Functional approach to the fermionic Casimir effect

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We use a functional approach to calculate the Casimir energy due to Dirac fields in interaction with thin, flat, parallel walls, which implement *imperfect* baglike boundary conditions. These are simulated by the introduction of δ-like interactions with the walls. We show that, with a proper choice for the corresponding coupling constants, bag-model boundary conditions are properly implemented. We obtain explicit expressions for the energies in $1+1$ and $3+1$ dimensions, for massless and massive fields.

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I. INTRODUCTION

In recent years, a renewed interest in the Casimir effect [1] emerged, as a consequence of new, refined experimental techniques [2], which raised the required standards of the theoretical calculations. They were usually based on gross simplifying assumptions, mostly about the properties of the mirrors. To explain the experimental results, it is then important to use more accurate models, including corrections due, for example, to the imperfect nature of the mirrors.

In this paper we are concerned with the calculation of the fermionic Casimir effect for imperfect "mirrors," in a functional integral framework. In a fermionic context, the mirrors are meant to partially reflect the fermionic current; including as a particular case the "bag" boundary conditions, where the current is completely reflected.

From a technical point of view, we shall use an adapted version of a previously used functional approach [3], to cope with the case of fermionic fields and imperfect boundaries. In the fermionic context, this method had already been applied to the case of fermions in $2+1$ dimensions satisfying bag conditions on a curve and coupled to a gauge field [4].

To cope with imperfect mirrors, which will occupy parallel planes, we follow [5] to introduce a coupling of the bilinear $\bar{\psi}\psi$ to a δ-function potential of the proper coordinate. We shall then calculate the resulting Casimir energy as a function of the distance between two mirrors, using to that effect a functional formalism which maps the original problem to a one-dimensional one, where the fields live on the surfaces of the mirrors. We show that exact results may be obtained by for $1+1$ and $3+1$ dimensions; for the $1+1$ dimensional case our result in the perfect bag case agrees with the one of [5], in spite of the fact that the treatment of the problem is quite different; in particular, the description of the interaction cannot be mapped in a straightforward manner from one calculation to the other.

This article is organized as follows. In Sec. II we introduce the method in general; in Sec. III we apply it to the calculation of the Casimir energy for two imperfect mirrors in $1+1$ and $3+1$ spacetime dimensions. In Sec. IV we analyze the relationship between this kind of interaction and the bag boundary conditions, which emerge when the coupling constant takes a particular value. Finally, in Sec. V we present our conclusions.

II. THE METHOD

In this section, we shall introduce the method used for the calculation of the vacuum energy, in the presence of "defects" which generate the approximate baglike boundary conditions, for the case of a fermionic field in $D+1$ spacetime dimensions.

Our approach is based on the well-known property that the vacuum energy, E_0, may be obtained from the Euclidean functional $\mathcal{Z} \equiv e^{-\Gamma}$, as

$$E_0 = \lim_{T\to\infty} \frac{\Gamma}{T}, \tag{1}$$

where T is the extension of the (imaginary) time coordinate. On the other hand, in the functional integral representation, $\mathcal{Z}$ is given, for a Dirac field, by

$$\mathcal{Z} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S}, \tag{2}$$

where S, the Euclidean action, will be such that $S = S_0 + S_I$, with

$$\begin{aligned} S_0 &= \int d^{D+1}x \bar{\psi}(x)(\not{\partial} + m)\psi(x), \\ S_I &= \int d^{D+1}x \bar{\psi}(x) V(x) \psi(x), \end{aligned} \tag{3}$$

which are the free and interaction actions, respectively.

In the last equation, $V(x)$ denotes a "potential", introduced in order to simulate the boundary conditions. Although we shall mostly consider just the cases of one or two defects, we write (for the sake of generality) its expression for a system with N equally-spaced defects:

$$V(x) = g \sum_{\alpha=0}^{N-1} \delta(x_D - a_\alpha), \tag{4}$$

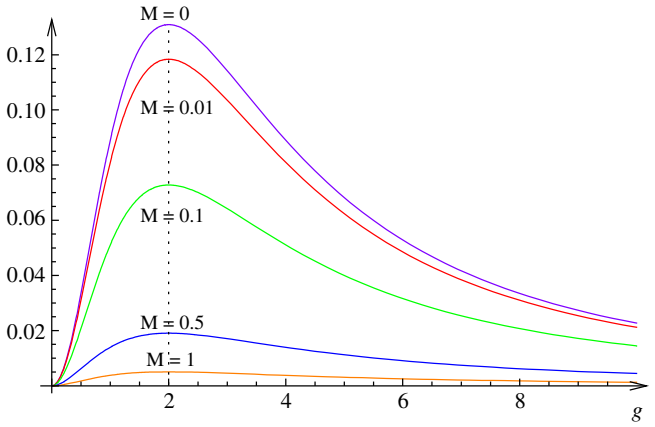

FIG. 1 (color online). c_1 as a function of g, for different values of $M = mL$.

where $a_\alpha = \alpha L$ and $\alpha = 0, 1, \ldots, N-1$. g is the coupling constant, whose role in imposing an approximate boundary condition is discussed in Sec. IV. It is worth noting here that, as mentioned in [5], Dirac's equation with this kind of potential is not well defined. However, also in [5], a consistent calculation of the Casimir energy for a Dirac field in $1+1$ dimensions could still be performed, since one can set up the problem in terms that avoid the calculation of the eigenstates of the Dirac Hamiltonian. We shall here use an approach that also bypasses the eigenproblem for the Dirac Hamiltonian, relying instead on the corresponding Euclidean propagator. The reason this can be done is, of course, that the vacuum energy may be expressed as a function of the expectation values of fermionic bilinears which are, at least in this case, well defined even when the eigensystem is not.

Of course, the total energy for this kind of potential will be, since there is translation invariance along the coordinates parallel to the surface of the mirrors, proportional to the "area" of each defect; thus we will be interested in evaluating energy densities, or pressures, rather than the extensive quantity E_0 (except the $D = 1$ case where the two quantities coincide).

In D spatial dimensions, the defects are codimension 1 hyperplanes. Putting the system in a $d \equiv D-1$-dimensional "box" of side a, we obtain the vacuum energy density $\mathcal{E}_0$ dividing E_0 (calculated for such a box) by a^d, and then taking the $a \to \infty$ limit:

$$\mathcal{E}_0 = \lim_{T,a\to\infty}\left(\frac{\Gamma}{a^d T}\right). \tag{5}$$

We have denoted by x the coordinates in $D+1$-dimensional spacetime $x = (x_0, \ldots, x_D)$. It will turn out to be convenient to introduce also coordinates $y \in \mathbb{R}^{d+1}$, for the subset of the $d+1$ spacetime components that parametrize the αth defect's world volume, as $x = (y, a_\alpha)$. Analogously, momentum-space coordinates in $D+1$ dimensions are denoted by p, while q is reserved for its $d+1$-dimensional restriction:

$$p = (q, p_D), \tag{6}$$

where p_D is the momentum component along the normal direction to the mirrors.

In terms of these conventions, we have

$$e^{-S_I} = \prod_{\alpha=0}^{N-1} e^{-g\int d^{d+1}y\bar{\psi}(y,a_\alpha)\psi(y,a_\alpha)} \tag{7}$$

which, following the methods of [3], may be written in an equivalent way by introducing a pair of auxiliary Grassmann fields $\bar{\chi}_\alpha(y)$ and $\chi_\alpha(y)$ for each defect. The vacuum functional then adopts the form:

$$\mathcal{Z} = \mathcal{Z}_0 \int\Big[\prod_{\alpha=0}^{N-1}\mathcal{D}\chi_\alpha\mathcal{D}\bar{\chi}_\alpha\Big]\exp\Big[\sum_{\alpha=0}^{N-1}\int d^{d+1}y\chi_\alpha^2(y) + g\int d^{D+1}x\int d^{D+1}x'\bar{\eta}(x)G^{(0)}(x,x')\eta(x')\Big], \tag{8}$$

where

$$\eta(x) \equiv \sum_{\alpha=0}^{N-1}\chi_\alpha(y)\delta(x_d - a_\alpha), \tag{9}$$

while $\mathcal{Z}_0$ is the vacuum functional in the absence of defects:

$$\mathcal{Z}_0 = \int\mathcal{D}\psi\mathcal{D}\bar{\psi}e^{-S_0} = \det(\not{\partial} + m). \tag{10}$$

On the other hand,

$$G^{(0)}(x,x') = \langle x|(\not{\partial} + m)^{-1}|x'\rangle, \tag{11}$$

is the free fermion propagator. Note that the auxiliary fields are naturally defined on each one of the mirrors' world volumes.

Since we are interested in calculating the dependence of $\mathcal{E}_0$ on the positions, a_α, of the defects, we shall discard any constant which is independent of those variables. Constant factors in the functional integral, like $\mathcal{Z}_0$, generate precisely that kind of constant, which we shall therefore ignore (without bothering to rename the corresponding subtracted quantity).

Replacing (9) into (8), we may write the latter in the more explicit form:

$$\mathcal{Z} = \int\Big[\prod_{\alpha=0}^{N-1}\mathcal{D}\chi_\alpha\mathcal{D}\bar{\chi}_\alpha\Big] \times e^{\sum_{\alpha,\beta=0}^{N-1}\int d^{d+1}y\int d^{d+1}y'\bar{\chi}_\alpha(y)\mathcal{K}_{\alpha\beta}(y,y')\chi_\beta(y')}, \tag{12}$$

where we introduced the matrix kernel

$$\mathcal{K}_{\alpha\beta}(y,y') = \delta_{\alpha\beta}\delta(y-y') + gG^{(0)}(y,a_\alpha,y',a_\beta), \tag{13}$$

where each α, β element is also a matrix in Dirac space.

The formal result of evaluating the integral above is then

$$\mathcal{Z} = \det[\mathcal{K}_{\alpha\beta}(y, y')], \tag{14}$$

where the determinant is meant to affect all, continuum and discrete, variables. This means that

$$\mathcal{E}_0 = -\lim_{T,a\to\infty}\left(\frac{1}{a^d T}\,\mathrm{Tr}\ln\mathcal{K}\right), \tag{15}$$

where "Tr" is the trace over continuous and discrete indices.[1]

Since there is invariance under translations in the y coordinates, we may Fourier transform with respect to them. The kernel becomes then block diagonal in the continuous variables; besides, a proper counting of the modes in the box shows that the $a^d T$ factor is cancelled by an identical one coming from the numerator; thus, in the limit,

$$\mathcal{E}_0 = -\int \frac{d^{d+1}q}{(2\pi)^{d+1}}\,\mathrm{tr}[\ln\tilde{\mathcal{K}}(q)], \tag{16}$$

where the tilde has been used to denote Fourier transformation, and "tr" denotes trace over the α, β and Dirac indices only.

In the following section, we deal with the evaluation of the previous expression for $\mathcal{E}_0$ in the most relevant case, i.e., $N = 2$.

III. CASIMIR ENERGY

Besides the subtraction of the vacuum energy in the absence of the defects, there is another constant to get rid of: the self-energy of the defects. It may be identified as the result of evaluating $\mathcal{E}_0$ for $L \to \infty$. Thus the prescription for such a subtraction, which will render a finite quantity as a result, is to subtract from $\mathcal{E}_0$ its limit when $L \to \infty$. Of course, to give meaning to the expression at intermediate steps, a regularization is introduced.

Since the last subtraction brings into play the self-energy of the defects, we present first $N = 1$, where the self-energy first emerges, and then the $N = 2$ case where we evaluate the energy for the case of two defects, identifying and subtracting the contributions of the self-energies of the two mirrors.

A. One defect

For $N = 1$, the $\tilde{\mathcal{K}}$ matrix has only one element ($\alpha = \beta = 0$), thus there is no need to use α, β indices:

$$\tilde{\mathcal{K}} = 1 + \frac{g}{2}W(q), \qquad W(q) \equiv \frac{m + i\not{q}}{\sqrt{m^2 + q^2}}, \tag{17}$$

so that $\mathcal{E}_0$ becomes

$$\mathcal{E}_0 = \int \frac{d^{d+1}q}{(2\pi)^{d+1}}\,\mathrm{tr}\left\{\ln\left[1 + \frac{g}{2}W(q)\right]\right\}. \tag{18}$$

When $D = 1$, the Euclidean gamma matrices have the single eigenvalues ± 1. Then the vacuum persistence amplitude is

$$\mathcal{E}_0 = -\int \frac{dp_0}{2\pi}\,\ln\left(1 + \frac{g^2}{4} + \frac{gm}{\sqrt{m^2 + p_0^2}}\right), \tag{19}$$

which has a logarithmic UV divergence, easily regularized by using a frequency cutoff.

A lengthier, but quite straightforward calculation shows that, in $D = 3$

$$\mathcal{E} = -2\int \frac{d^3p}{(2\pi)^3}\,\ln\left[1 + \frac{g^2}{4} + \frac{gm}{\sqrt{m^2 + q^2}}\right], \tag{20}$$

which is quadratically divergent. Again, a Euclidean can be used to give meaning to the integral, whose explicit form we do not need: indeed, it will emerge in the next subsection only to be subtracted in order to fix the energy to zero when $L \to \infty$.

B. Two defects

When $N = 2$ the matrix $\tilde{\mathcal{K}}(q)$ is given by

$$\tilde{\mathcal{K}}(q) = \begin{bmatrix} 1 + \frac{g}{2}W(q) & \frac{g}{2}(W(q) - \gamma_D)e^{-L\sqrt{m^2+q^2}} \\ \frac{g}{2}(W(q) + \gamma_D)e^{-L\sqrt{m^2+q^2}} & 1 + \frac{g}{2}W(q) \end{bmatrix}, \tag{21}$$

with W as defined in (17).

Using the property

$$\det\tilde{\mathcal{K}}(q) = \det\tilde{\mathcal{K}}_{00}\,\det\tilde{\mathcal{K}}_{11}\,\det[\mathcal{I} - \tilde{\mathcal{K}}_{00}^{-1}\tilde{\mathcal{K}}_{01}\tilde{\mathcal{K}}_{11}^{-1}\tilde{\mathcal{K}}_{10}] \tag{22}$$

for the determinant of the matrix in (21), we may identify the first two factors above as yielding the self-energies for the defects, as studied in Sec. III A. We first introduce an UV cutoff to give meaning to those two factors (the third factor is convergent) before subtracting the corresponding self-energies. The resulting pressure is

$$\mathcal{E}_0 = -\int \frac{d^{d+1}q}{(2\pi)^{d+1}}\,\mathrm{tr}[\ln\tilde{\mathcal{M}}(q)], \tag{23}$$

with

$$\tilde{\mathcal{M}}(q) \equiv \mathcal{I} - \tilde{\mathcal{K}}_{00}^{-1}\tilde{\mathcal{K}}_{01}\tilde{\mathcal{K}}_{11}^{-1}\tilde{\mathcal{K}}_{10}. \tag{24}$$

[1] Again, a finite spacetime box is introduced before tacking the limit.

Let us now evaluate $\mathcal{E}_0$ above for $D = 1$ and $D = 3$. In one spatial dimension, if we take into account Dirac's indices, $\tilde{\mathcal{K}}_{\alpha\beta}$ is a matrix with two 2×2 blocks, while $\tilde{M}$ is just a 2×2 matrix. A somewhat lengthy but otherwise straightforward calculation shows that the latter has eigenvalues λ_1, λ_2 given by

$$\lambda_1 = 1, \qquad \lambda_2 = 1 + \frac{16g^2q_0^2}{[(4+g^2)\omega(q_0) + 4gm]^2} e^{-2L|q_0|}, \tag{25}$$

where $\omega(q_0) \equiv \sqrt{q_0^2 + m^2}$. Therefore, the trace in the expression for the vacuum energy can be calculated exactly. In the massless case, we can obtain the exact expression for the energy as a function of g, since

$$\mathcal{E}_0(L) = -\frac{1}{\pi}\int_0^\infty dq_0 \ln\left[1 + \frac{16g^2}{(4+g^2)^2} e^{-2Lq_0}\right], \tag{26}$$

or,

$$\mathcal{E}_0 = -\frac{c_1(g)}{L}, \tag{27}$$

where

$$c_1(g) \equiv -\frac{1}{2\pi}\mathrm{Li}_2\left[\frac{-16g^2}{(4+g^2)^2}\right] \tag{28}$$

is a dimensionless constant which determines the strength of the Casimir interaction, and $\mathrm{Li}_n(x)$ denotes the polylogarithm function. In Fig. 1, we plot the dimensionless combination $c_1(g) = -L\mathcal{E}_0$ as a function of the (also dimensionless) variable g, for the massless case, and for different values of $M \equiv mL$, corresponding to the massive case (where there is no closed expression for the energy). Note that there is a maximum at $g = 2$. That value, as explained in the next section, corresponds to exact bag boundary conditions, where the boundaries are more effective and as a consequence the energy reaches it maximum possible value:

$$[\mathcal{E}_0]_{g=2} = -\frac{\pi}{24L}. \tag{29}$$

Note that in the approach of Ref. [5], bag conditions correspond to an infinite coupling constant. The difference is explained, of course, by the different meaning of the coupling constants in both cases. For example, the propagator in our approach includes tadpolelike propagation between the wall and itself, which naturally change the effective value of the coupling constant. Nevertheless, our $g = 2$ agrees with $\lambda \to \infty$.

A fuller explanation of this fact is given in Sec. IV, where we derive the form of the fermion propagator in the presence of the defects, studying its behavior close to the boundary.

As already mentioned, for the massive case the corresponding integral cannot be performed exactly. In Fig. 2

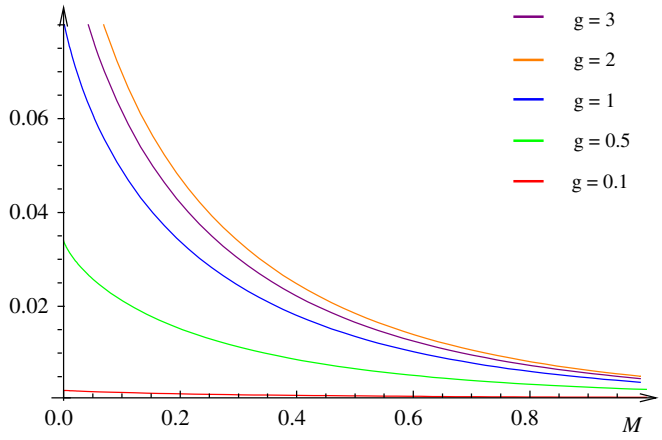

FIG. 2 (color online). c_1 as a function of $M \equiv mL$; $D = 1$.

we plot the result of numerically evaluating that integral for $c_1(g, mL) \equiv |\mathcal{E}_0|L$ as a function of $M = mL$, for different values of g.

In 3 spatial dimensions, each block in $\tilde{\mathcal{M}}$ is a 4×4 matrix. For a massive field, and using the notation $\omega(q) \equiv \sqrt{m^2 + q^2}$, we find that

$$\begin{aligned}\tilde{\mathcal{K}}_{00}^{-1}\tilde{\mathcal{K}}_{01}\tilde{\mathcal{K}}_{11}^{-1}\tilde{\mathcal{K}}_{10} = {} & \frac{4g^2e^{-2\omega(q)L}}{[(4+g^2)\omega(q) + 4gm]^2} \\ & \times [-2q^2\mathcal{I} + gq^2\gamma_D \\ & + i(2m + g\omega(q))\not{q} + i(2\omega(q) \\ & + gm)\not{q}\gamma_D],\end{aligned} \tag{30}$$

with $q = (p_0, p_1, p_2)$, $p_D = p_3$, and $\gamma_D = \gamma_3$.

The exact eigenvalues of $\tilde{\mathcal{M}}$ can be found after some algebra, the result being

$$\lambda_1 = 1, \qquad \lambda_2 = 1 + \frac{16g^2q^2}{[(4+g^2)\omega(q) + 4gm]^2} e^{-2\omega(q)L}, \tag{31}$$

each one with multiplicity equal to two. Then, after integrating out the angular variables, the Casimir energy per unit area may be written as follows:

$$\mathcal{E}_0(g, m, L) = -\frac{c_3(g, mL)}{L^3}, \tag{32}$$

where

$$\begin{aligned}c_3(g, M) \equiv {} & \frac{1}{\pi^2}\int_0^\infty dr r^2 \\ & \times \ln\left\{1 + \frac{16g^2r^2e^{-2\sqrt{r^2+M^2}}}{[4gM + (4+g^2)\sqrt{r^2+M^2}]^2}\right\},\end{aligned} \tag{33}$$

which converges in both the IR and UV regions.

It is convenient to introduce first the result corresponding to the case $m = 0$ and $g = 2$, where the energy reaches a maximum:

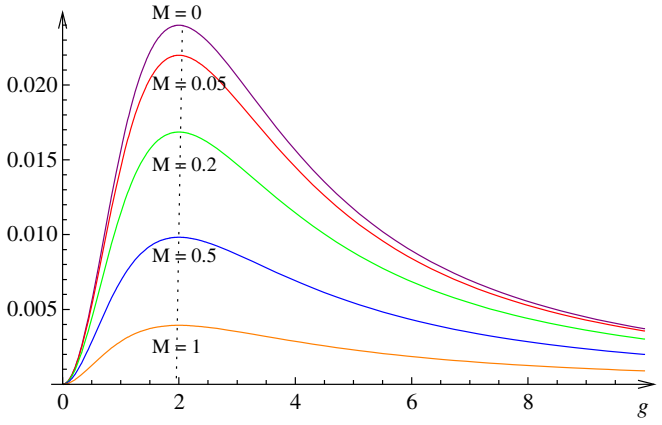

FIG. 3 (color online). c_3 as a function of g, for different values of $M = mL$.

$$\mathcal{E}_0(2, 0, L) = -\frac{7\pi^2}{2880L^3}, \tag{34}$$

in agreement with [6] (see also [7]).

When $m \neq 0$, and introducing $M = mL \neq 0$,

$$L^3\mathcal{E}_0 = \frac{1}{\pi^2}\int_0^\infty dr r^2 \ln\left[1 + \frac{16g^2r^2e^{-2\sqrt{r^2+M^2}}}{[4gM + (4+g^2)\sqrt{r^2+M^2}]^2}\right]. \tag{35}$$

This integral cannot be found analytically, but it has a convenient form for performing it numerically. In Fig. 3, we plot $c_3(g, M) = L^3|\mathcal{E}_0|$ as a function of g for different values of $M = mL$ between 0.1 and 1. We see the expected decreasing behavior of the energy when m grows, regardless of the value of g. This happens because the fermionic field propagator has a faster decay with distance the larger is the mass, reducing the interaction energy between them. On the other hand, the energy always has a maximum when $g = 2$, for any value of the mass. As we mentioned for the $D = 1$ case, this comes from the bag boundary conditions, which are met precisely at this value.

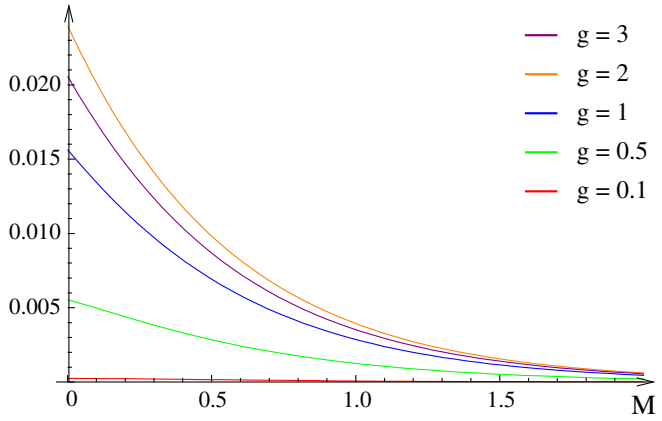

FIG. 4 (color online). c_3 as a function of $M = mL$, for different values of g.

In Fig. 4, we present the complementary view, by plotting c_3 as a function of M, for different values of g between 01. to 3. It can be seen that the energy becomes negligible when $M \simeq 2$ which means that the distance between the plates is of the order of the decay length of the propagator.

IV. THE δ-INTERACTION AND BAG BOUNDARY CONDITIONS

Bag boundary conditions are usually formulated in terms of the fermionic propagator $G(x, x') = \langle\psi(x)\bar{\psi}(x')\rangle$, in the presence of boundaries. For the case of just one defect located at $x_d = 0$, one says that it imposes bag conditions on its two, "right" and "left" faces, when

$$\lim_{x_D \to 0^\pm}(1 \pm \gamma_D)G(x, x') = 0, \tag{36}$$

respectively. They imply that the vacuum expectation value of the normal component of the fermionic current vanishes when approaching the wall from each side. Similar conditions may be imposed, of course, on more than one wall.

Following an analogous procedure to the one used for the calculation of $\mathcal{Z}$, but now for the propagator,

$$\langle\psi(x)\bar{\psi}(x')\rangle = \frac{\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\psi(x)\bar{\psi}(x')e^{-S}}{\int \mathcal{D}\psi\mathcal{D}\bar{\psi}e^{-S}} \tag{37}$$

we obtain

$$G(x, x') = G^{(0)}(x, x') + T(x, x'), \tag{38}$$

where

$$T(x, x') = -g\sum_{\alpha,\beta}\int \frac{d^dq}{(2\pi)^d}e^{-iq(y-y')}\tilde{G}^{(0)}(q, x_D, a_\alpha) \times [\tilde{K}^{-1}]_{\alpha\beta}(q)\tilde{G}^{(0)}(q, a_\beta, x'_D), \tag{39}$$

with

$$\tilde{G}^{(0)}(q, x_D, x'_D) = \int \frac{dp_D}{2\pi}e^{-ip_D(x_D - x'_D)}G^{(0)}(x, x'). \tag{40}$$

The matrix elements of the inverse of $\tilde{K}$ may be written in terms of the matrix elements of $\tilde{K}$:

$$\tilde{\mathcal{K}}^{-1} = \begin{pmatrix} C_0^{-1} & -[\tilde{\mathcal{K}}_{00}]^{-1}\tilde{\mathcal{K}}_{01}C_1^{-1} \\ -C_1^{-1}\tilde{\mathcal{K}}_{10}[\tilde{\mathcal{K}}_{00}]^{-1} & C_1^{-1} \end{pmatrix}, \tag{41}$$

where $[\tilde{\mathcal{K}}_{00}]^{-1}$ and $[\tilde{\mathcal{K}}_{11}]^{-1}$ denote the inverses of the respective matrix elements (not to be confused with the matrix elements of the inverse). We have used the definitions $C_0 \equiv \tilde{\mathcal{K}}_{00} - \tilde{\mathcal{K}}_{01}[\tilde{\mathcal{K}}_{11}]^{-1}\tilde{\mathcal{K}}_{10}$ and $C_1 \equiv \tilde{\mathcal{K}}_{11} - \tilde{\mathcal{K}}_{10}[\tilde{\mathcal{K}}_{00}]^{-1}\tilde{\mathcal{K}}_{01}$.

Let us apply the formulas above, for the sake of simplicity, to the case of massless fermions and only one defect in $D = 1$; we see that, when $x_1 \to 0^+$ and $x'_1 > 0$ we obtain

$$(1 \pm \gamma_1)[\tilde{G}(q,0,x_1') + \tilde{G}(q,0,x_1')] = e^{-|q||x_1'|}\left(\frac{2g}{4+g^2} \mp \frac{1}{2}\right)[\mathcal{I} \mp i\,\mathrm{sg}(q)\gamma_0 \pm \gamma_1 + i\,\mathrm{sg}(q)\gamma_0\gamma_1], \quad (42)$$

where sg is the sign function. Since we are interested in the $+$ sign when approaching the defect from the right, we see that the bag condition is fulfilled when

$$\frac{2g}{4+g^2} = \frac{1}{2} \quad (43)$$

thus, $g = 2$.

On the other hand, when one approaches the defect from the left, one also obtains $g = 2$ to satisfy the corresponding bag condition. An entirely analogous derivation yields the same value for g when the fermions are massive, or when more dimensions are considered. For example, in $2 + 1$ dimensions, and for $m = 0$, the propagator reduces to the one of [8].

V. CONCLUSIONS

We derived general expressions for the vacuum energy in the presence of N "defects," whose role is to impose *imperfect* baglike boundary conditions on parallel planes. Standard bag conditions may be obtained as a particular case. For the case of two plates, we obtain more explicit formulas, which yield the Casimir energy as an integral over a single variable. That integral is both UV and IR finite.

We show that, at any given distance between the plates, the energy reaches a maximum precisely when the bag condition is satisfied.

Our way of implementing the calculation relies on the calculation of the exact fermion propagator in the presence of the defects. This propagator is a perfectly well-defined object, in spite of the fact that the eigensystem for the associated Dirac Hamiltonian is not well defined. This ambiguity has the consequence of allowing for different parametrizations of the strength of the coupling between the Dirac field and the mirrors. In particular, our method produces a fermionic propagator which satisfies bag boundary conditions when the coupling constant $g = 2$. That value corresponds, in the setting of Ref. [5], to an infinite coupling constant.

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