Noether analysis of the twisted Hopf symmetries of canonical noncommutative spacetimes

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We study the twisted Hopf-algebra symmetries of observer-independent canonical spacetime noncommutativity, for which the commutators of the spacetime coordinates take the form $[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$ with observer-independent (and coordinate-independent) $\theta^{\mu\nu}$. We find that it is necessary to introduce nontrivial commutators between transformation parameters and spacetime coordinates, and that the form of these commutators implies that all symmetry transformations must include a translation component. We show that with our noncommutative transformation parameters the Noether analysis of the symmetries is straightforward, and we compare our canonical-noncommutativity results with the structure of the conserved charges and the "no-pure-boost" requirement derived in a previous study of κ-Minkowski noncommutativity. We also verify that, while at intermediate stages of the analysis we do find terms that depend on the ordering convention adopted in setting up the Weyl map, the final result for the conserved charges is reassuringly independent of the choice of Weyl map and (the corresponding choice of) star product.

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I. INTRODUCTION

Over these past few years there has been strong interest in the study of theories formulated in noncommutative versions of the Minkowski spacetime. The most studied possibility is the one of spacetime noncommutativity of "canonical" [1] form,

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \tag{1}$$

where $\hat{x}^\mu$ are the spacetime coordinates ($\mu \in \{0, 1, 2, 3\}$, time coordinate $\hat{x}^0$) and $\theta^{\mu\nu}$ is coordinate independent. The literature on this possibility is extremely large, because the same formula (1) can actually represent rather different physical scenarios, depending on the properties attributed to $\theta^{\mu\nu}$. The earliest studies we are aware of are actually the ones [2] in which richer properties were attributed to $\theta^{\mu\nu}$, including some restrictions [2] on the admissible forms of $\theta^{\mu\nu}$ and the possibility of nontrivial algebraic properties [3]. And this picture can find valuable motivation in the outcome of certain heuristic analyses of limitations on the localization of a spacetime point in the quantum-gravity realm [2]. The formalism is much simpler if one analyzes (1) assuming that $\theta^{\mu\nu}$ is a (dimensionful) number-valued tensor [4–6], and this gives rise to a picture which could be rather valuable, since it is believed to provide an accurate, effective-theory description of string theory in the presence of a certain tensor background [4–6]. The tensor background breaks the spacetime symmetries in just the way codified by the tensor $\theta^{\mu\nu}$: the laws of physics are different in different frames because $\theta^{\mu\nu}$ (transforming like a Lorentz-Poincaré tensor) takes different values in different frames. The third possibility is for $\theta^{\mu\nu}$ to be a number-valued observer-independent matrix. This would, of course, require the laws of transformation between inertial observers to be modified in a θ-dependent manner [7–9]. Preliminary results [10–12] suggest that this might be accomplished by introducing a description of translations, boosts, and space-rotation transformations based on the formalism of Hopf algebras.

We intend to focus here on this third possibility, looking for a deeper understanding of the structure of the Hopf-algebra symmetry transformations and hoping to set the stage for a more physical characterization of this novel concept. In particular, we are interested in establishing similarities and differences between the Hopf-algebra symmetries of canonical spacetimes and the Hopf-algebra symmetries of the so-called κ-Minkowski spacetime, for which some of us recently reported a Noether analysis [13,14].

The key ingredient which allowed [13,14] the completion (after more than a decade of failed attempts) of some Noether analyses in the κ-Minkowski case was the introduction of "noncommutative transformation parameters" with appropriate nontrivial commutators with the spacetime coordinates. And interestingly, the form of the commutators between transformation parameters and spacetime coordinates turns out to be incompatible with the possibility of a pure boost. We show here that analogous structures appear in the analysis of the Hopf-algebra symmetries of observer-independent canonical spacetime noncommutativity. In this case we find that neither a pure boost nor a pure rotation is allowed, and, combining these results with the ones previously obtained for the κ-Minkowski case, we conjecture a (limited) universality of a no-pure-boost uncertainty principle for Hopf-algebra symmetries of noncommutative Minkowski-like spacetimes.

We also stress the significance of the fact that our Noether analysis derives 10 conserved charges from the Poincaré-like Hopf-algebra symmetries. This provides encouragement for the idea that these Hopf-algebra symmetries are truly meaningful in characterizing observable aspects of the relevant theories, contrary to what was feared by some authors (see, e.g., Ref. [15]), who had argued that the Hopf-algebra structures encountered in the study of canonical noncommutative spacetimes might be just fancy mathematical formalizations of a rather trivial breakdown of symmetry.

Guided again by intuition developed in our previous studies of κ-Minkowski spacetime [13,14,16], we also expose an ordering issue for the so-called classical-action description of the generators of symmetry transformations in canonical noncommutative spacetime. While this issue should be carefully monitored in future analyses of other aspects of theories in canonical noncommutative spacetimes, we reassuringly find that our result for the charges has no dependence on this choice of ordering prescription.

II. TWISTED HOPF SYMMETRY ALGEBRA AND ORDERING ISSUES

Our first task is to show that the much studied [10–12] "twisted" Hopf algebra of (candidate) symmetries of canonical noncommutative spacetime can be obtained by introducing rules of "classical action" [16] for the generators of the symmetry algebra. We start by observing that the fields one considers in constructing theories in a canonical noncommutative spacetime can be written in the form [1]

$$\Phi(\hat{x}) = \int d^4k \tilde{\Phi}_w(k) e^{ik\hat{x}} \tag{2}$$

by introducing ordinary (commutative) "Fourier parameters" k_μ.[1]

This provides for any given noncommutative function $\Phi(\hat{x})$ a "Fourier transform" $\tilde{\Phi}_w(k)$. In turn, this provides the basis for introducing a "Weyl map" Ω_w, which specifies an auxiliary commutative function $\Phi_w^{(\mathrm{comm})}(x)$ for any given noncommutative function $\Phi(\hat{x})$:

$$\begin{aligned}\Phi(\hat{x}) &= \Omega_w(\Phi_w^{(\mathrm{comm})}(x)) \equiv \Omega_w\left(\int d^4k \tilde{\Phi}_w(k) e^{ikx}\right)\\ &= \int d^4k \tilde{\Phi}_w(k) e^{ik\hat{x}}.\end{aligned} \tag{3}$$

It is easy to verify that this definition of the Weyl map Ω_w acts on a given commutative function by giving a noncommutative function with full symmetrization ("Weyl ordering") on the noncommutative spacetime coordinates [e.g., $\Omega_w(e^{ikx}) = e^{ik\hat{x}}$ and $\Omega_w(x_1 x_2^2) = \frac{1}{3}(\hat{x}_2^2\hat{x}_1 + \hat{x}_2\hat{x}_1\hat{x}_2 + \hat{x}_1\hat{x}_2^2)$].

We shall stress that it is also legitimate to consider Weyl maps with other ordering prescriptions, but before we do that let us first use Ω_w for our description of the relevant twisted Hopf algebra. This comes about by introducing rules of "classical action" for the generators of translations and space rotations and boosts[2]:

$$P_\mu^{(w)} e^{ik\hat{x}} \equiv P_\mu^{(w)} \Omega_w(e^{ikx}) \equiv \Omega_w(P_\mu e^{ikx}) = \Omega_w(i\partial_\mu e^{ikx}), \tag{4}$$

$$\begin{aligned}M_{\mu\nu}^{(w)} e^{ik\hat{x}} &\equiv M_{\mu\nu}^{(w)} \Omega_w(e^{ikx}) \equiv \Omega_w(M_{\mu\nu} e^{ikx})\\ &= \Omega_w(ix_{[\mu}\partial_{\nu]} e^{ikx}).\end{aligned} \tag{5}$$

Here the antisymmetric "Lorentz-sector" matrix of operators $M_{\mu\nu}^{(w)}$ is composed as usual by the space-rotation generators $R_i^{(w)} = \frac{1}{2}\epsilon_{ijk}M_{jk}^{(w)}$ and the boost generators $N_i^{(w)} = M_{0i}^{(w)}$. The rules of action codified in (4) and (5) are said to be "classical actions according to the Weyl map Ω_w" since they indeed reproduce the corresponding classical rules of action within the Weyl map.

It is easy to verify that the generators introduced in (4) and (5) satisfy the same commutation relations of the classical Poincaré algebra:

$$\begin{aligned}&[P_\mu^{(w)}, P_\nu^{(w)}] = 0, \qquad [P_\alpha^{(w)}, M_{\mu\nu}^{(w)}] = i\eta_{\alpha[\mu}P_{\nu]}^{(w)},\\ &[M_{\mu\nu}^{(w)}, M_{\alpha\beta}^{(w)}] = i(\eta_{\alpha[\nu}M_{\mu]\beta}^{(w)} + \eta_{\beta[\mu}M_{\nu]\alpha}^{(w)}).\end{aligned} \tag{6}$$

However, the action of Lorentz-sector generators does not comply with the Leibniz rule,

$$\begin{aligned}M_{\mu\nu}^{(w)}(e^{ik\hat{x}}e^{iq\hat{x}}) &= (M_{\mu\nu}^{(w)}e^{ik\hat{x}})e^{iq\hat{x}} + e^{ik\hat{x}}(M_{\mu\nu}^{(w)}e^{iq\hat{x}})\\ &\quad - \tfrac{1}{2}\theta^{\alpha\beta}[\eta_{\alpha[\mu}(P_{\nu]}^{(w)}e^{ik\hat{x}})(P_\beta^{(w)}e^{iq\hat{x}})\\ &\quad + (P_\alpha^{(w)}e^{ik\hat{x}})\eta_{\beta[\mu}(P_{\nu]}^{(w)}e^{iq\hat{x}})],\end{aligned} \tag{7}$$

as one easily verifies using the fact that, from (1), it follows that

$$\begin{aligned}e^{ik\hat{x}}e^{iq\hat{x}} &= e^{i(k+q)\hat{x}}e^{-(i/2)k^\mu\theta_{\mu\nu}q^\nu}\\ &\equiv \Omega_w(e^{i(k+q)x}e^{-(i/2)k^\mu\theta_{\mu\nu}q^\nu}).\end{aligned} \tag{8}$$

The translation generators, instead, satisfy the Leibniz rule

$$P_\mu^{(w)}(e^{ik\hat{x}}e^{iq\hat{x}}) = (P_\mu^{(w)}e^{ik\hat{x}})e^{iq\hat{x}} + e^{ik\hat{x}}(P_\mu^{(w)}e^{iq\hat{x}}), \tag{9}$$

as one could have expected from the form of the commu-

[1]We use $\hat{x}$ for noncommuting coordinates and x for the auxiliary commuting ones.

[2]In light of (2) one obtains a fully general rule of action of operators by specifying their action only on the exponentials $e^{ik\hat{x}}$. Also note that we adopt a standard compact notation for antisymmetrized indices: $A_{[\alpha\beta]} \equiv A_{\alpha\beta} - A_{\beta\alpha}$.

tators (1), which is evidently compatible with classical translation symmetry (while, for observer-independent $\theta^{\mu\nu}$, it clearly requires an adaptation of the Lorentz sector.)

In the relevant literature, observations of the type reported in (7) and (9) are often described in terms of a Hopf algebraic structure, specifying the coproduct

$$\Delta P_\mu^{(w)} = P_\mu^{(w)} \otimes \mathbb{1} + \mathbb{1} \otimes P_\mu^{(w)},$$
$$\Delta M_{\mu\nu}^{(w)} = M_{\mu\nu}^{(w)} \otimes \mathbb{1} + \mathbb{1} \otimes M_{\mu\nu}^{(w)} - \frac{1}{2}\theta^{\alpha\beta}[\eta_{\alpha[\mu}P_{\nu]}^{(w)} \otimes P_\beta^{(w)} + P_\alpha^{(w)} \otimes \eta_{\beta[\mu}P_{\nu]}^{(w)}]. \quad (10)$$

Antipode and counit, the other two building blocks needed[3] for a Hopf algebra, can also be straightforwardly introduced [18], but do not play a role in the analysis we are reporting here.

It turns out that the coproducts (10) are describable as a deformation of the classical Poincaré Lie algebra by the following twist element:

$$\mathcal{F} = e^{(i/2)\theta^{\mu\nu}P_\mu^{(w)}\otimes P_\nu^{(w)}}. \quad (11)$$

The form of the twist element is most easily obtained from the structure of the star product, which is a way to reproduce the rule of product of noncommutative functions within the Weyl map: $e^{ik\hat{x}}e^{iq\hat{x}} \equiv \Omega_w(e^{ikx} \star e^{iqx})$. From (8) we see that our star product must be such that $e^{ikx} \star e^{iqx} = e^{i(k+q)x}e^{-(i/2)k^\mu\theta_{\mu\nu}q^\nu}$, and denoting by $\bar{\mathcal{F}} \equiv \sum(\bar{f}_1 \otimes \bar{f}_2)$ the representation of the inverse of the twist element $\mathcal{F}^{-1}$ on $\mathcal{A} \otimes \mathcal{A}$ [where $\mathcal{A}$ is the algebra of commutative functions $f(x)$], we must have [10] $\Omega_w(g(x) \star h(x)) = \Omega_w(\sum(\bar{f}_1(g))(\bar{f}_2(h)))$, from which (11) follows.

Hopf algebras that are obtained from a given Lie algebra by exclusively acting with a twist element preserve the form of the commutators among generators, so that all the structure of the deformation is codified in the coproducts. And these coproducts are structured in such a way that for a generator G_θ, obtained by twisting G, the coproduct Δ_θ has the form $\Delta_\theta(G_\theta) = \mathcal{F}\Delta(G)\mathcal{F}^{-1}$.

Having established that by introducing classical action according to Ω_w for translations, space rotations, and boosts one obtains a certain set of generators for a twisted Hopf algebra, it is natural to ask if something different is encountered if these generators are introduced with classical action according to a different Weyl map, such as the Weyl map Ω_1 defined by $\Omega_1(e^{ikx}) = e^{ik^A\hat{x}_A}e^{ik^1\hat{x}_1}$, where $A = 0, 2, 3$.

A given field $\Phi(\hat{x})$ with Ω_w-Fourier transform $\tilde{\Phi}_w(k)$ [in the sense of (2)] has different Fourier transform, $\tilde{\Phi}_1(k)$, according to the Weyl map Ω_1:

$$\Phi(\hat{x}) = \int d^4k\tilde{\Phi}_1(k)e^{ik^A\hat{x}_A}e^{ik^1\hat{x}_1}, \quad (12)$$

and, since $e^{ik\hat{x}} = e^{ik^A\hat{x}_A}e^{ik^1\hat{x}_1}e^{(i/2)k^Ak^1\theta_{A1}}$, the two Fourier transforms are simply related:

$$\tilde{\Phi}_1(k) = \tilde{\Phi}_w(k)e^{-(i/2)k^Ak^1\theta_{A1}}. \quad (13)$$

Denoting by $P_\mu^{(1)}$ and $M_{\mu\nu}^{(1)}$ the generators with classical action according to Ω_1, one easily finds that they also leave invariant the commutation relations (1). And, as most easily verified [18] through a simple analysis of the action of these generators on $e^{ik\hat{x}} = \Omega_w(e^{ikx}) = \Omega_1(e^{ikx}e^{(i/2)k^Ak^1\theta_{A1}})$, the following relations hold:

$$P_\mu^{(1)} = P_\mu^{(w)} \equiv P_\mu,$$
$$M_{\mu\nu}^{(1)} = M_{\mu\nu}^{(w)} + \frac{1}{2}\theta^{A1}[\eta_{1[\mu}P_{\nu]}P_A + \eta_{A[\mu}P_{\nu]}P_1]. \quad (14)$$

Setting aside the difference between $M_{\mu\nu}^{(1)}$ and $M_{\mu\nu}^{(w)}$, one could say that the construction based on the two Weyl maps Ω_w and Ω_1 leads to completely analogous structures. Again, one easily uncovers the structure of a twisted Hopf algebra, the commutators of generators are undeformed, and all the structure of the deformation is in a coproduct relation, which in the case of the Ω_1 map takes the form

$$\Delta M_{\mu\nu}^{(1)} = M_{\mu\nu}^{(1)} \otimes \mathbb{1} + \mathbb{1} \otimes M_{\mu\nu}^{(1)} - \frac{1}{2}\theta^{\alpha\beta}[\eta_{\alpha[\mu}P_{\nu]} \otimes P_\beta + P_\alpha \otimes \eta_{\beta[\mu}P_{\nu]}] + \frac{1}{2}\theta^{A1}[\eta_{A[\mu}P_{\nu]} \otimes P_1 + \eta_{1[\mu}P_{\nu]} \otimes P_A + P_1 \otimes P_{[\nu}\eta_{\mu]A} + P_A \otimes P_{[\nu}\eta_{\mu]1}]. \quad (15)$$

This may be viewed again as the result of "twisting," which in this case would be due to the following twist element:

$$\mathcal{F}_1 = e^{(i/2)\theta_{AB}P^A\otimes P^B}e^{-i\theta_{A1}P^1\otimes P^A}, \quad (16)$$

where $A, B = 0, 2, 3$.

The two sets of generators $\{P_\mu, M_{\mu\nu}^{(1)}\}$ and $\{P_\mu, M_{\mu\nu}^{(w)}\}$ can be meaningfully described as two bases of generators for the same twisted Hopf algebra. However, we shall keep track of the structures we encounter as a result of the difference between $M_{\mu\nu}^{(1)}$ and $M_{\mu\nu}^{(w)}$, which, since these differences merely amount to a choice of ordering convention, we expect not to affect the observable features of our theory.

III. NONCOMMUTATIVE TRANSFORMATION PARAMETERS

Our analysis of canonical noncommutativity will be guided by the description of symmetry transformations for κ-Minkowski spacetime noncommutativity reported by some of us in Refs. [13,14].

[3] We are dealing with a Hopf algebra obtained by the "twist" of a known Lie algebra [10,12], and this twist gives automatic prescriptions [17] for the coproduct, antipode, and counit that must be introduced in order to obtain a closed Hopf algebra.

After the failures of several other attempts, the criteria adopted in Refs. [13,14] finally allowed us to complete successfully the Noether analysis, including the identification of some conserved (time-independent) charges associated with the symmetries. We shall therefore assume that those criteria should also be adopted in the case of canonical noncommutativity.

In Refs. [13,14] κ-Poincaré symmetry transformations of a function $f(\hat{x})$ of the κ-Minkowski spacetime coordinates were described as $f(\hat{x}) \to f(\hat{x}) + df(\hat{x})$, where $df(\hat{x})$ was parametrized as follows[4]:

$$df(\hat{x}) = i(\gamma^\mu P_\mu + \sigma_j R_j + \tau_k N_k) f(\hat{x}), \qquad (17)$$

where γ_μ, σ_j, τ_k are the transformation parameters (respectively, translation, space-rotation, and boost parameters), and P_μ, R_j, N_k are, respectively, translation, space-rotation, and boost generators.

The properties of the transformation parameters γ_μ, σ_j, and τ_k were derived [13,14] by imposing the Leibniz rule on d,

$$d(f(\hat{x})g(\hat{x})) = (df(\hat{x}))g(\hat{x}) + f(\hat{x})(dg(\hat{x})). \qquad (18)$$

It turned out that this requirement cannot be satisfied by standard (commutative) transformation parameters, so Refs. [13,14] introduced the concept of "noncommutative transformation parameters" as a generalization of the standard concept of transformation parameters that would allow one to satisfy the Leibniz rule. These noncommutative transformation parameters were required to still act only by (associative) multiplication on the spacetime coordinates, but were allowed to be subject to nontrivial rules of commutation with the spacetime coordinates. An intriguing aspect of the commutators between transformation parameters and spacetime coordinates derived in Refs. [13,14] is that they turn out to be incompatible with the possibility of a pure boost. The structure of κ-Minkowski spacetime does allow pure translations and pure space rotations, but when the boost parameters are not set to zero then the space-rotation parameters also must not all be zero.

We intend to introduce here an analogous description of the twisted Hopf symmetry transformations of canonical spacetimes. For the laws of transformation we adopt Ansätze that are completely analogous to the one we reported here in Eq. (17), which was adopted in Ref. [14] and proved to be up to the task of derivation of κ-Minkowski conserved charges. Of course, at this early stage of development of techniques for the Noether analysis of theories with Hopf-algebra spacetime symmetries, it would be interesting to explore other possible Ansätze, at least for what concerns the possibility (which some of us already stressed in Ref. [13]) of some dependence of the results on the choice of ordering convention chosen in Eq. (17). Rather than the form $df \sim \epsilon_a(T_a f)$, with all transformation parameters, ϵ_a, to the left and the actions of symmetry generators on fields, $(T_a f)$, to the right, one could adopt [13] Ansätze of the type $df \sim (T_a f)\epsilon_a$ or perhaps $df \sim [(T_a f)\epsilon_a + \epsilon_a(T_a f)]/2$. But in this first paper on conserved charges for observer-independent canonical noncommutativity, we are satisfied with just showing that such charges do exist (contrary to some conjectures in the literature [15]), and for that purpose the use of Ansätze of the type in Eq. (17) will turn out to be sufficient.

We start by analyzing the case of a pure translation transformation:

$$d_P f(\hat{x}) = i\gamma^\mu_{(w)} P_\mu f(\hat{x}). \qquad (19)$$

Imposing the Leibniz rule, because of the triviality of the coproduct of the translation generators (see previous section), for this case of a pure translation transformation one easily verifies that the condition imposed by compliance with the Leibniz rule,

$$[f(\hat{x}), \gamma^\mu_{(w)}] P_\mu g(\hat{x}) = 0, \qquad (20)$$

is also trivial and is satisfied by ordinary commutative transformation parameters.

For the case of a pure Lorentz-sector transformation, we start by considering the generators $M^{(w)}_{\mu\nu}$, with classical action according to the Weyl map Ω_w, and introduce once again transformation parameters analogous to the ones in Eq. (17):

$$d_L f(\hat{x}) = i\omega^{\mu\nu}_{(w)} M^{(w)}_{\mu\nu} f(\hat{x}). \qquad (21)$$

In this case, from the Leibniz-rule requirement (18), one obtains

$$d_L(f(\hat{x})g(\hat{x})) = i\omega^{\mu\nu}_{(w)}(M^{(w)}_{\mu\nu} f(\hat{x}))g(\hat{x}) + if(\hat{x})\omega^{\mu\nu}_{(w)}(M^{(w)}_{\mu\nu} g(\hat{x})), \qquad (22)$$

while from the coproduct rule $\Delta M^{(w)}_{\mu\nu}$ codified in Eq. (10), one obtains

$$d_L(f(\hat{x})g(\hat{x})) = i\omega^{\mu\nu}_{(w)}[(M^{(w)}_{\mu\nu} f(\hat{x}))g(\hat{x}) + f(\hat{x})(M^{(w)}_{\mu\nu} g(\hat{x})) - \tfrac{1}{2}(\theta_{[\mu}{}^{\sigma}\delta_{\nu]}{}^{\rho} + \theta^{\rho}{}_{[\mu}\delta_{\nu]}{}^{\sigma})(P_\rho f(\hat{x})) \times (P_\sigma g(\hat{x}))]. \qquad (23)$$

Requiring that (22) be consistent with (23), one finds

$$[f(\hat{x}), \omega^{\mu\nu}_{(w)}] M^{(w)}_{\mu\nu} g(\hat{x}) = -\tfrac{1}{2}\omega^{\mu\nu}_{(w)}(\theta_{[\mu}{}^{\sigma}\delta_{\nu]}{}^{\rho} + \theta^{\rho}{}_{[\mu}\delta_{\nu]}{}^{\sigma}) \times (P_\rho f(\hat{x}))(P_\sigma g(\hat{x})). \qquad (24)$$

This does not admit any solution of the type we are allowing for the transformation parameters. In fact, in order to be

[4]The operator d, which implements a map from functions of the noncommutative coordinates to objects of the form (17) (which are products of transformation parameters with functions of the noncommutative coordinates), is a "differential" in the sense of Ref. [19], if it complies with the Leibniz rule.

solutions of (24) the $\omega^{\mu\nu}_{(w)}$ should be operators with a highly nontrivial action on functions of the spacetime coordinates, rather than being "noncommutative parameters," acting by simple (associative) multiplication on the spacetime coordinates.

We conclude that, whereas pure translations are allowed in canonical spacetimes, the possibility of a pure Lorentz-sector transformation is excluded.

We find however that, while pure Lorentz-sector transformations are not allowed, it is possible to combine Lorentz-sector and translation transformations. In fact, if we consider a transformation with

$$df(\hat{x}) = i[\gamma^{\alpha}_{(w)}P_{\alpha} + \omega^{\mu\nu}_{(w)}M^{(w)}_{\mu\nu}]f(\hat{x}), \tag{25}$$

then from the Leibniz-rule requirement, one obtains

$$[[f(\hat{x}), \gamma^{\alpha}_{(w)}] + \tfrac{1}{2}\omega^{\mu\nu}_{(w)}(\theta_{[\mu}{}^{\alpha}\delta_{\nu]}{}^{\rho} + \theta^{\rho}{}_{[\mu}\delta_{\nu]}{}^{\alpha})(P_{\rho}f(\hat{x}))] \times P_{\alpha}g(\hat{x}) + [f(\hat{x}), \omega^{\mu\nu}_{(w)}]M^{(w)}_{\mu\nu}g(\hat{x}) = 0, \tag{26}$$

which can be satisfied by transformation parameters of the type we are allowing. The properties of these transformation parameters can be inferred observing that this relation (26) must be valid for every choice of the functions $f(\hat{x})$ and $g(\hat{x})$, which leads us to impose that the term proportional to $P_{\alpha}g(\hat{x})$ and the term proportional to $M^{(w)}_{\mu\nu}g(\hat{x})$ be separately null, thus obtaining

$$[f(\hat{x}), \gamma^{\alpha}_{(w)}] = -\tfrac{1}{2}\omega^{\mu\nu}_{(w)}(\theta_{[\mu}{}^{\alpha}\delta_{\nu]}{}^{\rho} + \theta^{\rho}{}_{[\mu}\delta_{\nu]}{}^{\alpha})P_{\rho}f(\hat{x}),$$
$$[f(\hat{x}), \omega^{\mu\nu}_{(w)}] = 0. \tag{27}$$

These requirements imply the following properties of the transformation parameters:

$$[\hat{x}^{\beta}, \gamma^{\alpha}_{(w)}] = -\frac{i}{2}\omega^{\mu\nu}_{(w)}(\theta_{[\mu}{}^{\alpha}\delta_{\nu]}{}^{\beta} + \theta^{\beta}{}_{[\mu}\delta_{\nu]}{}^{\alpha}), \tag{28}$$

$$[\hat{x}^{\beta}, \omega^{\mu\nu}_{(w)}] = 0, \tag{29}$$

which are consistent with our criterion for noncommutative transformation parameters, since they introduce indeed a noncommutativity between transformation parameters and spacetime coordinates, but in a way that is compatible with our requirement that the transformation parameters act only by (associative) multiplication on the spacetime coordinates.

We conclude that Lorentz-sector transformations are allowed but only in combination with translation transformations. Indeed (28) is such that whenever $\omega_{(w)} \neq 0$ then also $\gamma_{(w)} \neq 0$. And interestingly, the translation-transformation parameters, which can be commutative in the case of a pure translation transformation, must comply with (28), and therefore be noncommutative parameters, in the general case of a transformation that combines a translation component and a Lorentz-sector component.

Since in the preceding section we raised the issue of possible alternatives to the $\{P_{\mu}, M^{(w)}_{\mu\nu}\}$ basis, such as the basis $\{P_{\mu}, M^{(1)}_{\mu\nu}\}$ obtained by a different ordering prescription in the Weyl map used to introduce the "classical action" of the generators, we should stress here that the analysis of transformation parameters proceeds in exactly the same way if one works with the basis $\{P_{\mu}, M^{(1)}_{\mu\nu}\}$; however, the noncommutativity properties of the transformation parameters are somewhat different. In the case $\{P_{\mu}, M^{(1)}_{\mu\nu}\}$ one ends up considering transformations of the form

$$d^{(1)}f(\hat{x}) = i[\gamma^{\alpha}_{(1)}P_{\alpha} + \omega^{\mu\nu}_{(1)}M^{(1)}_{\mu\nu}]f(\hat{x}), \tag{30}$$

and it is easy to verify that the transformation parameters must satisfy the following noncommutativity requirements:

$$[\hat{x}^{\beta}, \gamma^{\alpha}_{(1)}] = -\frac{i}{2}\omega^{\mu\nu}_{(1)}\Upsilon^{\alpha\beta}_{\mu\nu} \qquad [\hat{x}^{\beta}, \omega^{\mu\nu}_{(1)}] = 0, \tag{31}$$

where $\Upsilon^{\alpha\rho}_{\mu\nu} \equiv (\theta_{[\mu}{}^{\alpha}\delta_{\nu]}{}^{\rho} + \theta^{\rho}{}_{[\mu}\delta_{\nu]}{}^{\alpha}) - \theta^{A1}[\eta_{A[\mu}\delta_{\nu]}{}^{\rho}\delta_{1}{}^{\alpha} + \eta_{1[\mu}\delta_{\nu]}{}^{\rho}\delta_{A}{}^{\alpha} + \eta_{A[\mu}\delta_{\nu]}{}^{\alpha}\delta_{1}{}^{\rho} + \eta_{1[\mu}\delta_{\nu]}{}^{\alpha}\delta_{A}{}^{\rho}]$.

We shall show that, even though the differences between $M^{(w)}_{\mu\nu}$ and $M^{(1)}_{\mu\nu}$ require different forms of the commutators between transformation parameters and spacetime coordinates, these two possible choices of convention for the description of the symmetry Hopf algebra lead to the same conserved charges.

IV. CONSERVED CHARGES

We now test our formulation of twisted Hopf-algebra symmetry transformations in the context of a Noether analysis of the simplest and most studied theory formulated in canonical noncommutative spacetime: a theory for a massless scalar field $\phi(\hat{x})$ governed by the following Klein-Gordon-like equation of motion:

$$\Box\phi(\hat{x}) \equiv P_{\mu}P^{\mu}\phi(\hat{x}) = 0. \tag{32}$$

Consistently with the analysis reported in the previous section, we want to obtain conserved charges associated with the transformations of the form

$$\delta\phi(\hat{x}) = -d\phi(\hat{x}) = -i[\gamma^{\alpha}_{(w)}P_{\alpha} + \omega^{\mu\nu}_{(w)}M^{(w)}_{\mu\nu}]\phi(\hat{x}), \tag{33}$$

where the first equality holds because the field we are considering is a scalar.

We take as a starting point for the Noether analysis the action

$$S = \frac{1}{2}\int d^4\hat{x}\,\phi(\hat{x})\Box\phi(\hat{x}), \tag{34}$$

which (as one can easily verify [18]) generates the equation of motion (32) and is invariant under the transformation (33):

$$\delta S = \frac{1}{2}\int d^4\hat{x}(\delta\phi(\hat{x})\Box\phi(\hat{x}) + \phi(\hat{x})\Box\delta\phi(\hat{x}) - d(\phi(\hat{x})\Box\phi(\hat{x}))) = \frac{1}{2}\int d^4\hat{x}\phi(\hat{x})[\Box, \delta]\phi(\hat{x}) = 0. \qquad (35)$$

Of course, the charges are to be obtained for field solutions of the equation of motion, and therefore we can use (32) to rewrite (35) in the following way:

$$\delta S = \frac{1}{2}\int d^4\hat{x}\phi(\hat{x})\Box\delta\phi(\hat{x}) = \frac{1}{2}\int d^4\hat{x}P_\mu[\phi(\hat{x})P^\mu\delta\phi(\hat{x}) - (P^\mu\phi(\hat{x}))\delta\phi(\hat{x})]. \qquad (36)$$

Then using the commutation relations of the infinitesimal parameters obtained in Eq. (27), one can further rewrite δS in the following insightful manner:

$$\delta S = -i\int d^4\hat{x}(\gamma_\nu^{(w)}P_\mu T^{\mu\nu} + \omega_{(w)}^{\rho\sigma}P_\mu J^\mu_{\rho\sigma}), \qquad (37)$$

with

$$T^{\mu\nu} = \tfrac{1}{2}(\phi(\hat{x})P^\mu P^\nu\phi(\hat{x}) - (P^\mu\phi(\hat{x}))P^\nu\phi(\hat{x})),$$
$$J^\mu_{\rho\sigma} = \tfrac{1}{2}(\phi(\hat{x})P^\mu M^{(w)}_{\rho\sigma}\phi(\hat{x}) - (P^\mu\phi(\hat{x}))M^{(w)}_{\rho\sigma}\phi(\hat{x})) - \tfrac{1}{4}(\theta_{[\rho}{}^{\nu}\delta_{\sigma]}{}^{\lambda} + \theta^{\lambda}{}_{[\rho}\delta_{\sigma]}{}^{\nu})[(P_\lambda\phi(\hat{x}))P^\mu P_\nu\phi(\hat{x}) - (P^\mu P_\lambda\phi(\hat{x}))P_\nu\phi(\hat{x})]. \qquad (38)$$

It is rather easy to verify that by spatial integration of the zeroth components of the "currents" $T^{\mu\nu}$ and $J^\mu_{\rho\sigma}$, one obtains time-independent charges. Denoting these charges with Q_μ, $K_{\rho\sigma}$,

$$Q_\mu = \int d^3\hat{x}T^0_\mu, \qquad K_{\rho\sigma} = \int d^3\hat{x}J^0_{\rho\sigma}, \qquad (39)$$

and using the ordering convention (2) for the Fourier expansion of a generic field which is a solution of the equation of motion,

$$\phi(\hat{x}) = \int d^4k\delta(k^2)\tilde{\phi}_{(w)}(k)e^{ik\hat{x}}, \qquad (40)$$

upon integration over the spatial coordinates,[5] one finds

$$Q_\mu = \frac{1}{2}\int d^4kd^4q\delta(k^2)\delta(q^2)\tilde{\phi}_{(w)}(k)\tilde{\phi}_{(w)}(q)(q^0 - k^0)q_\mu \times \delta^{(3)}(\vec{k} + \vec{q})e^{i(k^0+q^0)\hat{x}_0}e^{(i/2)(k^0+q^0)(k^i+q^i)\theta_{i0}} \times e^{-(i/2)k^\mu q^\nu\theta_{\mu\nu}}, \qquad (41)$$

[5] Our spatial Dirac deltas are such that $\int d^3\hat{x}e^{ik^i\hat{x}_i} = \delta^{(3)}(\vec{k})$.

$$K_{\rho\sigma} = \frac{1}{2}\int d^4kd^4q\delta(k^2)\tilde{\phi}_{(w)}(k)\Big[iq_{[\rho}\frac{\partial}{\partial q^{\sigma]}}[\delta(q^2)\tilde{\phi}_{(w)}(q)] - \frac{1}{2}\delta(q^2)(\theta_{[\rho}{}^{\nu}\delta_{\sigma]}{}^{\lambda} + \theta^{\lambda}{}_{[\rho}\delta_{\sigma]}{}^{\nu})k_\lambda q_\nu\tilde{\phi}_{(w)}(q)\Big] \times (k^0 - q^0)\delta^{(3)}(\vec{k} + \vec{q})e^{i(k^0+q^0)\hat{x}_0}e^{(i/2)(k^0+q^0)(k^i+q^i)\theta_{i0}} \times e^{-(i/2)k^\mu q^\nu\theta_{\mu\nu}}. \qquad (42)$$

Then integrating in d^4k, and observing that in $K_{\rho\sigma}$ the term $-\frac{1}{2}(\theta_{[\rho}{}^{\nu}\delta_{\sigma]}{}^{\lambda} + \theta^{\lambda}{}_{[\rho}\delta_{\sigma]}{}^{\nu})k_\lambda q_\nu\tilde{\phi}(q)$ gives a null contribution, one obtains

$$Q_\mu = \frac{1}{2}\int\frac{d^4q}{2|\vec{q}|}\delta(q^2)\tilde{\phi}_{(w)}(q)q_\mu\{\tilde{\phi}_{(w)}(-\vec{q}, |\vec{q}|)(q^0 + |\vec{q}|) \times e^{i(q^0-|\vec{q}|)\hat{x}_0}e^{-(i/2)(q^0-|\vec{q}|)q^i\theta_{0i}} + \tilde{\phi}_{(w)}(-\vec{q}, -|\vec{q}|) \times (q^0 - |\vec{q}|)e^{i(q^0+|\vec{q}|)\hat{x}_0}e^{-(i/2)(q^0+|\vec{q}|)q^i\theta_{0i}}\}, \qquad (43)$$

$$K_{\rho\sigma} = \frac{i}{2}\int\frac{d^4q}{2|\vec{q}|}\delta(q^2)\tilde{\phi}_{(w)}(q)q_{[\rho}\Big\{(q^0 + |\vec{q}|) \times \Big[\frac{\partial}{\partial q^{\sigma]}}\tilde{\phi}_{(w)}(-\vec{q}, |\vec{q}|)\Big]e^{i(q^0-|\vec{q}|)\hat{x}_0}e^{-(i/2)(q^0-|\vec{q}|)q^i\theta_{0i}} + (q^0 - |\vec{q}|)\Big[\frac{\partial}{\partial q^{\sigma]}}\tilde{\phi}_{(w)}(-\vec{q}, -|\vec{q}|)\Big] \times e^{i(q^0+|\vec{q}|)\hat{x}_0}e^{-(i/2)(q^0+|\vec{q}|)q^i\theta_{0i}}\Big\}. \qquad (44)$$

One can then use the fact that $\delta(q^2)$ imposes $q_0 = \pm|\vec{q}|$, and the presence of factors of the types $(q^0 - |\vec{q}|)e^{\alpha(q^0+|\vec{q}|)}$ and $(q^0 + |\vec{q}|)e^{\alpha(q^0-|\vec{q}|)}$, to obtain the following explicitly time-independent formulas for the charges:

$$Q_\mu = \frac{1}{2}\int\frac{d^4q}{2|\vec{q}|}\delta(q^2)\tilde{\phi}_{(w)}(q)q_\mu\{\tilde{\phi}_{(w)}(-\vec{q}, |\vec{q}|)(q^0 + |\vec{q}|) + \tilde{\phi}_{(w)}(-\vec{q}, -|\vec{q}|)(q^0 - |\vec{q}|)\}, \qquad (45)$$

$$K_{\rho\sigma} = \frac{i}{2}\int\frac{d^4q}{2|\vec{q}|}\delta(q^2)\tilde{\phi}_{(w)}(q)q_{[\rho}\Big\{(q^0 + |\vec{q}|) \times \frac{\partial\tilde{\phi}_{(w)}(-\vec{q}, |\vec{q}|)}{\partial q^{\sigma]}} + (q^0 - |\vec{q}|)\frac{\partial\tilde{\phi}_{(w)}(-\vec{q}, -|\vec{q}|)}{\partial q^{\sigma]}}\Big\}. \qquad (46)$$

V. ORDERING-CONVENTION INDEPENDENCE OF THE CHARGES

In light of the "choice-of-ordering issue" we raised in Sec. II, which, in particular, led us to consider the examples of two possible bases of generators, the $\{P_\mu, M^{(w)}_{\mu\nu}\}$ basis and the $\{P_\mu, M^{(1)}_{\mu\nu}\}$ basis, and especially considering the fact that in Sec. III we found that in different bases the noncommutative transformation parameters should have somewhat different properties (different form of the commutators with the spacetime coordinates), it is interesting to verify whether or not the result for the charges obtained in the previous section working with the $\{P_\mu, M^{(w)}_{\mu\nu}\}$ basis is confirmed by a corresponding analysis based on the $\{P_\mu, M^{(1)}_{\mu\nu}\}$ basis.

When adopting the $\{P_\mu, M^{(1)}_{\mu\nu}\}$ basis, the symmetry variation of a field is described by

$$\delta\phi(\hat{x}) = -d^{(1)}\phi(\hat{x}) = -i[\gamma^\alpha_{(1)}P_\alpha + \omega^{\mu\nu}_{(1)}M^{(1)}_{\mu\nu}]\phi(\hat{x}), \tag{47}$$

rather than (33). And going through the same types of steps discussed in the previous section, the analysis of the symmetry variation of the action (34) then leads to [18] the following formulas for the currents:

$$T^{\mu\nu(1)} = \tfrac{1}{2}(\phi(\hat{x})P^\mu P^\nu\phi(\hat{x}) - P^\mu\phi(\hat{x})P^\nu\phi(\hat{x})), \tag{48}$$

$$\begin{aligned} J^{\mu(1)}_{\rho\sigma} &= \tfrac{1}{2}(\phi(\hat{x})P^\mu M^{(1)}_{\rho\sigma}\phi(\hat{x}) - P^\mu\phi(\hat{x})M^{(1)}_{\rho\sigma}\phi(\hat{x})) \\ &\quad + -\tfrac{1}{4}\Upsilon^{\nu\lambda}_{\rho\sigma}[P_\lambda\phi(\hat{x})P^\mu P_\nu\phi(\hat{x}) \\ &\quad - P^\mu P_\lambda\phi(\hat{x})P_\nu\phi(\hat{x})], \end{aligned} \tag{49}$$

where we used again the compact notation $\Upsilon^{\nu\lambda}_{\rho\sigma}$, introduced in Sec. III.

The current $T^{\mu\nu(1)}$ is manifestly equal to the current $T^{\mu\nu}$ obtained in the previous section using the P_μ, $M^{(w)}_{\mu\nu}$ basis. Therefore the corresponding charges also coincide:

$$Q^{(1)}_\mu \equiv \int d^3\hat{x}T^{0(1)}_\mu = \int d^3\hat{x}T^0_\mu = Q_\mu. \tag{50}$$

The current $J^{\mu(1)}_{\rho\sigma}$ does differ from $J^\mu_{\rho\sigma}$ of the previous section in two ways: in place of the factor $\Upsilon^{\nu\lambda}_{\rho\sigma}$ of $J^{\mu(1)}_{\rho\sigma}$ one finds in $J^\mu_{\rho\sigma}$ the factor $\theta_{[\rho}{}^\nu\delta_{\sigma]}{}^\lambda + \theta^\lambda{}_{[\rho}\delta_{\sigma]}{}^\nu$, and there are two (operator) factors $M^{(1)}_{\mu\nu}$ in places where in $J^\mu_{\rho\sigma}$ one, of course, has $M^{(w)}_{\mu\nu}$. Still, once again the final result for the charges is unaffected:

$$K^{(1)}_{\rho\sigma} \equiv \int d^3\hat{x}J^{0(1)}_{\rho\sigma} = K_{\rho\sigma}. \tag{51}$$

This is conveniently verified by following the ordering conventions of Eq. (12) in writing the generic solution of the equation of motion,

$$\phi(\hat{x}) = \int d^4k\delta(k^2)\tilde{\phi}_{(1)}(k)e^{ik^A\hat{x}_A}e^{ik^1\hat{x}_1}, \tag{52}$$

thereby obtaining, after spatial integration, the following formula for $K^{(1)}_{\rho\sigma}$:

$$\begin{aligned} K^{(1)}_{\rho\sigma} &= \frac{1}{2}\int d^4kd^4q\delta(k^2)\tilde{\phi}_{(1)}(k)\bigg[iq_{[\rho}\frac{\partial}{\partial q^{\sigma]}}[\delta(q^2)\tilde{\phi}_{(1)}(q)] \\ &\quad - \frac{1}{2}\delta(q^2)\Upsilon^{\nu\lambda}_{\rho\sigma}k_\lambda q_\nu\tilde{\phi}_{(1)}(q)\bigg](k^0 - q^0)\delta^{(3)}(\vec{k} + \vec{q}) \\ &\quad \times e^{i(k^0+q^0)\hat{x}_0}e^{(i/2)(k^0+q^0)(k^i+q^i)\theta_{i0}}e^{-(i/2)k^\mu q^\nu\theta_{\mu\nu}} \\ &\quad \times e^{-(i/2)(k^Ak^1+q^Aq^1)\theta_{A1}}. \end{aligned} \tag{53}$$

And, using observations that are completely analogous to some we discussed in the previous section, one easily manages [18] to rewrite $K^{(1)}_{\rho\sigma}$ as follows:

$$\begin{aligned} K^{(1)}_{\rho\sigma} &= \frac{i}{2}\int\frac{d^4q}{2|\vec{q}|}\tilde{\phi}_{(1)}(q)\delta(q^2)q_{[\rho}\bigg\{(q^0 + |\vec{q}|)\frac{\partial}{\partial q^{\sigma]}} \\ &\quad \times[\tilde{\phi}_{(1)}(-\vec{q}, |\vec{q}|)e^{-i(q^A\delta^j_Aq^1\theta_{j1}+(1/2)(|\vec{q}|+q^0)\theta_{01})}] \\ &\quad + (q^0 - |\vec{q}|)\frac{\partial}{\partial q^{\sigma]}}[\tilde{\phi}_{(1)}(-\vec{q}, -|\vec{q}|) \\ &\quad \times e^{-i(q^A\delta^j_Aq^1\theta_{j1}+(1/2)(-|\vec{q}|+q^0)\theta_{01})}]\bigg\}. \end{aligned} \tag{54}$$

This formula for $K^{(1)}_{\rho\sigma}$ is easily shown to reproduce the corresponding formula for $K_{\rho\sigma}$, using the fact that, as we showed in Sec. III, $\tilde{\phi}_{(1)}(k) = \tilde{\phi}_{(w)}(k)e^{-(i/2)k^Ak^1\theta_{A1}}$.

This result establishes that the values of the charges carried by a given noncommutative field can be treated as objective facts, independent of the choice of ordering prescription adopted in the analysis. Working with different ordering prescriptions one arrives at different formulas [for example, (46) and (54)] expressing the charges as functionals of the Fourier transform of the fields. However, these differences in the formulas are just such to compensate for the differences between the Fourier transforms of a given field that are found adopting different ordering conventions, and therefore the values of the charges carried by a given noncommutative field can be stated in an ordering-prescription-independent manner.

VI. CLOSING REMARKS

The fact that we managed to derive a full set of 10 conserved charges from the twisted Hopf-algebra symmetries that emerge from observer-independent canonical noncommutativity certainly provides some encouragement for the idea that these (contrary to some expectations formulated in the recent literature [15]) are genuine physical symmetries. And this viewpoint is strengthened by our result on the ordering-convention independence of the charges.

The characterization of "noncommutative transformation parameters" introduced by some of us in Refs. [13,14], for the analysis of theories in κ-Minkowski noncommutative spacetime, proved to be valuable also in the present study of canonical noncommutativity. This type of transformation parameters objectively does the job (without any need of "further intervention") of allowing one to derive conserved charges, but it still requires some work for what concerns establishing its physical implications and its realm of applicability. Is this only an appropriate recipe for deriving conserved charges? Or can we attribute to it all the roles that transformation parameters have in a classical-spacetime theory? A first step toward exploring these issues could be to consider alternative Ansätze for the description of the symmetry transformations, which may, in particular, adopt different choices of ordering convention (in the sense discussed here in parts of Sec. III) with respect to the one we chose for the Ansatz in Eq. (25) [inspired by its κ-Minkowski predecessor (17)].

It would be interesting to probe from alternative perspectives the obstruction we found for implementing pure boost (and pure space-rotation) transformations. Within our analysis this obstruction appears to be of the same type of the obstruction for the realization of pure boosts encountered in some previous studies [14] of theories in κ-Minkowski spacetime. Since, to our acknowledge, canonical and κ-Minkowski spacetimes are the only examples of noncommutative versions of Minkowski spacetime that one can single out with some reasonable physical criteria (see, e.g., Ref. [20] and references therein), if we could firmly establish that in both cases pure boosts are not allowed, it might then be natural to conjecture a universal "no-pure-boost principle" emerging from the general structure of spacetime noncommutativity.

A lot remains to be done for a proper characterization of the physical/observable implications of observer-independent canonical noncommutativity. The type of noncommutativity of transformation parameters which we encountered might imply that the concept of angle of rotation around a given axis is "fuzzy" in a canonical spacetime, a possibility which could be further explored by attempting to introduce explicit angular variables in these noncommutative geometries. And, of course, it is of paramount importance to establish by which measurements a theory with observer-independent canonical noncommutativity can be distinguished from a corresponding classical-spacetime theory. This issue would be most naturally addressed in the context of a theory of quantum fields in the noncommutative spacetime, which we have postponed to future work, but even within analyses of classical fields in canonical spacetime, such as the one we reported here, a preliminary investigation of "observability issues" could be attempted. New measurement-procedure ideas are needed in order to test the novel possibility of an obstruction for the realization of a pure Lorentz-sector transformation. And more work is also needed for a proper operative characterization of the differences between the charges obtained here for a theory with observer-independent canonical noncommutativity and the corresponding charges of a theory in classical spacetime.

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