PHYSICAL REVIEW D **78,** 025001 (2008)

Holographic glueball structure

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(Received 29 January 2008; published 1 July 2008)

I derive and systematically analyze scalar glueball correlation functions in both the hard-wall and dilaton soft-wall approximations to holographic QCD. The dynamical content of the holographic correlators is uncovered by examining their spectral density and by relating them to the operator product expansion, a dilatational low-energy theorem and a recently suggested two-dimensional power correction associated with the short-distance behavior of the heavy-quark potential. This approach provides holographic estimates for the three lowest-dimensional gluon condensates or alternatively their Wilson coefficients, the two leading moments of the instanton size distribution in the QCD vacuum and an effective UV gluon mass. A remarkable complementarity between the nonperturbative physics of the hard- and soft-wall correlators emerges, and their ability to describe detailed QCD results can be assessed quantitatively. I further provide the first holographic estimates for the decay constants of the 0^{++} glueball and its excitations. The hard-wall background turns out to encode more of the relevant QCD physics, and its prediction $f_S \simeq 0.8$–0.9 GeV for the phenomenologically important ground-state decay constant agrees inside errors with recent QCD sum rule and lattice results.

DOI: 10.1103/PhysRevD.78.025001 PACS numbers: 12.38.Lg, 11.25.Tq, 12.39.Mk

I. INTRODUCTION

Despite more than three decades of intense experimental and theoretical scrutiny the long predicted glueball states [1] of quantum chromodynamics (QCD) remain stubbornly elusive [2,3]. The slow pace of theoretical progress reflects the extraordinary complexity of the infrared Yang-Mills dynamics which generates both the gluonic bound states and their mixing with quarkonia. New analytical approaches for dealing with strongly coupled gauge theories, as they have recently emerged from gauge/gravity generalizations of the AdS/CFT correspondence [4,5], should therefore find rewarding and much needed applications in the glueball sector.

Until now such applications have focused on the glueball mass spectra, which were among the first holographically calculated observables in a variety of more or less QCD-like gauge theories [6] (for a review and current developments see e.g. Refs. [7]). More recently, glueball spectra were also obtained in the first bottom-up [8] proposals for the holographic QCD dual [9–12] as well as in back-reacted models [13,14].

In the present paper I am going to extend the holographic analysis of glueball properties beyond the spectrum, by focusing on the gauge physics content of the glueball correlation function and its spectral density. I will relate the holographic predictions to QCD information from the operator product expansion (OPE), a low-energy theorem based on the anomalous Ward identity for the dilatation current, and a recently advocated, effective UV gluon mass. The calculations will be based on two alternative AdS/QCD backgrounds, namely, the AdS_5 geometry with a "hard-wall" IR brane cutoff (of Randall-Sundrum type [15]) in the fifth dimension [16] and the dilaton-induced soft wall [17], which both proved phenomenologically successful in the meson sector [10,17,18].

A second major objective will be to provide the first holographic estimates for the decay constants of the scalar glueball and its excitations, i.e. for the glueball-to-vacuum matrix elements of the lowest-dimensional gluonic QCD interpolator. These on-shell observables are of particular interest because they contain fundamental information on glueball structure and govern the spacial extent of the glueball (Bethe-Salpeter) wave functions. Lattice indications for an exceptionally small size of the lowest-lying scalar glueball [19], for example, should translate into an unusually large value of its decay constant. Evidence for such an enhancement was indeed found in instanton vacuum models [20] as well as in those QCD sum-rule analyses which include instanton contributions to the OPE coefficients [21,22].

The decay constants, which are the first glueball observables besides the low-lying spectra for which direct (quenched) lattice results are now available [23], also play a crucial role in the theoretical analysis of glueball production and decay rates. For this reason, their accurate prediction will be instrumental in eventually meeting the two longstanding challenges of glueball physics, i.e. the establishment of unambiguous glueball signatures and their experimental identification. As a case in point, the decay constants provide critical nonperturbative input for the calculation of glueball production amplitudes in the "gluon-rich" radiative heavy-quarkonium decays which are currently measured at BES [24].

The paper is structured as follows: in Sec. II I define the dual bulk dynamics on which the following study will be based, and I derive general expressions for the scalar glueball correlator and the decay constants in IR-deformed AdS_5 duals with a nontrivial dilaton background. In Sec. III I focus on the two AdS/QCD backgrounds mentioned above (i.e. hard and soft wall) and derive exact analytical expressions for the corresponding correlators and their spectral functions. I then analyze the results by confronting them with the OPE of the QCD correlator (including nonperturbative contributions to the Wilson coefficients), the dilatational low-energy theorem which governs its low-momentum behavior, and the contributions of an effective UV gluon mass. This strategy provides holographic estimates for various QCD vacuum scales, i.e. three-gluon condensates (or alternatively their Wilson coefficients) and the two leading moments of the instanton size distribution, as well as for an effective UV gluon mass. In Sec. IV I obtain holographic predictions for the (ground and excited state) glueball decay constants and compare them to other available theoretical results. Section V, finally, contains a summary of the paper and my conclusions.

II. DUAL DYNAMICS OF THE SCALAR GLUEBALL

The gauge/string correspondence [4,5] maps string theories in curved, ten-dimensional spacetimes into gauge theories which live on the d dimensional boundaries. For UV-conformal gauge theories like QCD with $d = 4$, the dual spacetime metric factorizes into a five-dimensional noncompact manifold which close to its boundary approaches the anti–de Sitter space $\mathrm{AdS}_5(R)$ of curvature radius R, and a five-dimensional compact Einstein space X_5 (where e.g. $X_5 = S^5(R)$ for the maximally supersymmetric gauge theory) with the same intrinsic size scale. The corresponding line element is [16]

$$ds^2 = g_{MN}(x)dx^M dx^N = e^{2A(z)}\frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2) + R^2 ds_{X_5}^2 \quad (1)$$

(in conformal Poincaré coordinates) where $\eta_{\mu\nu}$ is the four-dimensional Minkowski metric. Conformal invariance of the dual gauge theory in the UV requires the absence of AdS deformations (i.e. $A(z) \to 0$) as $z \to 0$. Supergravity solutions suggest the additional presence of a nontrivial dilaton background $\Phi(x)$, and potentially of other background fields (including Ramond-Ramond axions, tachyons etc., see e.g. Ref. [14,25]) which do, however, not play an explicit role in the AdS/QCD duals considered below.

A. Bulk action and holographic glueball correlator

The scalar QCD glueballs are interpolated by the lowest-dimensional gluonic operator carrying vacuum quantum numbers,

$$\mathcal{O}_S(x) = G^a_{\mu\nu}(x)G^{a,\mu\nu}(x), \quad (2)$$

(where $G^a_{\mu\nu}$ is the gluon field strength) which also figures prominently in the anomalous dilatational Ward identity and in the corresponding low-energy theorems (cf. Appendix). Since the conformal dimension of $\mathcal{O}_S$ is $\Delta = 4$ (at the classical level), the AdS/CFT dictionary [5] prescribes its dual string modes $\varphi(x, z)$ to be the normalizable solutions of the scalar wave equation in the bulk geometry (1) (and potentially other background fields) with the UV behavior $\varphi(x, z) \xrightarrow{z\to 0} z^\Delta \phi(x)$. The latter implies that the square mass [26] $m_5^2 R^2 = \Delta(\Delta - d) = 0$ of the bulk field φ vanishes, and that its minimal action has the form

$$S[\varphi; g, \Phi] = \frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{|g|}e^{-\Phi}g^{MN}\partial_M\varphi\partial_N\varphi \quad (3)$$

(where κ^2 can be related to the five-dimensional Newton constant [25]) which breaks up as $S = S_M + S_{\partial M}$ into bulk and boundary contributions with

$$S_M[\varphi; g, \Phi] = -\frac{1}{2\kappa^2}\int_M d^{d+1}x\sqrt{|g|} \times e^{-\Phi}\varphi[e^\Phi\nabla_M e^{-\Phi}g^{MN}\partial_N]\varphi \quad (4)$$

($\nabla_M = \partial_M + |g|^{-1/2}\partial_M|g|^{1/2}$) and

$$S_{\partial M}[\varphi; g, \Phi] = \frac{1}{2\kappa^2}\int_{\partial M} d^d x[a^3(z)e^{-\Phi}\varphi\partial_z\varphi] \quad (5)$$

where $a^2(z) \equiv (R^2/z^2)\exp 2A(z)$ is the warp factor. The boundary ∂M consists of the UV brane $z = \varepsilon \to 0$ and of an additional IR brane at $z = z_m$ in the hard-wall geometry.

Variation of the bulk action (4) with respect to φ yields the field equation

$$e^\Phi\nabla_M e^{-\Phi}g^{MN}\partial_N\varphi(x, z) = [\Delta - (\partial_M\Phi)g^{MN}\partial_N]\varphi(x, z) = 0 \quad (6)$$

where $\Delta = \nabla_M\nabla^M$ is the Laplace-Beltrami operator of the metric (1). The action density of the solutions is finite only on the boundary ∂M while $S_M^{(\text{on-shell})} = 0$. I now specialize to dilaton fields Φ which depend exclusively on the fifth dimension, i.e. $\Phi = \Phi(z)$. The d-dimensional Fourier transform $\hat\varphi(q, z)$ of the normalizable dual modes then solves the reduced field equation

$$[\partial_z^2 + (d-1)(a^{-1}\partial_z a)\partial_z - (\partial_z\Phi)\partial_z + q^2]\hat\varphi(q, z) = 0 \quad (7)$$

with discrete on-shell momenta [27] $q^2 = m_n^2$ in both hard- and soft-wall backgrounds. The eigenvalues m_n^2 determine the glueball mass spectrum of the boundary gauge theory, and the corresponding orthonormalized solutions will be denoted $\psi_n(z) = N_n\hat\varphi(m_n, z)$.

Holographic glueball correlation functions are obtained by differentiating the bulk action of the solutions with respect to the boundary source [5]. The on shell action can be constructed with the help of the bulk-to-boundary propagator $\hat{K}(q, z)$ [5], which is the solution of the field Eq. (7) subject to the UV boundary condition $\hat{K}(q; \varepsilon \to 0) = 1$. Its spectral representation is therefore

$$\hat{K}(q, z) = -\frac{R^3}{\varepsilon^3} \sum_n \frac{\psi_n'(\varepsilon)\psi_n(z)}{q^2 - m_n^2 + i\varepsilon'} \tag{8}$$

(where the limit $\varepsilon \to 0$ at the end of the calculation is implied) and automatically satisfies the IR boundary condition imposed on the $\psi_n(z)$. Hence one can write the solution of Eq. (6) corresponding to a given boundary source $\varphi^{(s)}(x')$ as

$$\varphi(x, z) = \int \frac{d^4q}{(2\pi)^4} e^{-iqx} \hat{K}(q, z) \int d^4x' e^{iqx'} \varphi^{(s)}(x') \tag{9}$$

and obtain the associated on-shell action (which plays the role of a generating functional) by inserting Eq. (9) into the surface action (5). Taking two functional derivatives with respect to $\varphi^{(s)}$ then yields the two-point correlation function

$$\langle T\mathcal{O}_S(x)\mathcal{O}_S(y)\rangle = i \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \hat{\Pi}(-q^2) \tag{10}$$

of the scalar glueball where

$$\hat{\Pi}(-q^2) = -\frac{R^3}{\kappa^2} \left[\frac{e^{-\Phi(z)}}{z^3} \hat{K}(q, z) \partial_z \hat{K}(q, z) \right]_{z=\varepsilon \to 0}. \tag{11}$$

For Φ fields which vanish at the UV boundary (as does the soft-wall dilaton considered below), furthermore, the bulk-to-boundary propagator in the form (8) generates the spectral representation

$$\hat{\Pi}(-q^2) = -\left(\frac{R^3}{\kappa \varepsilon^3}\right)^2 \sum_n \frac{\psi_n'(\varepsilon)\psi_n'(\varepsilon)}{q^2 - m_n^2 + i\bar{\varepsilon}} = -\sum_n \frac{f_n^2 m_n^4}{q^2 - m_n^2 + i\bar{\varepsilon}} \tag{12}$$

of the correlator [where a prime denotes differentiation with respect to z and (divergent) contact terms are not written explicitly]. The pole residues of Eq. (12) at $q^2 = m_n^2$ define the decay constants of the nth 0^{++} glueball excitation as

$$f_n := \frac{1}{m_n^2} \langle 0|\mathcal{O}_S(0)|0_n^{++}\rangle = \frac{R^3}{\kappa m_n^2} \frac{\psi_n'(\varepsilon)}{\varepsilon^3}. \tag{13}$$

The physical role of the decay constants as the glueball "wave functions at the origin" becomes more transparent when viewing them as the coincidence limit of the Bethe-Salpeter amplitudes

$$\chi_n(x) = \langle 0| 2\mathrm{tr}\left\{ G_{\mu\nu}\left(-\frac{x}{2}\right) U\left(-\frac{x}{2}, \frac{x}{2}\right) G^{\mu\nu}\left(\frac{x}{2}\right) \right\} |0_n^{++}\rangle \tag{14}$$

(where the adjoint color parallel transporter $U(x, y)$ ensures gauge invariance and proper renormalization of the operators is understood). A smaller glueball size implies a higher concentration of the wave function at the origin and consequently a larger value of f_n. Since the decay constants are on-shell observables related to the bilinear part of the bulk action, one expects them to be reasonably well predicted by Eq. (13) even though the dual dynamics (3) contains operators of minimal dimension only.

B. Comments on the scalar dual dynamics

I have restricted the action (3), i.e. the dynamics of fluctuations dual to scalar glueballs in a metric plus dilaton background, to contain only operators with the minimal number of fields and derivatives. This is appropriate for the AdS/QCD candidates under consideration and will permit me to derive analytical expressions for the exact correlators. These restrictions also entail several typical limitations of contemporary bottom-up models, however, which I now discuss in view of their potential impact on the glueball sector.

A first obvious limitation is the treatment of the metric as a nondynamical background field, reflected in the fact that the action (3) contains neither the Einstein-Hilbert term nor higher-derivative corrections to it [28]. Graviton fluctuations around the bulk metric are generally neglected as well in bottom-up duals. If included, they could have a direct bearing on the scalar glueball dynamics since fluctuations around a nonconformal metric would generate a scalar "radion" mode (related to the fifth or radial dimension component of the graviton) as it appears e.g. in distance fluctuations between the branes of Randall-Sundrum models [15,29] and in dynamical dilaton-gravity models (see e.g. Ref. [13]). By mixing with the scalar field in the bulk action (3) the radion would then modify the glueball mass spectrum and the diagonalized correlator. Since the mixing strength decreases with increasing mass of the excitation dual to the radion (or mixed radion-scalar), the standard neglect of the graviton dynamics e.g. in hard-wall models is equivalent to the tacit assumption that an *a priori* unspecified stabilization mechanism pushes this mass up far enough for radion admixtures to become negligible (ideally by breaking conformal symmetry according to the QCD trace anomaly).

A more realistic holographic description of the glueball sector would probably require the inclusion of further operators, potentially of higher dimension, into the action (3). Those may contain additional bulk fields, with promising candidates including spin-zero background fields encoding condensates of relevant QCD operators and flavor-carrying gauge fields [18] for the description of quarkonium-gluonium mixing effects and for specific glueball decay channels (see e.g. Ref. [30]). Operators containing a higher number of derivatives are potentially important as well. They typically arise from stringy α'

corrections in bulk regions where the curvature radius R of the geometry becomes comparable to the string length l_s. In holographic duals of large-N_c gauge theories such regions are expected to describe the UV regime where the 't Hooft coupling $\lambda = g_{\rm YM}^2 N_c$ becomes small, i.e. where $\lambda \sim (R^2/\alpha')^2 \lesssim 1$ [31]. The lack of asymptotic freedom in current bottom-up duals (as well as in supergravity approximations), i.e. the fact that AdS/QCD models remain strongly coupled in the UV (although they approach a conformal fixed point) [32], is therefore closely related to the absence of higher-dimensional operators.

This discussion indicates that improvements of the AdS/QCD approach will depend in no small measure on whether a quantitative understanding for the impact of the strongly coupled UV regime on holographic predictions can be developed. In the present paper I propose a strategy towards clarifying this issue which is based on the comparison of holographic model predictions for hadronic correlation functions with the QCD OPE. The OPE lends itself particularly well to a systematic diagnosis of the UV sector since it factorizes gauge-theory amplitudes into short-distance mode contributions to the Wilson coefficients and long-distance physics contributions to local operators. I am going to exploit this factorization property below when searching for specific traces of the strongly coupled UV regime in the two-point function of the scalar glueball channel and will indeed find evidence for deficiencies in the AdS/QCD description of the perturbative Wilson coefficients (beyond the leading conformal logarithm). Moreover, the results will suggest systematic improvement strategies for bottom-up duals.

In view of the issues raised above one may wonder whether a decent holographic description of asymptotically free Yang-Mills theories at large N_c could at all be achieved on the basis of a *local* five-dimensional action (which may include a few higher-dimensional operators). Fortunately, there are several indications for an affirmative answer. A general argument due to Witten implies that the locality of the five-dimensional bulk dynamics is ensured by the large-N_c limit [33]. The bulk action may then be viewed as an effective string *field* theory which contains an elementary field for each string excitation (including those of arbitrarily high spin) while higher-dimensional operators are suppressed by powers of $1/N_c$ [17]. Moreover, α' corrections may be partially resummed e.g. into the dilaton potential [14], and extensive QCD sum-rule [34] analyses have shown that already a few leading OPE power corrections, and hence hopefully the few corresponding light bulk fields with controllably small α' corrections, can capture at least the essential properties of most hadronic ground states.

III. HOLOGRAPHIC GLUEBALL CORRELATORS

The expressions derived above hold for all geometries of the form (1) and for general dilaton backgrounds $\Phi(z)$ with $\Phi(0) = 0$. In order to gain dynamical insight into the holographic glueball correlator and to obtain quantitative estimates for the decay constants, I will now consider two specific AdS/QCD backgrounds, i.e. the AdS_5 slice of the hard IR wall geometry [16] and the dilaton soft wall of Ref. [17]. In particular, I will derive analytical expressions for the glueball correlator (11) and its spectral density in the hard- and soft-wall backgrounds. Those will then be analyzed by comparison with the QCD operator product expansion, a dilatational low-energy theorem which governs the correlator at zero momentum, and an effective UV gluon mass contribution of the type suggested in Ref. [35]. The pertinent QCD information is summarized in the appendix.

A. Conformal symmetry breaking by an IR brane

A substantial part of the successful recent AdS/QCD phenomenology (see e.g. [10,16,18,36,37]) was obtained on the basis of the so-called "hard-wall" geometry [16]. This rather minimal deformation of the AdS_5 metric approximately describes IR effects including confinement by a sudden onset of conformal symmetry breaking in the form of an IR brane at $z = z_m$, i.e.

$$e^{2A^{(\rm hw)}(z)} = \theta(z_m - z), \qquad z_m \simeq \Lambda_{\rm QCD}^{-1}, \qquad \Phi^{(\rm hw)} \equiv 0, \tag{15}$$

which reduces the five-dimensional bulk spacetime to an AdS_5 slice.

In this highly symmetric background an analytical expression for the holographic glueball correlator is straightforward to obtain. The bulk-to-boundary propagator $\hat{K}(q, z)$, in particular, can be found by solving the field equation (7) in the geometry (15), subject to the UV boundary condition $\hat{K}(q; \varepsilon) = 1$ (with $\varepsilon \to 0$) and the Neumann IR boundary condition $\partial_z \hat{K}(q; z_m) = 0$. The result is [38]

$$\hat{K}(q, z) = \frac{\pi}{4}(qz)^2\left[\frac{Y_1(qz_m)}{J_1(qz_m)}J_2(qz) - Y_2(qz)\right] \tag{16}$$

where J_ν, Y_ν are Bessel functions and $\hat{K}(0, z) = 1$. After plugging Eq. (16) into the general expression (11) and analytically continuing to spacelike momenta $Q^2 = -q^2$, one ends up with the hard-wall glueball correlator

$$\hat{\Pi}(Q^2) = \frac{R^3}{8\kappa^2}Q^4\left[2\frac{K_1(Qz_m)}{I_1(Qz_m)} - \ln\left(\frac{Q^2}{\mu^2}\right)\right] \tag{17}$$

(K_ν, I_ν are McDonald functions [39]) where two contact terms associated with UV divergent subtraction constants were discarded.

It is instructive to find the spectral density $\rho(s)$ of the correlator (17), which is defined by means of the dispersion relation

$$\hat{\Pi}(Q^2) = \int_{m_1^2}^{\infty} ds \frac{\rho(s)}{s+Q^2} \tag{18}$$

(where the necessary subtraction terms are again implied but not written explicitly) and can be derived e.g. from the well-known analyticity properties of the McDonald functions [39] (and the causal pole definition) as the imaginary part of $\hat{\Pi}/\pi$ at timelike momenta. The result is

$$\rho(s) = \frac{R^3}{2\kappa^2 z_m^2} s^2 \sum_{n=1}^{\infty} \frac{\delta(s-m_n^2)}{J_0^2(j_{1,n})} \tag{19}$$

from which one can read off the hard-wall mass spectrum $m_n = j_{1,n}/z_m$ [cf. Equation (42)]. The spectral weight (19) is non-negative, in agreement with general principles, and consists of a sum of zero-width poles, as expected at large N_c where the (infinitely many) glueballs become stable against strong decay. The leading large-s behavior of the density (19) necessitates subtractions in Eq. (18) and ensures the leading logarithmic Q^2 dependence of the correlator (17).

The holographic result (17) can be compared to the QCD short-distance expansion (A1) for $Q \gg \mu^2 > z_m^{-2}$. A standard procedure [18] for fixing the overall normalization R^3/κ^2 is to match the coefficients of the leading conformal logarithm in Eqs. (17) and (A2), which yields

$$\frac{R^3}{\kappa^2} = \frac{2(N_c^2-1)}{\pi^2}. \tag{20}$$

(For a discussion of the accuracy of such estimates see Ref. [40].) Below I will specialize Eq. (20) to the phenomenologically relevant $N_c = 3$ which seems—at least as far as glueball properties are concerned—to be a surprisingly good approximation to large N_c [41]. The nonconformal part of the holographic correlator (17) describes nonperturbative contributions of the boundary gauge theory and becomes

$$\hat{\Pi}^{(\mathrm{np})}(Q^2) \equiv \frac{R^3}{4\kappa^2}\frac{K_1(Qz_m)}{I_1(Qz_m)}Q^4 \overset{Qz_m\gg 1}{\rightarrow} \frac{4}{\pi} \times \left[1 + \frac{3}{4}\frac{1}{Qz_m} + O\left(\frac{1}{(Qz_m)^2}\right)\right]Q^4 e^{-2Qz_m} \tag{21}$$

in the OPE limit $Q^2 \gg \Lambda_{\mathrm{QCD}}^2 \sim z_m^{-2}$. Equation (21) reveals that the hard-wall glueball correlator contains no power corrections and that all of its nonperturbative content has an exponential Q^2 dependence (times powers of Q^2). In the OPE (A1) this exponential behavior originates from small-size instanton contributions to the Wilson coefficients. Indeed, for $Q \gg \bar{\rho}^{-1}$ the direct instanton contribution (A6) becomes

$$\hat{\Pi}^{(I+\bar{I})}(Q^2) \overset{Q\bar{\rho}\gg 1}{\rightarrow} 2^4 5^2 \pi \zeta \bar{n} (Q\bar{\rho})^3 e^{-2Q\bar{\rho}} \tag{22}$$

which has exactly the momentum dependence of the first subleading term in the nonperturbative hard-wall correlator (21). As shown in instanton vacuum models [20] and directly from the IOPE in QCD sum rules [21,22], these instanton-induced correlations are attractive and of relatively short range $\sim\bar{\rho}$. Hence they reduce the mass and size of the scalar glueball while increasing its decay constant.

For a quantitative comparison of holographic and instanton-induced contributions one may approximately equate Eq. (22) with the second term in Eq. (21). This yields the expressions

$$\bar{\rho} \simeq z_m, \qquad \bar{n} \simeq \frac{3}{2^4 5^2 \pi^2 \zeta}\frac{1}{z_m^4}, \tag{23}$$

for the average instanton size $\bar{\rho}$ and the overall instanton density $\bar{n}$ in terms of the hard-wall IR scale z_m. The relation $\bar{\rho} \simeq z_m$ is consistent with the duality between gauge-theory instantons of size ρ and pointlike bulk objects (D instantons or $D(-1)$ branes in the supersymmetric case [42]) localized at a distance $z = \rho$ from the UV boundary. However, it also identifies the instanton's average size $\bar{\rho}$ with the maximal size z_m in the AdS_5 slice, which is likely to result in an overestimate. Indeed, the standard identification $z_m^{-1} \sim \Lambda_{\mathrm{QCD}} \simeq 0.33$ GeV would imply $\bar{\rho} \sim 0.6$ fm, i.e. almost twice the instanton-liquid model (ILM) value $\bar{\rho}_{\mathrm{ILM}} \sim 0.33$ fm [43]. As a consequence, $\bar{n}_{\mathrm{ILM}} \simeq 0.5\ \mathrm{fm}^{-4}$ [43] would be underestimated by the second relation in Eq. (23).

Besides other likely limitations of the hard-wall background including the strongly coupled UV dynamics, the large estimate for $\bar{\rho}$ may also reflect the absence of fundamental quark flavors in the simple dual dynamics (3). The relations (23) (as well as other results below) may therefore apply more accurately to pure Yang-Mills theory for which several lattice studies indeed find larger average instanton sizes $\bar{\rho} \simeq 0.4$–0.5 fm [44]. In any case, one would not expect the instanton scales of the QCD vacuum to be precisely encoded in the hard-wall approximation. In fact, it seems remarkable that this minimal background can even semiquantitatively reproduce the key instanton contribution to the short-distance expansion. For a fully quantitative study of such corrections one should resort to top-down gravity duals in which the relation between bulk and boundary instantons can be traced exactly [42]. Such investigations may also shed light on the interpretation of the leading exponential contribution to Eq. (21) in terms of gauge-theory physics.

It is interesting to confront the hard-wall correlator with the QCD low-energy theorem (A7). The correlator (17) vanishes at $Q^2 = 0$ since the removal of the contact terms amounts to subtractions at $Q^2 = 0$. Even the contact terms do not contain a finite (or infinite) contribution to $\hat{\Pi}(0)$, however, and neither does the nonperturbative part (21) alone which would remain after subtracting the perturbative contributions from the spectral density, as suggested in the original definition [45]. (This is in contrast to the one-instanton contribution (A6) which contains a subtraction term $\hat{\Pi}^{(I+\bar{I})}(0) = 2^7 5^2 \zeta \bar{n}$. The one-instanton approximation is not reliable at small Q^2, however, where multi-instanton and other long-wavelength vacuum field contributions are likely to dominate the correlator.) From the low-energy theorem (LET) perspective this is consistent with the absence of power corrections and gluon condensates in the hard-wall background. As a consequence, both sides of Eq. (A7) vanish identically and the LET is trivially satisfied. A more complex situation will be encountered in the soft-wall background below.

The absence of condensate effects in the hard-wall approximation is not surprising because their purely geometrical encoding is known to require power-law deformations [46] of the warp factor $A(z)$ in the infrared [47]. Since large instantons generate finite gluon condensates, this furthermore indicates that the correlator (17) receives small-size instanton contributions only, in perfect agreement with our above discussion which indeed implies

$$\rho^{(\mathrm{hw})} \leq z_m \sim \mu^{-1} \tag{24}$$

because the instanton size cannot exceed the extension of the AdS_5 slice in the fifth dimension. Hence the simple hard-wall approximation seems to capture the fact that an essential part of the nonperturbative contributions to the 0^{++} glueball correlator is hard compared to the OPE scale μ and therefore resides in the Wilson coefficients [21,22,48]. Since the power corrections of the OPE (A2) are suppressed by unusually small Wilson coefficients, furthermore, the hard-wall background may indeed provide a reasonable first approximation to the scalar glueball correlator.

B. Dilaton-induced conformal symmetry breaking

A well-known shortcoming of the hard-wall background (15) is that it predicts squared hadron masses to grow quadratically with high radial, spin and orbital excitation quantum numbers [10,36,49], in contrast to the linear trajectories expected from semiclassical flux-tube models [50]. This problem manifests itself also in the hard-wall glueball spectra (cf. Eqs. (41) and (42)) which do not reproduce the expected linear Pomeron trajectory [51,52].

The presence of a nontrivial dilaton background field $\Phi(z) \propto z^2$ was recently proposed as an economical remedy for this problem in the meson [17] and glueball [12] sectors. In the simplest version of the resulting gravity dual, conformal symmetry breaking in the IR is the exclusive task of the dilaton while the geometry (1) remains undeformed AdS_5, i.e.

$$A^{(\mathrm{sw})}(z) \equiv 0, \qquad \Phi^{(\mathrm{sw})}(z) = \lambda^2 z^2. \tag{25}$$

In the present section I derive and analyze the scalar glueball correlator in this "dilaton soft-wall" background. (Alternative holographic realizations of linear trajectories have been obtained for mesons or glueballs in Refs. [53–56] and for both mesons and baryons in Ref. [57].)

Although the background (25) is somewhat more complex than the minimal hard-wall geometry (15), one can still find a closed integral representation for the corresponding scalar bulk-to-boundary propagator (8),

$$\hat{K}(q;z) = \frac{q^2}{4\lambda^2}\left(\frac{q^2}{4\lambda^2} - 1\right)\int_0^1 dx(1-x)x^{-[q^2/(4\lambda^2)+1]} \times e^{-(x/(1-x))\lambda^2 z^2}, \tag{26}$$

which may be rewritten in terms of confluent hypergeometric functions. Equation (26) is easily shown to be the solution of the field equation (7) in the background (25) which satisfies the UV boundary condition $\hat{K}(q;0) = 1$ and additionally $\hat{K}(0;z) = 1$. Inserting the expression (26) into Eq. (11) leads to

$$\hat{\Pi}(Q^2) = -2\frac{R^3\lambda^2}{\kappa^2}\frac{Q^2}{4\lambda^2}\left(\frac{Q^2}{4\lambda^2} + 1\right)\lim_{\varepsilon\to 0}\frac{1}{\varepsilon^2} \times \int_0^1 dx x^{Q^2/(4\lambda^2)} e^{-(x/(1-x))\lambda^2\varepsilon^2} \tag{27}$$

which is the exact soft-wall correlator at spacelike momenta $q^2 = -Q^2$. The remaining integral can be performed analytically. This is conveniently done by absorbing the small-ε singularity into the integrand such that the branch cut structure becomes manifest. One then obtains

$$\hat{\Pi}(Q^2) = -\frac{2R^3\lambda^4}{\kappa^2}\frac{Q^2}{4\lambda^2}\left(\frac{Q^2}{4\lambda^2} + 1\right)\Gamma\left(\frac{Q^2}{4\lambda^2} + 1\right) \times \lim_{\varepsilon\to 0} U\left(\frac{Q^2}{4\lambda^2} + 2, 2, \lambda^2\varepsilon^2\right) \tag{28}$$

where $U(a, b, z)$ is the (multivalued) confluent hypergeometric function [39]. After taking the $\varepsilon \to 0$ limit and discarding two divergent contact terms, one finally ends up with

$$\hat{\Pi}(Q^2) = -\frac{2R^3}{\kappa^2}\lambda^4\left[1 + \frac{Q^2}{4\lambda^2}\left(1 + \frac{Q^2}{4\lambda^2}\right)\psi\left(\frac{Q^2}{4\lambda^2}\right)\right] \tag{29}$$

in terms of the digamma function $\psi(z) = \Gamma'(z)/\Gamma(z)$ [39].

As in the hard-wall case, I begin the analysis of the correlator (29) by deriving its spectral density from the dispersion relation (18) (where the lower boundary of the integration region is now $s_{\mathrm{min}} = m_0^2$; see below) as the imaginary part of $\hat{\Pi}/\pi$ at timelike momenta. The analyticity structure of the digamma function [39] and the causal pole definition then imply

$$\rho(s) = \frac{\lambda^2 R^3}{2\kappa^2} s(s - m_0^2/2)\sum_{n=0}^{\infty}\delta(s - m_n^2). \tag{30}$$

The spectral density (30) is non-negative for $s \geq m_0^2/2$ and consists, as its hard-wall counterpart (19) and as expected at large N_c, of a sum of zero-width poles at the soft-wall masses $m_n^2 = 4(n+2)\lambda^2$ (cf. Eq. (46)). The leading large-s behavior again encodes the conformal large-Q^2 behavior of the correlator.

In order to compare the holographic soft-wall correlator to the OPE (A1) at $Q^2 \gg \Lambda_{\rm QCD}^2$, I rewrite Eq. (29) for $Q^2 \gg 4\lambda^2$ by means of the asymptotic expansion for the digamma function [39] and the Bernoulli numbers $B_{2n} = (-1)^{n-1}2(2n)!\zeta(2n)/(2\pi)^{2n}$ ($\zeta(z)$ is Riemann's zeta function) as

$$\begin{aligned}\hat{\Pi}(Q^2) &= -\frac{2R^3}{\kappa^2}\lambda^4\left[1+\frac{Q^2}{4\lambda^2}\left(1+\frac{Q^2}{4\lambda^2}\right)\right.\\ &\quad\times\left.\left(\ln\frac{Q^2}{4\lambda^2}-\frac{2\lambda^2}{Q^2}-\sum_{n=1}^{\infty}\frac{B_{2n}}{2n}\left(\frac{4\lambda^2}{Q^2}\right)^{2n}\right)\right]\\ &= -\frac{2}{\pi^2}Q^4\left[\ln\frac{Q^2}{\mu^2}+\frac{4\lambda^2}{Q^2}\ln\frac{Q^2}{\mu^2}+\frac{2^2 5}{3}\frac{\lambda^4}{Q^4}-\frac{2^4}{3}\frac{\lambda^6}{Q^6}\right.\\ &\quad\left.+\frac{2^5}{15}\frac{\lambda^8}{Q^8}+\ldots\right].\end{aligned} \tag{31}$$

(In the last line I have adapted the correlator to the OPE scale μ by absorbing additional, finite pieces into the contact terms.) Note that the expansion coefficients grow factorially with the power of λ^2/Q^2, as expected from QCD. The coefficients of the conformal logarithm in Eq. (31) and in the hard-wall correlator (17) are identical. This is because large momenta Q probe the $z \to 0$ region where neither the dilaton nor the IR brane affect the correlator, so that the same AdS_5-induced logarithm governs its behavior in both hard- and soft-wall backgrounds. Hence comparison with the perturbative gluon loop of the OPE (A2) fixes the normalization R^3/κ^2 as in Eq. (20) and as anticipated in the second line of Eq. (31).

In addition to the leading conformal logarithm, the expansion (31) contains an infinite tower of power corrections. Comparison with the OPE (A2) suggests them to be related to the gauge-theory condensates

$$\langle \mathcal{O}_D\rangle \sim \lambda^D \sim \Lambda_{\rm QCD}^D \tag{32}$$

of $D = 4, 6, 8, \ldots$ dimensional (local, gauge-invariant) composite operators. The appearance of the scale factor λ^D shows that the soft-wall power corrections are entirely dilaton-induced, in contrast to those arising from (hadron channel dependent) deformations of the metric in the geometric approach [46] or from additional scalar background fields. Tentatively equating the coefficients of the $D = 4, 6$ and 8 terms (without $O(\alpha_s)$ corrections) to their OPE counterparts in Eq. (A2) allows for a more quantitative check of the holographic expansion (31). The resulting relations for the three lowest-dimensional gluon condensates (defined at the OPE scale $\mu \sim 1$ GeV) are

$$\langle G^2\rangle \simeq -\frac{10}{3\pi^2}\lambda^4, \tag{33}$$

$$\langle gG^3\rangle \simeq \frac{4}{3\pi^2}\lambda^6, \tag{34}$$

$$\langle G^4\rangle \simeq -\frac{8}{15\pi^3\alpha_s}\lambda^8. \tag{35}$$

These holographic estimates indeed reproduce the order of magnitude expected from QCD. This is mostly because their scale is set by the dilaton IR parameter $\lambda \sim \sqrt{2}\Lambda_{\rm QCD}$ [57] which generates the mass gap and because the coefficients in Eqs. (33) and (34) are more or less of order unity. The sign of the most reliably determined four-dimensional QCD gluon condensate $\langle G^2\rangle \sim 0.4$–$1.2\ {\rm GeV}^4$ is positive, however, in contrast to Eq. (33). QCD estimates of both signs exist for the three-gluon condensate, namely, the lattice prediction $\langle gG^3\rangle \simeq -1.5\langle\alpha_s G^2\rangle^{3/2}$ [58] and the single-instanton value $\langle gG^3\rangle \simeq 0.27\ {\rm GeV}^2\langle\alpha_s G^2\rangle$. The signs of Eqs. (33) and (35), furthermore, are at odds with the factorization approximation [59]

$$\langle G^4\rangle \simeq \frac{9}{16}\langle G^2\rangle^2 \tag{36}$$

for the four-gluon condensate combination (A4). These shortcomings indicate that the tentative adoption of the (leading-order) perturbative QCD Wilson coefficients for the analysis of the holographic power corrections (31) is questionable. It will be revised on more physical grounds below. (Recall, furthermore, that the scalar background field of the soft wall (25) does not correspond to a $\Delta = 4$ operator.)

In addition to the OPE-type power corrections of Eq. (A2), the holographic soft-wall correlator (31) contains a two-dimensional power correction (times a logarithm) which cannot appear in the OPE since QCD lacks a corresponding (gauge-invariant and local) composite operator. However, a two-dimensional power correction of exactly this type was advocated some time ago and argued to improve QCD sum-rule results in several hadron channels [35]. More specifically, when (possibly renormalon-related) linear contributions to the heavy-quark potential at *short* distances are approximately accounted for by an effective gluon mass $\bar{\lambda}$, the latter produces the correction [35]

$$\hat{\Pi}^{(\rm CNZ)}(Q^2) = -\frac{2}{\pi^2}Q^4\ln\frac{Q^2}{\mu^2}\left(1+6\frac{\bar{\lambda}^2}{Q^2}+\ldots\right) \tag{37}$$

to the leading logarithm of the glueball correlator which has precisely the form of the second term in Eq. (31). The appearance of this term supports previous arguments which tentatively relate the quadratic behavior of the soft-wall dilaton background field (25) or alternatively of $A(z)$ [13,53,57,60] to a two-dimensional power correction and possibly to a two-dimensional nonlocal gluon condensate [61]. Comparison of the $\bar{\lambda}^2$ correction in Eq. (37) with its counterpart in Eq. (31) leads to the holographic estimate

$$\bar{\lambda}^2 \simeq \frac{2}{3}\lambda^2 \tag{38}$$

and with the approximate identification $\lambda \simeq \sqrt{2}\Lambda_{\rm QCD}$ further to $\bar{\lambda}^2 \simeq 0.15$ GeV^2 which is indeed of the expected magnitude [35]. However, as in the case of the leading OPE power corrections the sign turns out to be opposite to QCD expectations, i.e. the square mass (38) is not tachyonic.

The complete reproduction of the Q^2 dependence contained in the QCD short-distance expansion (to leading order in α_s) by the soft-wall dynamics, albeit with the signs of at least the leading power corrections opposite to QCD expectations, suggests an interpretation which may help to disentangle the holographic predictions for Wilson coefficients and condensates even though they appear as products in the power corrections. Indeed, the dimensions of the condensates are generated by the operators of the OPE which in turn are renormalized at relatively small scales $\mu \sim 1$ GeV and hence IR dominated. This makes it likely that the condensate part of the OPE and consequently the form of the power corrections and their scaling behavior are better reproduced by the strong-coupling dynamics of the soft-wall model, and that the deviations from the OPE should reside mainly in the Wilson coefficients (cf. Sec. II B). The lack of perturbative Q^2-dependence due to radiative $O(\alpha_s)$ corrections (cf. Appendix) in the soft-wall correlator provides additional support for this interpretation. It could be further tested by extending the comparison of holographic correlators with the OPE to other hadron channels. Indeed, since the condensates are universal (i.e. channel independent) while the Wilson coefficients are not, one would expect inconsistent soft-wall condensate predictions in different hadron correlator channels when relying on the questionable assumption that the soft-wall dynamics approximates their Wilson coefficients.

Tentatively assuming that the soft-wall dynamics approximately reproduces the values of the QCD (or Yang-Mills) condensates, on the other hand, one may obtain holographic estimates for the Wilson coefficients. The soft-wall prediction for the (leading-order) perturbative gluon condensate coefficient $C^{\rm (QCD,lo)}_{\langle G^2\rangle} \equiv B_0$, e.g., becomes with $\langle G^2\rangle \simeq (20/3)\Lambda^4_{\rm QCD}$ [22] and $\lambda \simeq \sqrt{2}\Lambda_{\rm QCD}$, $\Lambda_{\rm QCD} \simeq 0.33$ GeV [3]

$$C^{\rm (sw)}_{\langle G^2\rangle} \simeq -\frac{8}{\pi^2} = -\frac{2}{\pi^2} C^{\rm (QCD,lo)}_{\langle G^2\rangle}. \tag{39}$$

This prediction is of smaller size than the QCD value and has the opposite sign. As discussed above, it is suggestive to attribute at least part of these discrepancies to the strongly coupled UV regime of the soft-wall model, although the estimate (39) is prone to additional error sources including the current uncertainties in the QCD value of the gluon condensate and its sensitivity to the presence of light quark flavors. The uncertainties in the analogous predictions for the Wilson coefficients of higher-dimensional operators would be further increased by the less reliably known QCD values of the corresponding condensates. One should note, finally, that the above approximate separation of hard and soft (i.e. $k \gtrless \mu$) contributions to the holographic predictions would not work for the gluon mass term since both the mass $\bar{\lambda}$ and its coefficient receive UV contributions.

The soft-wall correlator in its subtracted form (29) fails to satisfy the low-energy theorem (A7): Eq. (33) (if taken literally) implies a finite right-hand side (RHS) while Eq. (29) gives $\hat{\Pi}(0) = 0$ (even before discarding the contact terms), i.e. a vanishing left-hand side. Of course this comparison should be considered naive since contact terms are renormalization scheme dependent and devoid of intrinsic physical meaning. However, other subtraction procedures including the subtraction of the conformal logarithm suggested in the original LET definition [45] would lead to the same result. In fact, the simple soft-wall background does not correctly represent the physics of the QCD trace anomaly on which the LET (A7) is based: the AdS$_5$ metric (which is dual to the energy-momentum tensor $T_{\mu\nu}$ of the gauge theory [62] on the flat boundary) implies $\langle T^\mu_\mu\rangle_{\rm metric} = 0$ since the AdS$_5$ Weyl anomaly vanishes [63], and there is no scalar background dual to the $\Delta = 4$ gluon condensate operator which appears on the RHS of the LET and in the matter anomaly contribution to $\langle T^\mu_\mu\rangle$. [The soft-wall dilaton would naively correspond to a local $\Delta = 2$ operator which does not exist in QCD but arises in (e.g. effective dual color [64]) theories with spontaneously broken gauge symmetry.]

To summarize, it is remarkable that the soft-wall background reproduces all qualitative features of the short-distance QCD correlator, i.e. exactly those powers and logarithms which appear in QCD, and even the hypothetical logarithmic corrections due to an UV gluon mass. The signs (and sizes) of both leading power corrections differ from those preferred in QCD, however, which I expect to be at least partly due to the failure of the strongly coupled UV regime to describe the perturbative QCD Wilson coefficients. Since QCD sum-rule analyses show that results for ground-state masses and couplings (decay constants) depend sensitively on magnitude and sign of the leading power corrections, it is likely that the soft-wall predictions will be contaminated by this shortcoming.

The addition of stringy corrections to the minimal bulk action (3) may be a promising direction for improving the soft-wall description in the UV. Indeed, first attempts to allow for such higher-dimensional operators in the action of holographic models [65–67] show that they can generate substantial contributions to the power corrections. Similar operators of stringy origin, including e.g. tachyon fields or α' corrections analogous to those considered in the vector meson sector [66], can therefore be expected to improve the soft-wall prediction for the short-distance correlator in the scalar glueball channel.

The comparison of the above results with those from the hard-wall correlator in Sec. III A shows that the whole nonperturbative momentum dependence of the known IOPE (up to radiative corrections) is reproduced by the holographic hard- and soft-wall correlators in a fully complementary fashion: while the soft-wall correlator contains all OPE power corrections of the types induced either by gluon condensates or by an effective UV gluon mass, the nonpertubative physics in the hard-wall correlator is exponential and includes a term which reproduces the behavior of the leading instanton contributions. This complementarity of the nonperturbative physics represented by both dual backgrounds is likely to persist in other hadron correlators as well (at least at distances smaller than the inverse QCD scale) and can be exploited for diagnostic purposes, e.g. by tracing the impact of different parts of the gauge dynamics on hadron observables (see below).

IV. GLUEBALL DECAY CONSTANTS

In the following section I obtain quantitative holographic predictions for the glueball decay constants (13) in both hard-wall and dilaton soft-wall backgrounds and discuss the underlying physics.

A. Hard-wall IR brane

The values of the glueball decay constants in the hard-wall approximation may serve as a benchmark for the results of more elaborate holographic duals. I calculate them directly from the normalizable solutions [9,10]

$$\psi_n(z) = N_n(m_n z)^2 J_2(m_n z) \tag{40}$$

(where $n = 1, 2, 3, \ldots$) of the massless field equation (7) in the AdS_5 slice (15), which I require to satisfy (in addition to the AdS/CFT boundary condition $\psi_n(z) \to z^\Delta$ at $z = \varepsilon \to 0$) either Dirichlet (D) or Neumann (N) boundary conditions on the IR brane. The normalization constants N_n are determined by the inner product of the eigenmodes, i.e. by requiring $\int_0^{z_m} dz(R/z)^3 \psi_n^2 = 1$. For Dirichlet boundary conditions $\psi_n(z_m) = 0$ one then obtains the masses [9,10] and normalizations

$$m_n^{(D)} = \frac{j_{2,n}}{z_m}, \qquad N_n^{(D)} = \frac{\sqrt{2}}{m_n^{(D)2} R^{3/2} z_m |J_1(j_{2,n})|} \tag{41}$$

while the alternative Neumann boundary conditions $\psi_n'(z_m) = 0$ yield the spectrum [11] and normalization constants

$$m_n^{(N)} = \frac{j_{1,n}}{z_m}, \qquad N_n^{(N)} = \frac{\sqrt{2}}{m_n^{(N)2} R^{3/2} z_m |J_0(j_{1,n})|}. \tag{42}$$

Here $j_{m,n}$ denotes the nth zero of the mth Bessel function [39]. Although the normalization constants do not affect the mass spectra, they provide a crucial overall scale for the decay constants.

From the general expression (13) for the decay constants and the hard-wall eigenmodes (40) one then finds

$$f_n = \lim_{\varepsilon \to 0} \frac{R^3}{\kappa m_n^2} \frac{\psi_n'(\varepsilon)}{\varepsilon^3} = \frac{N_n}{2} \frac{R^3}{\kappa} m_n^2 \tag{43}$$

or more specifically for the above two IR boundary conditions

$$f_n^{(D)} = \frac{1}{\sqrt{2}|J_1(j_{2,n})|} \frac{R^{3/2}}{\kappa z_m}, \qquad f_n^{(N)} = \frac{1}{\sqrt{2}|J_0(j_{1,n})|} \frac{R^{3/2}}{\kappa z_m}. \tag{44}$$

The expression for $f_n^{(N)}$ can alternatively be obtained by comparing the spectral density (19) of the Neumann hard-wall correlator to the general spectral representation (12). This provides a useful cross-check on the calculations.

After fixing the overall normalization factor $R^{3/2}/\kappa$ by comparison with the QCD gluon loop contribution according to Eq. (20), both masses and decay constants are given (by Eqs. (41), (42), and (44)) in terms of only one adjustable parameter, i.e. the IR scale $z_m^{-1} \sim \Lambda_{QCD}$ of the hard-wall geometry which has to be determined from independent input. The resulting quantitative predictions for f_n will be discussed in Sec. IV C.

B. Dilaton-induced soft wall

In the AdS_5-dilaton background (25), the solutions of the scalar field equation (7) turn into Kummer's confluent hypergeometric functions [12]. The spectrum-generating normalizable modes then form the subset of Kummer functions whose power series expansion truncates to a finite polynomial which turns out to be of generalized Laguerre type $L_n^{(2)}$ [39], i.e.

$$\psi_n(z) = N_n \lambda^4 z^4_1 F_1(-n, 3, z^2\lambda^2) = N_n \lambda^4 z^4 \frac{n!}{(3)_n} L_n^{(2)}(\lambda^2 z^2) \tag{45}$$

where $n = 0, 1, 2, \ldots,$ $(a)_n \equiv a(a+1)(a+2)\ldots(a+n-1)$ and ${}_1F_1$ is a confluent hypergeometric function [39]. The ensuing restriction to discrete eigenvalues $q^2 = m_n^2$ yields the glueball mass spectrum [12]

$$m_n^2 = 4(n+2)\lambda^2 \tag{46}$$

and relates the mass gap $m_0 = 2\sqrt{2}\lambda$ to the dilaton background scale. In contrast to its hard-wall counterparts (41) and (42), the soft-wall spectrum (46) grows linearly with n and thus generates a Pomeron-type trajectory [51,52]. The normalization constants N_n are obtained from the inner product in the eigenmode space by demanding

$$\int_0^\infty dz \left(\frac{R}{z}\right)^3 e^{-\lambda^2 z^2} \psi_n^2(z) = 1 \tag{47}$$

which yields

$$N_n = \lambda^{-1} R^{-3/2} (I_n)^{-1/2} \tag{48}$$

in terms of the integrals

$$I_n := \int_0^\infty d\xi e^{-\xi^2} \xi^5 {}_1F_1^2(-n, 3, \xi^2) = \frac{n!}{(3)_n} = \frac{2}{(n+1)(n+2)}.$$

(Note that $N_n \propto (I_n)^{-1/2} \to 2^{-1/2} n$ for $n \gg 3$, and to a rather good approximation already for $n \gtrsim 3$.)

From the general expression (13) one then obtains the glueball decay constants in the soft-wall background as

$$f_n^{(\mathrm{sw})} = 4 I_n^{-1/2} \frac{\lambda^3 R^{3/2}}{m_n^2 \kappa} = \frac{1}{\sqrt{2}} \sqrt{\frac{n+1}{n+2}} \frac{\lambda R^{3/2}}{\kappa}. \tag{49}$$

This expression shows that the $f_n^{(\mathrm{sw})}$ increase by only about 40% from $n = 0$ to $n = \infty$ and approach the universal value $f_\infty^{(\mathrm{sw})} = \lambda R^{3/2} / (\sqrt{2} \kappa)$ towards higher excitation levels rather fast, in contrast to the weak but unbounded increase of their hard-wall counterparts (44).

Equation (49) can be checked by alternatively deriving it from the spectral density (30), and the factor $R^{3/2}/\kappa$ can again be estimated by Eq. (20) which continues to hold in the soft-wall background. The dilaton scale λ will be approximately determined in Sec. IV C.

C. Quantitative analysis

I restrict the quantitative decay-constant estimates to the glueball ground state, i.e. to $f_1^{(\mathrm{hw})} \equiv f_S^{(\mathrm{hw})}$ and $f_0^{(\mathrm{sw})} \equiv f_S^{(\mathrm{sw})}$, since only f_S will be of phenomenological relevance in the foreseeable future and since independent theoretical information on it is currently available. (The extension to higher resonances by means of formulas (44) and (49) is of course immediate.) After having fixed the correlator normalization R^3/κ^2 according to Eq. (20) in both backgrounds, it remains to determine the IR scale $z_m^{-1}(\lambda)$ of the hard- (soft-) wall gravity dual. In order to get an idea of how the uncertainties involved in different scale-setting approaches affect the decay-constant predictions, I will discuss several alternative possibilities.

A commonly adopted strategy for fixing the IR scale is to match the holographic ground-state mass to lattice results. Uncertainties of this method include the still rather large scale-setting ambiguity of quenched lattice predictions [68] and the neglected light-quark effects (including quarkonium mixing and decay channels) which may substantially reduce the quenched scalar glueball masses [69]. Nevertheless, the quenched masses can serve as a useful benchmark for scale-setting purposes, especially because it is not clear how far quark effects are accounted for in the simple dual dynamics which I consider here.

I therefore base my first estimate on a typical quenched glueball mass $m_S \simeq 1.5$ GeV [23,52,70], which coincides with the mass of the experimental glueball candidate $f(1500)$ and fixes the IR scale of the Dirichlet (Neumann) hard wall at $z_m^{(\mathrm{D})-1} = 0.29$ GeV ($z_m^{(\mathrm{N})-1} = 0.39$ GeV) and that of the soft wall at $\lambda = 0.43$ GeV. (Note that the values for z_m and $\lambda/\sqrt{2}$ are indeed rather close to Λ_{QCD}, as assumed in the qualitative estimates of Sec. III.) When inserted into Eqs. (44) and (49), these scales lead to the predictions

$$f_S^{(\mathrm{D})} = 0.77 \text{ GeV}, \tag{50}$$

$$f_S^{(\mathrm{N})} = 0.87 \text{ GeV} \tag{51}$$

in the hard-wall geometry and to the about 3 times smaller value

$$f_S^{(\mathrm{sw})} = 0.28 \text{ GeV} \tag{52}$$

in the soft-wall background. Since both of the parameters which underlie these results were fixed in the glueball sector and in the absence of quarks (recall that the estimate (20) is based on the free gluon loop), the above values are probably best associated with pure Yang-Mills theory.

Alternatively, one can determine the value of the hard IR wall cutoff in the classical hadron sector, e.g. from a fit to π and ρ meson properties as in Refs. [10,18]. The typical result is $z_m^{-1} \simeq 0.35$ GeV and yields

$$f_S^{(\mathrm{D})} = 0.93 \text{ GeV}, \tag{53}$$

$$f_S^{(\mathrm{N})} = 0.78 \text{ GeV}. \tag{54}$$

The corresponding ground-state glueball mass predictions are then $m_S^{(\mathrm{D})} = 1.80$ GeV and $m_S^{(\mathrm{N})} = 1.34$ GeV (where $m_S \equiv m_1$). The latter is significantly smaller than most quenched lattice results but close to the $f(1270)$ and to results of K-matrix analyses of scalar resonance data [71], mixing schemes with only one 0^{++} multiplet below 1.8 GeV [72], a topological knot model [73] and the QCD sum-rule prediction $m_S = 1.25 \pm 0.2$ GeV [22]. One might speculate that fixing z_m^{-1} in the flavored meson sector takes some light-quark effects into account and hence corresponds to a lower, unquenched value of the scalar glueball mass (at least under Neumann IR boundary conditions). For an alternative estimate of the soft-wall IR mass scale λ (which has not yet been determined in the meson sector), finally, one can use its approximate relation $\lambda \simeq \sqrt{2} \Lambda_{\mathrm{QCD}} \simeq 0.49$ GeV (cf. e.g. Ref. [57]) to the QCD scale $\Lambda_{\mathrm{QCD}} \sim 0.33$ GeV [3] (for three light-quark flavors). This yields the soft-wall prediction

$$f_S^{(\mathrm{sw})} = 0.31 \text{ GeV} \tag{55}$$

which is similar to the first soft-wall estimate (52) but corresponds to a significantly smaller glueball mass $m_S^{(\mathrm{sw})} = 1.37$ GeV.

The above results may be summarized as follows: (i) whereas the hard-wall results for the ground-state mass can differ by more than 30% for Dirichlet vs Neumann IR boundary conditions, the decay-constant predictions remain in the smaller range

$$f_S^{(\mathrm{hw})} \simeq 0.8\text{–}0.9 \text{ GeV}, \tag{56}$$

and (ii) the soft-wall results for the ground-state decay constant center consistently around less than half of the hard-wall value,

$$f_S^{(\mathrm{sw})} \simeq 0.3 \text{ GeV}. \tag{57}$$

The substantial difference between the hard- and soft-wall predictions can be traced to the different slope of the normalized dual modes at the UV brane (i.e. for $z = \varepsilon \to 0$). (An analogous but less pronounced difference between the slopes of hard- and soft-wall modes was found in the rho meson sector [74].) The larger slope of the hard-wall mode translates into a larger Bethe-Salpeter amplitude at the origin and hence into a smaller size of the scalar glueball.

In view of the sign problem which afflicts the leading nonperturbative contributions to the soft-wall glueball correlator at distances larger than the inverse QCD scale (cf. Sec. III B), and because of the exceptional size of the missing exponential contributions, one would expect the soft-wall results in the spin-0 glueball sector to be less reliable than their hard-wall counterparts. This expectation is corroborated by the first (quenched) lattice simulation of glueball decay constants [23] which finds $f_S^{(\mathrm{lat})} = 0.86 \pm 0.18$ GeV. This lattice result is inside errors fully consistent with the IOPE sum-rule value $f_S^{(\mathrm{IOPE})} = 1.050 \pm 0.1$ GeV [22], the instanton-liquid model result $f_S^{(\mathrm{ILM})} = 0.8$ GeV [20] and our above holographic hard-wall result (56). The soft-wall result (57), on the other hand, is clearly incompatible with the lattice prediction.

Further insight into the holographic glueball dynamics can be gained by interpreting the above results on the basis of the structural complementarity between the nonperturbative physics accounted for in the soft- and hard-wall correlators (i.e. power vs exponential contributions, cf. Sec. III). Since the large exponential contributions to the hard-wall correlator can at least partially be associated with small-scale instantons and are absent in the soft-wall correlator, one infers that the instanton contribution can more than double the value of the decay constant. The mentioned IOPE sum-rule analyses [21,22] arrived at the same conclusion. Moreover, even the perturbative and hard instanton contributions alone (i.e. without the unusually small power corrections and thus comparable to the hard-wall physics) were found to provide reasonable approximations to the 0^{++} QCD glueball sum-rule results for the ground-state mass and decay constant [21]. The neglect of the hard instanton contributions, on the other hand, leads to the substantially smaller prediction $f_S^{(\mathrm{OPE})} = 0.390 \pm 0.145$ GeV [75] which is consistent with the soft-wall result (57) but not with the lattice value.

V. SUMMARY AND CONCLUSIONS

I have analyzed the scalar glueball dynamics contained in two approximate holographic QCD duals, viz. the hard-wall IR brane geometry and the dilaton soft-wall background. The article focuses on the 0^{++} glueball correlation function and its spectral density for which I have obtained closed analytical expressions in both gravity duals. A systematic comparison with the QCD physics content of the instanton-improved operator product expansion, a dilatational low-energy theorem and an additional, two-dimensional power correction then provides several new insights into the holographic representation of hadron physics as well as estimates for various bulk parameters of the QCD vacuum and predictions for the glueball decay constants.

In both dual backgrounds the spectral densities are found to be non-negative, in agreement with general principles, and to consist of an infinite sum of zero-width glueball poles, as expected in the limit of a large number of colors. In their representation of specific nonperturbative glueball physics (at momenta larger than the QCD scale), however, both holographic duals turn out to complement each other in a mutually exclusive fashion: the soft-wall correlator contains all known types of QCD power corrections (to leading order in the strong coupling), generated either by condensates or by an effective UV gluon mass, while sizeable exponential corrections as induced by small-scale instantons are found in the hard-wall correlator. (This complementarity may in fact suggest to combine brane- and dilaton-induced IR physics into improved QCD duals.)

As a consequence, the soft-wall correlator provides holographic estimates for either the three lowest-dimensional gluon condensates or their Wilson coefficients, as well as for the effective gluon mass (potentially associated with a two-dimensional nonlocal "condensate"), whereas the hard-wall correlator allows for predictions of the two leading moments of the instanton size distribution. All holographic estimates turn out to be of the order of magnitude expected from QCD, which is at least partly a consequence of the fact that the IR scale of both dual backgrounds is set by Λ_{QCD}. The predicted signs of the two leading dilaton-induced power corrections, however, are opposite to those of standard QCD estimates (and in conflict with the factorization approximation for the four-gluon condensate). I have argued that these shortcomings provide evidence for the short-distance physics in the OPE Wilson coefficients to be inadequately reproduced (beyond the leading conformal logarithm) by the strongly coupled UV regime of bottom-up models. In conjunction with the absence of the sizeable exponential contributions, this casts particular doubts on soft-wall results for glueball observables.

A second main objective of the analysis was to provide first holographic estimates for the decay constants of the 0^{++} glueball and its excitations, which contain valuable size information and are of direct importance for experimental glueball searches. The analysis shows that the decay constants probe aspects of the dual dynamics to which the mass spectrum is less sensitive, and thus provide a new testing ground for the development of improved QCD duals. The hard- and soft-wall predictions for the ground-state decay constant f_S differ by more than a factor of 2, as do the corresponding QCD sum-rule results with and without hard instanton contributions. In fact, as in the sum-rule analyses the enhancement of f_S and the consequently reduced size of the scalar glueball in the hard-wall background can be traced to the strong instanton-induced attraction (over relatively short distances of the order of the average instanton size) which the exponential contributions to the hard-wall correlator generate. It is remarkable that the simple hard-wall approximation can reproduce these small-instanton effects, which are known to be exceptionally strong in the 0^{++} glueball correlator. Their absence and the other shortcomings mentioned above render the soft-wall predictions for the glueball decay constants unreliable, while the hard-wall prediction $f_S^{(\mathrm{hw})} \simeq 0.8$–$0.9$ GeV agrees inside errors with IOPE sum-rule and lattice results.

The above arguments for the instanton-induced origin of the decay constant enhancement provide an example for how the complementary nonperturbative physics in the hard- and soft-wall backgrounds, which should for the most part generalize to other hadron channels, may be exploited to trace differences in the holographic predictions of both backgrounds to different origins in the soft gauge dynamics. The absence of instanton contributions to the soft-wall correlator provides another example: since the soft-wall background was designed to reproduce the linear trajectories of excited mesons, it indicates that instanton effects are not directly involved in the underlying flux-tube formation, in agreement with QCD expectations.

The above results demonstrate that the comparison of holographic predictions with QCD information at the correlator level can provide very specific and quantitative insights into the gauge dynamics which different dual backgrounds encode. This holds, in particular, for comparisons with the QCD operator product expansion. Owing to its ability to factorize contributions from short- and long-distance physics to gauge-theory amplitudes, the OPE allows for a transparent analysis and systematic improvement of several typical shortcomings of holographic models, including those which are rooted in their strongly coupled UV sector. These limitations notwithstanding, the amount of glueball dynamics found to be represented in even the simplest holographic duals is encouraging and indicates that the bottom-up approach may indeed provide a viable and systematically improvable approximation to holographic QCD.

ACKNOWLEDGMENTS

This work was supported by the Brazilian funding agency Fundação de Amparo a Pesquisa do Estado de São Paulo (FAPESP).

APPENDIX: SYNOPSIS OF QCD RESULTS

The instanton-improved operator product expansion (IOPE)

$$\hat{\Pi}^{(\mathrm{IOPE})}(Q^2) = \hat{\Pi}^{(\mathrm{OPE})}(Q^2) + \hat{\Pi}^{(I+\bar{I})}(Q^2) \qquad \text{(A1)}$$

of the scalar QCD glueball correlator, which holds at spacelike momenta $Q^2 = -q^2 \gg \Lambda_{\mathrm{QCD}}^2$, is currently known up to operators of dimension eight, radiative corrections to the Wilson coefficients up to $O(\alpha_s^2)$, and small-size (or "direct") instanton contributions of $O(\hbar^0)$ to the Wilson coefficient of the unit operator [21,22]. The standard part, with purely perturbative coefficients, has therefore the form (cf. [22,75] and references therein)

$$\hat{\Pi}^{(\mathrm{OPE})}(Q^2) = \left[A_0 + A_1 \ln\left(\frac{Q^2}{\mu^2}\right) + A_2 \ln^2\left(\frac{Q^2}{\mu^2}\right)\right] Q^4 \ln\left(\frac{Q^2}{\mu^2}\right) + \left[B_0 + B_1 \ln\left(\frac{Q^2}{\mu^2}\right)\right]\langle G^2\rangle + \left[C_0 + C_1 \ln\left(\frac{Q^2}{\mu^2}\right)\right]\frac{\langle gG^3\rangle}{Q^2} + D_0 \frac{\langle G^4\rangle}{Q^4}. \qquad \text{(A2)}$$

The full set of coefficients A_i–D_i can be found in Ref. [22]. Those needed for comparison with the holographic results below are $A_0 = -(N_c^2 - 1)/(4\pi^2)$ and (for the number of colors (light flavors) $N_c(N_f) = 3$ [76]) $B_0 = 4 + 49\alpha_s/(3\pi)$, $C_0 = 8$ (where a small anomalous dimension correction has been neglected) and $D_0 = 8\pi\alpha_s$. The gluon condensates are defined at the OPE scale μ as

$$\langle G^2\rangle := \langle G^a_{\mu\nu} G^{a,\mu\nu}\rangle, \qquad \langle gG^3\rangle := \langle g f_{abc} G^a_{\mu\nu} G^{b\nu}_{\rho} G^{c\rho\mu}\rangle, \qquad \text{(A3)}$$

$$\langle G^4\rangle := 14\langle (f_{abc} G^b_{\mu\rho} G^{\rho c}_{\nu})^2\rangle - \langle (f_{abc} G^b_{\mu\nu} G^c_{\rho\lambda})^2\rangle. \qquad \text{(A4)}$$

Contributions from instantons larger than the inverse OPE scale are accounted for in the condensates. Small-scale (or direct) instantons (and anti-instantons) contribute to the Wilson coefficients, on the other hand, and affect dominantly the coefficient of the unit operator [21,22,48]. In the glueball channel, the latter is given by [22,48]

$$\hat{\Pi}^{(I+\bar{I})}(Q^2) = (4\pi)^2 \alpha_s^{-2} \sum_{I+\bar{I}} \int d\rho n_{\rm dir}(\rho)[(Q\rho)^2 K_2(Q\rho)]^2 \quad (A5)$$

(K_2 is a McDonald function [39]) where ρ and $n_{\rm dir}(\rho)$ denote the size and density of small instantons with $\rho \leq \mu^{-1}$ in the vacuum. The nonperturbative contributions (A5) are known to be particularly important in the spin-0 glueball channels, i.e. comparable to the contributions from the perturbative coefficient and of equal or larger size than the power terms at $Q^2 \gtrsim \Lambda_{\rm QCD}^2$. The expression (A5) can be approximated as

$$\hat{\Pi}^{(I+\bar{I})}(Q^2) \simeq 2^5 5^2 \zeta \bar{n}[(Q\bar{\rho})^2 K_2(Q\bar{\rho})]^2 \quad (A6)$$

where the instanton density is approximated by the spike distribution $n_{\rm dir}(\rho) = \zeta\bar{n}\delta(\rho - \bar{\rho})$ which becomes exact at large N_c and where $\bar{\rho}$ and $\bar{n}$ are the average instanton size and density in the vacuum. The coupling $\alpha_s/\pi \simeq 0.2$ is fixed at a typical instanton scale and the factor $\zeta \simeq 0.66$ excludes contributions from instantons with $\rho > \mu^{-1}$ [22].

Further information on the behavior of the QCD glueball correlator is available in the opposite limit $Q^2 \to 0$. Indeed, the value of the correlator at zero momentum transfer is governed by the low-energy theorem (LET) [45]

$$\hat{\Pi}(0) = \frac{32\pi}{\alpha_s b_0}\langle G^2\rangle + O(m_q) \quad (A7)$$

where $b_0 = 11N_c/3 - 2N_f/3$, m_q are the light quark masses for flavor q, and UV renormalization of both sides by a dispersive subtraction of high-frequency field contributions is implied [45]. The appearance of the gluon condensate in Eq. (A7) reflects the fact that the LET is a consequence of the anomalous Ward identity for the QCD dilatation current. Additional information on the glueball correlator has been obtained from several versions of the instanton-liquid vacuum model (ILM) in Ref. [20], whereas direct lattice information on the (point-to-point) correlator seems currently not to exist.

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