Ambiguity of spontaneously broken gauge symmetry

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Local gauge symmetries cannot break spontaneously, according to Elitzur's theorem, but this leaves open the possibility of breaking some global subgroup of the local gauge symmetry, which is typically the gauge symmetry remaining after certain (e.g. Coulomb or Landau) gauge choices. We show that in an SU(2) gauge-Higgs system such symmetries do indeed break spontaneously, but the location of the breaking in the phase diagram depends on the choice of global subgroup. The implication is that there is no unique broken gauge symmetry, but rather many symmetries which break in different places. The problem is to decide which, if any, of these gauge-symmetry breakings is associated with a transition between physically different, confining and nonconfining phases. Several proposals—Kugo-Ojima, Coulomb, and monopole condensate—are discussed.

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I. INTRODUCTION

Most introductory treatments of the Higgs mechanism teach that the spontaneous breaking of a gauge symmetry is signaled by the nonvanishing expectation value of a Higgs field. Such introductory discussions occasionally overlook the fact that local gauge symmetries *cannot* break spontaneously, according to a celebrated theorem by Elitzur [1], and in the absence of gauge fixing the vacuum expectation value (VEV) of the Higgs field ϕ is rigorously zero, no matter what the form of the Higgs potential. In contrast, in a unitary gauge which fixes the gauge symmetry completely, it can happen instead that $\langle\phi\rangle \neq 0$, again irrespective of the Higgs potential. The point is that only a global subgroup of a local gauge symmetry can break spontaneously, and the order parameter for this symmetry breaking must transform nontrivially under the global subgroup, but remain invariant under arbitrary local gauge transformations. Local gauge transformations, of course, can vary independently at each spacetime point; this is the feature which is crucial to the Elitzur theorem, and the number of parameters specifying a local gauge transformation grows with spacetime volume. By a "global" subgroup we mean only that gauge transformations in the subgroup depend on a finite and fixed number of parameters which is independent of volume; such global transformations are not necessarily constant in spacetime.

One way of constructing appropriate order parameters is via a gauge choice, which leaves the desired global symmetry unfixed. Coulomb and Landau gauges are examples of such gauge choices. Since the local but not the global gauge freedom has been gauged away, the Higgs field (and other local observables) can serve as order parameters for breaking of the remaining gauge symmetry. An alternative approach is to build the gauge choice into the definition of the order parameter, rendering it invariant under local, but not global, transformations. For example, instead of computing the VEV of the Higgs field in, e.g., Coulomb gauge, one could compute the VEV of the nonlocal operator

$$\Phi(x;A) = g(x;A)\phi(x), \tag{1.1}$$

where $g(x;A)$ is a field-dependent gauge transformation which takes the given A field into Coulomb gauge. In an Abelian theory with an infinite spatial volume this transformation can be derived explicitly, and the result is

$$\Phi(\mathbf{x},t;A) = \exp\left[i\int d^3y A_k(\mathbf{y},t)\partial_k \frac{1}{4\pi|\mathbf{x}-\mathbf{y}|}\right]\phi(\mathbf{x},t). \tag{1.2}$$

This is the Dirac construction [2]. The operator Φ is invariant under local gauge transformations which go to the identity at spatial infinity, but transforms as a charged operator under spatially constant gauge transformations. It is easy to see that the VEV of ϕ in Coulomb gauge is the same as the VEV of Φ evaluated without gauge fixing. A similar construction can be made in the Landau gauge.

It is important to recognize that different gauge choices, and even different order parameters in the same gauge, single out different global subgroups of the full gauge symmetry. In Landau gauge there is of course a remnant gauge symmetry under spacetime-constant gauge transformations $g(x) = g$. There is also, in this gauge, an invariance with respect to certain spacetime-dependent transformations. For the SU(2) group, with

$$g(x) = \exp[i\Lambda^a(\epsilon;x)\tfrac{1}{2}\sigma_a] \tag{1.3}$$

and $\Lambda(\epsilon;x)$ linear in the infinitesmal parameters ϵ^a_μ, these spacetime-dependent transformations can be worked out in a power series expansion in the coupling $\mathbf{g}$ [3]. To first order, Λ is given by

$$\Lambda^a(\epsilon;x) = \epsilon^a_\mu x^\mu - \mathbf{g}\frac{1}{\partial^2}(A_\mu \times \epsilon_\mu)^a + O(\mathbf{g}^2). \tag{1.4}$$

If the Higgs field has an expectation value in Landau gauge, both the spacetime-constant and spacetime-dependent global symmetries are broken. The spacetime-dependent global symmetry (1.4) singled out in Landau gauge is not a global symmetry in Coulomb gauge (although a different but analogous symmetry could be constructed). Symmetry with respect to the spacetime-constant transformations $g(x) = g$ is a remnant symmetry in both Landau and Coulomb gauges, but in Coulomb gauge there is a much larger invariance with respect to transformations which are constant in space, but not in time, i.e. $g(\mathbf{x}, t) = g(t)$. Suppose we single out two specific times, e.g. $t = 0$ and $t = T$. The trace $\mathrm{Tr}[L]$ of a timelike Wilson line

$$L(\mathbf{x}, T) = P\exp\left[i\int_0^T dtA_0(\mathbf{x}, t)\right] \tag{1.5}$$

is invariant under gauge transformations which are constant in space and time,

$$\mathrm{Tr}[L(\mathbf{x}, T)] = \mathrm{Tr}[gL(\mathbf{x}, T)g^\dagger] \tag{1.6}$$

and is therefore insensitive to the spontaneous breaking of that symmetry. But this observable is not invariant under the group of transformations which are constant in space, but independent at times $t = 0$ and $t = T$

$$\mathrm{Tr}[L(\mathbf{x}, T)] \neq \mathrm{Tr}[g(0)L(\mathbf{x}, T)g^\dagger(T)]. \tag{1.7}$$

This means that $\langle\mathrm{Tr}[L]\rangle$ in Coulomb gauge probes the breaking of a global gauge symmetry which is different from the symmetry probed by $\langle\phi\rangle$ in Landau gauge.[1]

The question which naturally arises is whether the spontaneous breaking of different global subgroups of the local gauge symmetry, associated with different gauge choices and/or order parameters, occurs at the same location in the space of coupling constants. If not, then there is a certain ambiguity in the notion of gauge-symmetry breaking; precision requires specifying the particular global subgroup which is actually broken.

Assuming that different subgroups break in different places, the next question is which (if any) of the various global subgroups is associated, upon symmetry breaking, with a transition to a physically different phase. In particular, the breaking or restoration of which subgroup is associated with the transition from a confinement phase to some nonconfining phase? As it happens, a number of different approaches to the confinement problem, discussed below, associate confinement with the symmetric (or broken) realization of different global gauge symmetries. If these symmetries break in different places, it raises the obvious question of which global gauge symmetry is the "correct" way to characterize confinement, particularly when global center symmetry (which is *not* a gauge symmetry) is broken by matter fields

In this article we will investigate the possible ambiguity of gauge-symmetry breaking in the context of a gauge-Higgs theory on the lattice, with a fixed-modulus Higgs field in the fundamental color representation. For the SU(2) gauge group, the Lagrangian can be written in the form [4]

$$S = \beta\sum_{\mathrm{plaq}}\frac{1}{2}\mathrm{Tr}[UUU^\dagger U^\dagger] + \gamma\sum_{x,\mu}\frac{1}{2}\mathrm{Tr}[\phi^\dagger(x)U_\mu(x)\phi(x+\hat{\mu})] \tag{1.8}$$

with ϕ an SU(2) group-valued field. Investigations [5,6] of this model, carried out many years ago, revealed an important and surprising feature: Consider two points (β_1, γ_1) and (β_2, γ_2) in the $\beta - \gamma$ phase diagram, with $(\beta_1, \gamma_1) \ll 1$ deep in the "confinement" (strong-coupling) regime, and $(\beta_2, \gamma_2) \gg 1$ deep in the Higgs regime. Then according to a result due to Fradkin and Shenker [5] (which was based on an earlier theorem of Osterwalder and Seiler [6]), there is a path in the phase diagram connecting the two points, such that the expectation value of any local gauge-invariant observable, or product of such observables, varies analytically along the path. This means that there is no thermodynamic phase transition which entirely isolates the Higgs phase from a confinement phase. Subsequent numerical work [4,7] ruled out a massless phase, and indicated the phase structure sketched in Fig. 1, with a line of first-order transitions (or possibly just a line of rapid crossover) which ends at around $\beta = 2$, $\gamma = 1$, consistent with the Fradkin-Shenker-Osterwalder-Seiler theorem. Above the transition line, at large β, the

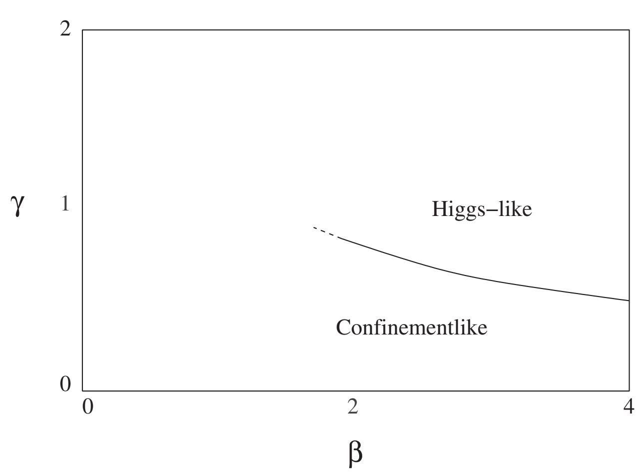

FIG. 1. Schematic phase diagram of the SU(2) gauge-Higgs system. The solid line is a line of weak first-order phase transitions.

[1]The Coulomb gauge remnant symmetry $g(t)$ is local in time, and if we consider timelike Wilson lines running from $t = t_0$ to $t = t_0 + T$, then the Elitzur theorem guarantees that $\mathrm{Tr}[L]$ would vanish if averaged over all t_0, as well as all 3-space positions $\mathbf{x}$. What happens is that Wilson lines can have a nonvanishing average at fixed t_0, because on a time slice the symmetry is global and can break spontaneously, but these spatial averages are in general different at different t_0, and must cancel upon averaging over t_0.

dynamics is clearly that of a Higgs phase, with a massive spectrum, no linear Regge trajectories, no flux-tube formation, and only Yukawa-type potentials between static color charges. On the other hand, at small values of γ, the theory is reminiscent of real QCD with dynamical fermions. In this coupling regime we have flux-tube formation and a linear potential over some finite distance range, followed by string breaking via scalar particle production.

One of the things that we learn from the Fradkin-Shenker work is that the Higgs phase cannot be distinguished from the confinement phase by so-called "color confinement" in the asymptotic particle spectrum. It is always possible to choose a path, from the confinement to the Higgs regime, such that all local gauge-invariant observables, products of such observables, and (in particular) the free energy, vary analytically along the path, and this behavior is incompatible with an abrupt, qualitative change in the spectrum. Asymptotic particle states are therefore color singlets throughout the phase diagram. In the absence of a massless, Coulombic regime, color is *always* screened by the fundamental-representation Higgs field, whether this screening is viewed as a string-breaking effect, or as the rearrangement of a condensate in the neighborhood of a color charge.

We then return to the basic question: In the absence of a thermodynamic separation, can the spontaneous breaking of a gauge symmetry distinguish unambiguously between the Higgs and confinement phases? To address this question, we will map out the location of the breaking of remnant global gauge symmetries in the Coulomb and Landau gauges. It will be found that these transitions coincide, within the accuracy of our data, along the thermodynamic transition line at $\beta > 2$. But away from that line, at $\beta < 2$, the transitions are found to diverge from one another. This result ties in with an earlier work [8] in the gauge-Higgs theory, comparing the line of gauge-symmetry breaking in Coulomb gauge with the line of center vortex percolation/depercolation (a "Kertész" line [9]), which was thought to be identical [10]. In fact, the Coulomb gauge and percolation transition lines also coincide with the thermodynamic transition line at $\beta > 2$, but diverge from one another at lower β. Percolation transitions at finite temperature, for other types of topological objects in electroweak gauge theory and QCD, were also discussed in Ref. [11], where it was pointed out (in the second article cited) that the precise location of the Kertész line depends on the type of object studied. Of course, spontaneous breaking of a gauge symmetry and a percolation transition are in principle very different things, and the result in Ref. [8] leaves open the question of whether or not the spontaneous symmetry breakings of different global gauge symmetries coincide.

In the next section we will discuss the order parameters for confinement in three different approaches: (i) the Kugo-Ojima criterion (covariant gauges); (ii) Coulomb confinement (Coulomb gauge); and (iii) dual superconductivity. Each of these order parameters is sensitive to the breaking of a different global gauge symmetry. In Sec. III we present our data for global gauge-symmetry breaking, in Landau and Coulomb gauges, in the SU(2) gauge-Higgs model. Symmetry breaking associated with the third order parameter, which is less straightforward to implement numerically, will be reserved for a later study. Section IV contains discussion and conclusions.

II. ORDER PARAMETERS FOR CONFINEMENT

In gauge theories with a nontrivial center symmetry, there is no difficulty in distinguishing qualitatively between the confinement phase and the Higgs phase, or between confinement and a high temperature deconfined phase. The vanishing of Polyakov lines, the large-volume behavior of the vortex free energy, and the nonvanishing of string tensions extracted from fundamental-representation Wilson loops all serve as appropriate, consistent, and gauge-invariant signals of the confinement phase [12]. A transition away from the confinement phase is always accompanied by the spontaneous breaking of the global center symmetry, and nonanalytic behavior in the free energy. But the situation is much less clear when there are dynamical matter fields in the fundamental representation of the gauge group, as in real QCD. When global center symmetry is broken explicitly, Polyakov lines are nonzero, and Wilson loops fall off asymptotically with a perimeter-law behavior, as in a Higgs phase. The question is whether there is some other symmetry which can distinguish the confinement phase from other massive phases. We will discuss three proposals, each of which could potentially identify the confined phase even in the absence of a global center symmetry in the Lagrangian.

A. The Kugo-Ojima criterion

Kugo and Ojima [13] begin with an equation satisfied by the conserved color current J^a_μ in covariant gauges

$$J^a_\mu = \partial^\nu F^a_{\mu\nu} + \{Q_B, D^{ab}_\mu \bar{c}^b\}, \tag{2.1}$$

where c and $\bar{c}$ are the ghost-antighost fields with Q_B the Becchi-Rouet-Stora-Tyutin charge, and also introduce the function $u^{ab}(p^2)$, defined by the expression

$$u^{ab}(p^2)\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) = \int d^4x e^{ip(x-y)} \langle 0|T[D_\mu c^a(x) \times g(A_\nu \times \bar{c})^b(y)|0\rangle. \tag{2.2}$$

They then show that the expectation value of color charge in any physical state vanishes

$$\langle \text{phys}|Q^a|\text{phys}\rangle = 0 \tag{2.3}$$

providing that (i) remnant symmetry with respect to spacetime-independent gauge transformations is unbroken; and (ii) the following condition is satisfied:

$$u^{ab}(0) = -\delta^{ab}. \tag{2.4}$$

This latter condition is the Kugo-Ojima confinement criterion, and it implies that the ghost propagator is more singular, and the gluon propagator less singular, than a simple pole at $p^2 = 0$ [14]. A number of efforts have focused on verifying this condition (or its corollaries) both analytically [15] and numerically [16].

It turns out that the Kugo-Ojima condition (2.4) is itself tied to the unbroken realization of remnant gauge symmetry in covariant gauges (such as Landau gauge). We have already noted that in Landau gauge there is a remnant group of spacetime-dependent gauge transformations, given in Eq. (1.4), which preserves the Landau gauge condition. It was shown by Hata in Ref. [3] (see also Kugo in Ref. [14]) that the condition (2.4) is a necessary (and probably sufficient) condition for the unbroken realization of the residual spacetime-dependent symmetry (1.4), while an unbroken, spacetime-independent symmetry is required, in *addition* to (2.4), for the vanishing of $\langle\psi|Q^a|\psi\rangle$ in physical states.

Thus the Kugo-Ojima scenario requires the full remnant gauge symmetry in Landau gauge, i.e. both the spacetime-dependent and the spacetime-independent residual gauge symmetries must be unbroken. Both of these symmetries are necessarily broken if a Higgs field aquires a VEV in Landau gauge.

B. The Coulomb gauge criterion

The criterion for confinement as the unbroken realization of remnant gauge symmetry in Coulomb gauge was first put forward by Marinari *et al.* in Ref. [17]; the idea was elaborated and studied numerically in Ref. [10]. The criterion can be motivated as follows: In Coulomb gauge it is simple to construct color nonsinglet physical states; an example is

$$\Psi_q^a = q^a(x)\Psi_0, \tag{2.5}$$

where Ψ_0 is the vacuum state in Coulomb gauge, and $q^a(x)$ is a heavy quark operator. Whereas the aim of the Kugo-Ojima approach is to prove that the space of physical states consists of only color singlets, the goal in Coulomb gauge is to prove that color nonsinglet states have an energy which is infinite above the vacuum. For heavy quarks, with a lattice regularization understood, we define

$$G(T) = \langle\Psi_q^a|e^{-(H-E_0)T}|\Psi_q^a\rangle \propto \langle\mathrm{Tr}[L(\mathbf{x}, T)]\rangle. \tag{2.6}$$

The energy of the charged state Ψ_q is infinite if $G(T) = 0$, i.e. $\langle\mathrm{Tr}[L]\rangle = 0$, and finite otherwise. This means that the Coulombic field energy of an isolated charge is infinite if the remnant global gauge symmetry associated with the pair of spatially homogeneous transformations $g(0)$, $g(T)$ is unbroken. Conversely, an isolated color charge has finite energy if this remnant symmetry is spontaneously broken.

One can also show that the instantaneous color Coulomb potential between quark-antiquark color charges is given by the logarithmic derivative of the correlator of timelike lines [18]

$$V_{\mathrm{coul}}(R) = -\lim_{T\to 0}\frac{d}{dT}\log[\mathrm{Tr}[L(\mathbf{x}, T)L^\dagger(\mathbf{y}, T)]] \tag{2.7}$$

($R = |\mathbf{x} - \mathbf{y}|$), and this potential is an upper bound on the static quark potential [19]. If $\langle\mathrm{Tr}[L]\rangle \neq 0$, then $V_{\mathrm{coul}}(R)$ is R independent as $R \to \infty$, and therefore nonconfining. This is a further motivation for the use of timelike Wilson lines, in Coulomb gauge, as an order parameter for confinement.

In principle, the color Coulomb potential can reveal the confining nature of the vacuum even in the presence of dynamical matter fields, because of its instantaneous nature. The color Coulomb potential derives from the nonlocal term in the Coulomb gauge Hamiltonian. When the VEV of this term is evaluated in a state such as

$$\Psi_{q\bar{q}} = \bar{q}^a(\mathbf{x})q^a(\mathbf{y})\Psi_0 \tag{2.8}$$

containing isolated quark-antiquark charges, it accounts for the energy of the associated Coulomb field before the quark-antiquark system has evolved in time, and screening effects due to matter and/or transverse gluon fields have set in. This means that $\mathrm{Tr}[L]$ can work as an order parameter for confinement even when, as in real QCD, there exist dynamical matter fields which break the global center symmetry. Confinement, in this approach, is identified with the phase in which the energy of the Coulomb field due to isolated color-charge sources diverges as the charge separation is taken to infinity. That also means that confinement is tied to the unbroken realization of a specific global subgroup of the gauge symmetry, which remains after fixing to Coulomb gauge.

C. Dual superconductivity

It is an old idea, due originally to 't Hooft and Mandelstam, that the Yang-Mills vacuum is a kind of dual superconductor, in which the roles of the E and B fields are interchanged. It is then electric, rather than magnetic, charges which are confined, and magnetic, rather than electric, charges which are condensed. Magnetic monopoles can exist in gauge theories with compact Abelian gauge groups, and an order parameter for monopole condensation, breaking the dual U(1) gauge symmetry associated with magnetic charge conservation, was introduced in Ref. [20]. The order parameter $\mu(\mathbf{x})$ is a monopole creation operator, which acts on states in the Schrödinger representation by inserting a monopole field configuration $A_i^M(y)$, centered at $\mathbf{y} = \mathbf{x}$ i.e.

$$\mu(\mathbf{x})|A_i\rangle = |A_i + A_i^M\rangle. \tag{2.9}$$

Explicitly, the operator

$$\mu(\mathbf{x}) = \exp\left[i \int d^3y A_i^M(y) E_i(y)\right] \tag{2.10}$$

performs the required insertion. In a non-Abelian SU(N) gauge theory, an Abelian projection gauge must be introduced to single out an Abelian $U(1)^{N-1}$ subgroup, and μ is defined in terms of the gauge fields associated with that subgroup. Details concerning this construction on the lattice, and the numerical computation of $\langle\mu\rangle$, can be found in Ref. [21].

The dual U(1) gauge symmetry, in an Abelian theory containing magnetic charge, is evident from the existence of a conserved magnetic current. Let

$$\tilde{F}_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta} \tag{2.11}$$

be the dual field strength tensor. Then

$$j_\mu^M = \partial^\nu \tilde{F}_{\mu\nu} \tag{2.12}$$

is the conserved magnetic current associated with the dual gauge symmetry. A global U(1) subgroup of this local symmetry is generated by the total magnetic charge operator, and it is shown in Ref. [20] that the μ operator transforms as a magnetically charged object under these global symmetry transformations. Thus, according to Ref. [22], the μ operator is in some sense the dual of the Dirac construction of electrically charged operators in Eq. (1.2). If $\langle\mu\rangle \neq 0$, this signals both monopole condensation, and the associated breaking of a global U(1) gauge symmetry in the dual gauge theory.

As with the Kugo-Ojima and Coulomb conditions, monopole condensation can be put forward as a confinement criterion whether or not there are dynamical matter fields in the theory, and whether or not global center symmetry is broken. Like the other two criteria, the condition that $\langle\mu\rangle \neq 0$ is tied to the spontaneous breaking of a global subgroup of some local gauge symmetry. Although we will not directly investigate the μ operator here, we believe that the general issues we raise in connection with spontaneous gauge-symmetry breaking apply to this approach as well. The Kugo-Ojima, Coulomb gauge, and dual-superconductor order parameters for confinement are all very well motivated, and each is associated with the way in which some global gauge symmetry is realized. But what if, in practice, these criteria disagree with one another in identifying the boundary between the Higgs and the confinement phases? Which symmetry is the "right" one, in terms of identifying physically distinct phases? This question is reserved for the concluding section; we first need to show that the location of gauge-symmetry breaking is, in fact, dependent on the choice of the global subgroup.

III. REMNANT SYMMETRY BREAKING IN COULOMB AND LANDAU GAUGES

The order parameter for remnant symmetry breaking in Landau gauge is straightforward. In Landau gauge, the remnant symmetry is broken if the magnitude of the spatial average of the Higgs field is nonzero in the infinite volume limit. Denoting the spatial average as

$$\tilde{\phi} = \frac{1}{V}\sum_x \phi(x), \tag{3.1}$$

we define[2]

$$\tilde{Q}_L = \tfrac{1}{2}\mathrm{Tr}[\tilde{\phi}\tilde{\phi}^\dagger], \qquad Q_L = \langle\tilde{Q}_L\rangle, \tag{3.2}$$

where V is the lattice 4-volume. The global remnant symmetry is unbroken if and only if $Q_L \to 0$ as $V \to \infty$. In fact, it is easy to see that if the symmetry is unbroken, and the Higgs field has a finite correlation length in Landau gauge, then

$$Q_L \propto \frac{1}{V}, \tag{3.3}$$

whereas $Q_L \to \mathrm{const} > 0$ as $V \to \infty$ in the broken phase.

In Coulomb gauge there is a larger remnant gauge symmetry, in which gauge transformations $g(\mathbf{x}, t) = g(t)$ which are constant in the spatial directions can nevertheless vary in time. We can use the timelike lattice link variables $U_0(x)$ as order parameters for this symmetry breaking, as previously proposed in [10], since $\mathrm{Tr}[U_0]$ is sensitive to symmetry transformations $g(t)$ which depend on t, but is invariant with respect to transformations which are also constant in the time direction. On the lattice, the logarithm of the U_0 correlator has also been used, in accordance with Eq. (2.7), to calculate the color Coulomb potential [18]. Denoting the spatial average of timelike links on a time slice as

$$\tilde{U}(t) = \frac{1}{V_3}\sum_{\mathbf{x}} U_0(\mathbf{x}, t), \tag{3.4}$$

where V_3 is the 3-volume of a time slice, we define[3]

$$\tilde{Q}_C = \frac{1}{L_t}\sum_{t=1}^{L_t}\frac{1}{2}\mathrm{Tr}[\tilde{U}(t)\tilde{U}^\dagger(t)], \qquad Q_C = \langle\tilde{Q}_C\rangle. \tag{3.5}$$

In the unbroken phase, assuming finite-range correlations among the timelike links at constant t,

[2] This operator was applied previously by Langfeld [23] to determine global gauge-symmetry breaking transitions in SU(2) and SU(3) gauge-Higgs theories, fixed to Landau gauge. The models studied in that work used Higgs fields of variable modulus, so the transition points are not directly comparable to our data.

[3] Note that this differs slightly from the observable proposed in [10], which defines Q_C by taking the square root of the trace.

$$Q_C \propto \frac{1}{V_3}, \tag{3.6}$$

while Q_C converges to a nonzero constant, in the broken phase, in the infinite volume limit.

The phase structure of the SU(2) gauge-Higgs model, sketched in Fig. 1, is reflected in plots of the plaquette expectation value P vs γ, as shown in Figs. 2(a) and 2(b), which are taken from Ref. [8]. For $\beta > 2$, we find a sudden rise in P at some value of γ, as seen, e.g., in Fig. 2(a) for $\beta = 2.2$. The data at this coupling indicate either a weak first-order transition, at $\beta = 2.2$, $\gamma = 0.84$, or possibly just a sharp crossover. The evidence for the first-order nature of the transition, for β values above this coupling, is given in Ref. [7]. Below $\beta \approx 2$ [see Fig. 2(b) at $\beta = 1.2$] there is no indication, in the P vs γ data, of any nonanalytic behavior in the observable, as expected from the Fradkin-Shenker-Osterwalder-Seiler theorem.

We will now display our evidence that, for fixed $\beta < 2$, there is a transition in Q_C and Q_L away from zero, in the infinite volume limit, to some nonzero value, but that this transition happens at different couplings γ for the Coulomb and Landau order parameters.

Figure 3 is a plot of Q_L and Q_C vs γ at $\beta = 1.2$, on a hypercubic lattice of volume 14^4. At low γ both Q_C and Q_L are very small, and cannot be distinguished from zero on the scale of the graph. At some γ both Q_C and Q_L rise rapidly away from zero, indicating a nonzero value in the infinite volume limit. However, this rise begins at *different* values of γ for the two observables.

Figure 4(a) is a log-log plot showing the dependence of Q_L on the lattice extension L, with $L = 6$, 8, 10, 12, and 14. The coupling $\beta = 1.2$ is fixed, and we show results for several γ values. The straight lines are a best fit of the data to

$$Q_L = \frac{\kappa}{L^4}. \tag{3.7}$$

We see that the fit is quite good at the lower γ values, which supports the extrapolation to $Q_L = 0$ in the infinite volume limit. On the other hand, at $\gamma = 1.45$, there is very little falloff in Q_L with lattice volume, indicating a nonzero infinite volume limit. This means that somewhere there is a transition from a phase of unbroken Landau gauge remnant symmetry to a broken phase. Figure 4(b) shows the same type of data for Q_C at $\beta = 1.2$ computed in Coulomb gauge. This time the straight lines on the log-log plot are a best fit to

$$Q_C = \frac{\kappa'}{L^3}. \tag{3.8}$$

Once again, the evidence supports an extrapolation to $Q_C = 0$ at low γ, and a nonzero value at higher γ, implying a transition from an unbroken to a broken phase of Coulomb gauge remnant symmetry. However, the actual Coulomb and Landau gauge transition points must be different. This is illustrated in Fig. 5, where we compute

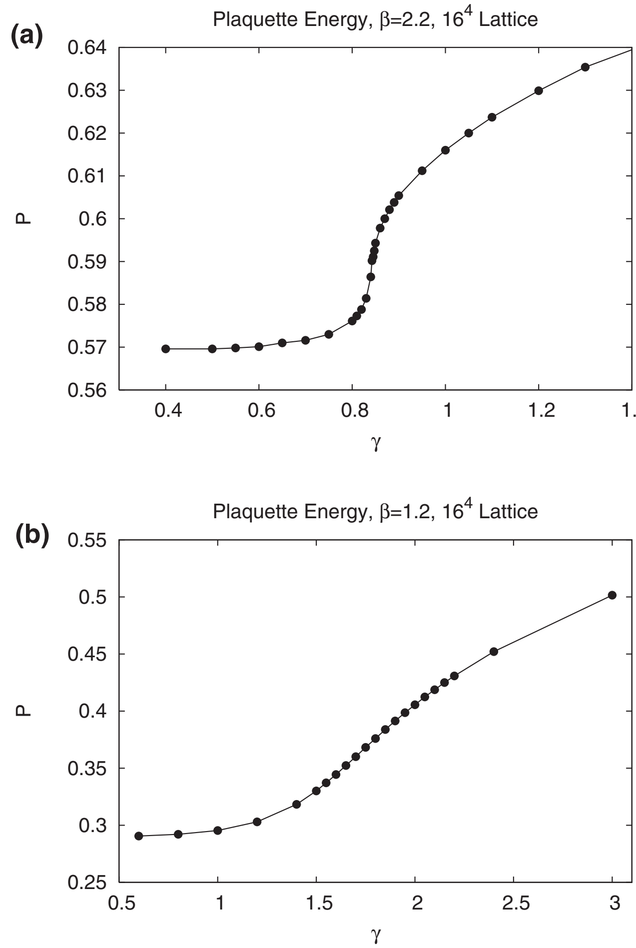

FIG. 2. Plaquette expectation value P vs Higgs coupling γ at gauge couplings: (a) $\beta = 2.2$, either a sharp crossover or a weak first-order transition is seen at $\gamma = 0.84$, and (b) $\beta = 1.2$, no transition is evident. The figures are taken from Ref. [8].

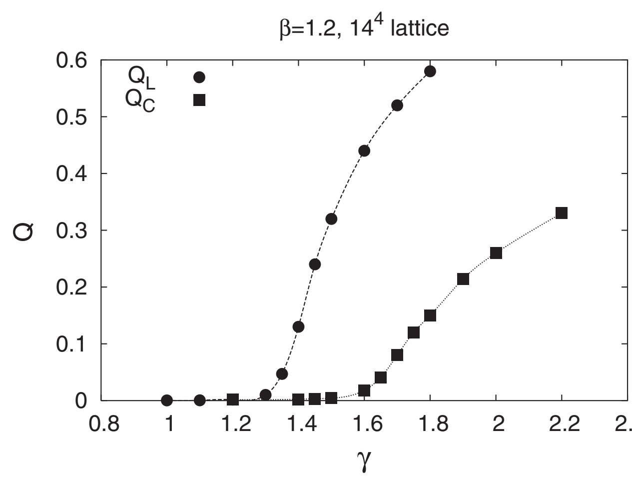

FIG. 3. Order parameters Q_L and Q_C, for global gauge-symmetry breaking in Landau and Coulomb gauge, respectively, vs gauge-Higgs coupling γ at fixed lattice volume 14^4 and gauge coupling $\beta = 1.2$.

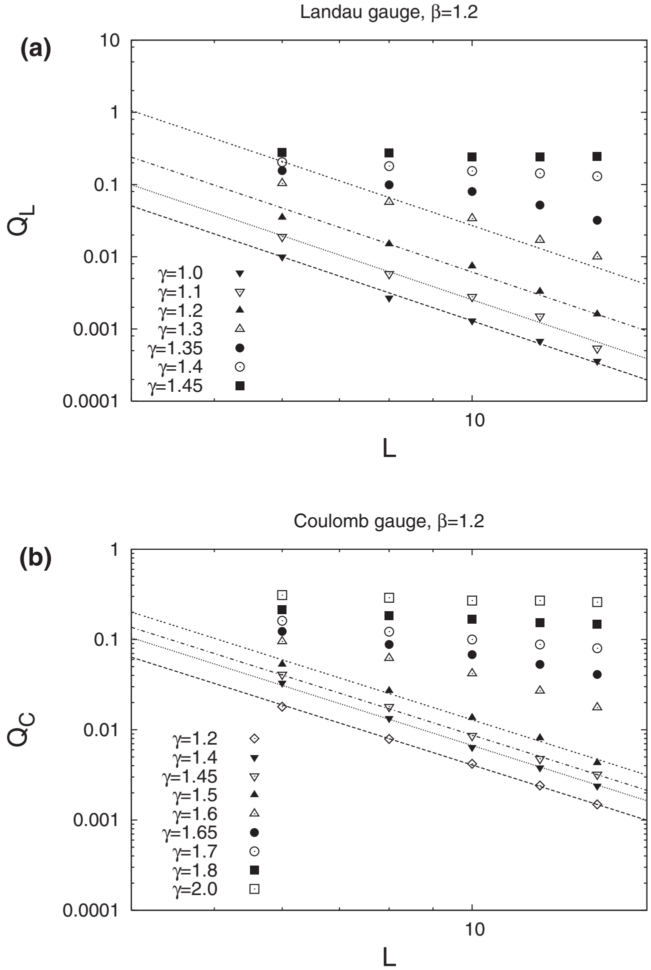

FIG. 4. Log-log plot of the gauge-symmetry breaking order parameters Q_L and Q_C vs lattice extension L, at $\beta = 1.2$ and a variety of gauge-Higgs couplings γ, in (a) Landau and (b) Coulomb gauges. In the Landau and Coulomb gauges the straight lines are a best fit to Eqs. (3.7) and (3.8), respectively.

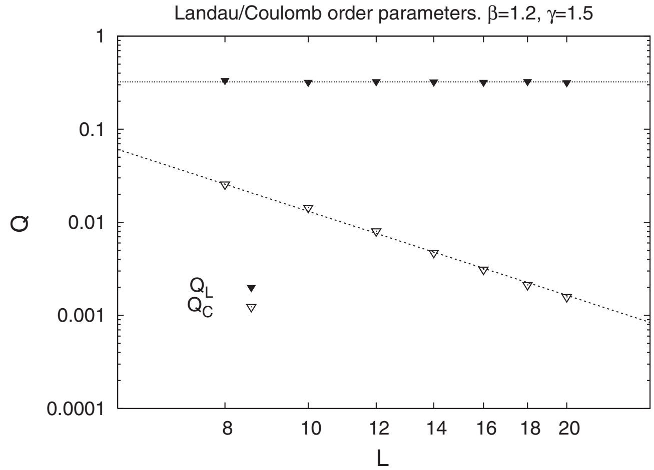

FIG. 5. Q_L and Q_C vs lattice extension L at $\beta = 1.2$ and $\gamma = 1.5$. The upper and lower lines are best fits, the upper to $Q_L = \text{const}$, and the lower to $Q_C = \kappa'/L^3$.

Q_L and Q_C vs lattice extension L at $\beta = 1.2$ and $\gamma = 1.5$, for lattice sizes up to 20^4. Here we see that the observable Q_L is very nearly volume independent, with fluctuations of less than 2% around the average value $Q_L = 0.323$, and the Landau remnant gauge symmetry is broken. On the other hand, the data for Q_C at these same couplings are very well fit by a $1/L^3$ falloff, which implies an unbroken Coulomb gauge remnant symmetry.

In order to improve the accuracy of our determination of the transition point, we follow the procedure of looking for the value of γ where fluctuations in the order parameter are largest.[4] For this we define

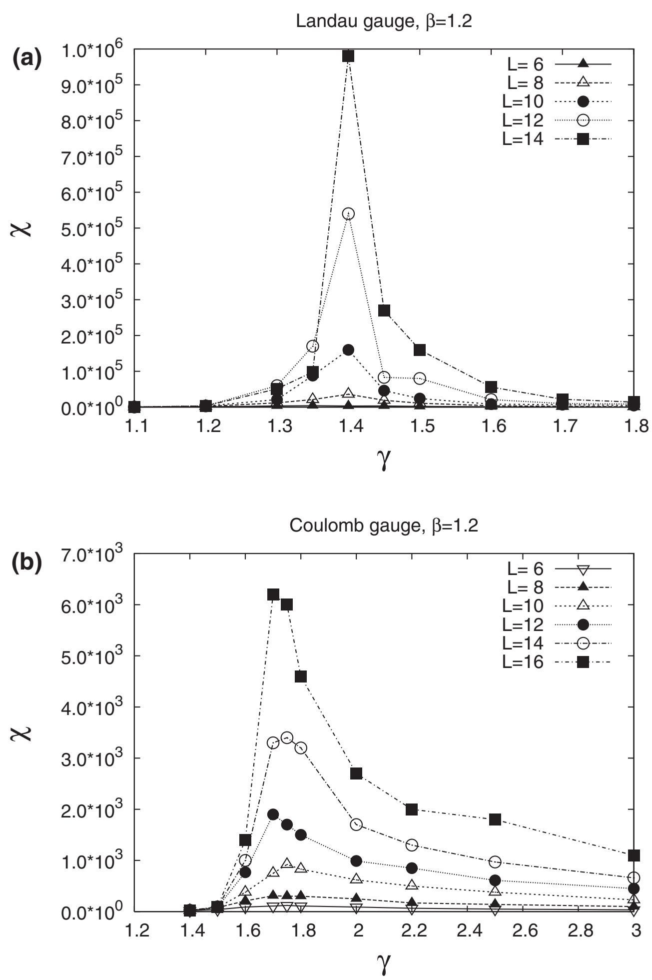

FIG. 6. Susceptibilities χ vs gauge-Higgs coupling γ at fixed $\beta = 1.2$ and a variety of lattice volumes L^4. (a) Landau gauge; (b) Coulomb gauge.

[4]We will not, however, attempt a finite size scaling analysis. The order of the transition is not especially important to us, particularly because there is, at $\beta < 2$, no actual thermodynamic transition. It is enough, for our purposes, to establish that a transition exists, in which $Q \to 0$ below the critical γ, and $Q > 0$ above the critical γ, in the infinite volume limit.

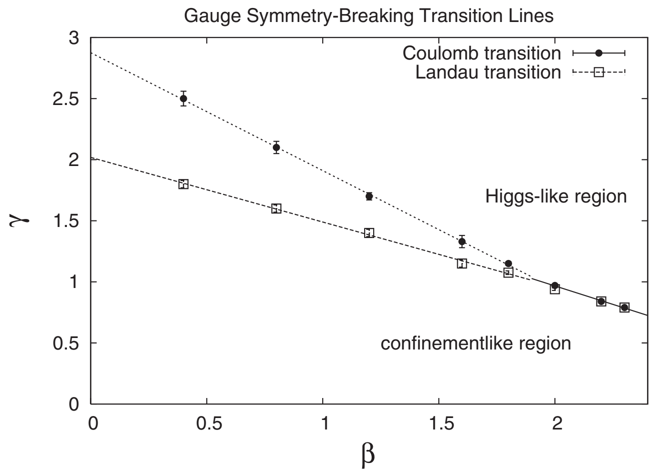

FIG. 7. The location of remnant global gauge-symmetry breaking in Landau and Coulomb gauges, in the $\beta - \gamma$ coupling plane.

$$\chi_L = V^2(\langle \tilde{Q}_L^2 \rangle - Q_L^2), \qquad \chi_C = V_3^2(\langle \tilde{Q}_C^2 \rangle - Q_C^2). \quad (3.9)$$

The overall volume-squared factor in these expressions is chosen so that χ_L and χ_C diverge, in the infinite volume limit, at their respective transition points, but go to a nonzero constant, in the same limit, in the unbroken phase. The unconventional power of the volume is explained in a brief Appendix. Briefly, it is due to the fact that we have defined $\tilde{Q}_{L,C}$ as products of spatially averaged observables, rather than being linear in spatially averaged observables. The results for χ_L and χ_C, respectively, at $\beta = 1.2$, are shown in Figs. 6(a) and 6(b). From this data we locate the remnant symmetry breaking transition points at $\gamma = 1.4$ for Landau gauge, and $\gamma = 1.7$ for Coulomb gauge, with uncertainties on the order of 0.03.

We have applied these methods to determine the Landau and Coulomb remnant symmetry breaking transition points at $\beta = 0.4$, 0.8, 1.2, 1.6, 1.8, 2.0, 2.2, and 2.3, with the results shown in Fig. 7. There is a clear separation of the two transition lines for $\beta < 2$, where there is no thermodynamic transition, while at $\beta > 2$ the symmetry breakings coincide with each other and with the thermodynamic transition/crossover points, within the accuracy of our measurements. This is the central result of our paper. Center vortex percolation/depercolation transitions in the SU(2) gauge-Higgs model were investigated in Ref. [8], and it was found that at $\beta \geq 2$ the percolation transition points also coincide with the thermodynamic transitions, while at $\beta < 2$ the percolation transitions lie above the Coulomb transition line [8].

IV. DISCUSSION AND CONCLUSIONS

We have shown that in the SU(2) gauge-Higgs model there is no unique transition line between unbroken and spontaneously broken gauge symmetry; instead there are different transition lines corresponding to different global subgroups of the local symmetry. Two subgroups, in particular, one associated with the Kugo-Ojima confinement criterion, and the other with the confining color Coulomb potential, are found to have distinct transitions. The order parameters for these two symmetries cannot both be order parameters for the transition from a confinement to a Higgs phase; this seems to be a firm conclusion of our study. In fact, since the particle spectrum consists of only color singlets throughout the phase diagram, and the asymptotic string tension is zero (except at $\gamma = 0$) throughout the phase diagram, it is unclear in exactly what sense a transition in either of these order parameters is associated with a transition to or from a confined phase.

The larger question is whether the breaking of these or *any* global gauge symmetries necessarily indicates a transition between physically different phases in non-Abelian gauge theory. Of course, gauge-symmetry breaking may accompany a change of state when there is a thermodynamic phase transition. But the question is whether gauge-symmetry breaking is *always* accompanied by a change of physical state, even when the thermodynamic transition is absent.

On the basis of the Fradkin-Shenker-Osterwalder-Seiler theorem, there is a compelling case that no transition exists in the SU(2) gauge-Higgs model from a Higgs phase to a physically distinct "confinementlike" phase, which includes the strong-coupling region. If we consider any two points $(\beta_1, \gamma_1) \ll 1$ and $(\beta_2, \gamma_2) \gg 1$ in the coupling-constant plane, then there is always a path between them along which the VEV of all local gauge-invariant observables vary analytically, and Green's functions constructed from such observables vary analytically. As a consequence, the free energy and the spectrum vary analytically. Moreover, the usual order parameters for confinement, i.e. the asymptotic string tension (which vanishes) and Polyakov lines (which do not vanish) exhibit nonconfining behavior throughout the coupling-constant plane, for any $\gamma > 0$. There is simply no evidence for, and strong evidence against, any abrupt change separating the Higgs region from the strong-coupling confinementlike region. So the fact that global gauge symmetries do break spontaneously in gauge-Higgs theory at small β, with different symmetries breaking in different places in the coupling-constant plane, makes it very unlikely that spontaneous breaking of these global gauge symmetries necessarily correspond to a change in physical state.

It is worth noting, in passing, that the absence of an isolated Higgs phase in SU(2) gauge-Higgs theory also makes it clear that there is no fundamental distinction between string breaking by pair production of scalar particles, and the screening of color charge by a scalar field "condensate." Along a path in the $\beta - \gamma$ plane which continuously interpolates between the confinementlike and Higgs-like regions, the two effects must smoothly morph into one another.[5]

The dual Abelian global gauge symmetry, probed by the monopole operator (2.9) associated with dual superconductivity, has not yet been investigated in SU(2) gauge-Higgs theory. However, there are already some indications, in G(2) gauge theory, that spontaneous breaking of the dual gauge symmetry is not necessarily accompanied by a change of physical state. In G(2) lattice gauge theory there is known to be a point of rapid crossover, where the plaquette action rises very sharply as β increases, but which does not appear to be accompanied by an actual thermodynamic transition [24]. The monopole operator μ, or more precisely the logarithmic derivative $\rho = d\log(\mu)/d\beta$ of that operator, has been studied in G(2) gauge theory by Cossu *et al.* [25], and preliminary numerical evidence suggests that the dual global gauge symmetry breaks at the crossover point, despite the absence of any actual change in the physical state at that coupling. The signal of a transition in the monopole operator, according to previous studies [21], is a large negative peak in ρ at the transition point, which grows with lattice volume, and this is found to be the case at the crossover point at $\beta = 7/g^2 = 9.44$. There is also a slight peak in ρ found at the deconfinement transition ($\beta = 9.765$ for $L_t = 6$ lattice spacings in the time direction), but this is tiny compared to the peak at the crossover point. If there is indeed a transition in μ at the G(2) crossover coupling, that would be in line with what we have found for remnant gauge symmetries in Landau and Coulomb gauges: these symmetries break at points where there is no actual change of phase.

There is still the question of whether there is any other symmetry which distinguishes confined from unconfined phases. The answer hinges on what is meant by the word confinement (cf. Ref. [26]). If all it means is that the asymptotic particle states are color singlets, then there is really no "unconfined" phase in gauge-Higgs theory, at any coupling, whose symmetry could be contrasted with the confined phase. If one chooses to define confinement in this way, then the existence of a linear static quark potential is a separate, and to some extent independent, issue. There is, however an alternative definition of confinement, which we prefer: *Confinement is the phase of magnetic disorder.* "Magnetic disorder" means the existence of vacuum fluctuations strong enough to disorder, i.e. induce an area-law falloff in, Wilson loops at arbitrarily large scales. SU(2) gauge-Higgs theory is not in a magnetically disordered phase at any $\gamma > 0$. There is always some cutoff length scale beyond which the large vacuum fluctuations, required for the area-law falloff, are no longer found, and the vacuum state is magnetically ordered in the infrared. (This is analogous to the concept of a massless phase: the phase does not exist if the Euclidean propagator of the lightest particle state falls off exponentially at large distances, even if the falloff appears to follow a power law up to some very large, but still finite scale.) A true magnetically disordered vacuum state, with magnetic disorder throughout the infrared region, is only found at $\gamma = 0$, and there is indeed a nongauge symmetry which distinguishes the magnetically disordered phase at $\gamma = 0$ from the ordered phase at $\gamma > 0$. This is the well-known global center symmetry. The linear potential, linear Regge trajectories, and electric flux-tube formation are only found, up to some finite distance scale, at small γ, where the center symmetry is only weakly broken (a situation labeled "temporary confinement" in Refs. [8,26]). As $\gamma \to 0$ and center symmetry is restored, this finite scale goes off to infinity, and magnetic disorder reigns throughout the infrared regime. In theories where the center of the gauge group is trivial, such as G(2) gauge-Higgs theory, a state of true magnetic disorder is never reached, even at $\gamma = 0$.

Let us finally consider an SU(2) gauge-Higgs theory with the Higgs field in the adjoint representation. In this case the Lagrangian is invariant under center symmetry transformations, the symmetry is not broken explicitly by the Higgs field, and this symmetry can break spontaneously in certain regions of the coupling-constant space [27]. The Fradkin-Shenker-Osterwalder-Seiler theorem does not apply in this case, and spontaneous center symmetry breaking is a transition between two physically different phases, only one of which is magnetically disordered. The example is instructive. Center symmetry breaks spontaneously *only* when there is a change in the physical state of the system, and confinement—understood as magnetic disorder at all large scales—is the phase of unbroken center symmetry. Global subgroups of a local gauge symmetry, on the other hand, can break spontaneously even when there appears to be no change of phase whatever, and their relevance to the confinement problem, in our opinion, remains to be firmly established.

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APPENDIX

In order to accurately determine the transition points for breaking Landau and Coulomb global gauge symmetries we have introduced χ_L and χ_C, which, from their defini-

[5]This fact may have implications for the screening of adjoint representation (e.g. gluon) color charge in pure-gauge theories, since that effect is not essentially different from the screening of fundamental-representation color charge by a dynamical matter field. Perhaps adjoint string breaking by gluon pair production can also be thought of as the screening effect of a gluon condensate.

tions, are simply the mean square uncertainties $(\Delta Q_L)^2$ and $(\Delta Q_C)^2$, multiplied by a factor of volume squared. Here we would like to explain why the volume is squared in this definition, while in, e.g., plaquette susceptibility or Polyakov line susceptibility, only a single factor of volume appears, e.g.

$$\chi_{\rm pol} = V_3[\langle P^2\rangle - \langle P\rangle^2], \qquad P = \frac{1}{V_3}\sum_{\mathbf{x}} \mathrm{Tr}[P(\mathbf{x})], \quad \text{(A1)}$$

where $P(\mathbf{x})$ is a Polyakov line at spatial position $\mathbf{x}$ and V_3 is the spatial volume of a time slice. Stated briefly, the reason for the differing powers of volume is that P is defined as the spatial average of a local observable, while $\tilde{Q}_C$ and $\tilde{Q}_L$ are defined as the *product* of spatially averaged observables. It is this fact which leads to the unusual volume squared factors in χ_L and χ_C.

It would be possible, in our case, to define order parameters such as $\mathrm{Tr}[\tilde{\phi}]$ for the Landau gauge, or $\mathrm{Tr}[\tilde{U}_0(t)]$ in Coulomb gauge, which are linear in the spatially averaged observables. However, this seems to us a little unnatural since, in the broken phase, there is no particular preferred direction for $\phi(x)$ or $U_0(\mathbf{x}, t)$ in group space. This is in contrast to Polyakov lines, which cluster around center elements, or plaquette actions, which are simple scalar quantities. For the purpose of mapping the spatially averaged matrix $\tilde{\phi}$ to a scalar, we think it is more natural to consider the squared magnitude $\mathrm{Tr}[\tilde{\phi}\tilde{\phi}^\dagger]$, which has the minor cost of requiring a nonstandard power of volume in the definition of χ_L.

In order to see where this power is coming from, let us begin by considering $\langle P^2\rangle$ which appears in the Polyakov line susceptibility. Suppose we are in the unbroken phase, well away from the transition, and Polyakov lines have some finite correlation length l. Then

$$\begin{aligned}\langle P^2\rangle &= \frac{1}{V_3^2}\sum_{\mathbf{x}}\sum_{\mathbf{y}}\langle \mathrm{Tr}[P(\mathbf{x})]\,\mathrm{Tr}[P(\mathbf{y})]\rangle \\ &= \frac{1}{V_3}\sum_{\mathbf{x}}\langle \mathrm{Tr}[P(\mathbf{0})]\,\mathrm{Tr}[P(\mathbf{x})]\rangle \propto \frac{l^p}{V_3}, \end{aligned} \quad \text{(A2)}$$

where p is some positive power. Therefore, if we multiply this quantity by a single power of V_3, it will go to a finite positive constant in the infinite volume limit. At the phase transition point where $l \to \infty$ in the infinite volume limit, the susceptibility $\chi_{\rm pol}$ also tends to infinity, which pinpoints the transition point. Now we do a similar analysis of $\langle Q_L^2\rangle$

$$\langle Q_L^2\rangle = \frac{1}{V^4}\sum_{x_1}\sum_{x_2}\sum_{x_3}\sum_{x_4}\langle \phi^{ab}(x_1)\phi^{\dagger ba}(x_2)\phi^{cd}(x_3)\phi^{\dagger dc}(x_4)\rangle. \quad \text{(A3)}$$

If the scalar fields have only finite-range correlations, then the main contribution to the positional sums comes from having pairs of points within a correlation length, e.g. $|x_1 - x_2| < l$, $|x_3 - x_4| < l$, or $|x_1 - x_4| < l$, $|x_2 - x_3| < l$. Then

$$\begin{aligned}\langle Q_L^2\rangle &\approx \frac{1}{V^4}\sum_{x_1}\sum_{x_2}\langle \phi^{ab}(x_1)\phi^{\dagger ba}(x_2)\rangle \sum_{x_3}\sum_{x_4}\langle \phi^{cd}(x_3)\phi^{\dagger dc}(x_4)\rangle \\ &\quad + \text{similar terms} \\ &\propto \frac{1}{V^4}(Vl^q)(Vl^q) \\ &\propto \frac{l^{2q}}{V^2}, \end{aligned} \quad \text{(A4)}$$

where q is a positive power. The same considerations show that $\langle Q_L\rangle^2$ is also $O(l^{2q}/V^2)$. This time, multiplying these quantities by volume *squared* in the definition of χ_L results in a quantity which is finite and nonzero in the infinite volume limit, and only tends to infinity in the disordered phase, as $V \to \infty$, as one approaches the transition point where $l \to \infty$. Similar arguments apply to χ_C.

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