Black hole mimickers: Regular versus singular behavior

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Black hole mimickers are possible alternatives to black holes; they would look observationally almost like black holes but would have no horizon. The properties in the near-horizon region where gravity is strong can be quite different for both types of objects, but at infinity it could be difficult to discern black holes from their mimickers. To disentangle this possible confusion, we examine the near-horizon properties, and their connection with far away asymptotic properties, of some candidates to black mimickers. We study spherically symmetric uncharged or charged but nonextremal objects, as well as spherically symmetric charged extremal objects. Within the uncharged or charged but nonextremal black hole mimickers, we study nonextremal ε-wormholes on the threshold of the formation of an event horizon, of which a subclass are called black foils, and gravastars. Within the charged extremal black hole mimickers we study extremal ε-wormholes on the threshold of the formation of an event horizon, quasi-black holes, and wormholes on the basis of quasi-black holes from Bonnor stars. We elucidate whether or not the objects belonging to these two classes remain regular in the near-horizon limit. The requirement of full regularity, i.e., finite curvature and absence of naked behavior, up to an arbitrary neighborhood of the gravitational radius of the object enables one to rule out potential mimickers in most of the cases. A list ranking the best black hole mimickers up to the worst, both nonextremal and extremal, is as follows: wormholes on the basis of extremal black holes or on the basis of quasi-black holes, quasi-black holes, wormholes on the basis of nonextremal black holes (black foils), and gravastars. Since in observational astrophysics it is difficult to find extremal configurations (the best mimickers in the ranking), whereas nonextremal configurations are really bad mimickers, the task of distinguishing black holes from their mimickers seems to be less difficult than one could think of it.

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I. INTRODUCTION

In recent years, it has been debated in the literature about possible alternatives to black holes, the black hole mimickers, which would look observationally almost like black holes but would have no horizon. The existence of such objects can, in principle, put in doubt astrophysical data which otherwise are considered as observational confirmation in favor of black holes [1]. On one hand, it is clear that the properties in the near-horizon region where gravity is strong can be quite different for both types of objects. On the other hand, the statements about the difficulties in discerning black holes from their mimickers are usually related to measurements at spatial infinity. Thus, one should insist on the question: Can an observer at infinity catch the difference between both types of objects in some indirect way, or even rule out some possible mimicker? In our view, the answer is positive and is connected with key properties, namely, regularity or singularity, of the corresponding geometries. It turns out that the requirement of full regularity up to an arbitrary neighborhood of the gravitational radius of the object enables one to rule out the potential mimickers in most of the cases.

The goal of the present work is to examine the near-horizon properties, and their connection with far away asymptotic properties, of some candidates to black mimickers. We study spherically symmetric configurations, and make two major divisions, or classes, on those candidates. First, uncharged or charged but nonextremal objects, and second extremal objects. Within the uncharged or charged but nonextremal one can invoke as black hole mimickers, nonextremal ε-wormholes on the threshold of the formation of an event horizon, some of which are called black foils [2] (see [3] for the construction with other purposes of ε-wormholes, which actually can also act as mimickers), and gravastars [4]. Within the extremal charged class one can invoke extremal ε-wormholes on the threshold of the formation of an event horizon, quasi-black holes [5] (see also [6]), and wormholes on the basis of quasi-black holes from Bonnor stars, to name a few. We want to elucidate whether or not the objects belonging to these two classes remain regular in the near-horizon limit. The arguments of [5] which rule out nonextremal limiting configurations as

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becoming singular do not apply to the wormhole case [2]. Thus, we carry out the corresponding analysis anew for both classes of objects.

II. EQUATIONS AND SETUP FOR MIMICKERS

For our purposes we write a generic spherically symmetric metric as

$$ds^2 = -\exp(2\Phi(r,\lambda_i))dt^2 + \frac{dr^2}{V(r,\lambda_i)} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where r is the radial coordinate, and the λ_i are generic parameters that depend on each situation we are studying. For instance, when treating wormholes one has $i = 1, \ldots, 4$, and $\lambda_i = (r_+, r_-, \varepsilon, r_0)$, such that $\exp(2\Phi(r,\lambda_i)) = \exp(2\Phi(r, r_+, r_-, \varepsilon, r_0))$ and $V(r,\lambda_i) = V(r, r_+, r_-, \varepsilon, r_0)$. Here r_+ is the radius of the would-be horizon when ε is zero, r_- is the radius of the other possible horizon, ε is in principle a small quantity, and r_0 is the radius of a possible matter shell, satisfying $r_0 \geq r_+$. In other situations, e.g., for gravastars, one has $\lambda_i = r_0$ and, possibly, the energy density and the pressure should be somehow included. For the metric (1) the components of the Riemann tensor in an orthonormal frame, the hat frame, are equal to

$$K(r) \equiv R^{\hat{t}\hat{r}}_{\hat{t}\hat{r}} = -V(\Phi'' + \Phi'^2) - \frac{V'}{2}\Phi', \tag{2}$$

$$N(r) \equiv R^{\hat{t}\hat{\theta}}_{\hat{t}\hat{\theta}} = -\frac{V}{r}\Phi', \tag{3}$$

$$F(r) \equiv R^{\hat{\phi}\hat{\theta}}_{\hat{\phi}\hat{\theta}} = \frac{1}{r^2}(1 - V), \tag{4}$$

$$H(r) \equiv R^{\hat{\theta}\hat{r}}_{\hat{\theta}\hat{r}} = -\frac{V'}{2r}, \tag{5}$$

where a prime denotes derivative with respect to r. Here these components of the Riemann tensor have a simple physical meaning. The $K(r)$ component in Eq. (2) yields the radial geodesic deviation, the $N(r)$ component in Eq. (3) yields the angular deviation, and analogously for the $F(r)$ and $H(r)$ components in Eqs. (4) and (5). In summary, they describe the deviation of geodesics in the corresponding directions. In general, forcing a matching at r_0, with $r_0 > r_+$, surface stresses S_a^b appear, which, in a coordinate frame, are equal to [7,8]

$$8\pi\Sigma \equiv -8\pi S_t^t = -\frac{2}{r_0}\left[\left(\frac{dr}{dl}\right)_+ - \left(\frac{dr}{dl}\right)_-\right], \tag{6}$$

$$8\pi S \equiv 8\pi S_\theta^\theta = \frac{1}{r_0}\left[\left(\frac{dr}{dl}\right)_+ - \left(\frac{dr}{dl}\right)_-\right] + \left(\frac{d\Phi}{dl}\right)_+ - \left(\frac{d\Phi}{dl}\right)_-, \tag{7}$$

S_ϕ^ϕ being equal to S_θ^θ, and l being the proper radial distance. Now, if metric (1) represents a wormhole, then the areal radius $r(l)$ should have a local minimum at the throat. Thus, we have two branches emerging out of the minimum radius, one with $(\frac{dr}{dl})_+ = \sqrt{V(r)}$ and the other with $(\frac{dr}{dl})_- = -\sqrt{V(r)}$. Then,

$$8\pi\Sigma = -\frac{4}{r_0}\sqrt{V(r_0)}, \tag{8}$$

$$8\pi S = \frac{2\sqrt{V(r_0)}}{r_0} + \left(\frac{d\Phi}{dl}\right)_+ - \left(\frac{d\Phi}{dl}\right)_-. \tag{9}$$

There are also the bulk stress-energy components, but those do not interest us here and do not need computation.

In some situations one has to deal here with naked behavior. This means there are cases in which the Kretschmann scalar and other curvature quantities are finite on the horizon in a static coordinate system, but some of those quantities may blow up in a freely falling frame. Such a kind of behavior is called naked behavior, and many instances of it have been found [9–16]. Another example is with quasi-black holes [5]. One of the features typical of quasi-black holes consists of precisely showing naked behavior on and beyond the quasihorizon surface. In addition, metrics obtained by gluing two spacetimes can have a similar behavior, but now the surface stresses, which are finite in a static coordinate frame, blow up in a free-falling frame. Thus, since we have found in Eqs. (2)–(7) the curvature and surface stresses in a static frame for the spacetime in question, we now examine the behavior of the same quantities for a free-falling frame. Consider then a radial local boost from a static frame with four-velocity u^μ to a free-falling frame with the velocity $\bar{u}^\mu$. Under a boost the four-velocity transforms according to $\bar{u}^\mu = u^\mu \cosh\alpha - n^\mu \sinh\alpha$, where the orthonormal vector n^μ is pointing in the radial outward direction, and α is the velocity boost parameter. In relation to the tidal forces in the bulk, the curvature components (2) and (3) in the orthonormal basis responsible for tidal forces transform according to

$$\bar{K} = K, \tag{10}$$

$$\bar{N} = N - Z\sinh^2\alpha = H + E^2\exp(-2\Phi)(N - H), \tag{11}$$

where a bar means a quantity evaluated in the freely falling frame, $Z = H - N$ [see Eq. (5) for the definition of H], $\cosh\alpha = \exp(-\Phi)E$, and E is the energy of the particle frame (see e.g. [5] for more details). The most interesting situation arises when K is finite (so, the Kretschmann scalar is also finite) but $\bar{N}$ diverges. The corresponding horizons can be called truly naked [13,14]. In relation to the surface stresses, it is useful to define the quantity

$$\bar{\Sigma} = S_{\mu\nu}\bar{u}^{\mu}\bar{u}^{\nu}, \tag{12}$$

which represents the energy density of the shell as observed by the observer with the four-velocity $\bar{u}^{\mu}$. In a static frame $\bar{\Sigma} = \Sigma = -S_t^t$. Then, considering a boosted motion along a radial geodesic with energy E, one obtains

$$\bar{\Sigma} = -S_t^t \exp(-2\Phi)E^2. \tag{13}$$

This is a useful expression for analyzing naked behavior of wormholes and other objects. For the wormhole case it reduces to

$$\bar{\Sigma} = -\frac{\sqrt{V(r_0)}}{2\pi r_0}\exp(-2\Phi)E^2, \tag{14}$$

where r_0 is the radius at which the shell is located and we took into account (8).

III. MIMICKERS OF NONEXTREMAL BLACK HOLES

A. Non-extremal wormholes on the basis of ε-metrics with surgery

1. Basics

There are many ways of making wormholes [17,18]. In this section we are interested in making wormholes from charged metrics, more general than the Reissner-Nordström metric, but for certain choices the metrics can be reduced to the Reissner-Nordström metric. Even for this metric one probably can think of many manners of making wormholes. We are interested in two different ways that easily lead to the threshold of black hole formation, and the discussion of how they mimic black holes. Then we compound both ways into one single way.

The first way is the surgery approach (see, in particular, Sec. 15.2.1 of [18]). Pick up a spherically symmetric metric of the form $ds^2 = -\exp(2\Phi(r, r_+, r_-, r_0))dt^2 + \frac{dr^2}{V(r, r_+, r_-, r_0)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$, where r_+ is the radius of the would-be horizon, r_- is the radius of the other possible horizon, and r_0 is the radius of a possible matter shell, satisfying $r_0 \geq r_+$. Take for instance the nonextremal Reissner-Nordström metric, where $\exp(\Phi) = \sqrt{V} = \sqrt{(1 - \frac{r_+}{r})(1 - \frac{r_-}{r})}$, with $r_\pm = GM \pm \sqrt{G^2M^2 - GQ^2}$ and $r_+ \neq r_-$, M and Q being the mass and electrical charge of the object, respectively, and G is Newton's constant (we use $c = 1$). Cut the metric at some r_0 and join the resulting spacetime with a symmetric branch. This is a nonextremal Reissner-Nordström surgery (the Schwarzschild surgery, with no charge and so $r_- = 0$, being a particular case of this), resulting in a nonextremal wormhole with a thin shell of matter at r_0, the throat. In brief, one places at some radius, r_0, a thin shell which separates two regions, with nonextremal geometries, with r_0 also defining the throat. Then, one introduces another radial coordinate l, such that $r = r(l)$ with $r_0 = r(0)$, and which covers the whole of the manifold, $-\infty < l < \infty$. The function $r(l)$ is monotonically decreasing for the branch $l < 0$, which we call the " $-$" branch, and monotonically increasing for $l > 0$, the " $+$" branch. In general, giving this construction, surface stresses S_a^b appear.

A second way, i.e., another approach, to build wormholes, is through metrics of the type $ds^2 = -\exp(2\Phi(r, r_+, r_-, \varepsilon))dt^2 + \frac{dr^2}{V(r, r_+, r_-, \varepsilon)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$, where ε is a small quantity, r_+ is the radius of the would-be horizon when ε is zero, and r_- is the radius of the other possible horizon [2,3]. Metrics of this type, depending on the parameter ε, can be generically call ε-spacetimes, which in special cases can become wormholes, i.e., ε-wormholes. In [2] the model with metric $ds^2 = -(V + \varepsilon^2)dt^2 + \frac{dr^2}{V} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$ was considered, where thus $\exp(\Phi(r, \varepsilon)) = \sqrt{V + \varepsilon^2}$, with V being chosen appropriately. In turn, in [3] the model with metric $ds^2 = -(\lambda\sqrt{V} + \varepsilon)^2dt^2 + \frac{dr^2}{V} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$ was considered, where thus $\exp(\Phi) = \lambda\sqrt{V} + \varepsilon$, with V being chosen appropriately and with λ being an additional parameter. Thus, a generic ε-metric of the type given above yields a generic spacetime that comprehends the two cited models, one model studied in [2], the other in [3]. One can calculate the Riemann tensor for the ε-metric, and of course, since spacetime is not empty, there is a smooth energy-momentum tensor associated to the metric but we do not need to calculate it here. All these ε-spacetimes are smooth. Now, take the ε-metric to construct a wormhole, i.e., an ε-wormhole. Since for the construction we need to impose some more conditions, in particular, on the potentials Φ and V of the ε-metric, let us adopt the following approach. First, as above, one introduces the radial coordinate l, such that $r = r(l)$ and $-\infty < l < \infty$. The function $r(l)$ is monotonically decreasing for the " $-$" branch, $l < 0$, and monotonically increasing for the " $+$" branch, $l > 0$. Second, the dependence of the function Φ on the parameter ε, $\Phi = \Phi(r, \varepsilon)$, which can be of the type of the models considered above [2,3], is such that $\exp(\Phi(r_+, 0)) = 0$, and the dependence of the function V on the parameter ε is also such that $V(r_+, 0) = 0$, so in the limit $\varepsilon \to 0$ the original wormhole configuration indeed approaches a black hole. Third, if the first derivative $\frac{dr}{dl}$ is continuous at the throat, we have $\frac{dr}{dl} = 0$ (see [17]). When V does not depend on ε at all, the throat is situated on the possible would-be horizon. This is the approach used in [2,3] to build a wormhole. This approach is smooth as long as $\varepsilon \neq 0$. As a particular instance of this approach, one can choose V as being Reissner-Nordström, $V \equiv (1 - \frac{r_+}{r})(1 - \frac{r_-}{r})$, as usual. For $r_+ \neq r_-$ one has a nonextremal choice for V, the case $r_- = 0$, i.e., $Q = 0$, yielding the Schwarzschild potential V as a particular case, the one chosen in [2,3]. Such ε-wormholes have been called foils in [2]. When $\varepsilon = 0$ we have the full nonextremal Reissner-Nordström metric.

So, let us compound both approaches, the surgery approach of [17,18], and the ε approach of [2,3]. Write then a generic ε-metric with surgery as $ds^2 = -\exp(2\Phi(r, r_+, r_-, \varepsilon, r_0))dt^2 + \frac{dr^2}{V(r,r_+,r_-,\varepsilon,r_0)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$, i.e., from (1) one chooses

$$\exp(2\Phi(r, \lambda_i)) = \exp(2\Phi(r, r_+, r_-, \varepsilon, r_0)), \quad V(r, \lambda_i) = V(r, r_+, r_-, \varepsilon, r_0), \tag{15}$$

where ε is a small quantity, r_+ is the radius of the would-be horizon (if $\varepsilon = 0$), and r_0 is the radius of a possible matter shell, satisfying $r_0 \geq r_+$. Since we are studying here nonextremal metrics we have, when $\varepsilon = 0$, that $V(r_+) = 0$ and $V'(r_+) \neq 0$. Essentially, what we have done is a surgery on ε-metrics, nonextremal Reissner-Nordström (with Schwarzschild included) metrics being particular $\varepsilon = 0$ instances. This can also be thought of as a one-parametric deformation of the original ε-wormhole metric by gluing two branches at the throat r_0, with $r_0 > r_+$. Now, given the general ε-metric with surgery, (1) and (15), and the correspondent wormhole construction, we are interested in getting a spacetime that mimics a black hole. It is then not hard to understand that there are two distinct situations to obtain spacetimes on the threshold of being black holes. If there is no shell, then wormholes approach black holes when $\varepsilon \to 0$. If there is a shell but $\varepsilon = 0$ then the wormhole throat approaches the horizon when $r_0 \to r_+$. Therefore, there is a play of two small parameters ε and $r_0 - r_+$ and the limiting procedure should be considered with great care. It gives rise to two distinct situations, depending on the order one takes the limiting procedures. Situation BT: This situation is achieved by in the end turning the wormhole metric into the metric of a black hole (B) (i.e., taking $\varepsilon \to 0$ as the last operation), after first having moved the shell towards the minimum throat (T) radius (i.e., taking $r_0 \to r_+$ as the initial operation). Formally, this means taking the limits in the following order BT $\equiv \lim_{\varepsilon\to 0} \lim_{r_0 \to r_+}$. Situation TB: This situation is achieved by, in the end the location of the shell approaches the throat (T) (i.e., taking $r_0 \to r_+$ as the last operation), after first turning the wormhole metric into a black hole (B) (i.e., taking $\varepsilon \to 0$ as the initial operation). Formally, this means taking the limits in the following order TB $\equiv \lim_{r_0\to r_+} \lim_{\varepsilon \to 0}$. Note the case considered in [2] for the metric (1) is a particular instance of the BT situation, since there $r_0 = r_+$ always, and one only takes the $\varepsilon \to 0$ limit. So the situation BT is the one that yields black foils, following the nomenclature of [2]. One can calculate from Eqs. (3)–(5) that the components $N(r)$, $F(r)$, and $H(r)$ of the Riemann tensor are always finite, and from Eq. (6) that both limits when applied to $\Sigma \equiv -S_t^t$ give zero, i.e., $\lim_{\varepsilon\to 0} \lim_{r_0\to r_+} \Sigma = 0 = \lim_{r_0\to r_+} \lim_{\varepsilon\to 0} \Sigma$. But for the quantities K, $S \equiv S_\theta^\theta$ and $\bar{\Sigma}$ [see Eqs. (2), (7), and (13)] the situation may be different depending on the order one takes the limits.

Now, as we have been seeing, in treating this problem there are many levels of distinction. First, we can specify two models of ε-metrics with surgery which depend on the parameter ε and r_0, namely, the model considered in [2], where $\exp(\Phi) = \sqrt{V + \varepsilon^2}$, and V is nonextremal, or some appropriate generalization of it, which we will call Model 1, and the model given in [3], where $\exp(\Phi) = \lambda\sqrt{V} + \varepsilon$ and V nonextremal, or some appropriate generalization of it, which we will call Model 2. Second, within each of the two cases provided by Model 1 and Model 2, we should study the situations BT and TB. Furthermore, as we want to examine the regularity of the system under discussion, the relevant quantities which we are going to calculate are the spacetime curvature components, and the surface stresses which appear on the glued boundary, i.e., the shell. We will also study the naked behavior of each case. So, within each situation we have to study the behavior of the scalars, and in addition the naked behavior. Thus we have eight distinct cases to analyze. We consider these eight cases, each in turn.

2. Models

Here we consider the one-parametric deformation, Eqs. (1) and (15), such that for $\varepsilon = 0$ our metric represents the gluing of two nonextremal black holes. Note that in this nonextremal case the function V of the metric (1) does not need to contain the parameter ε, so we put $V = V(r_+, r_0, r)$.

a. Model 1

Let the metric have the form (1) together with (15). For Model 1 choose the metric potentials as

$$\exp(\Phi) = \sqrt{V + \varepsilon^2}, \tag{16}$$

where $V(r)$ can be any function that satisfies $V(r_+) = 0$ and $V'(r_+) \neq 0$. For instance V can be Reissner-Nordström, $V(r) \equiv (1 - \frac{r_+}{r})(1 - \frac{r_-}{r})$ with $r_\pm = GM \pm \sqrt{G^2M^2 - GQ^2}$ and $r_+ \neq r_-$, with the case $r_- = 0$ being the Schwarzschild case, the one chosen in [2]. Let us now work out generically the general behavior of this model, i.e., how the curvature and stress-tensor quantities behave, and also work out generically the naked behavior. Then we apply these behaviors to the two situations BT and TB. In doing so, we will display the properties of the quantities K, S, which characterize regular or singular general behavior, and the properties of the quantities $\bar{N}$, $\bar{\Sigma}$ which characterize non-naked or naked behavior. As for understanding the general behavior note that, from Eqs. (2) and (7) explicit calculations give

$$K = -\frac{V}{2}\frac{V''}{V+\varepsilon^2} - \frac{1}{4}\frac{\varepsilon^2 V'^2}{(V+\varepsilon^2)^2}, \quad (17)$$

and

$$8\pi S = \frac{2}{r_0}\sqrt{V(r_0)} + \frac{V'(r_0)\sqrt{V(r_0)}}{\varepsilon^2 + V(r_0)}. \quad (18)$$

In relation to naked behavior, note that since from (3) one has $N = -\frac{V}{r}\Phi' = -\frac{VV'}{2r(\varepsilon^2+V)}$, and from (5) one has $H = -\frac{V'}{2r}$, one finds $Z = H - N = -\varepsilon^2\frac{V'}{2r(V+\varepsilon^2)}$. Thus, from (11)

$$\bar{N} = \frac{V'}{2r}\left[\frac{E^2\varepsilon^2}{(V+\varepsilon^2)^2} - 1\right]. \quad (19)$$

With this we can now study the situations BT and TB.

Situation BT: As for the general behavior, one has that for a nonextremal system $V(r_+) = 0$ and $V'(r_+) \neq 0$. Then, it follows from Eq. (17) that

$$K(r_+, \varepsilon) = -\frac{V'^2(r_+)}{4\varepsilon^2}, \quad (20)$$

and so,

$$\lim_{\varepsilon\to 0}\lim_{r_0\to r_+} K(r_0, \varepsilon) = -\infty. \quad (21)$$

Correspondingly, the Kretschmann scalar $\mathrm{Kr} = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ also diverges. It was shown in [2] for the choice $V = 1 - \frac{r_+}{r}$ that for $\varepsilon \neq 0$ there exist geodesics which have no analogue for the Schwarzschild black hole metric. The timelike particles which move along them oscillate between turning points, which are situated at different sides of the throat. However, the problem is that in the limit $\varepsilon \to 0$ these geodesics pass through a region of a strong gravitational field. This gives rise to tidal forces in the radial direction which are of order $\varepsilon^{-2}r_+^{-2}$. If ε is exponentially small [2], the tidal forces are exponentially large. In addition, from Eq. (18), it follows that

$$\lim_{\varepsilon\to 0}\lim_{r_0\to r_+} 8\pi S(r_0, \varepsilon) = 0. \quad (22)$$

Now let us analyze the naked behavior. Here one has, $\bar{N} \sim \varepsilon^{-2}$, $\bar{K} \sim -\varepsilon^{-2} \to$. So, $\bar{N} \to \infty$, $\bar{K} \to -\infty$. Thus, there is infinite contraction in the longitudinal direction and infinite transversal stretching. Moreover, since $V(r_+) = 0$ and $\exp(\Phi(r_+, \varepsilon)) \neq 0$, we obtain immediately from (14) and (16) that in the situation BT one has

$$\bar{\Sigma} = 0. \quad (23)$$

Situation TB: As for the general behavior, now one finds, $K(r_0, 0) = -\frac{1}{2}V''(r_0)$. So,

$$\lim_{r_0\to r_+}\lim_{\varepsilon\to 0} K(r_0, \varepsilon) = -\frac{V''(r_+)}{2}, \quad (24)$$

a result equal to that of a black hole. Also, $8\pi S(r_0, 0) = \frac{2}{r_0}\sqrt{V(r_0)} + \frac{V'(r_0)}{\sqrt{V(r_0)}}$, so

$$\lim_{r_0\to r_+}\lim_{\varepsilon\to 0} 8\pi S(r_0, \varepsilon) = +\infty. \quad (25)$$

Note that Eq. (25) is in agreement with the behavior of surface stresses of a wormhole obtained by gluing two copies of the Schwarzschild metric (see Eq. 15.46 of [18]). Now let us analyze the naked behavior. One has, $\bar{N} = N = -\frac{V'(r_+)}{2r}$, so $\bar{N}$ is finite and negative. Thus, one obtains finite deformation in both directions. Moreover, it also follows from (13) that in the situation TB

$$\bar{\Sigma} \to -\infty, \quad (26)$$

i.e., $\bar{\Sigma}$ diverges. Thus, a free-falling observer encounters diverging surface energy density. The same conclusion applies to the flux $J = S_{\mu\nu}\bar{u}^\mu\bar{e}^\nu$.

Concluding here Model 1, we can say that there are two nonequivalent limits, but each of them is "bad" in that in the BT situation the Kretschmann scalar diverges, whereas in the TB situation it is the surface stresses that diverge.

b. Model 2

Let the metric have the form (1) together with (15). For Model 2 choose the metric potentials as

$$\exp(\Phi) = \lambda\sqrt{V} + \varepsilon, \quad (27)$$

with λ and ε being parameters. In [3] the model with $V = \sqrt{1 - \frac{r_+}{r}}$, was considered, in which case, when $\varepsilon = 0$ one has the Schwarzschild metric see also [15]. Here, $V(r)$ can be any function that satisfies $V(r_+) = 0$ and $V(r_+)' \neq 0$, a typical example being the Reissner-Nordström V potential. As for the general behavior, again after some calculations, we obtain

$$K(r, \varepsilon) = -\frac{\lambda}{2}\frac{\sqrt{V}V''}{\varepsilon + \lambda\sqrt{V}}, \quad (28)$$

and

$$8\pi S = \frac{2}{r_0}\sqrt{V(r_0)} + \frac{\lambda V'(r_0)}{\varepsilon + \lambda\sqrt{V(r_0)}}. \quad (29)$$

Now let us analyze the naked behavior. Also, again after some calculations, we obtain $N(r) = -\frac{V}{r}\Phi' = -\frac{\lambda\sqrt{V}V'}{2r(\varepsilon+\lambda\sqrt{V})}$, so it follows from (11) that

$$\bar{N} = \frac{V'}{2r}\left[\frac{E^2\varepsilon}{(\varepsilon + \lambda\sqrt{V})^3} - 1\right] \quad (30)$$

With this we can now study the situations BT and TB.

Situation BT: As for the general behavior, in this situation one finds

$$\lim_{\varepsilon\to 0}\lim_{r_0\to r_+} K(r_0, \varepsilon) = 0, \quad (31)$$

and

$$\lim_{\varepsilon\to 0}\lim_{r_0\to r_+} 8\pi S(r_0, \varepsilon) = \infty. \quad (32)$$

Now let us analyze the naked behavior. One has $\bar{K} \to 0$, and

$$\bar{N} \to -\infty. \quad (33)$$

So one finds no longitudinal deformation, and an infinite transversal stretching. Taking into account that $V(r_+) = 0$, $\exp[\Phi(r_+, \varepsilon)] \neq 0$, we obtain immediately from (14) that in situation BT one finds $\bar{\Sigma} = 0$.

Situation TB: As for the general behavior in this situation, one finds

$$\lim_{r_0\to r_+}\lim_{\varepsilon\to 0} K(r_0, \varepsilon) = -\frac{V''(r_+)}{2}, \quad (34)$$

and

$$\lim_{r_0\to r_+}\lim_{\varepsilon\to 0} 8\pi S(r_0, \varepsilon) = \infty. \quad (35)$$

Now let us analyze the naked behavior. One finds, finite transversal stretching and finite longitudinal contraction ($V''(r_+) > 0$) or stretching ($V''(r_+) < 0$). It follows from (14) and (16) that in the situation TB,

$$\bar{\Sigma} \to \infty, \quad (36)$$

i.e., it diverges. Thus, a free-falling observer encounters diverging surface energy density in the situation TB. The same conclusion applies to the flux $J = S_{\mu\nu}\bar{u}^\mu \bar{e}^\nu$.

Concluding here Model 2, we can say that in both situations, BT and TB, the curvature components remain finite but the limiting values of K do not coincide. Moreover, in both situations the surface stresses diverge.

c. Overall conclusions for Models 1 and 2

As an overall conclusion for the situation TB in Models 1 and 2, we find the results agree for both models. This can be explained and generalized as follows. Since, in the situation TB, the limit $\varepsilon \to 0$ is taken first, the dependence of the metric on ε drops out in the final expressions for the curvature and surface stresses, so any model gives the same result. In addition, assuming that for $\varepsilon = 0$ one has $\exp(2\Phi) = V$, and taking into account that $V(r_+) = 0$ and $\exp(\Phi(r_+, \varepsilon)) \neq 0$, we obtain immediately from (14) that in the situation BT, $\bar{\Sigma} = 0$, and in the situation TB, $\bar{\Sigma}$ diverges. This holds independently of the kind of the model used. Thus, a free-falling observer encounters diverging surface energy density in the situation TB. The same conclusion applies to the flux $J = S_{\mu\nu}\bar{u}^\mu \bar{n}^\nu$. In the nonextremal cases just studied, it turned out that each of the limits under discussion is singular: either the Kretschmann scalar or surface stresses on the throat (or both) diverge. Thus, the limit is singular. In other words, a black hole mimicker made from a wormhole, and, in particular, a black hole foil, is not smooth. It is convenient to summarize the results in a table shown in Fig. 1.

3. Remarks: naked behavior and observable differences between nonextremal black holes and nonextremal ε-wormholes

Although the singular or almost singular behavior of these black hole mimickers based on ε-wormholes casts doubts on their real existence, it is worth our while to study a little more on the effects of such mimickers on infalling sources and their detection by far away observers. Intuitively it is clear that there should be some observational effects if infalling sources are distorted by stronger than normal tidal fields.

First, we point out that indeed the fact that tidal forces grow unbound when $\varepsilon \to 0$ can be used, in principle, to distinguish a black hole from an ε-wormhole which mimics it. Suppose a small mass falling freely into a massive body, such as an ε-wormhole. Consider, for example, the situation BT. One can compare two approaches. In the first approach, one neglects the size of the small mass and considers the geodesic along which such a pointlike small mass moves [2]. Then, if the throat is very close to the would-be horizon and subsequent pulses are emitted near the throat, the intervals of time measured at infinity,

Configuration	Kr	Curvature in free-falling frame	Energy density in free-falling frame $\bar{\Sigma}$	Surface stress S
Model 1, situation BT	infinite	$\bar{N} \to \infty$, $\bar{K} \to -\infty$	0	0
Model 1, situation TB	finite	finite	infinite	infinite
Model 2, situation BT	finite	$\bar{N} \to \infty$	0	infinite
Model 2, situation TB	finite	finite	infinite	infinite

FIG. 1. Table summarizing the main features of the Models 1 and 2 for nonextreme mimickers, in each situation BT or TB, studied in the text.

grow unbound as $\Delta t \sim -\ln\varepsilon$ in the limit $\varepsilon \to 0$. So an observer at infinity cannot distinguish the fall of matter into a wormhole with vanishingly small ε from absorption of matter by a black hole, if the interval of observation time is less than Δt, by construction a very long interval, see [2] for details. In the second approach, however, one takes the finiteness of the size of the free-falling small mass into account. Then, the overall picture changes since, due to the growing tidal forces, the small mass gets deformed. If the small mass is a luminescent source, such a change can in principle be detected by an observer at infinity. Moreover, such changes happen much quicker than the typical times needed to penetrate the immediate vicinity of a horizon. The intervals of time during which the size of a small mass changes can be estimated from Eq. (20) and the geodesic deviation equation. For a process occurring near the throat it yields for the proper time the value $\Delta\tau \sim r_+\varepsilon$, so that $\Delta\tau \to 0$, as $\varepsilon \to 0$. The corresponding interval of time Δt at infinity is $\Delta t = \frac{\Delta\tau}{\sqrt{-g_{00}}} \sim r_+$ is finite. Therefore, in case of stretching, an observer at infinity would see an extended distorted image of an infalling small mass. If the tidal forces are infinite, as they can be in some of the cases shown above for ε-wormholes, then the stretching is correspondingly large. Of course, it is also possible that infinite contraction in some direction converts an extended small mass to zero size in this direction, see [11]. For the usual black holes, tidal forces are also present but such forces are much weaker than for ε-wormholes, where near the throat they can be as large as one likes for sufficiently small ε. Thus, the key point is to look not to single geodesics as in [2], but to the separation between geodesics of the same congruence. Such a separation delivers, to an observer at infinity, meaningful information about the region of strong gravity where a black hole or a ε-wormhole can be situated, and such information has very different properties, which depend on the massive body in question.

Second, we indicate what further physical changes could be expected by performing some interesting estimates of the effects. Here, one can take advantage of known results (see [8], section 32.6). Suppose then that a box of small proper mass m and proper length l is freely falling towards an object of mass M. To simplify we consider an uncharged object, $Q = 0$. From [8] one finds that the radial tidal force exerted on the box is equal to $F = -\frac{1}{4}mlK$ where K has the meaning of a tidal radial acceleration. Indeed, K is given precisely in Eq. (2). For the Schwarzschild metric one has $K = \frac{2M}{r^3}$, so that near the horizon of a Schwarzschild black hole, where $r \approx r_+ = 2M$, one obtains the value $K_{\rm bh} \approx \frac{1}{4M^2}$. However, for Model 1, if we correspondingly choose the metric potential as $V(r) = 1 - \frac{2M}{r}$ in the situation BT, it follows from (20) that near the would-be horizon one has $K \approx \frac{1}{4\varepsilon^2 M^2} = \frac{K_{\rm bh}}{\varepsilon^2}$. Let us suppose that a distant observer is able to recover from observational data the value of K. If the observer thinks that the measured value of K is due to a Schwarzschild black hole, he should ascribe a mass $M_{\rm bh}$ to it. However, if the object turns out to be a black hole mimicker, then, for the same K, the value of the actual mass will be much greater, given by $M_{\rm m} \sim \varepsilon^{-1} M_{\rm bh}$. If, in addition, the observer, knowing the mass M of the object, in this case $M = M_{\rm m}$, insists on explaining it in terms of the usual black hole metric, he will find that $M_{\rm bh} \ll M_{\rm m}$, and will certainly start asking about the "hidden mass." This example shows that "hidden mass" in some situations may arise simply because the metrics of a black hole or of a black hole mimicker were not properly discerned.

B. Gravastars

As far as gravastars are concerned, they contain, by construction, a thin layer of normal matter with positive density ρ and positive radial pressure p on the border of the tension matter with vacuum. In the model suggested in [4] the stiff matter equation of state with $p = \rho$ was chosen. Then, it follows immediately from the field equations and the conservation laws that, as the border approaches the gravitational radius, the gradient of the pressure becomes infinite. One can, in general, admit discontinuous radial pressure, giving rise to a surface pressure on the boundary between matter and vacuum. However, this surface pressure and other surface stresses also grow unbounded in the horizon limit. Surely the more the border approaches the horizon, the better a black hole mimicker it becomes, but, at the same time, the closer the system approaches the singular state. This does not exclude in advance the astrophysical significance of gravastars as compact vacuumlike geometries, but it shows that they can hardly pretend to be good black hole mimickers.

In more detail, to see that the surface stresses go unbounded, we introduce the quantity $b = \exp(\Phi)$, so that we can rewrite Eq. (7) as $8\pi S = \frac{1}{r_0}[\sqrt{V(r_0+0)} - \sqrt{V(r_0-0)}] + \frac{1}{b(r_0)}[(\frac{db}{dl})_+ - (\frac{db}{dl})_-]$, where we have taken into account that the continuity of the first fundamental form demands $b_+ = b_- = b(r_0)$. In the outer region we have the Schwarzschild metric, so $(\frac{db}{dl})_+ > 0$. Let the radius of the shell approach that of the would-be horizon, i.e., $r_0 \to r_+$, and $b(r_+) = 0$. By definition of a gravastar, actually there is no horizon in the system, so the function b cannot cross r at r_+ at all. In the inner region, either $(\frac{db}{dl})_- = 0$ or $(\frac{db}{dl})_- < 0$. Thus, since there is a $1/b$ ($r_0 \to r_+$) term in $8\pi S$, and the other terms remain finite and nonzero, we find that $8\pi S \to \infty$. This makes the gravastar unphysical in the near-horizon limit. It is worth noting that this conclusion and its derivation are very close to the statement that quasi-black holes cannot be nonextremal, if only finite surface stresses are allowed (see Sec. IV of [5]). The only difference is that in the whole inner region $(\frac{db}{dl})_- \to 0$ everywhere for quasi-black holes whereas for gravastars $(\frac{db}{dl})_-$ can be nonzero there. However, in the present context, only the vicinity of the would-be horizon is relevant, so the conclusions are similar.

IV. MIMICKERS OF EXTREMAL BLACK HOLES

A. Wormholes on the basis of extremal ε-metrics with surgery

1. Basics

In Sec. III A on wormholes on the basis of nonextremal ε-metrics with surgery, we have discussed wormhole configurations which mimic nonextremal black holes. Here we consider the one-parametric deformation (1) such that for $\varepsilon = 0$ our metric represents the gluing of two extremal Reissner-Nordstöm black holes. Note that in the nonextremal case, when using metric (1) we have chosen a function V which itself does not contain the parameter ε. However, now such a simple construction cannot be implemented. Our goal is to trace the relationship between a wormhole metric with a generic ε parameter ($\varepsilon \neq 0$), and a black hole metric ($\varepsilon = 0$). If we start from Eq. (1) with $r_+ = r_-$, i.e., $V = (1 - \frac{r_+}{r})^2$, we encounter the immediate difficulty that spacetime is geodesically complete and represents an infinitely long horn, so one cannot speak about a wormhole at all. Therefore, we should deform the extremal Reissner-Nordström metric in a somewhat different way and include the parameter ε not only into the function Φ but into V as well. Let us make the simplest choice for $V(r, \varepsilon)$, namely,

$$V(r) = \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right), \quad \text{with} \quad r_- = r_+(1 - \delta(\varepsilon)), \tag{37}$$

$\delta(\varepsilon)$ being such that $0 \leq \delta(\varepsilon) \leq 1$, and $\delta(0) = 0$. At some $r_0 > r_+$, we glue two copies of the spacetime, the " +" branch to the " −" branch. Then, the behavior of K and S follows from Eqs. (17) and (18). Note that if $\delta =$ constant and $\delta \leq 1$ we return to the deformed nonextremal Reissner-Nordström case and the results (21) and (22) are reproduced, $\delta = 1$ being the Schwarzschild case.

2. Models

a. Model 1

Using (16), i.e., $\exp(\Phi) = \sqrt{V + \varepsilon^2}$, and (37), we can study Model 1 in the extremal case.

Situation BT: As for the general behavior one finds

$$\lim_{\varepsilon\to 0}\lim_{r_0\to r_+} K(r_0, \varepsilon) = -\frac{\alpha}{4r_+^2}, \quad \text{with} \quad \alpha = \lim_{\varepsilon\to 0}\left(\frac{\delta}{\varepsilon}\right)^2, \tag{38}$$

and

$$\lim_{\varepsilon\to 0}\lim_{r_0\to r_+} 8\pi S(r_0, \varepsilon) = 0. \tag{39}$$

Let us analyze the naked behavior. In principle, the quantity α may be finite or infinite depending on the model for $\delta(\varepsilon)$. From Eq. (38), longitudinal contraction is finite if α is finite, or infinite if α is infinite. The value of $\bar{N}$ which determines transverse deformation can be found from Eq. (19). Then, we obtain that in the limit under discussion

$$\bar{N} = \frac{\beta E^2}{2r_+^2}, \tag{40}$$

where $\beta = \lim_{\varepsilon\to 0}\frac{\delta}{\varepsilon^2}$. Then we have transverse stretching which is finite if β is finite, or infinite if β is infinite. The stress component $\bar{\Sigma}$ is finite, indeed zero, in the limit.

Situation TB: As for the general behavior one finds

$$\lim_{r_0\to r_+}\lim_{\varepsilon\to 0} K(r_0, \varepsilon) = -\frac{1}{r_+^2}, \tag{41}$$

$$\lim_{r_0\to r_+}\lim_{\varepsilon\to 0} S(r_0, \varepsilon) = \frac{1}{4\pi r_+}. \tag{42}$$

Note that when $\alpha = 4$ in the situation BT, the quantities K for two situations coincide. Let us analyze the naked behavior. Here there is finite longitudinal contraction, and no transverse deformation since, according to Eq. (19), $\bar{N} \to 0$ in the limit under consideration. Thus, the only manifestation of naked behavior is connected with the surface stresses. According to (14), the quantity $\bar{\Sigma}$ behaves as

$$\bar{\Sigma} \to -\infty, \tag{43}$$

i.e., it diverges in situation TB.

b. Model 2

Using Eq. (27), i.e., $\exp(\Phi) = \lambda\sqrt{V} + \varepsilon$, and (37), we can study Model 2 in the extremal case.

Situation BT: As for the general behavior one finds

$$\lim_{\varepsilon\to 0}\lim_{r_0\to r_+} K(r_0, \varepsilon) = 0, \tag{44}$$

$$\lim_{\varepsilon\to 0}\lim_{r_0\to r_+} S(r_0, \varepsilon) = \frac{\lambda\sqrt{\alpha}}{4\pi r_+}. \tag{45}$$

Let us analyze the naked behavior. There is no deformation in the radial direction. The behavior of $\bar{N}$, which is responsible for transverse deformation, can be obtained from (30) and coincides with (40) in the limit under discussion. Thus, again transverse stretching is finite if β is finite, or infinite if β is infinite. Note that it follows from the definitions of α and β that $\alpha = \lim_{\varepsilon\to 0}\beta\delta$. According to the definition (37) of $\delta(\varepsilon)$, $\lim_{\varepsilon\to 0}\delta(\varepsilon) = 0$. Therefore, if β is finite, $\alpha = 0$, then K, S, $\bar{N}$, and $\bar{\Sigma}$ are finite (moreover, $S = \bar{\Sigma} = 0$), so naked behavior is absent in this case. On the other hand, as $\beta = \lim_{\varepsilon\to 0}\frac{\alpha}{\delta}$, in case $\alpha \neq 0$ the quantity β diverges and so does $\bar{N}$. This means transverse stretching is infinite.

Situation TB: As for the general behavior one finds,

$$\lim_{r_0\to r_+}\lim_{\varepsilon\to 0} K(r_0, \varepsilon) = -\frac{1}{r_+^2}, \tag{46}$$

$$\lim_{r_0\to r_+}\lim_{\varepsilon\to 0} S(r_0,\varepsilon) = \frac{1}{4\pi r_+}. \tag{47}$$

Let us analyze the naked behavior. One finds finite contraction in the longitudinal direction and no transverse deformation. However,

$$\bar{\Sigma} \to -\infty, \tag{48}$$

i.e., it diverges.

c. Overall conclusions for Models 1 and 2

As an overall conclusion, assuming that for $\varepsilon = 0$, one has $\exp(2\Phi) = V$, and taking into account that $V(r_+) = 0$, $\exp[\Phi(r_+, \varepsilon)] \neq 0$, we obtain immediately from (14) that in the situation BT, $\bar{\Sigma} = 0$, and in the situation TB, $\bar{\Sigma}$ diverges. This holds independently of the kind of the model, and is valid for the nonextremal spacetimes, as well as for the extremal spacetimes. Thus, a free-falling observer encounters diverging surface energy density in the situation TB. The same conclusion applies to the flux $J = S_{\mu\nu}\bar{u}^\mu\bar{n}^\nu$.

3. Remarks: Lightlike shells and classical electron models

a. Lightlike shells

One essential feature of the almost extremal configurations under discussion consists of the fact that we deal with shells which are timelike but become lightlike in the limit when they are approaching the would-be horizon. A natural question arises: what happens if we start out our analysis with two extremal black hole spacetimes, i.e., putting $\varepsilon = 0$ and $r_0 = r_+$ and the shell in-between the two spacetimes lies on the event horizon and so is lightlike? More precisely, we are interested whether the stresses on the shell remain finite or become infinite. One can expect from our previous results that they are finite but this is not so obvious in advance. We have seen already that taking different limiting procedures in the near-horizon limit is a rather delicate issue. Moreover, the formalism for lightlike shells [19,20] somewhat differs from that for timelike ones, so we perform our analysis anew. We restrict ourselves to the case of taking two extremal Reissner-Nordström black holes. The reason for this choice is that only extremal black holes are good candidates for gluing without severe singularities, i.e., the gluing procedure maintains a finite Kretschmann scalar throughout the spacetime as well as finite stresses on the glued surface. This case is also a Majumdar-Papapetrou case, and could be analyzed in the next section as well, but since it is also a limit of what has been done above, namely, the null limit of a timelike wormhole throat with two extremal external vacuum spacetimes, we discuss it now. To match two extremal Reissner-Nordström spacetimes at the lightlike surface $r_0 = r_+$ we follow the general formalism for lightlike shells [19,20]. We write the metric in Kruskal-type coordinates $ds^2 = -H(U,V)dUdV + r^2(U,V)(d\theta^2 + \sin^2\theta d\phi^2)$. Let the surface $r_0 = r_+$ correspond, say, to $U = 0$. Then, the effective energy density is $\mu = -\frac{\gamma_{\theta\theta}}{16\pi r^2}$, (see, e.g., equation (3.99) of [19]). Here $\gamma_{\theta\theta} = [(\frac{\partial g_{\theta\theta}}{\partial x^\alpha})_+ - (\frac{\partial g_{\theta\theta}}{\partial x^\alpha})_-]N^\alpha$, where the indexes "+" and "−" refer to the different sides of the shell, and the null vector N^α is such that $k_\alpha N^\alpha \neq 0$, with k^α being a null tangent vector. Now, the only nonvanishing components of k^α and N^α, are by construction k^V and N^U. Since there is no rest frame in the null case, the measured energy density μ depends on the chosen observer. To check that it is finite and nonzero, it is sufficient to check that $[(\frac{\partial r}{\partial U})_+ - (\frac{\partial r}{\partial U})_-]$ is finite and nonzero. For this purpose, it is sufficient to exploit the result of [21] where it is shown that $\frac{\partial r}{\partial U} = -1$ on the horizon. In our case, as we deal with two black holes instead of a single one, the coordinate r increases on both sides of the shell. Thus, $\frac{\partial r}{\partial U}$ has different signs and takes the values ± 1 on each side. So indeed, the difference is equal to 2 and is finite and nonzero. In general, there are other contributions to the effective stress-energy tensor of the shell due to effective pressures and currents, but it is easy to show that in our case they are absent. It is worth noting that the gluing in our case differs from the gluing between different extremal Reissner-Nordström black holes considered in [22] in that we replaced the usual metric inside the horizon by their "−" branch. It also differs from the wormhole construction used in [23] where two spheres were cut out from a vacuum Majumdar-Papapetrou system, or more precisely, they were cut from a single spacetime containing two extremal Reissner-Nordström black holes.

b. Classical electron models

As a by-product of this lightlike shell construction, and a very interesting one, we have just found a configuration that represents a regular wormhole configuration which is also a black hole. More important perhaps, in addition, it can serve as a classical model for an elementary particle in that (i) the system is characterized by a minimum number of fixed parameters like mass and charge, and (ii) it is free of a central singularity inside. We are aware that it is not entirely of electromagnetic nature because it has stresses on the horizon, a kind of Poincaré stress. But, anyway, such a surface stress, can be considered as a mild singularity when compared to the usual central singularity. In this model a free-falling observer can penetrate to the inside but, of course, cannot return back to the original asymptotic region due to the existence of a horizon. So the wormhole is an untraversable one. Thus, the body under discussion combines features of an untraversable wormhole and a regular black hole, and can be called a worm-black hole. Since the proper distance to the extremal horizon is infinite, such a hybrid construction is similar to the null wormholes, or N-wormholes for short, see [24]. Note that it differs from configurations representing quasi-black holes. For example, in some quasi-black hole models, see

[5], there is a region $r \leq r_+$ which becomes degenerate in the quasihorizon limit. This region is missing in our worm-black hole model, since the black hole metric beyond the horizon is replaced by a branch with an areal radius that grows away from the horizon. It also differs from the model considered in [21] where the external Reissner-Nordström part was glued to the Bertotti-Robinson metric that leads to surface stresses that vanish in the horizon limit in the static frame but grow unbounded in the free-falling one, and as a result, the inner region becomes impenetrable for a free-falling observer (see [5] for details).

B. Quasi-black holes

Another candidate for the role of a black hole mimicker is a quasi-black hole. Roughly speaking, it is an extremal object that appears when the system approaches the quasihorizon as nearly as one likes, along a family of quasi-static configuration. It was argued in [5] that such a limit can correspond to an extremal quasihorizon only if we restrict ourselves to static configurations which are regular in the strong sense. The latter means that the Kretschmann scalar should be finite everywhere in the system, and surface stresses at the quasihorizon should be finite as well. There are subtleties in the nontrivial relation between regular and singular features of quasi-black holes [5]. For example, the whole region can look degenerate from the viewpoint of a distant observer and, nevertheless, the Kretschmann scalar remains finite in that region.

In more detail, consider the static spherically symmetric metric (1), and let it represent a spacetime in which there is an inner matter configuration attached to an asymptotic flat exterior region. The λ_i in (1) stands for the radius r_0 of the configurations and possibly some other parameters connected with the particular object one is analyzing. For instance, the parameter ε also enters in the analysis, and here has a slightly altered meaning. It means a small deviation from a quasi-black hole, rather than from a black hole solution as in Sec. III A, see also below.

Suppose the spacetime in question has the following properties: (a) the function $V(r)$ in (1) attains a minimum at some $r^* \neq 0$, such that $V(r^*) = \varepsilon$, with $\varepsilon << 1$, this minimum being achieved either from both sides of r^* or from $r > r^*$ alone, (b) for such a small but nonzero ε the configuration is regular everywhere with a nonvanishing metric function $\exp(2\Phi)$, at most the metric contains only delta-function like shells, and (c) in the limit $\varepsilon \to 0$ the metric coefficient $\exp(2\Phi) \to 0$ for all $r \leq r^*$. These three features define a quasi-black hole. In turn, these three features imply that, there are infinite redshift whole regions when $\varepsilon \to 0$, a free-falling observer finds in his own frame infinitely large tidal forces in the whole inner region, showing thus naked behavior, although the curvature scalars are finite. Moreover it has some form of degeneracy since, although the spacetime curvature invariants remain perfectly regular everywhere, in the limit, outer and inner regions become mutually impenetrable and disjoint. For a free-falling external nearby observer it is as if a null singular horizon is being formed. For external far away observers the spacetime may be said to be naively indistinguishable from that of extremal black holes. However, if one makes experiments with infalling luminescent extended small masses, one might find differences, since as discussed previously [5], due to the naked behavior, quasi-black holes enlarge grossly the tidal forces on an infalling small mass when compared to the tiny effect of an extremal black hole on the same small mass. Thus, as with the extremal ε-metrics studied before, the naked behavior shows that quasi-black holes are not such good mimickers as was previously thought, but they are still better than black foils [2] where the singularity is more severe.

A further important property is that quasi-black holes must be extremal. For a quasi-black hole the metric is well defined and everywhere regular. However, when $\varepsilon = 0$, quasi-black hole spacetimes become degenerate, almost singular, see [5]. The quasi-black hole is on the verge of forming an event horizon, but it never forms one, instead a quasihorizon appears.

In summary, quasi-black holes have normal general behavior and singular naked behavior. Quasi-black holes may appear from Bonnor stars, i.e., systems composed of extremal charged dust and vacuum, from self-gravitating Higgs magnetic monopole systems, and from composite spacetimes even in the case of pure electrovacuum, in which these vacuum systems are composed of an exterior Reissner-Nordström part glued to an inner Bertotti-Robinson spacetime or of an exterior Reissner-Nordström part glued to an inner Minkowski spacetime, see [5] for a full discussion and references.

C. Wormholes on the basis of quasi-black holes from Bonnor stars

1. Basics

Bonnor stars are Majumdar-Papapetrou matter systems with either a sharp or smooth boundary to an exterior vacuum. Since Bonnor stars are paradigmatic to understand the formation of quasi-black holes (see [5]), it is interesting to use those stars on the threshold of forming a quasi-black hole to understand whether wormholes on the basis of quasi-black holes which can be formed from Bonnor stars can mimic extremal black holes or not. This is an interesting variant, although with similarities, to wormholes on the basis of extremal ε-metrics. We use Bonnor stars, both in their compact version [25,26] as well as in their extended one [27].

2. Wormholes on the basis of quasi-black holes from compact Bonnor stars

Now, we start from the configuration which contains Majumdar-Papapetrou matter inside and vacuum outside, a compact Bonnor star [25,26]. Let us have a compact object, a Bonnor star, with extremal dust for $r \leq r_0$, joined to an extremal Reissner-Nordström metric for $r \geq r_0$. The potential $V(r)$ can be written as

$$V(r) = \left(1 - \frac{\mu(r)}{r}\right)^2, \tag{49}$$

with the mass density ρ and the function $\mu(r)$ being connected through $4\pi\rho = \frac{\mu'}{r^2}(1 - \frac{\mu}{r})$. The function $\mu(r)$ can be interpreted as the proper mass enclosed within a sphere of a radius r. In addition, for extremal dust, one has $\mu(r) = e(r)$, where $e(r)$ is the electric charge within this sphere. At the boundary r_0, one has $\mu(r_0) = M$, and $e(r_0) = Q$, such that $M = Q$, M and Q being the total mass and charge, respectively.

To construct a wormhole, one can take the following procedure. Cut the solution somewhere in the interior at some radius $r_1 < r_0$ and discard the region $r < r_1$. One obtains a Majumdar-Papapetrou matter region for $r_1 \leq r < r_0$, and a vacuum region for $r_0 \leq r < \infty$. For definiteness, let this spacetime be situated on the left. A symmetric right branch, also containing interior and exterior, is again cut at the radius $r_1 < r_0$, and glued to the symmetric left branch, producing thus a boundary shell at r_1. Then, it follows from the field equations that the left branch is given by

$$\exp(\Phi) = \exp\left[\int_{r_0}^{r} dr \frac{\mu}{r^2(1 - \frac{\mu}{r})}\right], \quad r_1 \leq r \leq r_0,$$
$$\exp(\Phi) = 1 - \frac{M}{r}, \quad r_0 \leq r < \infty, \tag{50}$$

and that the proper radius l and the coordinate radius r are related by

$$\frac{dr}{dl} = -(1 - \frac{\mu}{r}). \tag{51}$$

As the Bonnor star is here a compact object, the proper distance from r_1 to r_0 is finite, and tends to zero in the limit $r_1 \to r_0$. Therefore, in this limit, the matter between right and left boundaries becomes negligible and the construction corresponds to gluing two extremal Reissner-Nordström black holes in the situation TB of Sec. III A. Thus in the limit of our interest, $r_1 \to r_0 \to M$, it is not surprising that the results coincide with (42) and (47) where $r_+ = M$. Indeed, one finds

$$K = \text{finite}, \tag{52}$$

and

$$8\pi S = \frac{2}{Q} = \frac{2}{M}, \tag{53}$$

being finite as well. We have put $M = Q$, as is the case for these systems.

There is yet another procedure to produce a wormhole. In the above considerations, we performed a symmetric construction, in the sense that the " $+$" and " $-$" branches differed by the sign of $\frac{dr}{dl}$ only. Now, we start again from a compact Bonnor star configuration which contains matter inside and vacuum outside. We want to preserve this feature, and so we have to make a nonsymmetric deformation. Thus, we change the procedure and consider the following construction. Again, a Bonnor star is made of Majumdar-Papapetrou matter for $r \leq r_0$, which in turn is joined to an extremal Reissner-Nordström metric for $r \geq r_0$. Now, in the region $r \leq r_0$, the left " $-$" branch, choose the distribution with $\frac{dr}{dl} \leq 0$. The potential $V(r)$ can be written again as in Eq. (49). Then, for the left branch, it follows from the field equations that

$$\exp(\Phi) = \exp\left[\int_{r_0}^{r} dr \frac{\mu}{r^2(1 - \frac{\mu}{r})}\right], \quad 0 \leq r \leq r_0, \tag{54}$$

and

$$\frac{dr}{dl} = -\left(1 - \frac{\mu}{r}\right). \tag{55}$$

It is worth paying attention that because of the property $\frac{dr}{dl} < 0$, the matter distribution which was originally compact turned after deformation into a noncompact one since at left infinity $l \to -\infty$, where $\mu \to 0$, $\frac{dr}{dl} \to -1$. For the right " $+$" branch we use the extremal Reissner-Nordström metric with the mass $M = Q$, which gives

$$\exp(\Phi) = 1 - \frac{M}{r}, \quad r_0 \leq r < \infty, \tag{56}$$

and

$$\frac{dr}{dl} = 1 - \frac{M}{r}. \tag{57}$$

In the limit $r_0 \to M$, $\mu \to M = Q$ one finds that

$$K = \text{finite}, \tag{58}$$

and

$$8\pi S = \frac{2}{Q} = \frac{2}{M}, \tag{59}$$

being finite as well. We have put $M = Q$, as is the case for these systems.

Thus, extremal quasi-black wormholes made of Majumdar-Papapetrou matter are possible. Their distinctive feature is the presence of finite nonzero surface stresses on the horizon. Curvature components remain finite. It is worth noting that, although in this subsection we did not introduce the parameter ε explicitly, actually its role is played, say, by the difference $r_0 - M$ in the sense that this quantity is responsible for the deviation of the spacetime from its limiting state (a quasi-black hole or two quasi-black holes glued together).

3. Wormholes on the basis of quasi-black holes from extended Bonnor stars

Here we exploit the distribution of extremal charged dust given in [27]. Near the quasi-black hole limit, the first order corrected quasihorizon has radius r^* given by

$$r^* = q\left[1 + \frac{3}{4}\left(\frac{2c^2}{q^2}\right)^{1/3} + \ldots\right], \tag{60}$$

where c is the parameter that yields the deviation from the Reissner-Nordström solution, and q is a quantity with units of electric charge, which is indeed the total charge Q when $c = 0$ from the outset (see [5,27] for details). The quasi-black hole limit is such that $c \ll q$, with $c \to 0$. In a sense, the dimensionless parameter c/q here corresponds to the ε parameter in the ε-wormhole construction of a previous section. Then the solution has asymptotics near the quasi-horizon r^* given by

$$V = \frac{9}{2^{4/3}}\left(\frac{c}{q}\right)^{4/3} + \frac{2(r - r_*)}{q^2} + \ldots, 2 \tag{61}$$

and

$$\exp(\Phi) = 2^{1/3}\left(\frac{c}{q}\right)^{2/3} + \frac{2}{3}\frac{(r - r_*)}{q} + \frac{2^{2/3}}{9c^{2/3}q^{4/3}}(r - r_*)^2 \ldots . \tag{62}$$

Consider again the " $+$" and " $-$" branches, symmetric relative to the first order corrected quasihorizon radius in the region $r \geq r^*$, but with different signs of $\frac{dr}{dl}$. Each branch has the same dependence of the metric potential, Eqs. (61) and (62), on r. Thus, for simplicity, we restrict ourselves to the analog of the situation BT considered previously, which translated to here means taking the limits as follows, $\lim_{\frac{c}{q}\to 0} \lim_{r_* \to r_+}$. Then, simple calculations show that

$$K = \text{finite}, \tag{63}$$

and

$$8\pi S = \frac{2}{Q} = \frac{2}{M}, \tag{64}$$

being finite as well. In (64) we have put $Q = q$ in the limit $c \to 0$ and $M = Q$ also in this limit, where M is the mass of the configuration. In the limit under consideration the metric in the region $r > r^*$ is given by the extremal Reissner-Nordström metric, whereas in the immediate vicinity of the quasihorizon the metric is described by the Bertotti-Robinson metric. Therefore, as in the preceding subsection, our construction gives two extremal Reissner-Nordström black holes glued along the quasihorizon with different signs of $\frac{dr}{dl}$ on opposite sides.

V. DISCUSSION AND CONCLUSIONS

We have studied wormhole and other configurations as possible mimickers of black holes. We have separated the configurations into nonextremal and extremal.

For wormholes, we have examined separately two limiting procedures in which the wormhole throat approaches the black hole horizon. In the first procedure, we fix the location of an observer (exactly on the throat). Then, we change the spacetime (making a wormhole on the verge of being a black hole). This is the situation BT. In the second, we change spacetime (making a wormhole on the verge of a black hole), then place the shell outside the throat and move it toward the throat (which coincides with the horizon). This is the situation TB. This procedure is carried out for nonextremal and extremal configurations separately. In the nonextremal case it turned out that each of the limits under discussion is singular: either the Kretschmann scalar or surface stresses on the throat (or both) diverge. Thus, the limit is singular. In other words, a mimicker of a nonextremal black hole, made from a nonextremal wormhole, including the black foil of [2], is not smooth. We have summarized the results for the nonextremal case in a table. For the extremal case both the Kretschmann scalar and surface stresses remain finite. This pronounced distinction between properties of the limiting configurations in the nonextremal and extremal cases is one counterpart of the conclusion made in [5] that quasi-black holes can be only extremal. However, one should not forget about some subtleties connected with the fact that singular behavior can, in general, manifest itself not only in the value of the Kretschmann scalar. Even if this scalar is finite, a naked behavior is possible or even inevitable as was shown in the present paper and in [5] (see, e.g., Sec. V of [5] for a discussion about other subtleties in which the singular features of quasi-black holes are revealed). There is also another candidate for the role of a nonextremal black hole mimicker, a gravastar [4]. However, the corresponding surface stresses grow unbounded when the radius approaches the gravitational radius, as we have seen.

From an astrophysical viewpoint, the situation BT in the nonextremal case, i.e., a black foil, is more interesting since it implies no necessity of making a shell by hand. It is the case considered in [2]. One of the questions raised in [2] is whether it is possible or not to distinguish between a true nonextremal black hole, a Schwarzschild black hole say, and a wormhole. The main conclusion of [2] is that it is impossible to distinguish for any finite time in the limit under discussion. This conclusion is reached on the basis of considering properties of bodies moving along separate fixed geodesics and emitting signals detected at infinity. However, if from single geodesics we shift our attention to a congruence of geodesics, it turns out that the strong gravity forces on the near-horizon region leave their imprint on the form of a moving body and, thus, on the properties of signals which an observer at infinity is detecting. If the surface of the body is luminescent, an observer at infinity would see either a finite width instead of a point, a continuous detection instead of separate pulses, and so on. It is essential that in the case discussed in [2], the corresponding proper time of deformation tends to zero when the curvature grows unbounded, with the time at infinity being finite. Thus, at least in principle, an observer can distinguish between a black hole and an almost singular wormhole. The singular nature of the limit in the nonextremal case makes also questionable the applicability of the membrane paradigm used in [2]. The key point in this paradigm consists in boundary conditions according to which a free-falling observer sees a finite value of physical fields on the horizon (see [17], Sec. II). However, in the problem under discussion, typically, this observer as well as the geometry itself become ill defined. Only in some cases (see Model 2, situation TB) the curvature components remain finite in the free-falling frame. But even in such situations the infinite surface stresses on the horizon surface make the physical meaning of the membrane paradigm unclear since this paradigm relies heavily on the concept of a regular surface.

Objects based on nearly extremal wormholes, although of less interest astrophysically perhaps, have a much better behavior in the sense that both the geometry and surface stresses remain finite. Moreover, typically there is no naked behavior. In this case, the effect of strong curvature is much less pronounced than in the case of quasi-black holes where a naked behavior is typical [5]. In this sense, a wormhole composed on the basis of two extremal black holes seems to be the best mimicker of an extremal black hole. As by-product, we have obtained a model of a regular black hole.

Thus, if we try to arrange a ranking of black hole mimickers, both nonextremal and extremal, the list looks as follows from top to bottom: wormholes on the basis of extremal black holes or on the basis of quasi-black holes, quasi-black holes, wormholes on the basis of nonextremal black holes (and within these the best are black foils), and gravastars. Bearing in mind that in observational astrophysics it is difficult to find extremal configurations (the would-be best mimickers), whereas nonextremal configurations are really bad mimickers, the task of distinguishing black holes from their mimickers seems to be less difficult than one could think of it.

In the present paper we have restricted ourselves to static spherically symmetric spacetimes. Meanwhile, in a recent work [28] the status of black hole mimickers is undermined in the rapidly rotating case as well since it is argued that they are unstable. We have also circumscribed our discussion to particular wormholes, gravastars and quasi-black holes [2–6], since these objects are well adapted to our goal of examining the near-horizon properties and their connection with far away asymptotic properties. However, there are many other objects with properties that make them also potential black hole mimickers (see, e.g., [29–32]) and which are worthy of study within our formalism.

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