribution of a Euclidean world sheet, which is localized at a point in $\mathbb{R}^{3,1} \times S^4$, and fills the region in the $(u, x^4)$ plane between the boundary at $u = u_{\max}$ and the $D8$-branes (see Fig. 1).

To leading order in $\alpha'$ the action of such a string is proportional to its area,

$$S_{\rm str} = \frac{1}{2\pi\alpha'}\int dx^4 \int_{u(x^4)}^{u_{\max}} du\sqrt{g_{uu}g_{44}} = \frac{1}{2\pi\alpha'}\int dx^4[u_{\max} - u(x^4)]. \tag{3.5}$$

Performing the integral one finds

$$S_{\rm str} = \frac{u_{\max}L}{2\pi\alpha'} - \frac{R_{D4}^{3/2}u_0^{1/2}}{8\pi\alpha'}B\left(\frac{7}{16}, \frac{1}{2}\right) = \frac{u_{\max}L}{2\pi\alpha'} - C_1\frac{\lambda_5}{L}, \tag{3.6}$$

where the constant $C_1$ is given by $C_1 = B(\frac{7}{16}, \frac{1}{2})B(\frac{9}{16}, \frac{1}{2})/128\pi^2 \simeq 0.0079$, and we neglected correc-

[5] The Sakai-Sugimoto model [7] is obtained by compactifying $x^4$ on a circle, with twisted boundary conditions for the fermions on the $D4$-branes [28]. We will comment on the generalization of our considerations to this case below.

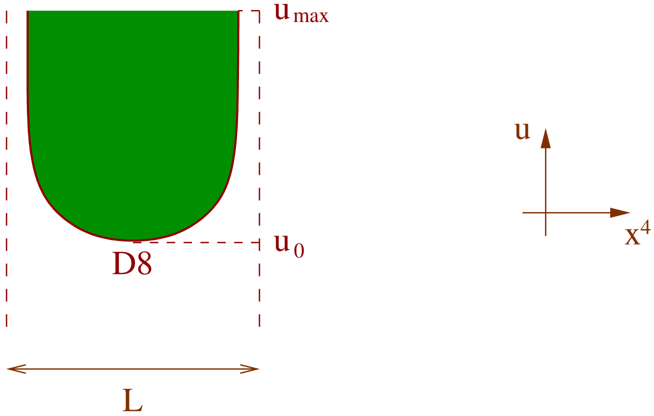


FIG. 1 (color online). The semiclassical world sheet which gives the chiral condensate in the $D4 - D8 - \bar{D}8$ model is drawn in green.

tions that go to zero in the limit $u_{\max} \to \infty$. The term proportional to the UV cutoff $u_{\max}$ on the right-hand side is independent of the coupling $\lambda_5$, and can be absorbed in the definition of the operator (1.1).[6]

Thus, we conclude that at strong coupling the expectation value of the operator (1.1) is given by

$$\langle OW_i^j \rangle \simeq \delta_{ij} \exp(-S_{\rm str}) \simeq \delta_{ij} \exp(C_1 \lambda_5 / L). \tag{3.7}$$

The calculation above captures the leading behavior of this one-point function at strong coupling. The first subleading corrections come from quadratic fluctuations around the Euclidean world sheet of Fig. 1, and from the coupling of the string to the varying dilaton. They are expected to give a polynomial prefactor in front of the exponential (3.7).

The nonvanishing expectation value (3.7) exhibits the expected pattern of chiral symmetry breaking, $U(N_f)_L \times U(N_f)_R \to U(N_f)_{\rm diag}$, in agreement with our earlier discussion. It grows exponentially with the coupling $\lambda_5/L$ in the region $\lambda_5 \gg L$ in which our calculation is reliable. At first sight this might seem surprising, since in the weakly coupled field theory regime the chiral condensate is closely related to the dynamically generated fermion mass, whereas here this is not the case—the fermion mass scales like $\lambda_5/L^2$, those of the mesons scale like $1/L$, while the condensate (3.7) is exponentially large.[7] The difference between the two regimes is that for strong coupling most of the contribution to (3.7) appears to be due to the Wilson line in (1.1) rather than to the fermion bilinear part of the operator, while for weak coupling this Wilson line gives a negligible contribution.

[6]As explained in [6], this term is naturally canceled by a Legendre transform which is part of the definition of locally supersymmetric Wilson line operators.

[7]The chiral condensate we find is also widely separated from the pion decay constant $f_\pi$, which will be discussed in the next section.

The fact that the exponential behavior of (3.7) at strong coupling is due to the Wilson line rather than to the fermions can be seen more quantitatively by studying its dependence on the contour $\tilde{C}$. Consider, for instance, the one-point function of an open Wilson line (2.2) connecting two points in $\mathbb{R}^{3,1}$, $x_0^\mu$, and $x_1^\mu$, (spacelike) separated by a distance much larger than $L$. A class of contours connecting these points that is useful for our purposes involves moving first in $x^4$ (at a fixed value of $x^\mu$, $x^\mu = x_0^\mu$) from $-L/2$ to some $x_0^4$, then varying $x^\mu$ from $x_0^\mu$ to $x_1^\mu$ at fixed $x^4$, and finally moving again in $x^4$ to $L/2$. Such contours have cusps, but these can be smoothed out (and in any case the divergences they lead to are well understood and can be subtracted out).

Finding the precise shape of the string world sheet which minimizes the action with these boundary condition is rather complicated. However, when the two points $x_0$ and $x_1$ are widely separated, we expect the main contribution to this expectation value to come from the part of the world sheet at $x^4 = x_0^4$. In the special case $x_0^4 = 0$, this part of the world sheet is easy to analyze. Its contribution to the regularized action is given by

$$S_{\rm str} = -\frac{u_0|\vec{x}_0 - \vec{x}_1|}{2\pi\alpha'} \propto -\frac{\lambda_5}{L^2}|\vec{x}_0 - \vec{x}_1|. \tag{3.8}$$

The proportionality constant on the right-hand side can be read off from (3.4). On the other hand, when $x_0^4$ approaches (say) $L/2$, the regularized action turns out to be proportional to $-\lambda_5|\vec{x}_0 - \vec{x}_1|/(x_0^4 - L/2)^2$.

We see that the expectation values of these operators, proportional to $\exp(-S_{\rm str})$, grow exponentially with the distance between the endpoints of the contour $\tilde{C}$ in $\mathbb{R}^{3,1}$, and the coefficient of the distance in the exponent depends on the precise contour we choose. We conclude that this exponential growth is a property of the contour rather than of the fermion bilinear at its ends. This also explains why the expectation value under consideration does not decay exponentially with the distance in $\mathbb{R}^{3,1}$, $|\vec{x}_0 - \vec{x}_1|$, as one might have expected due to the fact that the fermions are massive.

So far we have discussed the computation of the chiral condensate for the extremal $D4$-brane background, but it is easy to generalize the discussion to the case where $x^4$ lives on a circle of radius $R$, with antiperiodic boundary conditions for the fermions. The near-horizon $D4$-brane geometry is in this case a Wick-rotated Euclidean black hole geometry, in which the radius of the $x^4$ circle varies between its asymptotic value $R$ at large $u$ and zero at $u = u_\Lambda = \lambda_5\alpha'/9\pi R^2$ [28].

One can again analyze the shape of the $D8$-branes as a function of $L$ and $R$ and calculate the expectation value of OWL operators such as (1.1), by evaluating the area of the corresponding Euclidean string world sheet. There are some small differences in the precise form of the solutions for the $D8$-branes and for the strings, but the qualitative properties are not modified. There are now two independent operators of the form (1.1), one with the Wilson line going in the positive $x^4$ direction and the other in the negative $x^4$ direction. For generic $L$, $R$, the world sheets that determine the expectation values of the two operators have different areas, so one of the operators has a larger VEV. In the special antipodal case $L = \pi R$ considered in [7], the one-point functions of both operators are given by

$$\langle OW_i^j \rangle \simeq \delta_{ij} \exp(\lambda_5/18\pi R). \tag{3.9}$$

For general $L \ll \lambda_5$ one finds a result that smoothly interpolates between (3.9) for $L = \pi R$ and (3.7) for $L \ll R$, where the modification of the background at the location of the $D8$-branes due to the compactness of $x^4$ is negligible.

The discussion above was restricted to the strong coupling regime $\lambda_5 \gg L$. In the opposite limit, $\lambda_5 \ll L \sim R$, at energies much smaller than $1/R$ one expects the model to reduce to QCD with massless quarks. In this limit the gauge field $A_4$ and the scalar fields are expected to decouple [28], so (1.1) should go over to the usual chiral condensate of QCD, which scales as $\Lambda_{\rm QCD}^3 \simeq \frac{1}{R^3} \times \exp(-16\pi^3 R/\lambda_5)$. If there is no phase transition as one varies $\lambda_5/R$, we expect a smooth interpolation between this result and (3.9).

For $L \ll R$, and, in particular, in the limit $R \to \infty$ with fixed $L$, the situation is not completely clear. At strong coupling ($\lambda_5 \gg L$), one finds in this limit a theory which breaks chiral symmetry but does not confine, which can be thought of as a particular UV completion of the Nambu-Jona-Lasinio model [8]. Field theoretic intuition suggests that at weak coupling ($\lambda_5 \ll L$) chiral symmetry is not broken, and thus the theory undergoes a phase transition at some critical value of the coupling $\lambda_5/L$, but this has not been conclusively established yet.

In other closely related brane systems, discussed in [23,24], which give rise to $1 + 1$ dimensional intersections, such as the $D4 - D6 - \bar{D}6$ system, one can analyze the dynamics for both weak and strong coupling, and, in particular, calculate the expectation value of (1.1) in both limits (for any value of $R$). The strong coupling computation is very similar to that described above, and gives $\langle OW \rangle \sim \exp(\tilde{C}_1 \lambda_5/L)$ with some calculable constant $\tilde{C}_1$ that depends on $L/R$ and approaches a finite value as $L/R \to 0$.

For weak coupling and infinite $R$, one obtains in this case a generalized Gross-Neveu model which can be analyzed using field theoretic methods and gives $\langle OW \rangle \sim \exp(-L/\lambda_5)$. For finite $R$ one gets a generalization of the 't Hooft model of two dimensional QCD that includes four-Fermi interactions, and is solvable at large $N_c$, like its two extreme limits—the 't Hooft and Gross-Neveu models. It would be interesting to compute the chiral condensate in this model as a function of $L/R$, and compare it to the strong coupling calculation described above. We expect a smooth interpolation between the strong and weak coupling limits as one varies the parameters $\lambda_5/L$, $\lambda_5/R$ that govern chiral symmetry breaking and confinement, respectively.

### B. Correlation functions of open Wilson lines

The computation of the expectation value of a product of several OWLs (2.2) is also straightforward in principle, but in practice it is more difficult to find the appropriate semiclassical string world sheets (if they exist). As in correlation functions of closed Wilson loops, in some cases a correlation function of a product of OWLs is dominated by a single semiclassical world sheet; in other cases it is dominated by several semiclassical world sheets connected by propagators in the bulk (at leading order in $1/N_c$ they must be connected by propagators of open string fields); in yet other cases there may be no semiclassical contribution at all. In the supergravity limit, there can be sharp phase transitions between the first two possibilities, as in closed Wilson loop correlators [30].

A case where the dominant world sheets are easy to describe is the correlation function $F_2 \equiv \langle OW_i^i(x_0^\mu) \times (OW_i^i)^\dagger(x_1^\mu)\rangle$ (no sum over $i$ implied) in the $D4 - D8 - \bar{D}8$ system. There are two distinct semiclassical contributions to this correlation function. One involves the world sheets that appear in the computation of the one-point functions of $OW_i^i$ and $(OW_i^i)^\dagger$ (the world sheet corresponding to $OW^\dagger$ is the same as the one for $OW$, but with an opposite orientation), connected by a propagator of an open string field on the $D8$-brane. It is depicted in the left part of Fig. 2. The leading order contribution at large distances is due to massless pion exchange, and should be proportional to

$$F_2 \simeq \frac{\langle OW_{ii}\rangle \langle OW_{ii}\rangle^\dagger}{|\vec{x}_0 - \vec{x}_1|^2} \simeq \frac{\exp(2C_1\lambda_5/L)}{|\vec{x}_0 - \vec{x}_1|^2}. \tag{3.10}$$

The second semiclassical world sheet smoothly connects the two OWL's at $x_0^\mu$ and $x_1^\mu$, by extending into the bulk, as in the right part of Fig. 2.

If the $D8$ and $\bar{D}8$-branes were localized at fixed values of $x^4$, this configuration would be precisely the one that appears in the computation of the energy of a quark and an antiquark separated by a distance $|\vec{x}_0 - \vec{x}_1|$, with $x^4$ playing the role of time (the world sheet would simply stretch in this direction and end on the $D8$-branes at $x^4 = \pm L/2$). In the $D4$-brane background (3.1) and (3.2), this energy is given by [31] $(-\lambda_5/|\vec{x}_0 - \vec{x}_1|^2)$, so in this case we would obtain

$$F_2 \simeq \exp(\lambda_5 L/|\vec{x}_0 - \vec{x}_1|^2). \tag{3.11}$$

In the actual configuration we are interested in, the $D8$-branes bend in $x_4$, and (3.11) should be modified by taking their shape into account. When the extent of the string in the radial direction becomes comparable to $u_0$,

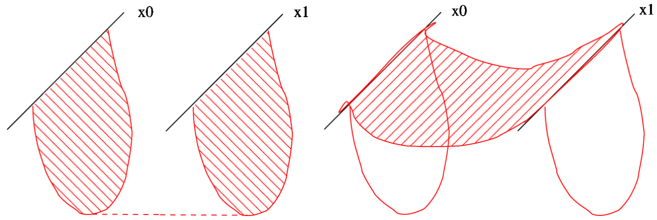


FIG. 2 (color online). The two semiclassical configurations that dominate the computation of $F_2$. On the left we have the two-string configuration, with one string (as in Fig. 1) ending on a $D8$-brane at $x_0$ and the other at $x_1$, connected by an open string propagator inside the $D8$-brane. On the right we have the single string configuration. The string world sheets are filled with diagonal lines, and the $D8$-brane lives everywhere but was only drawn at $x_0$ and $x_1$.

this modification is significant. However, for short distances, (3.11) is still reliable.[8]

As the distance increases, the area of the world sheet in the right part of Fig. 2 increases, and at some point it becomes larger than that in the left part. At that point, the correlation function in question makes a phase transition from (3.11) to (3.10). This transition is expected to occur at $|\vec{x}_0 - \vec{x}_1| \simeq L$.

An example of a correlator for which there is no obvious smooth world sheet configuration at short distances is $\langle OW_i^i(x_0^\mu) OW_i^i(x_1^\mu)\rangle$. The string ending at $x_0^\mu$ would have to change its orientation in the bulk before coming back to end at $x_1^\mu$. Thus, in this case it seems likely that the two-string configuration on the left of Fig. 2 always dominates and gives the behavior (3.10).

Another interesting correlator is $\langle \det(OW_i^j(x^\mu))\rangle$, which is a singlet of the non-Abelian $SU(N_f)_L \times SU(N_f)_R$ but carries axial $U(1)$ charge. In the case of finite $R$, due to the axial anomaly, this should be nonzero even in phases where the chiral symmetry is not spontaneously broken and the $D8$-branes and $\bar{D}8$-branes do not connect. The computation of $\langle \det(OW_i^j)\rangle$ involves in this case $N_f$ strings ending on the boundary, on the $D8$-branes and on the $\bar{D}8$-branes. Naively it vanishes when the $D8$-branes and $\bar{D}8$-branes do not connect, since the strings have nowhere to end in the IR. However, there are contributions from Euclidean $D0$-branes wrapped around the $x^4$ circle (which are instantons from the point of view of the $4+1$ dimensional gauge theory). Such Euclidean $D0$-branes should have $N_f$ fundamental strings ending on them between the $D8$-branes and the $\bar{D}8$-branes [32].[9] These strings can extend to the boundary and thus contribute to $\langle \det(OW_i^j)\rangle$. Being $D$-instanton effects, such contributions are exponentially suppressed in the 't Hooft large $N_c$ limit, but they are the leading contribution to $\langle \det(OW_i^j)\rangle$ in phases where the non-Abelian chiral symmetry is unbroken.

## IV. DEFORMING BY OPEN WILSON LINES

In the previous section we computed the expectation value of the OWL operator (1.1) in the generalized Sakai-Sugimoto model. In this section we will study a deformation of the model that corresponds to adding this operator to the Lagrangian,

$$\delta S = \kappa \int d^4x \sum_{j=1}^{N_f} OW_j^j(x) + \text{c.c.}. \qquad (4.1)$$

This leads to explicit breaking of the $U(N_f)_L \times U(N_f)_R$ chiral symmetry to the diagonal $U(N_f)$, in addition to the spontaneous breaking present at $\kappa = 0$. The deformation (4.1) can be thought of as a generalization to strong coupling of a "current mass" for the fermions, $\delta S = \kappa \int d^4x \sum_j \psi_L^{\dagger j}\psi_{Rj} + \text{c.c.}$, that plays a role in QCD. The generalization to nonequal "masses" $\kappa_j$ for different quark flavors is straightforward.

We will study the deformed theory semiclassically at strong coupling, in the hope that the strong coupling results are smoothly related to large $N_c$ QCD with massive quarks. We will work to first order in the mass parameter $\kappa$; this involves a single insertion of the perturbation, for which we can use our results from the previous section. In QCD this is a good approximation for the $u$ and $d$ quarks, whose current mass is much smaller than the QCD scale. It would be interesting to go beyond first order in $\kappa$. For this, one needs to evaluate $n > 1$ point functions of the operators $OW_i^j(x)$, which are complicated, as discussed in the previous section.

To first order in $\kappa$, the deformation (4.1) can be described by adding to the space-time action the term

$$\delta S = \frac{\kappa}{\mathrm{Vol}(S^4)} \int d^4x \int d^4\Omega \sum_i e^{-S_{\rm str}^{(i)}} + \text{c.c.}, \qquad (4.2)$$

where $S_{\rm str}^{(i)}$ is the action of the string ending on the $i$th $D8$-brane discussed in the previous section,[10] and the integral over the four-sphere implements an average over the scalar field that enters the definition of the operator (1.1), restoring the $SO(5)$ symmetry of the model.

The deformation (4.2) is nonlocal,[11] since the action $S_{\rm str}^{(i)}$ depends on the position of the $D8$-branes everywhere in the

[8]Note that $F_2$ diverges as $\vec{x}_0 \to \vec{x}_1$ (when the cutoff is sent to infinity).

[9]Recall that the $D8$-branes generate a tenform flux, which couples to the gauge field on the $D0$-branes.

[10]Of course, this only makes sense in the phase in which the $D8$ and $\bar{D}8$-branes are connected. In phases like the high-temperature phase of the Sakai-Sugimoto model in which the branes are not connected, $\langle OW\rangle$ vanishes, and we do not have a semiclassical description of the deformation.

[11]Similar nonlocal mass terms were also recently considered in [33].

radial coordinate (it also includes a coupling to the gauge field on the $D8$-branes, and to closed string fields). This is not surprising, since the field theory deformation (4.1) is nonlocal. In the dual string description, in addition to the explicit nonlocality in the direction of the Wilson line, we also have nonlocality in the radial direction.[12]

The deformation (4.1) and (4.2), is of order $N_c$ (or $1/g_s$), like any other open string deformation, so it is expected to influence open string fields (like the position of the $D8$-branes) at leading order, while the corrections to closed string fields (like the metric) are suppressed by a power of $g_s$. One thing that is relatively easy to compute is the mass of the Nambu-Goldstone bosons (the pions) due to the deformation (4.1) at leading order in $\kappa$.

For $\kappa = 0$ we have a $U(N_f)_L \times U(N_f)_R$ global symmetry spontaneously broken to $U(N_f)$. In the effective field theory on the $D8$-branes the order parameter for this breaking can be taken to be the holonomy matrix

$$U \equiv \mathcal{P}\exp\left(i\int_{-L/2}^{L/2} dx^4 \tilde{A}_{x^4}\right), \tag{4.3}$$

where $\tilde{A}$ is the gauge field on the $D8$-branes. The matrix $U$ transforms as a bifundamental of $U(N_f)_L \times U(N_f)_R$.[13] It is precisely the matrix appearing in the low-energy chiral Lagrangian,[14] which is usually written in terms of pion fields as $U(x) = \exp(i\pi(x)/f_\pi)$. Its low-energy effective Lagrangian is given by

$$L_{\mathrm{eff}} = (f_\pi^2/4)\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger), \tag{4.4}$$

with [7,35]

$$f_\pi^2 \simeq \lambda_5 N_c/L^3. \tag{4.5}$$

The $N_f^2$ pion fields in $U$ are massless Nambu-Goldstone bosons.[15]

The deformation (4.1) explicitly breaks the chiral symmetry to the diagonal subgroup, and is expected to give a mass to all the Nambu-Goldstone bosons. Indeed, the perturbation $\exp(-S_{\mathrm{str}}^{(i)})$ in (4.2) includes a coupling to the gauge field on the $D8$-branes, of the form $(\mathcal{P}\exp[-i\int_{-L/2}^{L/2} dx^4 \tilde{A}_{x^4}])_{ii}$. This coupling did not play a role in our evaluation of $\langle OW\rangle$, since we assumed that we were expanding around a configuration in which the gauge field on the $D8$-branes vanishes, but it is important in analyzing the perturbed theory (4.2). The effective Lagrangian for $U$, (4.4), is deformed at first order in $\kappa$ by

$$\delta L_{\mathrm{eff}} = |\langle OW\rangle|\kappa\mathrm{tr}(U) + \mathrm{c.c.}. \tag{4.6}$$

When $\kappa$ is real and positive, this is precisely the same as the change in the low-energy effective action of QCD when we add to the theory a quark mass proportional to $\kappa$ (the proportionality constant depends on the chiral condensate). It leads to a pion mass equal to (this is sometimes called the Gell-Mann-Oakes-Renner relation [38])

$$m_\pi^2 = \frac{4\kappa|\langle OW\rangle|}{f_\pi^2}. \tag{4.7}$$

Note that $\kappa$ and $f_\pi$ have dimensions of mass, while $OW$ has the dimension of a mass cubed.

When $\kappa$ has an imaginary part (or is negative), the minimum of the pion potential is no longer at $\pi(x) = 0$ and the deformation (4.1) leads to a change in the phase of the chiral condensate. We will assume that $\kappa$ is positive from here on (the other cases are classically equivalent to this, since they are related by the axial $U(1)$ symmetry).

In addition to giving a mass to the pions, the perturbation (4.2) changes the masses of the massive mesons as well. To calculate their mass shifts, one needs to determine the shape of the $D8$-branes in the presence of the perturbation. This shape is obtained by minimizing the deformed Lagrangian for the eight-branes,

$$L = T_{D8}\int dx^4 u^4\sqrt{1 + \left(\frac{R_{D4}}{u}\right)^3 u'^2} + 2\kappa B\exp\left(\frac{1}{2\pi\alpha'}\int dx^4 u\right), \tag{4.8}$$

where $B$ is the coefficient of the exponent in the computation of $\langle OW\rangle$, which is necessary to give $L$ the appropriate

[12]This nonlocality could be avoided if instead of deforming by ($OW$ + c.c.) we would deform by ($\ln(OW)$ + c.c.), since this would just shift the action by a multiple of the action $S_{\mathrm{str}}$ of the stretched string, which is an integral of a local function of the $D8$-brane fields (a similar perturbation for closed Wilson loops was recently considered in [34]). However, such a deformation does not have the same symmetry properties as (4.1) (in particular it does not break the axial $U(1)$ symmetry), and it is not obvious that $\langle\ln(OW)\rangle = \ln(\langle OW\rangle)$ semiclassically, so we will not consider it further here.

[13]Parametrizing the position of the $D8$-branes in the $(u, x^4)$ plane by a variable $z$ which goes from minus infinity at one boundary of the branes to plus infinity at the other boundary, we can write $U = \mathcal{P}\exp(i\int_{-\infty}^{\infty} dz\tilde{A}_z)$.

[14]Naively one might think that the holonomy matrix $U$ could serve as an order parameter for the chiral symmetry breaking in the full string theory as well. However, while the holonomy is gauge-invariant in the $D8$-brane gauge theory, it is not gauge invariant in the full string theory under gauge transformations of the NS-NS $B$ field. In order to obtain a gauge-invariant object we must multiply $U$ by $\exp(i\int B)$ where the integral is over a surface bounded by the $D8$-branes. The only way to construct an operator containing this phase in string theory is to put in a fundamental string (or another object with the same charges) ending on the $D8$-branes, giving precisely the OWL operators discussed above. Thus, one can think of $OW$ as a completion of $U$ to the full string theory; in the $N_f = 1$ case $U$ is the phase of $OW$.

[15]The axial $U(1)$ symmetry is anomalous, and the corresponding pion obtains a mass at order $1/N_c$ [7,32,36,37].

dimension and to make the second term have the same scaling $\mathcal{O}(N_c)$ as the first term.[16] In the second term in (4.8) we used (3.5).

The equation of motion corresponding to (4.8) is

$$\frac{T_{D8}R_{D4}^3}{[1+(\frac{R_{D4}}{u})^3u'^2]^{3/2}}\left[uu''-\frac{11}{2}u'^2-4\left(\frac{u}{R_{D4}}\right)^3\right] = \frac{\kappa B}{\pi\alpha'}\exp\left(\frac{1}{2\pi\alpha'}\int dx^4 u\right). \tag{4.9}$$

The right-hand side is independent of $x^4$, so the left-hand side is a constant. Denoting this constant by $A \equiv \kappa|\langle OW\rangle|/\pi\alpha'$, (4.9) is equivalent to the first order differential equation

$$H = \frac{T_{D8}u^4}{\sqrt{1+(\frac{R_{D4}}{u})^3u'^2}} - Au = \text{constant}, \tag{4.10}$$

associated with the symmetry $\{x^4 \to x^4 + \text{constant}\}$ of (4.8). The constant value of $H$ may be determined by requiring that $u$ goes to the UV cutoff $u = u_{\max}$ at $x^4 = \pm L/2$. It is related to the minimal position $u_0$ of the $D8$-branes in the $u$ direction by $H = T_{D8}u_0^4 - Au_0$.

Equation (4.10) enables us to compute the deformation in the position of the $D8$-branes at leading order in $\kappa|\langle OW\rangle|$. This may then be used to determine the shift of the meson masses, by analyzing the quadratic fluctuations of the deformed action around this new solution. It would be interesting to understand how to go to higher orders in $\kappa$.

Note that, unlike the QCD mass deformation, the deformation (4.2) in the strongly coupled $D4 - D8 - \bar{D}8$ theory is irrelevant, and its effect grows in the UV region; this is clear from (4.10). Thus, as for other irrelevant deformations, the perturbation expansion in the deformation is only meaningful if we put in a finite UV cutoff $u = u_{\max}$, and demand that the deformation is small at the cutoff scale.

It is easy to generalize the computations above to the Sakai-Sugimoto model in which the $x^4$ direction is compactified. One interesting difference is that, in the special case of $L = \pi R$, it seems natural to deform by the sum of the OWL operator (1.1) corresponding to the contour connecting the $D8$ and $\bar{D}8$-branes in the positive $x^4$ direction, and the one connecting them in the negative $x^4$ direction. In this case the shape of the $D8$-branes is not modified by the deformation, since the two semiclassical strings pull the $D8$-branes in opposite directions. Thus, in this special case, adding the "quark mass deformation" does not change the shape of the $D8$-branes, but it does give a mass to the pions as discussed above. In all other cases, the shape of the $D8$-branes is also modified; they are pulled to larger values of $u$ by the string. In this model the distance between the minimal position of the $D8$-branes, $u_0$, and the minimal value of the $u$ coordinate, $u_\Lambda$, may be interpreted as a constituent quark mass (at least in the context of high-spin mesons [39–41]). We find generically (except for the special case discussed above) that increasing the bare quark mass increases also the constituent quark mass, as expected.

[16]This coefficient also depends on $u(x^4)$ through the coupling of the world sheet to the varying dilaton in our background. However, this dependence is suppressed by a power of $\alpha'$ in the small curvature limit we are working in.

## V. $D$-BRANES IN THE BACKGROUND OF $NS5$-BRANES

In this section we study another example of a holographic description of operators corresponding to long strings stretched between two $D$-branes. This example is of interest for the study of $D$-brane dynamics near singularities of the bulk geometry. It also has the advantage that the relevant classical string background is under control, and can be analyzed exactly in $\alpha'$.

Consider the following brane configuration in type II string theory. We start with $k$ $NS5$-branes stretched in $\mathbb{R}^{5,1}$ labeled by $(x^0, x^1, x^2, x^3, x^4, x^5)$, and located at the origin in the transverse $\mathbb{R}^4$. As is well known from the brane construction of gauge theories (see [42] for a review), $Dp$-branes which have one direction transverse to the five-branes can end on them. Thus, we add a $Dp$-brane stretched in the directions $(x^0, x^1, x^2, \cdots, x^{p-1})$, and semi-infinite in the $x^6$ direction (i.e., it has $x^6 \geq 0$ and ends on the five-branes at $x^6 = 0$).

The above $D$-brane is localized in the $\mathbb{R}^{6-p}$ labeled by $(x^p, x^{p+1}, \cdots, x^5)$. We can add a second $D$-brane, which is parallel to the first one, but is displaced from it by a distance $L$ in $\mathbb{R}^{6-p}$, and has the opposite orientation, i.e., it is a $\bar{D}p$-brane. We will label the direction along which the $D$ and $\bar{D}$-brane are separated by $x$, with $x(D) = -\frac{L}{2}$ and $x(\bar{D}) = +\frac{L}{2}$. The brane configuration is depicted in Fig. 3.

We will be primarily interested in the physics associated with the two brane intersections in Fig. 3. As reviewed in [42], each of the two intersections separately preserves eight supercharges,[17] and carries no localized massless modes. One way to see this is to compactify some of the directions along the five-branes, and use $U$-duality to turn each of the intersections in the system in question to $k$ $D5$-branes stretched in (012345) intersecting a $D3$-brane stretched in (0126) along an $\mathbb{R}^{2,1}$. If the $D3$-brane is fully extended in $x^6$, 3–5 strings give a massless hypermultiplet in the fundamental representation of the low-energy $U(k)$ gauge symmetry on the five-branes, localized at the intersection. To reach the configuration of interest to us, one needs to separate the two halves of the $D3$-brane (those with positive and negative $x^6$) along the five-branes, and to

[17]The system with both branes and antibranes of course does not preserve any supersymmetry.

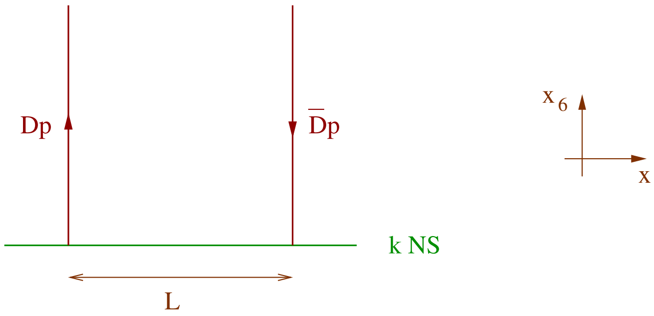


FIG. 3 (color online). The brane configuration: a $Dp$-brane and an $\bar{D}p$-brane ending on $NS5$-branes.

send the lower half to infinity. This corresponds to giving an infinite mass to the hypermultiplet.

The endpoint of the $Dp$-brane on the five-branes looks like a charged object in the five-brane theory. For example, for $p = 1$, the $D1$-brane ending on the $NS5$-branes gives rise to a static quark in the fundamental representation of the low-energy $U(k)$ gauge theory of $k$ $NS5$-branes in type IIB string theory. For $p = 3$, the $D$-brane is extended in two of the directions along the five-branes (12), and looks like a magnetic monopole in the remaining three.

While the system with just one intersection is uninteresting in the infrared,[18] when both branes and antibranes are present, as in Fig. 3, the situation is richer. Since we are interested in the physics near the intersections, we can replace the five-branes by their near-horizon geometry, the Callan-Harvey-Strominger geometry [43]

$$ds^2 = dx_\mu dx^\mu + d\phi^2 + d\Omega^2, \tag{5.1}$$

where $\phi$ is related to the radial coordinate in the transverse $\mathbb{R}^4$ as follows:

$$r = g_s\sqrt{k\alpha'}\exp\left(\frac{\phi}{\sqrt{k\alpha'}}\right), \tag{5.2}$$

and $\Omega$ parametrizes the angular three sphere in $\mathbb{R}^4$, whose radius is given by $\sqrt{k\alpha'}$. More precisely, the angular degrees of freedom are described by a supersymmetric $SU(2)$ Wess-Zumino-Witten model at level $k$. $g_s$ is the asymptotic string coupling, far from the five-branes. The geometry (5.1) is obtained from the full five-brane geometry by taking $g_s \to 0$ with $\phi$ held fixed; in this limit it describes a "little string theory" (LST) (see [44,45] for reviews). The dilaton behaves in this limit like

$$\Phi = -\frac{\phi}{\sqrt{k\alpha'}}. \tag{5.3}$$

A $Dp$-brane ending on the five-branes corresponds in the geometry (5.1), (5.2), and (5.3) to a brane stretched in $(x^0, x^1, \cdots, x^{p-1}, \phi)$, and localized on the three sphere and in $\mathbb{R}^{6-p}$ [46]; the $\bar{D}p$-brane is described similarly. As in the full geometry, the $D$ and $\bar{D}$-branes are a distance $L$ apart in $\mathbb{R}^{6-p}$. Note that unlike the previous cases we discussed, here this distance does not grow as we move out in the radial direction.

The $Dp$ and $\bar{D}p$-branes attract each other via exchange of closed string modes, but we will ignore this effect, and work just at leading order in the string coupling. We will view the distance between the $Dp$ and $\bar{D}p$-branes at $\phi \to \infty$, $L$, as a fixed ( = non-normalizable) boundary condition. Our focus here will be on the classical dynamics of normalizable open string modes.

It turns out that for $L$ larger than a certain critical value,

$$L_{\rm crit} = \pi\sqrt{k\alpha'}, \tag{5.4}$$

all such modes are massive. As $L \to L_{\rm crit}$, a light mode appears. For $L < L_{\rm crit}$ this mode becomes tachyonic and destabilizes the brane configuration of Fig. 3. A heuristic way of understanding this instability is the following. The endpoints of the $Dp$ and $\bar{D}p$-branes on the $NS5$-branes attract each other via exchange of modes localized on the five-branes (LST modes). However, since the tension of the $Dp$-branes goes like the inverse string coupling, while the attractive force due to exchange of a particular five-brane mode is of order one, this is a subleading effect in (the local) $g_s$.

A classical instability can only occur if the sum over the exchanges of all modes of the LST diverges. Such a divergence can only be due to the contributions of arbitrarily heavy LST states. The contribution to the attractive force of a given mode of mass $m$ decreases at large mass like $\exp(-mL)$, while the density of LST states is well-known to behave like $\rho(m) \sim \exp(2\pi\sqrt{\alpha' k}m)$. Thus, superficially it seems that the sum over states diverges for $L < 2L_{\rm crit}$ (5.4).

This factor of 2 discrepancy is familiar from another, closely related, context—closed string emission from accelerating branes in LST. It was argued in [47] that it is natural to expect that the density of states that can be emitted by $D$-branes in LST in fact goes like $\sqrt{\rho(m)}$. This would certainly be the case in ordinary (critical) string theory, since a $D$-brane can only emit left-right symmetric closed string states. Assuming that this is the case in LST as well, we conclude that the exchange of LST modes by the $D$-branes diverges precisely for $L < L_{\rm crit}$.

In the regime $k \gg 1$, $L_{\rm crit}$ is large in string units, and the above light mode is best described as a translational mode of the $D$-brane configuration (which will be described in detail below). For $k \sim 1$ or smaller,[19] a better description of this mode is as a fundamental string stretched between the

[18]In brane constructions of gauge theories, such systems do give interesting infrared physics when embedded in richer brane configurations; this will not play a role in our discussion below.

[19]Such values of $k$ in (5.3) cannot arise in the near-horizon limit of flat $NS5$-branes, but they can arise in other systems.

$D$ and $\bar{D}$-branes. We will consider the geometric regime $k \gg 1$, but will return to this stretched string below.

To exhibit the geometric massless mode for $k \gg 1$, consider the projection of the $D$-branes of Fig. 3 on the two dimensional space labeled by $(\phi, x)$. This corresponds to a $D$ string described by a curve $x = x(\phi)$. The configuration of Fig. 3 corresponds to $x = \pm L/2$; the light mode corresponds to deformations to a more general $x(\phi)$. The DBI action for such a $D$-brane is given by

$$S = -C \int dx \exp\left(\frac{\phi}{\sqrt{k\alpha'}}\right)\sqrt{1 + \phi'^2}. \tag{5.5}$$

Here $\phi = \phi(x)$, $\phi' = \partial_x \phi$, and $C$ is a known constant whose value will not be needed below.

The fact that the Lagrangian (5.5) does not depend explicitly on $x$ implies that one can integrate the Euler-Lagrange equation once. After squaring the resulting equation one gets

$$\exp\left(\frac{2\phi}{\sqrt{k\alpha'}}\right) = 1 + \phi'^2, \tag{5.6}$$

where we fixed a constant that appears in the integration to a particular value by shifting $\phi$. The solution of (5.6) is

$$\exp\left(-\frac{\phi}{\sqrt{k\alpha'}}\right) = \cos\left(\frac{x}{\sqrt{k\alpha'}}\right). \tag{5.7}$$

It describes a U-shaped connected brane, the hairpin brane of [12] (or, more precisely, its generalization to the fermionic string discussed in [13,14], and other papers). As $\phi \to \infty$, it approaches a brane and antibrane a distance $L_{\text{crit}}$ (5.4) apart. As $\phi$ decreases, the two $D$-branes bend towards each other; they smoothly connect at $\phi = 0$ (see Fig. 4).

As mentioned above, the position of the bottom of the brane depicted in Fig. 4 is a free parameter of the solution, as is clear from the form of the action (5.5). Moreover, the energy of the brane is independent of this parameter. Thus, when the distance between the $D$ and $\bar{D}$-branes at infinity is equal to the critical one (5.4), the mode corresponding to fluctuations of the bottom of the hairpin brane is massless and has a flat potential (at leading order in the string coupling).

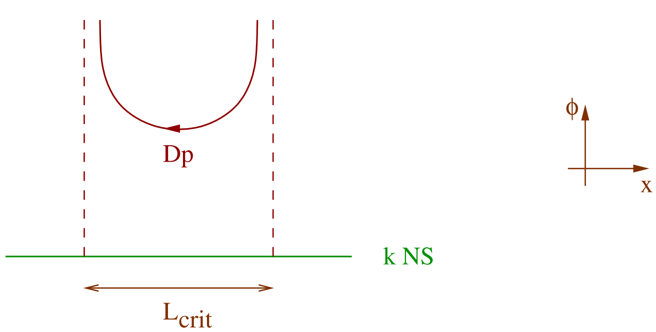


FIG. 4 (color online). The hairpin $D$-brane (5.7).

When $L > L_{\text{crit}}$, this mode is massive, and the hairpin tends to collapse back to the brane-antibrane configuration of Fig. 3. For $L < L_{\text{crit}}$ it is tachyonic and the bottom of the U-shape tends to run to large $\phi$. The resulting time-dependent solutions can be described using techniques similar to those of [48], who studied a closed string analog of this problem.

The original brane configuration of Fig. 3 has a $U(1) \times U(1)$ symmetry associated with the two $Dp$-branes. This is a local symmetry on the $D$-branes, but from the point of view of the LST it is a global one. This symmetry is broken to the diagonal $U(1)$ when the branes connect. It is interesting to ask whether there is an operator that is charged under the broken $U(1)$ and has a nonzero expectation value in the configuration of Fig. 4. Such an operator could serve as an order parameter for the symmetry breaking described above, as in our discussion of the $D4 - D8 - \bar{D}8$ system in Sec. III.

A natural candidate for such an operator is a string stretched between the $Dp$ and $\bar{D}p$-branes in Fig. 3. The lowest lying state of such a string is the open string "tachyon" stretched between the two branes. From studies of the hairpin brane, which turns out to be described by an exactly solvable boundary conformal theory, it is known that such an operator is indeed turned on in the vacuum. In the bosonic string this was discussed in [12,49], while in the fermionic case of interest to us here in [50].

Asymptotically, at large $\phi$, the world sheet Lagrangian contains a term corresponding to a boundary $\mathcal{N} = 2$ superpotential, which behaves like

$$\delta S_{\text{ws}} = \mu \int dt d\theta \exp\left[-\frac{1}{2}\sqrt{\frac{k}{\alpha'}}(\phi + i\tilde{x})\right] + \text{c.c.}. \tag{5.8}$$

Here $\tilde{x} = x_L - x_R$ is the $T$ dual of $x$, and the coupling $\mu$ is determined by $\phi_{\text{IR}}$, the location of the bottom of the hairpin brane. The dependence can be determined by a scaling argument of the kind familiar from Liouville theory. For the hairpin shape (5.7), one has $\phi_{\text{IR}} = 0$, which corresponds to some particular $\mu = \mu^{(*)}$. If we replace $\phi \to \phi - \phi_{\text{IR}}$, such that the bottom of the hairpin is at $\phi = \phi_{\text{IR}}$, we see from (5.8) that

$$\mu = \mu^{(*)} \exp\left(\frac{1}{2}\sqrt{\frac{k}{\alpha'}}\phi_{\text{IR}}\right). \tag{5.9}$$

When $\phi_{\text{IR}} \to -\infty$, the bottom of the hairpin brane descends into the strong coupling region and one smoothly approaches the parallel brane-antibrane configuration of Fig. 3. $\mu$ (5.9) also goes to zero in this limit.[20]

As mentioned above, the large symmetry of the problem ($\mathcal{N} = 2$ world sheet superconformal symmetry) allows

[20]When $\phi_{\text{IR}}$ becomes too small, we cannot trust the shape of the bottom of the hairpin due to strong quantum effects, but there is no reason to expect nonsmooth behavior there.

one to solve the boundary conformal field theory corresponding to the hairpin brane exactly, and, in particular, one can deduce the presence of the boundary $\mathcal{N} = 2$ superpotential (5.8). Thus, it is interesting to study this case in detail, in the hope of developing techniques which could be useful also in more general circumstances where the world sheet theory is not solvable, such as backgrounds with Ramond-Ramond fields turned on.

In particular, we would like to understand the origin of (5.8) at large $k$, where both the closed string background (5.1), (5.2), and (5.3), and the shape of the $D$-brane (5.7), are slowly varying, and we can expect semiclassical techniques to be valid. To do that it is useful to note that the boundary superpotential (5.8) is a normalizable operator at large $\phi$. As is familiar from holography in general, $\mu$ is proportional to the expectation value of the non-normalizable operator that creates a string stretched between the $D$ and $\bar{D}$-branes at the boundary. This operator, which is analogous to the OWL operators described in the previous sections, behaves at large $\phi$ like

$$T \simeq \exp\left[\left(\frac{1}{2}\sqrt{\frac{k}{\alpha'}} - \frac{1}{\sqrt{k\alpha'}}\right)\phi - \frac{i}{2}\sqrt{\frac{k}{\alpha'}}\tilde{x}\right]. \tag{5.10}$$

Thus, we need to calculate the expectation value of (5.10) in the hairpin state. A scaling argument similar to that described above implies that if this expectation value is nonzero, it is indeed proportional to $\mu$ (5.9).

To calculate this expectation value it is useful to note that the tachyon background (5.8) is a nonperturbative effect in the world sheet theory, whose loop expansion parameter is $1/\sqrt{k}$ (the curvature of the $D$-brane). Thus, it is natural to expect that it is due to a world sheet instanton effect, involving an open string ending on the boundary; this also follows from our general discussion in the previous sections of the holographic dual of long open strings. The instanton in question is a map from the world sheet disk $|z| \leq 1$ to the part of the two dimensional $(x, \phi)$ plane bounded by the hairpin,

$$\exp\left(-\frac{\phi}{\sqrt{k\alpha'}}\right) \leq \cos\left(\frac{x}{\sqrt{k\alpha'}}\right). \tag{5.11}$$

Near the boundary $\phi \to \infty$ this world sheet looks like a string stretched between the $D$-branes, which implies that this configuration contributes to the one-point function of the stretched string operator (5.10).

The instanton configuration can be constructed as follows. Start with the world sheet action

$$S = \frac{1}{\pi\alpha'}\int d^2z(\partial_z\phi\partial_{\bar{z}}\phi + \partial_z x\partial_{\bar{z}}x). \tag{5.12}$$

It is convenient to parametrize the $(x, \phi)$ plane by the coordinate

$$U = \exp\left(\frac{\phi - \phi_{\rm IR} + ix}{\sqrt{k\alpha'}}\right), \tag{5.13}$$

in terms of which the hairpin shape (5.7) takes the simple form

$$U + U^* = 2, \tag{5.14}$$

or, equivalently, $\mathrm{Re}(U - 1) = 0$.

The world sheet action (5.12) now takes the form

$$S = \frac{k}{\pi}\int d^2z\frac{1}{|U|^2}(\partial_z U\partial_{\bar{z}}U^* + \partial_{\bar{z}}U\partial_z U^*). \tag{5.15}$$

The disk instanton we are looking for is a holomorphic map from the disk to the half-plane bounded by (5.14), and is easy to write down

$$U - 1 = \frac{1 + z}{1 - z}. \tag{5.16}$$

Its action is proportional to the area $A$ of the Euclidean string world sheet (5.16)

$$S_{\rm inst} = \frac{A}{2\pi\alpha'}. \tag{5.17}$$

This area is infinite, since as $\phi \to \infty$ the hairpin looks like two $D$ strings a distance $L_{\rm crit}$ (5.4) apart, so there is a divergence from that region. This divergence can be regulated by introducing an upper bound on $\phi$, $\phi_{\rm UV}$, which can be thought of as a UV cutoff.

In any case, we are only interested in the dependence of the area on the position of the bottom of the hairpin, $\phi_{\rm IR}$, discussed around (5.9). We can isolate this dependence by differentiating the area with respect to $\phi_{\rm IR}$. A short calculation leads to

$$\frac{\partial A}{\partial\phi_{\rm IR}} = -L_{\rm crit} \tag{5.18}$$

in the limit $\phi_{\rm UV} \to \infty$. Therefore, after rescaling the operator (5.10) by a factor which depends on the UV cutoff, we conclude that

$$\langle T\rangle \sim \exp(-S_{\rm inst}) \sim \exp\left(\frac{L_{\rm crit}\phi_{\rm IR}}{2\pi\alpha'}\right) \sim \mu, \tag{5.19}$$

where we used (5.4), (5.9), (5.17), and (5.18). We see that indeed the instanton contribution scales in the right way with $\phi_{\rm IR}$ to give a nonzero one-point function to the long open string operator (5.10). Note that we have only computed the leading exponential contribution to the one-point function. The preexponential factor involves contributions from the dilaton coupling in the world sheet action, and the determinant of small fluctuations around the instanton (5.16). These are subleading in the large $k$ limit, and are expected to give rise to a constant contribution to (5.19) (independent of $\phi_{\rm IR}$).

So far we discussed the spontaneous breaking of the $U(1) \times U(1)$ symmetry of the brane configuration of Fig. 3 to the diagonal $U(1)$, by the brane configuration of Fig. 4. We have seen that the order parameter for this breaking can be taken to be the stretched string operator (5.10), and it indeed has a nonzero expectation value in the hairpin state (5.19). It is natural to ask what happens if we deform the system by adding to the world sheet Lagrangian the non-normalizable operator $T$ (5.10),

$$\delta S_{ws} = \frac{\kappa}{2}\int dt d\theta T(\phi, x) + \text{c.c.}. \tag{5.20}$$

This deformation breaks the $U(1)\times U(1)$ symmetry explicitly. It also breaks the $\mathcal{N}=2$ superconformal symmetry of the hairpin brane; therefore we do not expect the resulting theory to be exactly solvable. However, one can still ask how the shape of the $D$-brane and its low-lying spectrum change in the presence of this deformation.

To first order in $\kappa$ and in the semiclassical regime $k \gg 1$ one can answer this question by adding to the DBI action (5.5) the exponential of the Nambu-Goto (NG) action for the instanton string discussed above,

$$S = -C\int dx \exp\left(\frac{\phi}{\sqrt{k\alpha'}}\right)\sqrt{1+\phi'^2} - \kappa B e^{-S_{\text{NG}}}. \tag{5.21}$$

Here, as before [see (5.17)], $S_{\text{NG}} = \frac{A}{2\pi\alpha'}$, where $A$ is the area of a minimal world sheet enclosed by the deformed hairpin, and $B$ is the preexponential factor in the expectation value of $T$ above. It depends on the shape of the deformed hairpin, but for the purpose of the calculation below, to leading order in the $1/k$ expansion we can neglect this dependence.

To calculate the shape of the deformed hairpin to first order in $\kappa$ we need to solve the equation of motion of $\phi(x)$ with the deformed action (5.21). For this we need the dependence of $S_{\text{NG}}$ on the shape $\phi(x)$. It is easy to see that it is given by

$$S_{\text{NG}} = -\frac{1}{2\pi\alpha'}\int dx\phi(x) + \cdots, \tag{5.22}$$

where the ellipsis stand for terms that depend on the UV cutoff $\phi_{\text{UV}}$, but not on the shape $\phi(x)$. Varying (5.21) with respect to $\phi(x)$ and integrating once, we find the first order equation

$$C\frac{e^{(\phi/\sqrt{k\alpha'})}}{\sqrt{1+\phi'^2}} + \frac{\kappa\langle T\rangle}{2\pi\alpha'}\phi = D, \tag{5.23}$$

where $D$ is a function of $\phi_{\text{IR}}$ (or, equivalently, of the separation between the brane and antibrane at some UV cutoff $\phi_{\text{UV}}$). This equation generalizes (5.6) to nonzero $\kappa$, and it can be solved by expanding $\phi$ as $\phi = \phi_0 + \kappa\phi_1 + \cdots$, and keeping only first order terms in $\kappa$. For example, at large $\phi$, the leading deformation of the hairpin from its original form is given by

$$C\partial_\phi x = \left(D - \frac{\kappa\langle T\rangle}{2\pi\alpha'}\phi\right)e^{-(\phi/\sqrt{k\alpha'})}. \tag{5.24}$$

Of course, when $\phi$ becomes too large, one has to go beyond the linear approximation in $\kappa$ described above.

It is interesting to compare the deformed shape of the hairpin (5.24) which we found above, to the deformed shape implied by the effective action on the $Dp$-brane coupled to the tachyon field $T$. In curved space and for curved $D$-branes (as in the discussion of the previous sections) it is not known how to write down such an effective action, but for flat $D$-branes in flat space we know how to write it down, and this is the situation in the asymptotic region of the hairpin. In this region we know that, if we denote the distance between the brane and the antibrane by $L_{\text{crit}} - 2x(\phi)$ [where $x(\phi)$ is small in the UV], the mass of the open string ground state is given by

$$m^2(x(\phi)) = -\frac{1}{2\alpha'} + \left(\frac{L_{\text{crit}} - 2x}{2\pi\alpha'}\right)^2 \simeq m_0^2 - \sqrt{\frac{k}{\alpha'^3}}\frac{x}{\pi}. \tag{5.25}$$

The effective action of the tachyon stretched between the $D$ and $\bar{D}$-branes in Fig. 4 is given to quadratic order by

$$S = -C\int d\phi \exp\left(\frac{\phi}{\sqrt{k\alpha'}}\right)[(\partial_\phi x)^2 + (\partial_\phi T)^2 + m^2(x)T^2]. \tag{5.26}$$

We are looking for a configuration where the normalizable mode of the tachyon (5.8) is turned on with a coefficient $\langle T\rangle$, and the non-normalizable mode (5.10) is turned on with a coefficient $\kappa$, such that at leading order in $\kappa$ the tachyon field behaves asymptotically as

$$T^2 \simeq \beta\kappa\langle T\rangle \exp\left(-\frac{\phi}{\sqrt{k\alpha'}}\right), \tag{5.27}$$

where $\beta$ is a constant coming from carefully normalizing the normalizable and non-normalizable modes of the tachyon.

The equation of motion of $x(\phi)$ with this tachyon source, at leading order in $\kappa$ (and in the UV region where $x$ is small), then takes the form

$$\partial_\phi\left[\exp\left(\frac{\phi}{\sqrt{k\alpha'}}\right)\partial_\phi x\right] = -\frac{\beta\kappa\langle T\rangle}{2\pi\alpha'}\sqrt{\frac{k}{\alpha'}}. \tag{5.28}$$

For $1/\beta = C\sqrt{k/\alpha'}$, this precisely agrees with (5.24) above.

## VI. ADDITIONAL ISSUES

### A. A supersymmetric example

The main examples we focused on so far were non-supersymmetric, but one can also construct interesting examples of OWL operators (2.2) in supersymmetric theories, including examples which preserve some of the supersymmetry. We will describe here just one example, leaving a further investigation to future work.

Consider the $d=4$ $\mathcal{N}=4$ $SU(N_c)$ SYM theory coupled to $N_f$ three dimensional massless hypermultiplets living on the surface $x_3 = 0$. In the 't Hooft large $N_c$ limit with 't Hooft coupling $\lambda_4$ and with fixed $N_f$, this is described by type IIB string theory on $\mathrm{AdS}_5 \times S^5$, with $N_f$ $D5$-branes filling an $\mathrm{AdS}_4 \times S^2$ subspace [25,26]; if we use the Poincaré coordinates of $\mathrm{AdS}_5$ (with a boundary at $z \to 0$),

$$ds^2 = \sqrt{\lambda_4}\alpha' \frac{dx_\mu^2 + dz^2}{z^2}, \tag{6.1}$$

then the $D5$-branes are simply located at $x_3 = 0$ (and wrap some maximal $S^2$ inside the $S^5$). This theory breaks half of the supersymmetry of the $\mathcal{N} = 4$ SYM theory; it preserves a $d = 3$ $\mathcal{N} = 4$ superconformal symmetry.

Now, consider an OWL starting at a hypermultiplet at $x_0 = x_1 = x_2 = x_3 = 0$ and stretching to infinity in the $x_3$ direction. Such an operator is analogous to the "straight Wilson line" in the $\mathcal{N} = 4$ SYM theory; it is well-defined if we put appropriate boundary conditions at infinity. In the holographic dual description, the computation of the one-point function of this operator is dominated by a string sitting at $x_0 = x_1 = x_2 = 0$ and filling the $z$ axis and the positive $x_3$ axis in (6.1) (we assume that the OWL couples to a scalar such that the string lives at a point in the $S^2$ filled by the $D5$-branes). This operator breaks half of the supersymmetry (leaving eight unbroken supercharges, including both regular supercharges and superconformal charges), and the holographic computation of its VEV gives one, since the regularized area of the surface vanishes (just like for the "straight Wilson line").

This case is not very interesting, but suppose that we now perform a conformal transformation involving an inversion around a point $x_0 = x_1 = x_3 = 0$, $x_2 = a$. This transformation leaves the field theory described above invariant. However, the contour in the OWL now maps to a semicircle

$$\left(x_2 - a + \frac{1}{2a}\right)^2 + x_3^2 = \frac{1}{4a^2}, \qquad x_3 \geq 0. \tag{6.2}$$

This is a standard OWL connecting two hypermultiplets of the form (2.2), with a semicircular contour (6.2) between the two points ($x_2 = a$, $x_3 = 0$) and ($x_2 = a - 1/a$, $x_3 = 0$). Our derivation of this configuration by a conformal transformation ensures that this OWL still preserves eight supercharges, though these are now combinations of standard supersymmetries and superconformal symmetries.

The holographic computation of the one-point function of this OWL is straightforward; the dominant solution is just half of the solution for the circular Wilson line [6,51], with a string world sheet at

$$\left(x_2 - a + \frac{1}{2a}\right)^2 + x_3^2 + z^2 = \frac{1}{4a^2}, \qquad x_3 \geq 0. \tag{6.3}$$

Its area is thus half of that corresponding to the circular Wilson line, which is $\sqrt{\lambda_4}$, so the VEV of the OWL (at leading order in the $\alpha'$ expansion) is equal to $\exp(\sqrt{\lambda_4}/2)$.

In the case of the closed circular Wilson line case it has been conjectured [52,53] and recently proven [54] that the result is given by a zero-dimensional matrix model, since the conformal transformation can only change the result because a point is brought in from infinity. Similar arguments imply that the semicircular open Wilson line $\langle OW\rangle$ described above should also be computable by a zero-dimensional model of matrices and vectors; it would be interesting to verify this.

### B. Divergences in open Wilson line computations

Closed supersymmetric Wilson loops are known to have divergences at cusps, which can be computed both perturbatively and at strong coupling (with a qualitatively similar behavior found in both limits [6]). Similarly, in the case that the fields in the fundamental representation are localized on some subspace, the correlation functions of the open Wilson line observables (2.2) have a divergence whenever the contour $\tilde{C}$ ends on that subspace at an angle which is not a straight angle. In this section we describe this divergence both at weak coupling (using perturbation theory) and at strong coupling (using the mapping to string world sheets).

Let us consider a $D$-dimensional large $N$ gauge theory, in which some fields in the fundamental representation are localized on a $d$-dimensional subspace; without loss of generality we can take this subspace to be

$$x^{d+1} = x^{d+2} = \cdots = x^D = 0. \tag{6.4}$$

When we consider an open Wilson line operator of the form (2.2), starting at a fundamental field located at $x = 0$, there is now an angle associated with this operator, which is the angle $\theta$ between the direction of the Wilson line (near $x = 0$) and the subspace that the fundamental fields live on. For instance, again without loss of generality, we can assume that near $x = 0$ the Wilson line (parametrized by $t$) looks like

$$x^{d+1} = t\sin(\theta), \qquad x^d = t\cos(\theta), \qquad t \geq 0, \tag{6.5}$$

implying that the Wilson line couples to the gauge field components $\sin(\theta)A_{d+1} + \cos(\theta)A_d$. On the other hand, the fields in the fundamental representation couple just to $A_d$ and they do not couple to $A_{d+1}$. The one-loop diagram involving the exchange of a gauge field between the Wilson line and the propagator of the field in the fundamental representation then has a divergence as $t \to 0$, proportional (near $\theta = \pi/2$) to $\cos^2(\theta)$, going as $\int dt/t^{D-3}$. For $D = 4$ we have a logarithmic divergence (as for a cusp in a closed Wilson line), and for $D = 5$ a linear divergence. The only case in which there is no divergence is when the Wilson line intersects the surface (6.4) at a straight angle $\theta = \pi/2$.

When the fundamental representation fields couple also to scalar fields (note that this is not the case in the $D4 - D8 - \bar{D}8$ system), then, for a specific choice of the scalar field couplings of the open Wilson line, it may be possible to cancel this divergence. However, generally this divergence is present even for locally supersymmetric OWL operators (as is the case for the cusp divergence).

On the strong coupling side, for the purposes of computing the divergence in the open Wilson line correlators we can concentrate just on the region near the boundary, where the flavor $D$-brane just sits at $x^{d+1} = \cdots = x^D = 0$ and stretches in the radial direction. We need to find a minimal world sheet ending on the contour (6.5) at the boundary and transverse to the $D$-brane. It is easy to convince oneself that such a world sheet is the same as half of the closed string world sheet ending on the contour

$$x^{d+1} = t\sin(\theta), \qquad x^d = |t|\cos(\theta), \tag{6.6}$$

that we obtain by joining to (6.5) its reflection around the subspace that the fundamental fields live on (a similar trick was recently used in [17]). This contour has a cusp at $t = 0$ with an angle of $2\theta$, so it leads to a divergence which is similar to the cusp divergence occurring in closed Wilson loops (whenever $\theta \neq \pi/2$). For the case of $D = 4$ this is a logarithmic divergence, just as in the previous paragraph, but its precise dependence on the angle is different from the one found at weak coupling (this is also true for the closed Wilson loop cusp divergence) [6].

Note that in this computation we assumed that the end of the open Wilson line is at the same position as the $D$-brane in the compact directions (otherwise there is no semiclassical world sheet contributing to the computation of correlation functions of $OW$). If the $D$-brane is partially localized in the compact directions (so that the fundamental fields couple to some of the scalar fields of the gauge theory) then this implies that near the end of the open Wilson line, the Wilson line couples to different scalar fields than the ones which the fundamental fields couple to. Thus, for such Wilson lines there is no contribution from the scalar fields at leading order in perturbation theory, and their one-loop computation diverges as described above.

In any case, we showed that both at weak coupling and at strong coupling, when the fundamental representation fields are localized on a subspace, one has to choose the Wilson line operators (2.2) such that the direction of the open Wilson line is transverse to that subspace at its beginning and end, in order to avoid cusplike divergences in the computation.

## ACKNOWLEDGMENTS

We would like to thank O. Bergman, N. Drukker, S. Hartnoll, Z. Komargodski, O. Lunin, D. Reichmann, A. Schwimmer, J. Sonnenschein, and S. Yankielowicz for useful discussions. The work of O. A. was supported in part by the Israel-U.S. Binational Science Foundation, by a center of excellence supported by the Israel Science Foundation (Grant No. 1468/06), by a grant (DIP H52) of the German Israel Project Cooperation, by the European network MRTN-CT-2004-512194, and by Minerva. D. K. is supported in part by DOE Grant No. DE-FG02-90ER40560, by the National Science Foundation under Grant No. 0529954, and by the Israel-U.S. Binational Science Foundation. D. K. thanks the Weizmann Institute for hospitality during part of this work.

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); Int. J. Theor. Phys. **38**, 1113 (1999).
[2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998).
[3] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
[4] J. M. Maldacena, Phys. Rev. Lett. **80**, 4859 (1998).
[5] S. J. Rey and J. T. Yee, Eur. Phys. J. C **22**, 379 (2001).
[6] N. Drukker, D. J. Gross, and H. Ooguri, Phys. Rev. D **60**, 125006 (1999).
[7] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005).
[8] E. Antonyan, J. A. Harvey, S. Jensen, and D. Kutasov, arXiv:hep-th/0604017.
[9] O. Aharony, J. Sonnenschein, and S. Yankielowicz, Ann. Phys. (N.Y.) **322**, 1420 (2007).
[10] O. Bergman, S. Seki, and J. Sonnenschein, J. High Energy Phys. 12 (2007) 037.
[11] A. Dhar and P. Nag, J. High Energy Phys. 01 (2008) 055.
[12] S. L. Lukyanov, E. S. Vitchev, and A. B. Zamolodchikov, Nucl. Phys. **B683**, 423 (2004).
[13] D. Kutasov, arXiv:hep-th/0405058.
[14] Y. Nakayama, Y. Sugawara, and H. Takayanagi, J. High Energy Phys. 07 (2004) 020.
[15] L. F. Alday and J. M. Maldacena, J. High Energy Phys. 06 (2007) 064.
[16] Z. Komargodski and S. S. Razamat, J. High Energy Phys. 01 (2008) 044.
[17] J. McGreevy and A. Sever, J. High Energy Phys. 02 (2008) 015.
[18] N. Drukker and B. Fiol,J. High Energy Phys. 02 (2005) 010.
[19] S. Yamaguchi, J. High Energy Phys. 05 (2006) 037.
[20] J. Gomis and F. Passerini, J. High Energy Phys. 08 (2006) 074.
[21] G. 't Hooft, Nucl. Phys. **B72**, 461 (1974).
[22] O. Aharony, A. Fayyazuddin, and J. M. Maldacena, J. High Energy Phys. 07 (1998) 013.
[23] E. Antonyan, J. A. Harvey, and D. Kutasov, Nucl. Phys. **B776**, 93 (2007).
[24] E. Antonyan, J. A. Harvey, and D. Kutasov, Nucl. Phys.

B784, 1 (2007).
[25] A. Karch and L. Randall, J. High Energy Phys. 06 (2001) 063.
[26] O. DeWolfe, D. Z. Freedman, and H. Ooguri, Phys. Rev. D **66**, 025009 (2002).
[27] A. Karch and E. Katz, J. High Energy Phys. 06 (2002) 043.
[28] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
[29] N. Itzhaki, J. M. Maldacena, J. Sonnenschein, and S. Yankielowicz, Phys. Rev. D **58**, 046004 (1998).
[30] D. J. Gross and H. Ooguri, Phys. Rev. D **58**, 106002 (1998).
[31] A. Brandhuber, N. Itzhaki, J. Sonnenschein, and S. Yankielowicz, J. High Energy Phys. 06 (1998) 001.
[32] O. Bergman and G. Lifschytz, J. High Energy Phys. 04 (2007) 043.
[33] C. D. Carone, J. Erlich, and M. Sher, arXiv:0802.3702.
[34] M. Headrick, Phys. Rev. D **77**, 105017 (2008).
[35] O. Aharony, K. Peeters, J. Sonnenschein, and M. Zamaklar, J. High Energy Phys. 02 (2008) 071.
[36] E. Witten, Nucl. Phys. **B156**, 269 (1979).
[37] G. Veneziano, Nucl. Phys. **B159**, 213 (1979).
[38] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).
[39] M. Kruczenski, L. A. P. Zayas, J. Sonnenschein, and D. Vaman, J. High Energy Phys. 06 (2005) 046.
[40] K. Peeters, J. Sonnenschein, and M. Zamaklar, J. High Energy Phys. 02 (2006) 009.
[41] F. Bigazzi and A. L. Cotrone, J. High Energy Phys. 11 (2006) 066.
[42] A. Giveon and D. Kutasov, Rev. Mod. Phys. **71**, 983 (1999).
[43] C. G. Callan, J. A. Harvey, and A. Strominger, arXiv:hep-th/9112030.
[44] O. Aharony, Classical Quantum Gravity **17**, 929 (2000).
[45] D. Kutasov, Prepared for ICTP Spring School on Superstrings and Related Matters, Trieste, Italy, 2001.
[46] S. Elitzur, A. Giveon, D. Kutasov, E. Rabinovici, and G. Sarkissian, J. High Energy Phys. 08 (2000) 046.
[47] D. A. Sahakyan, J. High Energy Phys. 10 (2004) 008.
[48] N. Itzhaki, D. Kutasov, and N. Seiberg, J. High Energy Phys. 12 (2005) 035.
[49] S. L. Lukyanov and A. B. Zamolodchikov, Nucl. Phys. **B744**, 295 (2006).
[50] D. Kutasov, arXiv:hepth/0509170.
[51] D. E. Berenstein, R. Corrado, W. Fischler, and J. M. Maldacena, Phys. Rev. D **59**, 105023 (1999).
[52] J. K. Erickson, G. W. Semenoff, and K. Zarembo, Nucl. Phys. **B582**, 155 (2000).
[53] N. Drukker and D. J. Gross, J. Math. Phys. (N.Y.) **42**, 2896 (2001).
[54] V. Pestun, arXiv:0712.2824.