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Low-scale gaugino mass unification

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We study a new class of scenarios with the gaugino mass unification at the weak scale. The unification conditions are generally classified and then, the mirage gauge mediation is explored where the low-energy mass spectrum is governed by a mirage of unified gauge coupling which is seen by low-energy observers. The gaugino masses have natural and stable low-scale unification. The mass parameters of scalar quarks and leptons are given by gauge couplings but exhibit no large mass hierarchy. They are nonuniversal even when mediated at the gauge coupling unification scale. In addition, the gravitino is rather heavy and not the lightest superparticle. These facts are in contrast to existing gauge and mirage mediation models. We also present several explicit models for dynamically realizing the TeV-scale unification.

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I. INTRODUCTION

Supersymmetry is one of the most attractive frameworks for exploring theoretical and phenomenological aspects of possible extensions of the standard model (SM) [1]. Supersymmetry is expected to be broken around the electroweak scale. That is strongly suggested by the supersymmetric unification of the SM gauge coupling constants. The gauge coupling unification is obtained from the precise measurement of coupling constants in the low-energy regime [2] and then probes the existence of unification hypothesis in the high-energy fundamental theory.

The search for supersymmetry will be performed in various near future experiments, e.g., through the dark matter probe of the lightest superparticle. A more direct consequence of low-energy supersymmetry is the observation of superpartners of the SM particles in the forthcoming Large Hadron Collider, and the most important observable quantity is the mass spectrum of new particles. The masses of superparticles are generally expressed by soft breaking terms which do not introduce quadratic divergences [3]. These soft breaking terms consist of gaugino masses, scalar masses, and scalar trilinear couplings. They are induced by supersymmetry-breaking dynamics in high-energy fundamental theory and are forced to have some special properties in order to satisfy low-energy experimental constraints, e.g., from flavor-changing rare processes [4] and *CP* violation [5]. To this end, various scenarios of supersymmetry breaking have been proposed in the literature, and each of them predicts individual and distinctive signatures which would be observed in coming experiments.

In this paper, we explore a new type of low-energy mass spectrum of superparticles, where the spectrum around the electroweak scale is predictive and directly written down in terms of high-energy quantities, and under the hypothesis of gauge coupling unification in fundamental theory, gaugino masses are unified in the low-energy regime. Furthermore, scalar quarks and leptons have no mass hierarchy among them and also their masses are comparable in size to those of gauginos. The situation that the low-energy spectrum tends to be degenerate is similar to the supersymmetry-breaking model [6,7] related to the moduli stabilization in the string theory. However our approach in this paper is the field-theoretical construction of supersymmetry breaking and its mediation which induces such a type of mass spectrum. The scenario is basically the gauge-mediated supersymmetry breaking [8] in which the threshold of messenger fields is affected by the super-Weyl anomaly mediation [9,10] in supergravity. Therefore it shares the phenomenological virtues with the gauge mediation, for example, the suppression of serious higher-dimensional operators including supersymmetry-breaking fields. The mass spectrum is, however, rather different from existing scenarios and induces distinctive phenomenology in particle experiments and cosmology. In particular, the spectrum of scalar quarks and leptons is determined by gauge charges and not universal even when they are mediated at the gauge coupling unification scale. The low-energy gaugino mass unification is unaffected by changing the supersymmetry-breaking scale and also by the existence of multiple thresholds. These facts are in contrast to the model in Refs. [6,7]. On the other hand, the gravitino is heavy and not the lightest superparticle, which is different from typical gauge mediation models.

This paper is organized as follows. In Sec. II, we first study the low-energy unification of gaugino masses in the simplest case with the universality assumption. The general form of gaugino masses with the low-energy unification is derived in Sec. III and its pattern is briefly classified in Sec. IV. We also discuss in Sec. V the low-energy unification in the presence of multiple threshold scales of messenger fields and, in particular, examine whether the unification scale is destabilized or not. Section VI contains the formulas for supersymmetry-breaking parameters of scalar fields. From Sec. VII, we focus on the gauge mediation scenario with low-scale gaugino mass unification. In Sec. VII, we derive the general formula of mass spectrum and discuss phenomenological aspects of the scenario. In Sec. VIII, the unification scale is supposed to be a TeV scale, and various dynamical realizations of TeV-scale unification are investigated including the effect of uplifting the vacuum energy. The last section is devoted to summarizing the results and some discussion of phenomenology.

II. MIRAGE MEDIATION

Throughout this paper, M_X denotes the scale at which we have soft supersymmetry-breaking parameters generated by some high-energy dynamics, e.g., from supergravity interactions, strongly-coupled gauge sector, etc. In this section, we focus on the property of gaugino mass M_λ. Its general form at the scale M_X is parametrized as follows:

$$M_\lambda(M_X) = M_\lambda^X + \frac{bg^2(M_X)}{16\pi^2}F_\phi, \tag{2.1}$$

where g and b are the gauge coupling constant and the one-loop beta-function coefficient of the corresponding gauge theory, $dg/d\ln\mu = bg^3/16\pi^2$ where μ is the renormalization scale. The first term in (2.1) is the above-mentioned supersymmetry-breaking effect from high-energy dynamics. It is given at tree or loop level of coupling constants in the theory and generally depends on the energy scale: $M_\lambda^X = M_\lambda^X(M_X)$. The second term is called the anomaly mediation effect [9,10] and comes from the one-loop contribution of the super-Weyl anomaly in supergravity. The latter effect generally exists in any supersymmetry-breaking theory and must be taken into account. The F_ϕ contribution is usefully expressed in terms of the compensator chiral multiplet Φ in the conformal supergravity [11] and its value is given by fixing the superconformal gauge transformation such that $\Phi = 1 + F_\phi\theta^2$ in the conformal frame.

The one-loop renormalization group for the gaugino mass below the scale M_X is evolved down to the low-energy regime as

$$\begin{aligned} M_\lambda(\mu) &= M_\lambda(M_X)\frac{g^2(\mu)}{g^2(M_X)} \\ &= M_\lambda^X\left[1 + \frac{bg^2(\mu)}{16\pi^2}\ln\left(\frac{\mu^2}{M_X^2}\right) + \frac{bg^2(\mu)}{16\pi^2}\frac{F_\phi}{M_\lambda^X}\right]. \end{aligned} \tag{2.2}$$

Here an important scale μ_m is introduced at which the running effect [the second term in (2.2)] and the anomaly mediation effect [the third term in (2.2)] are cancelled out. Note that the complex phases of two contributions in (2.1) must be aligned in order to have a real-valued μ_m. From (2.2), we obtain

$$\mu_m = M_X \exp(-F_\phi/2M_\lambda^X), \tag{2.3}$$

and the gaugino mass at this scale reads

$$M_\lambda(\mu_m) = M_\lambda^X. \tag{2.4}$$

The scale μ_m is determined only by the ratio of two SUSY-breaking effects, and the gaugino mass at μ_m is given by the contribution of high-energy dynamics, exclusive of the anomaly mediation. It is interesting that we directly observe in low-energy particle experiments the high-energy effect of supersymmetry breaking in fundamental theory without being disturbed by ambiguous renormalization-group effects.

As in the supersymmetric extensions of SM, there are generally several numbers of gauge groups in a theory. The mirage mediation, the unification of gaugino masses (more generally, of superparticle masses) at a low scale, is derived by the assertion that the scale μ_m can be defined independently of the gauge groups considered. This condition is found from (2.3) to require that M_λ^X's are universal

$$M_{\lambda_a}^X = (a - \text{independent}). \tag{2.5}$$

That is, the mirage mediation scale μ_m can be obtained in the case that high-scale dynamics generate the universal boundary value for different gauginos. The condition does not need any details of M_λ^X. Furthermore, if the universality (2.5) is realized, Eq. (2.4) means that gaugino mass parameters at the mirage scale, $M_{\lambda_a}(\mu_m)$, take the unified value ($= M_\lambda^X$). Consequently, the mirage mediation is found to imply the mirage unification (of gaugino masses). The gauge coupling unification is not necessarily needed and the only assumption is to have the universal gaugino masses from some high-energy dynamics. For example, the universal contribution comes from moduli fields in supergravity or string theory. In particular, a recent scenario of string-theory moduli stabilization [12] is known to predict a suppressed value of M_λ^X relative to F_ϕ and then a hierarchically (exponentially) small scale μ_m can be naturally realized [see Eq. (2.3)], which provides a characteristic framework for low-energy phenomenology [13].

It seems however that the universality (2.5) is only a sufficient condition for the mirage mediation, where high-scale effects directly appear as if by a projected mirage in a low-energy regime. In what follows, we investigate more general situations for the mirage unification to occur.

III. GENERAL MIRAGE UNIFICATION

The nonuniversality of the superparticle spectrum is often generated by high-energy physics at the mediation scale M_X. In this case, it is a nontrivial issue to study what conditions are implied by asserting the mirage unification (of gaugino masses) at a low-energy scale. From the discussion in the previous section, it is a naive expectation that the mirage unification takes place if gaugino masses from high-energy physics "unify" at some scale (except for the anomaly mediation effect). Notice however that this unification scale is virtual and not the mediation scale M_X.

Let us consider the simplest situation that there is one threshold of supersymmetry-breaking dynamics at M_X. The general unification of gaugino masses, $M_{\lambda_a} = M_{\lambda_b}$, is derived from the low-energy renormalization-group evolution,

$$\left[M_{\lambda_a}^X + \frac{b_a g_a^2(M_X)}{16\pi^2}F_\phi\right]\frac{g_a^2(\bar{\mu}_m)}{g_a^2(M_X)} = \left[M_{\lambda_b}^X + \frac{b_b g_b^2(M_X)}{16\pi^2}F_\phi\right]\frac{g_b^2(\bar{\mu}_m)}{g_b^2(M_X)}, \tag{3.1}$$

where $\bar{\mu}_m$ is the gaugino mass unification scale and the gauge coupling constants at this scale are given by

$$\frac{1}{g_x^2(\mu_m)} = \frac{1}{g_x^2(M_X)} + \frac{b_x}{16\pi^2}\ln\left(\frac{M_X^2}{\mu_m^2}\right) \qquad (x = a, b). \tag{3.2}$$

Inserting these values into the above unification equation, we find the unification scale

$$\bar{\mu}_m = M_X \exp\left[\frac{16\pi^2(M_{\lambda_a}^X - M_{\lambda_b}^X) + [b_a g_a^2(M_X) - b_b g_b^2(M_X)]F_\phi}{2b_b g_b^2(M_X)M_{\lambda_a}^X - 2b_a g_a^2(M_X)M_{\lambda_b}^X}\right], \tag{3.3}$$

and the unified value of gaugino masses

$$M_{\lambda_a}(\bar{\mu}_m) = M_{\lambda_b}(\bar{\mu}_m) = \frac{b_a g_a^2(M_X)M_{\lambda_b}^X - b_b g_b^2(M_X)M_{\lambda_a}^X}{b_a g_a^2(M_X) - b_b g_b^2(M_X)}. \tag{3.4}$$

It may be interesting to notice that the anomaly mediation effects are dropped out in the expression of unified gaugino mass, though any cancellation is not assumed between these and renormalization-group effects.

We first study the unification condition that $\bar{\mu}_m$ is independent of the gauge indices a and b. If this condition is satisfied, the scale $\bar{\mu}_m$ is entitled to the mirage unification scale at which more than two gaugino masses take a common value. A trivial solution is the universal contribution $M_{\lambda_a}^X = M_{\lambda_b}^X$ at the threshold scale. In this case, the expressions (3.3) and (3.4) become equivalent to the result of the mirage mediation discussed in the previous section. To look for a more general solution, we rewrite the bracket in the right-handed side of (3.3) as

$$\frac{\frac{16\pi^2}{b_a b_b g_a^2(M_X) g_b^2(M_X)}(M_{\lambda_a}^X - M_{\lambda_b}^X) - [\frac{1}{b_a g_a^2(M_X)} - \frac{1}{b_b g_b^2(M_X)}]F_\phi}{M_{\lambda_a}^X/b_a g_a^2(M_X) - M_{\lambda_b}^X/b_b g_b^2(M_X)}. \tag{3.5}$$

Since the supergravity interactions are universal, the coefficient of F_ϕ must be a, b independent. The renormalization group running above the threshold scale are generally given by

$$\frac{1}{g_x^2(M_X)} = \frac{1}{g_U^2} + \frac{b_x + N_x}{16\pi^2}\ln\left(\frac{M_G^2}{M_X^2}\right) \qquad (x = a, b), \tag{3.6}$$

where $N_{a,b}$ denote the contribution of decoupled fields at the threshold and M_G is the scale at which the two running gauge couplings g_a and g_b meet $[g_a(M_G) = g_b(M_G) \equiv g_U]$. The coefficient of F_ϕ in (3.5) now becomes

$$\frac{1}{b_a g_a^2(M_X)} - \frac{1}{b_b g_b^2(M_X)} = \left(\frac{1}{b_a} - \frac{1}{b_b}\right)\frac{1}{g_U^2} + \frac{1}{16\pi^2} \times \left(\frac{N_a}{b_a} - \frac{N_b}{b_b}\right)\ln\left(\frac{M_G^2}{M_X^2}\right). \tag{3.7}$$

Therefore we should have $N_a = N_b$ ($\equiv N$) as the first condition for unification. With this condition at hand, we finally find that the gauge-factor independence of the exponent (3.5) leads to the common value of gaugino masses,

$$M_{\lambda_a}^X(M_G) = M_{\lambda_b}^X(M_G), \tag{3.8}$$

with which not only the coefficient of F_ϕ but the first term of (3.5) becomes a, b independent. Here the gaugino mass factors $M_{\lambda_x}^X$ above the supersymmetry-breaking scale M_X are virtually defined as if they obey the renormalization-group equations corresponding to (3.6). Namely, the second condition for unification to appear is that, at the scale of gauge coupling unification, the corresponding gaugino masses also (virtually) unify, except for the anomaly mediation effect. The form of the unified value $M_{\lambda_x}^X(M_G)$ has no restriction and is an arbitrary function of g_U and other universal couplings.

In summary, the general mirage unification (of gaugino masses) is achieved in theory with gauge coupling unification and satisfies two conditions: (i) the threshold of supersymmetry-breaking dynamics preserves the gauge coupling unification, and (ii) gaugino masses from supersymmetry-breaking dynamics virtually unify at the scale of gauge coupling unification. As a result, the general form of gaugino masses induced at the supersymmetry-breaking scale is consistent with the low-energy mirage when it satisfies

$$M_\lambda^X = c_0 + c_1 g^2(M_X). \tag{3.9}$$

The coefficients c_0 and c_1 are universal (gauge-factor independent) and, in particular, do not depend on $g(M_X)$. We have also included the trivial solution (the c_0 term), corresponding to the simple mirage mediation if $M_X = M_G$. The scale $\bar{\mu}_m$ of the general mirage unification is written in a parallel fashion to the previous case (2.3) as

$$\bar{\mu}_m = \bar{M}_X \exp(-F_\phi/2\bar{M}_\lambda^X). \tag{3.10}$$

The effective threshold scale $\bar{M}_X$ and the supersymmetry-breaking mass parameter $\bar{M}_\lambda^X$ are defined as

$$\bar{M}_X \equiv M_G\left(\frac{MX}{MG}\right)^{c_0/\bar{M}_\lambda^X}, \qquad \bar{M}_\lambda^X \equiv c_0 + c_1 g_G^2, \tag{3.11}$$

$$\frac{1}{g_G^2} \equiv \frac{1}{g_U^2} + \frac{N}{16\pi^2}\ln\left(\frac{M_G^2}{M_X^2}\right). \tag{3.12}$$

These three quantities, $\bar{M}_X$, $\bar{M}_\lambda^X$, g_G are found not to have gauge-group dependences and so $\bar{\mu}_m$ does denote the low-energy unification scale of gaugino masses. From (3.4) and (3.9), we evaluate the unified value of gaugino masses at this scale,

$$M_{\lambda_a}(\bar{\mu}_m) = M_{\lambda_b}(\bar{\mu}_m) = c_0 + c_1 g_G^2, \tag{3.13}$$

which is just equivalent to the effective boundary mass $\bar{M}_\lambda^X$. This fact implies that the effect of high-energy physics is directly observed in the low-energy regime as the projection of the mirage. It is also found, compared to (3.9), that the low-energy value $M_\lambda(\bar{\mu}_m)$ is equal to the dynamically induced mass M_λ^X by replacing the gauge coupling with g_G. The "coupling constant" g_G represents the effect of decoupled fields and naively seems to depend on the threshold scale M_X. However we can show from the running equations of gauge couplings that low-energy values of gauge couplings are related to g_G as

$$\frac{1}{g_G^2} = \frac{1}{g_x^2(\mu)} + \frac{b_x}{16\pi^2}\ln\left(\frac{\mu^2}{M_G^2}\right) \qquad (x = a, b). \tag{3.14}$$

This equation indicates an important property that g_G is interpreted as the (virtual) unified value of gauge couplings in the absence of any threshold and is determined only by (experimentally) observed values of g_x in the low-energy regime. In particular, g_G does not depend on M_X and therefore, the low-energy unified value of gaugino masses $M_{\lambda_x}(\bar{\mu}_m)$ is also insensitive to the threshold scale. Further, it is interesting to notice that, similarly to the gaugino masses, the gauge coupling constants are also mediated by the mirage from high to low-scale physics: in future particle experiments, we would directly probe the high-energy unified value g_G through the determination of superparticle masses. Probing high-energy physics without being disturbed by intermediate-scale unknown factors will clarify the mechanism of supersymmetry breaking as well as the grand unified theory.

As we have shown, the general mirage unification can be defined even when the spectrum is nonuniversal at the supersymmetry-breaking scale and the coupling unification scale. The general formulas are given by (3.9) and (3.10) with the mirage value of unified gauge coupling which is evaluated only by low-energy observables and independent of supersymmetry-breaking thresholds. One remark is that the unification scale $\bar{\mu}_m$ does not make sense unless the complex phases of c_0 and c_1 terms are aligned to that of the anomaly mediation F_ϕ. That may restrict the possible dynamics of supersymmetry breaking and its mediation.

IV. CLASSIFICATION

The gaugino masses in the scenario of general mirage unification have the form (3.9). We briefly discuss each case separately and comment on possible dynamics of the supersymmetry-breaking mediation sector.

A. $c_0 \neq 0$, $c_1 = 0$

The first simple case is that the c_0 term is dominant. In this case, gaugino masses are universal at the supersymmetry-breaking scale M_X. The universal contribution originates from, e.g., gravitational interactions, moduli fields in high-energy theory, and so on. That results in the simple mirage mediation discussed in Sec. II.

B. $c_0 = 0$, $c_1 \neq 0$

The second case is that the gauge threshold contribution is dominant: $c_0 = 0$ and $c_1 \neq 0$. That is understood as the situation that supersymmetry breaking is mediated by some gauge interactions at the loop level. In this case, M_λ^X's become universal if they were interpolated to the gauge coupling unification scale, and then the discussion returns to the simple mirage mediation with the threshold scale $M_X = M_G$. However, there is one difference that the gauge coupling in the interpolated mass $M_\lambda^X(M_G)$ is not the real value g_U but the virtual one g_G, which represents the deviation due to the presence of supersymmetry-breaking dynamics above M_X. In other words, if one uses the mirage scale formula (2.3) with the interpolated mass $M_\lambda^X(M_G)$, the threshold scale should be modified accordingly.

As an explicit example, let us see the following form of gaugino masses:

$$M_{\lambda_a}(M_X) = \frac{g_a^2(M_X)}{16\pi^2}F + \frac{b_a g_a^2(M_X)}{16\pi^2}F_\phi. \tag{4.1}$$

The superparticle mass spectrum is given by the sum of the gauge and super-Weyl anomaly contributions. The relative complex phase of two F terms, F and F_Φ, should be aligned from a phenomenological analysis of CP violation [14]. A simple dynamical example is the so-called deflected anomaly mediation [15]. The above expression means $c_0 = 0$ and $c_1 = F_X/16\pi^2$, and hence the mirage unification scale is found

$$\bar{\mu}_m = M_G \exp\left(\frac{-8\pi^2}{g_G^2}\frac{F_\phi}{F}\right), \tag{4.2}$$

where the mirage value of unified gauge coupling g_G is determined by the observed values of gauge couplings at a low-energy scale μ,

$$\frac{1}{g_G^2} = \frac{1}{g_x^2(\mu)} + \frac{b_x}{16\pi^2}\ln\left(\frac{\mu^2}{M_G^2}\right). \tag{4.3}$$

C. $c_0 \neq 0$, $c_1 \neq 0$

The last one is the most general case and normally needs two sources of supersymmetry breaking. A simple example is the coexistence of the contributions via supergravity and gauge interactions [16,17] from several supersymmetry-breaking sectors. It is also possible to realize this type of spectrum with a single source of supersymmetry breaking. For this purpose, let us assume the following schematic Lagrangian:

$$\int d^2\theta\left[\left(\frac{1}{4g^2} + \frac{X}{M_{\rm pl}}\right)W^\alpha W_\alpha + (M_\Psi + X)\bar{\Psi}\Psi\right] + \text{H.c.} + (\text{dynamics for } X), \tag{4.4}$$

where X is the representative field of supersymmetry breaking which has a nonvanishing F component, and Ψ, $\bar{\Psi}$ are the vectorlike messenger multiplets. The first term gives a tree-level gravity contribution to gaugino masses of the form of $F_X/M_{\rm pl}$. The second term induces a mass splitting in each messenger multiplet and gives the gauge contribution from a one-loop diagram involving the messenger fields. Thus, the gauge contribution takes the form of $(1/16\pi^2)(F_X/M_\Psi)$. If the messenger mass scale M_Ψ is smaller than the gravity scale $M_{\rm pl}$ by one-loop order quantity, the two contributions of supersymmetry breaking are comparable to each other and equally important for phenomenology such as the modification of low-energy unification scale and superparticle mass spectrum.

V. MULTITHRESHOLDS AND STABILITY OF MIRAGE

In this section, we study the case that there exists multiple threshold scales of supersymmetry-breaking dynamics. In addition to the scale M_X previously discussed, superparticles are supposed to receive the contribution of supersymmetry-breaking masses from different dynamics at M_X', which is assumed to be smaller than M_X without the loss of generality. In particular, we examine whether the mirage unification is spoiled or not in the presence of additional thresholds.

A. Simple mirage case ($c_0 \neq 0$, $c_1 = 0$)

Let us first consider the simple mirage case ($c_1 = 0$) analyzed in Sec. II. We have additional gaugino mass contribution M_λ' at the scale M_X'. It is noted that the net contribution at this threshold is the sum of M_λ' and the supersymmetric contribution which compensates the anomaly mediation. In the low-energy regime ($\mu < M_X'$), the gaugino mass is given by the one-loop renormalization-group flow,

$$\begin{aligned} M_\lambda(\mu) &= [M_\lambda(M_X') + M_\lambda']\frac{g^2(\mu)}{g^2(M_X')} \\ &= (M_\lambda^X + M_\lambda')\left[1 + \frac{b' g^2(\mu)}{16\pi^2}\ln\left(\frac{\mu^2}{M_X'^2}\right)\right] \\ &\quad + \frac{b' g^2(\mu)}{16\pi^2}F_\phi + \frac{b g^2(\mu)}{16\pi^2}M_\lambda^X \ln\left(\frac{M_X'^2}{M_X^2}\right), \end{aligned} \tag{5.1}$$

where b' is the beta-function coefficient of gauge coupling g below the threshold scale M_X'. Repeating the previous analysis, the new scale of low-energy unification is formally written down as

$$\bar{\mu}'_m = \bar{\mu}_m\left(\frac{M_X'}{M_X}\right)^{(b'-b/b')(M_\lambda^X/M_\lambda^X + M_\lambda')}\left(\frac{M_X'}{\bar{\mu}_m}\right)^{(M_\lambda'/M_\lambda^X + M_\lambda')}. \tag{5.2}$$

The unification scale $\bar{\mu}_m$ in the single threshold case has been defined in (2.3). It is found from this expression that, in order for $\bar{\mu}_m'$ to be the unification scale, the following two conditions are additionally required: (i) the threshold contribution M_λ' is universal, and (ii) the ratio of beta functions b/b' is independent of gauge groups. The latter condition is rather restrictive. The general solution to the latter condition is given by $b = b'$ which implies an unrealistic situation that decoupled fields at either threshold are only gauge singlets. Moreover, one notices that $\bar{\mu}_m'$ is no longer a mirage unification scale, even if the threshold contribution is supersymmetric ($M_\lambda' = 0$) or grand unificationlike ($b_a - b_a' =$ universal).

B. Gauge threshold case ($c_0 = 0$, $c_1 \neq 0$)

Another typical case has the contribution of gauge threshold only ($c_0 = 0$), i.e., the scenario with gauge and anomaly-mediated supersymmetry breaking. Let us consider an additional gauge threshold at M_X'. Its form is written down as $M_\lambda' = c_1' g^2(M_X')$ where the coefficient c_1' is universal for different gaugino masses. In the low-energy regime ($\mu < M_X'$), the gaugino mass is given by the one-loop renormalization-group flow,

$$\begin{aligned} M_\lambda(\mu) &= \left[\left(c_1 g^2(M_X) + \frac{b g^2(M_X)}{16\pi^2}F_\phi\right)\frac{g^2(M_X')}{g^2(M_X)} + c_1' g^2(M_X') + \Delta_{\rm AM}(M_X')\right]\frac{g^2(\mu)}{g^2(M_X')} \\ &= (c_1 + c_1')g^2(\mu) + \frac{b' g^2(\mu)}{16\pi^2}F_\phi, \end{aligned} \tag{5.3}$$

where b' is the beta-function coefficient for gauge coupling g below the threshold scale M_X'. The last quantity $\Delta_{\rm AM}$ denotes the supersymmetric threshold correction which preserves the ultraviolet insensitivity of super-Weyl anomaly mediation. It is found that, in the previous expressions for the single threshold case, c_1 should be shifted to $c_1 + c_1'$, and further, M_X and b are replaced with M_X' and b'. The last point we should take into account is the modification of the renormalization-group running of gauge couplings. In the case of multiple thresholds at M_X and M_X', the gauge couplings take the unified value g_U' at M_G',

$$\frac{1}{g_x^2(M_X')} = \frac{1}{g_U'^2} + \frac{b_x}{16\pi^2}\ln\left(\frac{M_X^2}{M_X'^2}\right) + \frac{b_x + N}{16\pi^2}\ln\left(\frac{M_G'^2}{M_X^2}\right) \quad (x = a, b). \tag{5.4}$$

Repeating the previous analysis of mirage unification with this modified running equation, we find that the mirage unification is preserved for the grand unificationlike threshold, that is, $b_a - b_a' = b_b - b_b'$ $(\equiv N')$. At the same time, the gauge coupling unification scale is not modified: $M_G = M_G'$. In the end, the mirage unification scale in the multithreshold case is given by

$$\bar{\mu}'_m = M_G \exp(-F_\phi/2\bar{M}_\lambda^{X'}). \tag{5.5}$$

The effective boundary mass $\bar{M}_\lambda^{X'}$ is defined as

$$\bar{M}_\lambda^{X'} \equiv (c_1 + c_1')g_G'^2, \tag{5.6}$$

$$\frac{1}{g_G'^2} \equiv \frac{1}{g_U'^2} + \frac{N'}{16\pi^2}\ln\left(\frac{M_X^2}{M_X'^2}\right) + \frac{N + N'}{16\pi^2}\ln\left(\frac{M_G^2}{M_X^2}\right). \tag{5.7}$$

Since $\bar{M}_\lambda^{X'}$ and g_G' do not have gauge-group dependences, the new scale $\bar{\mu}'_m$ is properly defined as the mirage unification scale (of gaugino masses). The unified value of gaugino masses at $\bar{\mu}'_m$ is evaluated as

$$M_{\lambda_a}(\bar{\mu}'_m) = M_{\lambda_b}(\bar{\mu}'_m) = (c_1 + c_1')g_G'^2, \tag{5.8}$$

which is equal to the effective boundary mass $\bar{M}_\lambda^{X'}$. Compared with the single threshold case, the virtual coupling g_G seems to be modified to g_G' due to the effect of the additional threshold. However, we can show from (5.4) and (5.7) that low-energy values of gauge couplings ($\mu < M'$) become

$$\frac{1}{g_G'^2} = \frac{1}{g_x^2(\mu)} + \frac{b_x'}{16\pi^2}\ln\left(\frac{\mu^2}{M_G^2}\right) \qquad (x = a, b). \tag{5.9}$$

This equation indicates that g_G' does not depend both on M_X and M_X', i.e., insensitive to the presence of supersymmetry-breaking dynamics. It is also interesting to find that, compared with the single threshold case (3.14), g_G' is equivalent to g_G for fixed low-energy observables, and so equal to the (mirage) unified gauge coupling without any thresholds

$$g_G = g_G'. \tag{5.10}$$

The real unified gauge coupling g_U', of course, becomes different from g_U and sensitive to the presence of thresholds.

In summary, for the gauge threshold case, the mirage unification is preserved even when there exists multiple thresholds of supersymmetry-breaking dynamics. The mirage scale does not explicitly depend on the threshold scales (the messenger mass scales). The only influence of multiple thresholds is the cumulative effect of c_1 terms in gaugino mass. These facts show that only the total number of messenger fields is relevant. Finally, the mirage unification is not spoiled by supersymmetric threshold ($c_1' = 0$), unlike the simple mirage case.

VI. SUPERSYMMETRY-BREAKING TERMS FOR SCALARS

We have discussed supersymmetry-breaking mass parameters for gauginos. Scalar superparticles also receive similar effects from their couplings to supersymmetry-breaking fields X and Φ. The result is expressed in terms of soft mass parameters: trilinear and bilinear holomorphic couplings and nonholomorphic scalar masses squared. In this section, we present the general formulas for scalar supersymmetry-breaking terms.

As seen above, the effect of the super-Weyl anomaly is important in discussing the gaugino mass unification and then, the compensator formalism of supergravity is useful for deriving the general form of supersymmetry-breaking terms for scalars. For scalar supermultiplets, the supergravity Lagrangian is given by two ingredients, i.e., the Kähler potential K and superpotential W,

$$\mathcal{L} = \int d^4\theta \Phi^\dagger \Phi f(Q_i, Q_i^\dagger, X, X^\dagger, \Phi, \Phi^\dagger) + \left[\int d^2\theta \Phi^3 W(Q_i, X) + \text{H.c.}\right], \tag{6.1}$$

where Q_i denote the scalar superfields for which we now want to derive the supersymmetry-breaking terms. The supergravity f function is related to the Kähler potential as $f = -3e^{-K/3}$. We have taken into account the fact that the Kähler potential has the quantum-level dependence on the compensator field Φ. Note that the superpotential is known to be protected from radiative corrections due to the nonrenormalization theorem and have no Φ dependence. As in the case of gaugino masses, the Φ-dependent pieces come out through the renormalization procedure and induce the anomaly-mediated contribution of supersymmetry breaking. When including quantum effects, it may be easier to analyze the scalar potential in the conformal frame of supergravity where the superconformal gauge symmetry is fixed by choosing $\Phi = 1 + F_\phi\theta^2$. The supergravity Lagrangian in the Einstein frame, where the (super) gravity kinetic terms are canonical, is obtained by the specific super-Weyl transformation [18] and the scalar potential analysis in this frame can be performed with the gauge fixing condition [19]: $\Phi = e^{K/6}[1 + (F_\phi + \frac{1}{3}K_iF_i)\theta^2]$. It is noted that there is no difference between these two gauge choices for deriving the leading order supersymmetry-breaking terms if $|K_i F_i| \ll |F_\phi|$. This condition is obviously satisfied when $|F_X/X| \sim |F_\phi|$ and $|X| \ll 1$ as in the case of mirage unification with gauge thresholds (and also satisfied in the case of simple mirage mediation where $|F_X/X| \ll |F_\phi|$ and $|X| \sim 1$). Therefore in the following we study the scalar supersymmetry-breaking terms in the conformal frame of supergravity.

To see the supersymmetry-breaking terms of scalar fields Q_i, we first integrate out the auxiliary components F_{Q_i} via their equations of motion,

$$F_\phi^* f_{Q_i} + W_{Q_i} + \sum_I F_I^\dagger f_{Q_i I^\dagger} = 0, \tag{6.2}$$

where the lower indices of f and W denote the field derivatives. The index I runs over all the chiral multiplet scalars in the theory, i.e., $I = Q_i, X, \Phi$ in the present case. After the integration, the resultant scalar potential is given by

$$\begin{aligned} V = {}& \left(\frac{f_Q f_{Q^\dagger}}{f_{QQ^\dagger}} - f\right) F_\phi^* F_\phi + \sum_{I,J\neq Q}\left(\frac{f_{QI^\dagger} f_{JQ^\dagger}}{f_{QQ^\dagger}} - f_{JI^\dagger}\right) F_I^\dagger F_J \\ & + \left[\sum_{I\neq Q}\left(\frac{f_Q f_{IQ^\dagger}}{f_{QQ^\dagger}} - f_I\right) F_\phi^* F_I + \text{H.c.}\right] \\ & + \left[f_{QQ^\dagger}^{-1} W_Q \left(\frac{1}{2} W_{Q^\dagger}^* + F_\phi f_{Q^\dagger} + \sum_{I\neq Q} F_I f_{IQ^\dagger}\right)\right. \\ & \left. - 3WF_\phi - W_X F_X + \text{H.c.}\right]. \end{aligned} \tag{6.3}$$

We have dropped the flavor index i of Q_i just for notational simplicity. Note that the derivative indices I, J contain the compensator field Φ which leads to radiative effects through the supergravity anomaly. It easily turns out that the first line in (6.3) generates nonholomorphic scalar mass terms and the second one holomorphic supersymmetry-breaking couplings as well as a possible supersymmetric mass term contained in $|W_Q|^2$.

A. Holomorphic scalar couplings

The scalars Q_i acquire supersymmetry-breaking holomorphic couplings, including trilinear and bilinear ones in scalar fields (usually called the A and B terms, respectively). They are induced in the presence of corresponding superpotential terms in W. The most general expression of holomorphic supersymmetry-breaking terms can be calculated from the second line of the supergravity scalar potential (6.3). For practical purposes, it is almost sufficient to know holomorphic scalar couplings for the minimal Kähler form $K = Z_Q Q^\dagger Q$ where the wave function factor depends on Φ through the renormalization: $Z_Q = Z_Q(X, X^\dagger, \Phi, \Phi^\dagger)$. In this case we find that the second line in the potential (6.3) induces

$$\begin{aligned} \mathcal{L}_A = {}& F_X\left[W_X - \sum_Q \frac{\partial \ln Z_Q}{\partial X}\frac{\partial W}{\partial \ln Q}\right] \\ & + F_\phi\left[3W - \sum_Q\left(1 + \frac{\partial \ln Z_Q}{\partial \phi}\right)\frac{\partial W}{\partial \ln Q}\right] + \text{H.c.} \end{aligned} \tag{6.4}$$

The first term (the $F_X W_X$ term) is irrelevant unless the scalar multiplets Q_i directly couple to X in the superpotential. As an example, let us consider the superpotential with Yukawa and mass terms; $W = y_{ijk} Q_i Q_j Q_k + \mu_{ij} Q_i Q_j$. The corresponding trilinear and bilinear supersymmetry-breaking couplings are read off from the general expression $\mathcal{L}_A$ and are given by

$$A_{ijk} = \sum_{I=X,\phi} \frac{\partial \ln(Z_{Q_i} Z_{Q_j} Z_{Q_k})}{\partial I} F_I, \tag{6.5}$$

$$B_{ij} = -F_\phi + \sum_{I=X,\phi} \frac{\partial \ln(Z_{Q_i} Z_{Q_j})}{\partial I} F_I, \tag{6.6}$$

for the definition of Lagrangian parameters: $\mathcal{L} = -A_{ijk} y_{ijk} Q_i Q_j Q_k - B_{ij}\mu_{ij} Q_i Q_j + \text{H.c.}$. The ϕ derivative is translated to the energy-scale dependence of wave function factors and the coefficients of F_ϕ are given by the anomalous dimensions of scalar fields. On the other hand, the X dependence of Z is fixed model dependently and its supersymmetry-breaking effects have some variety.

We have two brief comments on the phenomenological aspect of these formulas. First, it is noted that the supersymmetry-breaking parameters are described by F_X/X and F_ϕ with real coefficients. Therefore if the complex phases of these two F terms are aligned, phases of supersymmetry-breaking parameters including gaugino masses can be rotated away with one suitable R symmetry rotation, and the CP symmetry is not violated in the supersymmetry-breaking sector. Second, the above B-term formula, when applied to the minimal supersymmetric SM and beyond, causes a too large value of the B parameter to trigger the correct electroweak symmetry breaking, if the F_ϕ contribution is dominant. While there have been several proposed solutions to this problem [9,15,20], they are model dependent and generally predict different values of B according to how to develop μ parameters.

B. Nonholomorphic scalar masses

Scalar fields generally receive nonholomorphic supersymmetry-breaking masses from their couplings to supersymmetry-breaking fields. The mass spectrum of superpartners of quarks and leptons is sensitive to the detailed form of Kähler potential which, in turn, is restricted by phenomenological constraints. Here we suppose the minimal Kähler potential $K = Z_Q Q^\dagger Q$ as in the previous section. The possible X dependence of the wave function factor is determined, depending on the property of X, by claiming the absence of flavor-changing higher-dimensional operators [7]. We do not discuss further here and derive the general formula for supersymmetry-breaking scalar masses.

The nonholomorphic mass terms come from the first line of the potential (6.3). Expanding about Q, we obtain the general expression for the minimal Kähler form,

$$m_Q^2 = \sum_{I,J=X,\phi} \frac{\partial^2 \ln Z_Q^{-1}}{\partial I^\dagger \partial J} F_I^\dagger F_J, \tag{6.7}$$

for the canonical normalization of the Q_i field kinetic term. The second-order derivative with respect to the compensator Φ leads to the anomaly-mediated contribution to supersymmetry-breaking masses. A more essential ingredient is the cross term of two F-component effects F_X and F_ϕ. This part is found to play an important role in discussing the mirage behavior of superparticle masses.

VII. MIRAGE GAUGE MEDIATION

Among general mirage unification scenarios, the gauge threshold case is shown to have stable low-energy unification against possible but obscure intermediate thresholds. The gauge threshold scenario is also favored from phenomenological viewpoints such as the suppression of rare processes beyond the SM and the cosmology. In the rest of this paper, we focus on analyzing this class of scenario. We first present a simple gauge threshold model and discuss its mirage unification behavior and superparticle spectrum. The model is called here the mirage gauge mediation.

A. Setup and supersymmetry-breaking terms

We consider the following form of Lagrangian:

$$\begin{aligned}\mathcal{L} = &\int d^4\theta \Phi^\dagger \Phi Z_i(X, X^\dagger, \Phi, \Phi^\dagger) Q_i^\dagger Q_i \\ &+ \int d^2\theta S(X, \Phi) W^\alpha W_\alpha + \text{H.c.} + \int d^2\theta \Phi^3 X \bar{\Psi}\Psi \\ &+ \text{H.c.} + (\text{dynamics for } X),\end{aligned} \tag{7.1}$$

where Q_i and W^α denote the matter and gauge chiral superfields with the renormalization factors Z_i and S, respectively. The vectorlike messenger multiplets Ψ and $\bar{\Psi}$ belong to grand unificationlike representations, i.e., they give the universal contribution to the SM gauge beta functions in order to preserve the gauge coupling unification in the absence of threshold. The Kähler f function is expanded by Q_i and only the leading kinetic term is included. The leading constant (Q_i-independent) term was not explicitly written but its significance will be discussed in later sections. In what follows, we assume as an example that the messenger multiplets compose of N pairs of fiveplets and its conjugates of $SU(5)$. The compensator superfield Φ controls the Weyl invariance of the theory and its scalar and fermionic components are fixed by the superconformal gauge transformation. Finally, X is the representative chiral superfield of supersymmetry breaking and its expectation value is determined by high-energy dynamics of stabilizing X such that $X = M_X + F_X\theta^2$.[1] The basic building blocks of the model are parallel to the deflected anomaly mediation [15]. The dynamics for X field is unspecified here and will be explicitly discussed with various examples in supergravity towards constructing a fully viable theory.

The wave function factors Z_i and the gauge kinetic function S depend on the supersymmetry-breaking field X at the quantum level. The tree-level X dependences through higher-dimensional operators are suppressed by the cutoff scale $M_{\rm pl}$ which is much larger than the messenger scale M_X, otherwise these operators sometimes induce disastrous phenomenology such as flavor-changing rare processes and CP violations. This suppression is known to be one of the virtues of gauge-mediated supersymmetry breaking and our present model shares this excellent property. The compensator dependence also appears at the loop level due to the classical scale invariance. Therefore the supersymmetry-breaking effects are extracted by turning on the F components and by expanding the quantum-level dependence [21] with respect to F_X and F_ϕ. The renormalization factors in the low-energy region and their dependences on X and Φ are obtained from the solutions of one-loop renormalization-group equations in the superfield forms

$$Z_i(\mu) = Z_i(\Lambda)\left(\frac{\text{Re}S(X\Phi)}{\text{Re}S(\Lambda)}\right)^{(2C_i/b+N)}\left(\frac{\text{Re}S(\mu)}{\text{Re}S(X\Phi)}\right)^{(2C_i/b)}, \tag{7.2}$$

$$S(\mu) = S(\Lambda) + \frac{b+N}{32\pi^2}\ln\left(\frac{\Lambda}{X}\right) + \frac{b}{32\pi^2}\ln\left(\frac{X\Phi}{\mu}\right), \tag{7.3}$$

where b is the one-loop beta-function coefficient below the threshold scale and C_i denotes the quadratic Casimir, explicitly given below. The scale Λ means some high-energy initial point ($\Lambda > M_X$) above which no supersymmetry-breaking dynamics exists.[2]

The soft supersymmetry-breaking mass parameters, gaugino masses M_{λ_a}, scalar trilinear couplings A_i, nonholomorphic scalar masses m_i^2, are then derived from the general formulas given in the previous section

[1] Here M_X is a dimensionless parameter. The threshold mass scale is given by the expectation value of the scalar component of dimension-one superfield $\tilde{X} \equiv X\Phi$. If the following formulas are expressed in terms of $F_{\tilde{X}}$ instead of F_X, the anomaly-mediated contribution should be replaced with the one evaluated above the threshold scale.

[2] We assume that the messenger fields Ψ and $\bar{\Psi}$ do not have soft supersymmetry-breaking parameters above M_X. If not so, the more general formulas [22] should be utilized for deriving soft terms for low-energy fields.

$$M_{\lambda_a}(\mu) = \frac{-Ng_a^2(\mu)}{16\pi^2}\frac{F_X}{M_X} + \frac{b_a g_a^2(\mu)}{16\pi^2}F_\phi, \tag{7.4}$$

$$A_i(\mu) = \frac{NC_i^a}{8\pi^2 b_a}\Big[g_a^2(\mu) - g_a^2(M_X)\Big]\frac{F_X}{M_X} - \frac{C_i^a g_a^2(\mu)}{8\pi^2}F_\phi, \tag{7.5}$$

$$m_i^2(\mu) = \frac{NC_i^a}{128\pi^4 b_a}[(b_a + N)g_a^4(M_X) - Ng_a^4(\mu)] \times \left|\frac{F_X}{M_X}\right|^2 - \frac{C_i^a b_a g_a^4(\mu)}{128\pi^4}|F_\phi|^2 + \frac{NC_i^a g_a^4(\mu)}{128\pi^4}\left(\frac{F_X}{M_X}F_\phi^* + \text{H.c.}\right), \tag{7.6}$$

where the summations for the gauge index a are understood, and C_i^a is the quadratic Casimir operator of gauge-group G_a for the field Q_i, for example, $(N_c^2 - 1)/2N_c$ for the vectorial representation of $SU(N_c)$. For each formula, the first term is the contribution of the gauge threshold. This part determines the mirage unification scale and the mirage mass spectrum as previously shown for gaugino masses. The second term in each formula denotes the anomaly mediation. The third term in the scalar mass-squared m_i^2 is the mixed contribution of gauge and anomaly mediations. It is noted that the relative complex phase of F_X/M_X and F_Φ should be aligned from a phenomenological viewpoint of CP violation. If this is the case, the mirage unification does appear and further the third term in m_i^2 does not contain CP-violating complex phases.

B. Mirage unification

From the general formula (3.10) in Sec. III, the mirage unification scale for the present setup is found

$$\bar{\mu}_m = M_G \exp\left(\frac{-8\pi^2 R}{Ng_G^2}\right), \tag{7.7}$$

where g_G is the mirage value of unified gauge coupling at M_G and can be determined by evolving the low-energy observed values up to high energy. Therefore, g_G and M_G are insensitive to the threshold scale and so is the mirage scale $\bar{\mu}_m$. It is noted that this is the general and model-independent property of the theory with gauge coupling unification such as the minimal supersymmetric SM. The real value of unified gauge coupling, g_U, in the present model is related to g_G as $1/g_U^2 = 1/g_G^2 + (N/16\pi^2) \times \ln(M_X^2/M_G^2)$, but g_U itself does not appear explicitly in any formulas for mirages. The parameter R in (7.7) is defined as the ratio of two F terms,

$$R = \frac{-F_\phi}{F_X/M_X}, \tag{7.8}$$

which is real valued as mentioned above and is defined so that its sign becomes positive in most of the known dynamics for the X stabilization. In the limit $R \to 0$ ($R \to \infty$), the contribution of gauge (anomaly) mediation becomes dominant. It may be interesting to see from Eq. (7.7) that the low-energy mirage scale emerges as an analogy of dimensional transmutation: let us consider a virtually-defined gauge coupling g_m. It has an initial condition $g_m(M_G) = g_G$ and obeys the renormalization-group equation with the beta-function coefficient $b_m = -N/R$ which is negative in most cases, and hence g_m has the asymptotically free behavior.

Since the gauge couplings at $\bar{\mu}_m$ are related to g_G as

$$g_a^2(\bar{\mu}_m) = \frac{N}{N + b_a R}g_G^2, \tag{7.9}$$

the soft supersymmetry-breaking mass parameters at the mirage unification scale are found to be given by the following form:

$$M_{\lambda_a}(\bar{\mu}_m) = \frac{-Ng_G^2}{16\pi^2}\frac{F_X}{M_X}, \tag{7.10}$$

$$A_i(\bar{\mu}_m) = \frac{NC_i^a}{8\pi^2 b_a}[g_G^2 - g_a^2(M_X)]\frac{F_X}{M_X}, \tag{7.11}$$

$$m_i^2(\bar{\mu}_m) = \frac{NC_i^a}{128\pi^4 b_a}[(N + b_a)g_a^4(M_X) - Ng_G^4]\left|\frac{F_X}{M_X}\right|^2. \tag{7.12}$$

The mass spectrum is controlled by two parameters, the messenger contribution N and the threshold scale M_X, and is insensitive to the F-term ratio R. On the other hand, the mirage scale $\bar{\mu}_m$ is determined by N and R and is insensitive to the threshold scale M_X. These behaviors are important for studying phenomenological aspects of the model, especially for examining whether the mirage unification scale can be set to be observable in future collider experiments, which we will discuss in details in Sec. VIII.

C. Mirage spectrum

It is found from the above mass formula that the mirage gauge mediation has a complete correspondence to the gauge mediation scenario. That is, these two theories are traded to each other by interchanging the gauge coupling constants: the mirage spectrum is read off from the gauge-mediated one by simply replacing the gauge couplings $g_a(\mu)$ at a low-energy scale μ with the mirage unified value g_G which is evaluated from $g_a(\mu)$. Furthermore soft scalar masses (7.12) are found to generally satisfy two types of sum rules, as in gauge mediation [23]: $\sum Ym^2 = 0$ and $\sum(B - L)m^2 = 0$ where Y and $B - L$ are the hypercharge and the baryon minus lepton number, respectively. (It is noted that each term in (7.6), i.e., the gauge, anomaly, mixed term, separately satisfies the sum rules.)

The clear comparison to the gauge mediation model is summarized in Table I. Here we show several illustrative

TABLE I. The soft supersymmetry-breaking parameters in the gauge mediation (GM) and mirage GM. In both cases, the parameters are evaluated at the TeV scale. In this table, the gaugino masses M_{λ_a} and trilinear couplings A_i (scalar masses squared m_i^2) are normalized by $F_X/16\pi^2 M_X$ ($|F_X/16\pi^2 M_X|^2$).

	Low-scale mediation ($M_X = \text{TeV}$)	High-scale mediation ($M_X = M_G$)
GM	$M_{\lambda_a} = -Ng_a^2(\mu)$	$M_{\lambda_a} = -Ng_a^2(\mu)$
($\mu = \text{TeV}$)	$A_i = 0$	$A_i = \frac{2NC_i^a}{b_a}[g_a^2(\mu) - g_G^2]$
	$m_i^2 = 2NC_i^a g_a^4(\mu)$	$m_i^2 = \frac{2NC_i^a}{b_a}[(b_a + N)g_G^4 - Ng_a^4(\mu)]$
Mirage GM	$M_{\lambda_a} = -Ng_G^2$	$M_{\lambda_a} = -Ng_G^2$
($\bar{\mu}_m = \text{TeV}$)	$A_i = \frac{2NC_i^a}{b_a}[g_G^2 - g_a^2(M_X)]$	$A_i = 0$
	$m_i^2 = \frac{2NC_i^a}{b_a}[(b_a + N)g_a^4(M_X) - Ng_G^4]$	$m_i^2 = 2NC_i^a g_G^4$

limits that the mass spectra of two theories are evaluated at the same low-energy scale (= TeV) in the cases that the supersymmetry-breaking threshold scales are low ($M_X =$ TeV) and high ($M_X = M_G$). We have two typical spectra of the mirage unification:

(i) The first is the case that supersymmetry breaking is mediated at the gauge coupling unification scale (the lower-right panel in the table). The low-energy mass spectrum of gauginos is universal, the trilinear scalar couplings vanish, and the scalar masses squared are specified by the quadratic Casimir operators. The last fact means that scalar superparticles with the same quantum charge have the universality of mass spectrum, which leads to enough suppressions of rare processes involving flavor-changing neutral currents. This virtue of the gauge mediation also appears in the mirage gauge mediation. However, the mass spectrum is rather different from the gauge mediation: the low-energy spectrum is written only by the unified value of gauge couplings g_G, not the low-energy values. This fact leads to the low-energy unification of gaugino masses as well as almost degenerate scalar superparticles. The scalar lepton masses are of similar order of scalar quark masses and they differ only by $\mathcal{O}(1)$ coefficients C_i^a. Moreover (too) restrictive mass formulas generally imply several relations among observed mass values in future collider experiments. For example, in the minimal supersymmetric SM, the mirage spectrum is exactly given by

$$M_{\lambda_1}^2 : M_{\lambda_2}^2 : M_{\lambda_3}^2 : m_Q^2 : m_u^2 : m_d^2 : m_L^2 : m_e^2 = N : N : N : \tfrac{21}{5} : \tfrac{16}{5} : \tfrac{14}{5} : \tfrac{9}{5} : \tfrac{6}{5}, \tag{7.13}$$

without including Yukawa coupling effects. Therefore the whole superpartners are found to receive a similar size of supersymmetry-breaking masses.

(ii) The second limit is the low-scale threshold (the lower-left panel in the table). Here we discuss the situation $M_X \sim \bar{\mu}_m \sim$ TeV. The low-energy gaugino masses are universal and given by g_G, which is a robust prediction of the mirage gauge mediation. Unlike the usual (low-scale) gauge mediation, scalar trilinear couplings are generated at one-loop order of gauge couplings and naturally comparable to other supersymmetry-breaking parameters. For example, if $M_X = \bar{\mu}_m$, the trilinear couplings are found from the above formula and (7.9) that $A_i = \frac{g_G^2}{8\pi^2}\frac{NRC_i^a}{N+b_aR} \times \frac{F_X}{M_X} \sim M_\lambda$. Such sizable A parameters would be important for phenomenology around the electroweak symmetry breaking scale. The scalar soft mass parameters m_i^2 are also characteristic. In the gauge mediation with chiral messengers, m_i^2 is positive irrespectively of the threshold scale M_X, but the low-scale mirage gauge mediation sometimes predicts tachyonic scalar superpartners. For example, if $M_X = \bar{\mu}_m$, we find from the mass formula and the gauge coupling relation (7.9) that the positivity constraint of scalar masses squared ($m_i^2 > 0$) lead to an inequality

$$b_a R^2 < N(1 - 2R) \tag{7.14}$$

for all beta-function coefficients b_a. That implies that, for $R > 1$ ($R > 1/2$), asymptotically free (nonfree) gauge groups induce tachyonic contributions to scalar soft masses. It is therefore important for phenomenology of the model to satisfy some lower bound on the threshold scale M_X and/or an upper bound on the ratio R, the latter of which restricts possible dynamics for the X stabilization.

VIII. MIRAGE UNIFICATION AT TEV

Based on the formalism shown above, we investigate the model with mirage gauge mediation around the TeV-scale, i.e., $\bar{\mu}_m \sim$ TeV. The forthcoming Large Hadron Collider experiment will probe the TeV-scale physics, and, in particular, would observe the superpartners of SM fields with the mirage pattern of the mass spectrum. Such a characteristic spectrum provides distinctive experimental signatures from any other supersymmetry-breaking scenarios, and clearly suggests the existence of a specific mechanism in high-scale dynamics. In this section, we first derive the conditions for realizing TeV-scale unification. Next, we examine several models of X field dynamics discussed in the literature and show that it seems difficult for these models to satisfy the required conditions. Finally, possible dynamical mechanisms are presented to make the conditions unnecessary or weakened. We also point out that the hidden-sector contribution, which is generally needed to have the de Sitter vacuum in supergravity but is usually decoupled, may play an important role for constructing a full theory of TeV mirage unification.

A. TeV-scale mirage

For phenomenological discussions of mirage gauge mediation, there are three points to be taken into account: (i) the perturbative evolution of gauge coupling constants, (ii) nontachyonic scalar mass spectrum, and (iii) the low mirage scale.

As for the first point, the one-loop evolution of gauge couplings are solved as

$$\frac{1}{g_U^2} = \frac{1}{g_a^2(\mu)} + \frac{b_a}{16\pi^2}\ln\left(\frac{\mu^2}{M_X^2}\right) + \frac{b_a+N}{16\pi^2}\ln\left(\frac{M_X^2}{M_G^2}\right), \quad (8.1)$$

for $\mu < M_X < M_G$. The unification scale is determined by low-energy observed values $g_a^2(\bar{\mu}_m)$ and the requirement of gauge coupling unification, independently of other parameters. Therefore the running of gauge couplings, in particular, their high-energy values are controlled by the threshold scale M_X and the number of messenger fields N. A bound on these parameters is derived from the requirement of perturbative unification that the gauge couplings do not diverge below the unification scale (i.e., $g_U < \infty$),

$$N\ln\left(\frac{M_G}{M_X}\right) < \frac{8\pi^2}{g_G^2}. \quad (8.2)$$

This inequality implies the lower bound on the messenger mass M_X and the upper bound on its number N. For example, we obtain from (8.2)

$$N = 5:\ M_X > 4.5\times 10^2\ \text{GeV}, \quad (8.3)$$

$$N = 10:\ M_X > 3.0\times 10^9\ \text{GeV}, \quad (8.4)$$

$$N = 15:\ M_X > 5.7\times 10^{11}\ \text{GeV}, \quad (8.5)$$

for $M_G = 2.0\times 10^{16}$ GeV which is a typical scale of supersymmetric grand unification of the SM gauge couplings.

The second condition comes from the superparticle mass spectrum at a low-energy observable scale. In order that charged scalar superpartners do not develop condensations, their mass-squared terms in the potential must be positive. Here we consider the constraint that soft supersymmetry-breaking masses squared m_i^2 must be positive, as a conservative one without including the effects of Yukawa couplings and trilinear scalar parameters. The analysis in the previous section shows that the scalar masses squared become at the mirage scale

$$m_i^2(\bar{\mu}_m) = \frac{NC_i^a}{128\pi^4 b_a}\left[(N+b_a)g_a^4(M_X) - Ng_G^4\right]\left|\frac{F_X}{M_X}\right|^2. \quad (8.6)$$

The gauge couplings at the intermediate scale, $g_a(M_X)$, are determined by M_X for fixed values of low-energy gauge couplings. Therefore the scalar masses are controlled by the two parameters M_X and N. Roughly speaking, tachyonic scalars are avoided if $m_i^2(\bar{\mu}_m) > 0$, namely, the quantity in the bracket of (8.6) is negative (positive) for the asymptotically free (nonfree) gauge theory. For example, in the minimal supersymmetric SM, the right-handed scalar leptons usually give the most significant constraint. We find from (8.6) that $m_e^2(\bar{\mu}_m) > 0$ implies

$$b_1 R(M_X)^2 < N[1 - 2R(M_X)], \quad (8.7)$$

where $b_1 = 33/5$ is the beta-function coefficient for the hypercharge gauge coupling, and $R(M_X)$ has been introduced as a generalization of (7.14) and defined as $R(M_X) \equiv (Ng_G^2/8\pi^2)\ln(M_G/M_X)$. It is easily found that the number of messengers N has an upper bound for their masses fixed, and in other words, the threshold scale M_X has a lower bound. In Fig. 1, we show the numerical result of the positivity constraint $m_e^2(\bar{\mu}_m) > 0$. The parameter bounds are often more severe than (8.2) which is obtained from the perturbative gauge coupling unification.

The last point is whether the mirage unification takes place at a low-energy observable scale as one chooses. The low-energy unification scale in the mirage gauge mediation is found in the previous analysis [Eq. (7.7)],

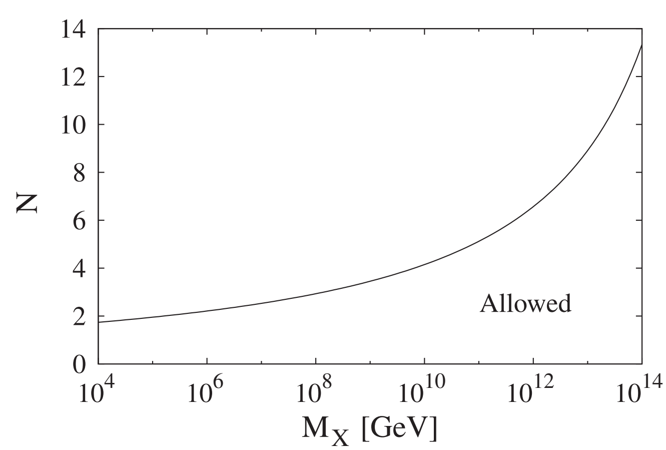

FIG. 1. The region for the messenger parameters M_X and N allowed by the positivity constraint on the right-handed scalar lepton mass in the minimal supersymmetric SM.

TABLE II. Typical dynamics for supersymmetry-breaking messenger mass splitting. The limit $R \to \infty$ corresponds to the anomaly mediation dominant (supersymmetric thresholds), and M_X and m_X denote the threshold scale and the X scalar mass in the vacuum, respectively.

Dynamics	R	M_X	m_X	
$K = \|X\|^2$ $W = X^n (n > 3)$	$\frac{n-1}{2}$	$F_\phi^{(1/(n-2))}$	$(n-3)\|F_\phi\|$	[15]
$K = Z(X, X^\dagger)\|X\|^2$ $W = X^3$	1	arbitrary	$(2-\text{loop}) \times \|F_\phi\|$	[15]
$K = \|X\|^2 + \|Y\|^2$ $W = X^n Y^m$	$\frac{m+n-1}{2}$	$F_\phi^{(1/(m+n-2))}$	$(m+n-3)\|F_\phi\|$	[24]
$K = Z(X, X^\dagger)\|X\|^2 - \|c\|\|X\|^4$ $W = 0$	1	$(1-\text{loop}) \times \Lambda$	$(2-\text{loop}) \times \|F_\phi\|$	[24]
No X $\Delta K = \Psi\bar{\Psi}$	$\frac{1}{2}$	$\mathcal{O}(F_\phi)$	$\cdots$	[25]

$$\bar{\mu}_m = M_G \exp\left(\frac{-8\pi^2 R}{N g_G^2}\right). \tag{8.8}$$

Since M_G and g_G are determined by low-energy theory and observations, the scale $\bar{\mu}_m$ is controlled by N and R. The latter is defined by the ratio of two supersymmetry-breaking F terms in the theory and then, possible dynamics of X are restricted for the TeV-scale mirage to be achieved,

$$\frac{R}{N} = \frac{g_G^2}{8\pi^2} \ln\left(\frac{M_G}{\text{TeV}}\right) \simeq 0.20. \tag{8.9}$$

Here we have used a typical unified value of SM gauge couplings, $g_G^2/4\pi^2 = 1/24.5$, which is obtained from the weak-scale experimental data and the renormalization-group evolution in the minimal supersymmetric SM [2]. The result (8.9) is deeply related to high-energy dynamics of supersymmetry breaking. As a simple example, let us consider the case that the Kähler potential is minimal and the superpotential contains a single polynomial term [15],

$$K = Z|X|^2, \qquad W = X^n (n \geq 3). \tag{8.10}$$

Turning on the compensator F term as a background in the Lagrangian[3] and minimizing the supergravity scalar potential, one obtains a nonvanishing F component of the supersymmetry-breaking field X,

$$\frac{F_X}{X} = \frac{2}{1-n} F_\phi, \qquad \left(R = \frac{n-1}{2}\right). \tag{8.11}$$

In order to satisfy (8.9), the correlation is found between the potential form ($W = X^n$) and the number of messenger fields, as given in the following:

n	3	4	5	6	7	8	9
N	5.0	7.5	10.0	12.6	15.1	17.6	20.1

An integer value of the messenger number may be reasonable. It is interesting that the simplest dynamics, $W = X^3$, are approximately consistent with the TeV-scale mirage for an integer number $N = 5$. A variety of other models have been proposed in the literature to stabilize X with the F_Φ effect and to provide nonsupersymmetric (nondecoupling) messenger thresholds. The potentials and their predictions are summarized in Table II, in which we also show the X scalar mass m_X in each supersymmetry-breaking vacuum. It is found from the table that, in most of models, the F-term ratio R is $\mathcal{O}(1)$ or sometimes becomes large. This fact generally means that the number of messenger fields is required to be large for the TeV-scale mirage. If this is the case, the messenger mass scale should be unfortunately high in order to have the perturbative gauge coupling unification or not to have any tachyonic scalar superpartners.

[3]The compensator F_ϕ is dynamically fixed, e.g., by including a constant (X-independent) superpotential term in supergravity. Such a detail is irrelevant to the result presented here.

B. Possible ways out

We have studied the phenomenological constraints in the simple case of mirage gauge mediation and found that it tends to need a large number of messenger fields and a high mediation scale. It is a natural amelioration to realize a low-scale mirage unification without introducing model complexity and/or without loosing observation feasibility.

If the messenger number N becomes large, supersymmetry-breaking masses squared of scalar superpartners become negative, as seen in the previous section. It is then possible to introduce some additional dynamics for stabilizing these tachyons in parallel ways to various proposed solutions of the tachyonic scalar lepton problem in the pure anomaly mediation [9]. It may be interesting to look for tachyon-stabilization dynamics which is characteristic to the mirage gauge mediation.

Another remedy is found from the expression of the mirage scale (7.7) that if the virtual unified gauge coupling g_G is increased, the messenger number N can be correspondingly reduced for a fixed value of the mirage scale. A high-energy gauge coupling is generally increased by introducing additional fields. It is however noted that, as shown in Sec. V B, the virtual gauge coupling g_G is insensitive to the existence of intermediate-scale thresholds and is fixed only by low-energy physics. Therefore one is lead to modifying low-energy physics by adding TeV-scale extra fields. We suppose that these fields belong to grand unificationlike and vectorlike representations in order to preserve the gauge coupling unification and to avoid the experimental constraints from precision electroweak-scale measurements. The extra fields are assumed to be irrelevant to supersymmetry breaking and their threshold is supersymmetric. In this case, g_G is increased as

$$\frac{g_G'^2}{g_G^2} = \frac{1}{1 - \frac{\Delta b g_G^2}{8\pi^2}\ln(\frac{M_G}{\text{TeV}})}, \tag{8.13}$$

where Δb denotes the universal extra-field contribution to beta-function coefficients ($\Delta b > 0$). We find that the number of supersymmetry-breaking messengers is reduced for the fixed mirage scale,

$$\frac{N'}{R'} = \frac{N}{R} - \Delta b. \tag{8.14}$$

The ratio of the messenger number and the F-term ratio is determined by low-energy physics, and $N/R \simeq 5.0$ in the minimal supersymmetric SM [Eq. (8.9)]. As a simple example, if we add one pair of 16 and 16^* representations of $SO(10)$ at the TeV scale [26], $\Delta b = 4$ and hence the minimal messenger ($N' = 1$ and $R' = 1$) is sufficient to obtain the mirage phenomenon, where tachyonic scalar superpartners do not emerge (see Fig. 1).

A more reasonable solution is to reduce R in a dynamical way. It is found from (8.9) that a smaller (positive) R implies a fewer messenger multiplets needed and the model becomes simplified. For example, if we have some dynamics which predict $R \simeq 1/5$, only a single pair of messengers is sufficient to realize the TeV-scale mirage unification. Since a smaller value of $|R|$ means a larger effect of F_X relative to F_ϕ, some mechanism of the X field is needed to amplify its supersymmetry-breaking effect a few times or so.

(i) Multiple X fields:
One may naively expect that the threshold contribution increases when several supersymmetry-breaking fields are introduced with nonvanishing F components. However, the total effect of supersymmetry breaking is not enhanced if these X fields have similar types of dynamics and then induce similar orders of F terms: the resulting effect from multiple X fields is not additive and is the same as the single X case. This behavior is confirmed for various types of X dynamics (e.g., see [24]). In the end, a viable model along this line must be constructed to have highly asymmetric property among multiple X fields. That generally makes the model complex and unrealistic.

(ii) Different X potentials:
In the above example of mirage gauge mediation, the Kähler and superpotential of X are minimal and simplest. A model with a different type of X potential may lead to increasing the supersymmetry-breaking effect $|F_X/X|$ and then reducing R. As we will show in details, the supergravity analysis of F terms leads to the following form of the R parameter in the vacuum

$$R = \frac{W_{XX} + \frac{4}{3}X^\dagger W_X + \frac{1}{3}X^{\dagger 2}W}{2W_X/X}, \tag{8.15}$$

for the minimal Kähler $K = |X|^2$ and general superpotential $W(X)$. This expression has been written down by neglecting higher-order terms in X and without including the hidden-sector effect, for simplicity. The exploration of $W(X)$ realizing $R \simeq 0.2N \lesssim 1$ is an interesting task to be performed. We will later discuss it in several examples including the hidden-sector contribution. The R parameter is sometimes determined by continuous model parameters. In this case, the mirage scale is set just by choosing these parameters nondynamically, while it is preferable that the mirage is described in terms of discrete parameters which define the dynamics of the model such as the power of polynomial potential.

(iii) Different messenger couplings:
The messenger supermultiplets Ψ and $\bar{\Psi}$ are coupled to X and the compensator Φ somewhere in the Lagrangian and receive supersymmetry-breaking mass splitting within each multiplet when the F components F_X and F_Φ are turned on.
A simple and direct way to modify the mass splitting is to introduce extra quadratic terms in the Kähler and superpotential,

$$\Delta K = \bar{\Psi}\Psi, \qquad \Delta W = M_\Psi \bar{\Psi}\Psi, \tag{8.16}$$

in addition to the basic Lagrangian of mirage gauge mediation (7.1). The messenger scalars receive additional supersymmetry-breaking masses induced from these terms as well as the supersymmetric mass M_Ψ. The modification of the model is easily found by noticing that the inclusion of additional supersymmetry breaking is effectively described by the field redefinition: $X \to X + M_\Psi + F_\phi^*/\Phi^2$. Therefore the modified R parameter is read off as

$$R = \frac{(X + M_\Psi)F_\phi + |F_\phi|^2}{2|F_\phi|^2 - F_X}. \tag{8.17}$$

The new contribution becomes significant only when the messenger mass scale is low: $M_X \sim M_\Psi \sim F_\phi$. If this application limit is acceptable, the TeV-scale mirage may be realized with a fewer number of messengers by taking appropriate values of the ΔK and ΔW contributions. For example, if the ΔK effect is dominant, R is reduced to $1/2$ and Eq. (8.9) requires $N \simeq 2.5$, which is the half of the previous result in the simplest case ($R = 1$).

The messenger coupling to X is another possible source of modifying the supersymmetry-breaking mass splitting and reducing the number of messengers. Let us consider the following form of messenger coupling

$$W = X^m \bar{\Psi}\Psi. \tag{8.18}$$

This form can be general by assigning suitable R symmetry charges. The $m = 1$ case is simplest and has been analyzed before. The superpotential coupling determines the messenger mass scale as well as the strength of supersymmetry-breaking mediation. Inserting $X = M_X + F_X\theta^2$, we find that the effective messenger mass $M_{\rm eff}$ and supersymmetry-breaking mass splitting $F_{X_{\rm eff}}$ are given by

$$M_{\rm eff} = (M_X)^m, \qquad \frac{F_{X_{\rm eff}}}{M_{\rm eff}} = m\frac{F_X}{M_X}. \tag{8.19}$$

The supersymmetry-breaking effect is thus enhanced by the factor m compared with the usual $m = 1$ case and so the R parameter is reduced by the same factor. In the end, the number of the messenger can be reduced. One price to pay is that the messenger mass scale is no longer free and is suppressed from the mediation scale M_X as $M_{\rm eff} = (\frac{M_X}{\Lambda})^{m-1}M_X$ where Λ is the ultraviolet cutoff. It should be noted that the mirage unification and its emergence scale are not affected by changing the mass scales of messenger fields and supersymmetry breaking, as we have shown. A phenomenological bound on the messenger mass scale, i.e., $M_{\rm eff} > {\rm TeV}$, leads to a restriction of messenger coupling, in particular, an upper bound on the index m as a function of the supersymmetry-breaking scale M_X. For example, if one takes $\Lambda = M_{\rm pl}$ and $M_X = M_G$, the index must satisfy $m < 7.6$. Therefore a favorable value, $m = 5$, for the TeV-scale mirage [see, (8.9)] is within the allowed range. In other words, the superpotential coupling with $m = 5$ generates the messenger mass around the 100 PeV scale, enough high to satisfy experimental constraints.

(iv) Extra sources of supersymmetry breaking:

The enhancement of the F_X effect is effectively done by introducing extra sources of supersymmetry breaking other than the X field. We here comment on several possibilities in order.

It is a natural expectation that a better way to modify a model involves fewer extensions of it. In this sense, a simple way is to consider the complete anomaly mediation, i.e., to include the supersymmetry-breaking effects induced not only from the super-Weyl anomaly but also from other anomalies in supergravity. The latter effects may be comparable in some framework to that of the conformal compensator and then our previous results for the R parameter may be changed.

Extra supersymmetry-breaking effects are supposed to have the property that the mirage unification is not disturbed. The general form of such supersymmetry breaking is parametrized as (3.9). That is, in addition to the gauge threshold effect (the c_1 term) analyzed before, some universal contribution (the c_0 term) can be included. If these two contributions are on similar orders of magnitude (and have the same sign), the supersymmetry-breaking effect is effectively enhanced and the number of messengers may be reduced to a reasonable level. A plausible possibility of the universal contribution comes from the gravity and related modulus fields, which have field-universal interactions. A well-known framework of moduli stabilization in string theory [12] provides such a possibility [17]. It is important to notice that in this framework the modulus contribution to supersymmetry breaking is found to be comparable to the anomaly-mediated one [6,7,27] and hence also comparable to the gauge threshold contribution. This is the property we just wanted in the above for suitably improving the mirage gauge mediation.

C. Hidden-sector and mirage gauge mediation

The above analysis has not been concerned about the vacuum energy (the cosmological constant). At the minimum of potential, the scalar component of the X field (related to the messenger mass scale) is taken to be suppressed and then the vacuum energy is negative. We therefore need to uplift the potential to make the cosmological constant zero or slightly positive. This point has not been discarded in the literature of deflected anomaly mediation or simply regarded as adding the hidden sector which decouples from X. In this section, we examine the possibility that the mirage gauge mediation, in particular, the R parameter, is modified by utilizing hidden-sector dynamics for uplifting the vacuum energy. It is better to realize that the modification is done such that the TeV-scale mirage naturally emerges in a simpler model and the mirage scale is controlled by discrete parameters. It has been known [28] that the TeV-scale unification is difficult to realize in the scenario of simple mirage mediation. It may be therefore interesting that the mirage gauge mediation solves this problem with the hidden sector uplifting which is experimentally required for the cosmological observation.

1. Hidden-sector contribution

We introduce a hidden-sector field Z with a nonvanishing vacuum expectation value of the F component. The general supergravity Lagrangian for Z and the supersymmetry-breaking field X has the following form:

$$\mathcal{L}_H = \int d^4\theta \Phi^\dagger \Phi f(X, X^\dagger, Z, Z^\dagger) + \left[\int d^2\theta \Phi^3 W(X, Z) + \text{H.c.}\right]. \quad (8.20)$$

The supergravity f function is related to the Kähler potential as $f = -3e^{-K/3}$. The loop-level dependence on the compensator Φ has been dropped since it is quantitatively irrelevant to the discussion in this section. One can incorporate in f the direct couplings between X and Z without conflicting with phenomenological observation in the visible sector. Integrating out the hidden sector, we obtain the supergravity scalar potential

$$V_H = e^{K/3}[(WK_X + W_X)K^{-1}_{XX^\dagger}(W^* K_{X^\dagger} + W^*_{X^\dagger}) - 3|W|^2] + f_{ZZ^\dagger}|F_Z|^2, \quad (8.21)$$

where the lower indices of f and W denote the field derivatives. We consider that the potential V_H is a function of the X field, and the hidden variable is treated as a background parameter which is determined by solving the Z dynamics in the hidden sector.

In this paper, we explore the mirage gauge mediation which has the parameter region: $|F_X/X| \sim |F_\phi|$ for the mirage to appear and $|X| \ll 1$ for the messengers to be lighter than the cutoff scale. It is noted that, in a complete contrast, $|F_X/X| \ll |F_\phi|$ and $|X| \sim 1$ in the scenario of string-theory moduli stabilization. Therefore the framework of supersymmetry-breaking and uplifting hidden dynamics is expected to be different from the string-theory scenario [29]. The smallness of the expectation value $|X| \ll 1$ may naturally lead to the conditions for the Kähler potential that $|XK_X| \ll 1$ and $|XX^\dagger K_{XX^\dagger}| \ll 1$ at the minimum. If this is the case, the vacuum energy is easily found

$$V_{H0} = -3e^{-K/3}|F_\phi|^2 + f_{ZZ^\dagger}|F_Z|^2. \quad (8.22)$$

The requirement of the vanishing cosmological constant is fulfilled with a nonvanishing F term of hidden-sector field Z in the vacuum. Minimizing the potential V_H with respect to the X scalar, we find the shifted vacuum by turning on F_Z. Substituting these results, we obtain the F components F_X and F_Φ in the shifted vacuum, in particular, the general formula of their ratio in the leading order,

$$R = \frac{W_{XX} + W_X K_X + W K_{XX} + (W_X + W K_X)[\frac{1}{3}K_X + (K^{-1}_{XX^\dagger})_X K_{XX^\dagger}]}{2W_X/X - 3W f_{XZZ^\dagger}/X f_{ZZ^\dagger}}. \quad (8.23)$$

For example, the R parameter is evaluated for the minimal form of Kähler potential $K = |X|^2 + |Z|^2$ as

$$R_{\text{min}} = \frac{W_{XX} + \frac{4}{3}X^\dagger W_X + \frac{1}{3}X^{\dagger 2}W}{2W_X/X + X^\dagger W/X}. \quad (8.24)$$

Since the second term in the denominator expresses the hidden-sector contribution, the uplifting of vacuum energy is found to multiply the F_X effect by the factor H:

$$H \equiv 1 + \frac{X^\dagger W}{2W_X}. \quad (8.25)$$

This formula of the enhancement is given only by the superpotential for the X field. If the dynamics for X stabilization satisfies $H > 1$, the hidden sector enhances the F_X effect which implies that the number of messenger fields is effectively reduced and tachyonic scalar mass spectrum is avoided. Moreover the H factor (8.25) indicates that the ratio of two F terms remains real and does not disturb the phase alignment of supersymmetry-breaking soft mass parameters.

2. Sample potentials

In this subsection, we assume that the Kähler potential has the minimal form: $K = |X|^2 + |Z|^2$ as the simplest case, and examine several forms of superpotential for X to have a suitable value of the enhancement factor H.

(i) $W = yX^n + c$ $(n > 3)$:
The first example is the polynomial superpotential discussed in Sec. VIII A. Here we also include a constant superpotential term to dynamically stabilize F_ϕ. The analysis of the supergravity potential is found to give the minimum at

$$\frac{X^n}{|X|^2} = \frac{3-n}{n(n-1)}\frac{c}{y}, \quad (8.26)$$

for $|y| \gg |c|$. From (8.25), we obtain the factor H as

$$H = \frac{n-5}{2n-6}. \quad (8.27)$$

While H becomes a real parameter, it generally takes $H < \frac{1}{2}$ and cannot be used to effectively enhance the F_X effect.

(ii) $W = yX + c$:
The second is the linear superpotential term (so to say, a low-scale Polonyi model). This model has a different type of minimum than the above polynomial superpotential with a higher power. For $|y| \ll |c|$, the supergravity potential is minimized at

$$X^3 = \frac{6cy^{*2}}{c^{*2}y}. \quad (8.28)$$

The model predicts $R = 1$ without taking into account the uplifting hidden sector. The hidden-sector enhancement factor is given by

$$H = 1 + \frac{cX^\dagger}{2y} \simeq \left(\frac{|c|}{|y|}\right)^{2/3}. \tag{8.29}$$

Since H becomes large and positive, the F_X effect is enhanced in the uplifted true vacuum. So the TeV-scale mirage can be made natural. It is however noted that the F-term ratio R is no longer a discrete value and depends on the continuous coupling constants of the model.

(iii) $W = y_n X^n + y_m X^m$:
The third model is the racetracklike superpotential. That is, the two similar superpotential terms work in cooperation to stabilize the X field. Analyzing the supergravity potential, we obtain the minimum at

$$X^{m-n} = -\frac{ny_n}{my_m}, \tag{8.30}$$

where the relative size of y_m and y_n is assumed to have $|X| \ll 1$. The model predicts $R = 1$ without the hidden-sector contribution. It is noticed that, with this expectation value of X (8.30), the first derivative of the superpotential vanishes and the previous formula (8.25) cannot be used. In this case, the general (re)analysis of potential minimization and the vacuum energy uplifting lead to

$$R = \frac{1}{1 + ff_{XZZ^\dagger}W/f_{ZZ^\dagger}XW_{XX}}. \tag{8.31}$$

From this formula, the H factor is evaluated for the minimal Kähler potential,

$$H = 1 + \frac{X^\dagger W}{XW_{XX}} = 1 - \frac{|X|^2}{mn}. \tag{8.32}$$

In the end, we find $H \simeq 1$ and the hidden-sector effect is negligible in this model.

We have investigated three types of models and found three different conclusions. All of these models unfortunately have somewhat unsatisfied points. The improvement and construction of realistic models are left for future study.

IX. SUMMARY AND DISCUSSION

In this paper, we investigated a new class of supersymmetry-breaking mediation models, where gaugino masses are unified in the low-energy regime. We first classified the conditions of gaugino mass unification, and then studied the gauge threshold case. The mirage gauge mediation scenario is basically the gauge-mediated supersymmetry breaking, but at the low-energy unification scale, the virtual high-energy unified gauge coupling behaves as the one at the renormalization scale in gauge mediation. Thus, under the hypothesis of gauge coupling unification, gaugino masses become naturally unified at the weak scale. On the other hand, it is nontrivial to dynamically realize the mirage unification at the TeV scale. We also discussed several possible ways out in the last part of this paper.

The mirage gauge mediation possesses the characteristic mass spectrum of superparticles and various virtues from the phenomenological points of view. Compared with the gauge mediation, the masses of superparticles tend to be degenerate at the weak scale. Also the gravitino is not the lightest superparticle anymore, but is rather heavy to have sizable corrections from the anomaly mediation. On the other hand, unlike the simple mirage case such as the string-theory framework of moduli stabilization, the low-scale gaugino mass unification is not an assumption but is a natural prediction of the mirage gauge mediation. In addition, thanks to the virtues of gauge mediation, the flavor-changing rare processes and CP violations are automatically suppressed.

The mirage gauge mediation is favored as well from the cosmological points of view. The scenario contains a singlet scalar field, X, coupled to supersymmetry-breaking messenger fields. Since X has a rather flat potential, it is considered to dominate the energy of the Universe. Then the X scalar decays into superparticles and gravitinos, diluting the preexisting particles and producing radiations. The produced gravitinos often easily spoil the successes of the standard cosmology such as the big-bang nucleosynthesis, or overclose the Universe. However, it is expected in our scenario that the branching ratio of the gravitino production becomes suppressed since the vacuum expectation value of X is much smaller than the Planck scale (see, e.g., [30]). This feature is contrasted to the string-related mirage models, where a light modulus field is involved and causes a serious problem of the gravitino overproduction [31]. The dark matter candidates in the mirage gauge mediation are the (degenerate) gauginos and the superpartner of X. They are produced from the decay of X scalar, while their relic abundance is quite model dependent. We need further studies on the phenomenological and cosmological aspects of the scenario, and they will be discussed in the future.

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