

Torsion cosmology and the accelerating universeKun-Feng Shie,¹ James M. Nester,^{1,2,3,*} and Hwei-Jang Yo^{4,+}¹*Department of Physics, National Central University, Chungli 320, Taiwan*²*Graduate Institute of Astronomy, National Central University, Chungli 320, Taiwan*³*Center for Mathematics and Theoretical Physics, National Central University, Chungli 320, Taiwan*⁴*Department of Physics, National Cheng-Kung University, Tainan 701, Taiwan*

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Investigations of the dynamic modes of the Poincaré gauge theory of gravity found only two good propagating torsion modes; they are effectively a scalar and a pseudoscalar. Cosmology affords a natural situation where one might see observational effects of these modes. Here, we consider only the “scalar torsion” mode. This mode has certain distinctive and interesting qualities. In particular, this type of torsion does not interact directly with any known matter, and it allows a critical nonzero value for the affine scalar curvature. Via numerical evolution of the coupled nonlinear equations we show that this mode can contribute an oscillating aspect to the expansion rate of the Universe. From the examination of specific cases of the parameters and initial conditions we show that for suitable ranges of the parameters the dynamic “scalar torsion” model can display features similar to those of the presently observed accelerating universe.

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I. INTRODUCTION

One of the outstanding successes of theoretical physics in the latter part of the last century that led to a much deepened understanding was the recognition that all the known fundamental physical interactions, the strong, weak, and electromagnetic—*not excepting gravity*—can be well described in terms of a single unifying principle: that of local gauge theory. Although there are other possible gauge approaches, for gravity it seems highly appropriate to regard it as a gauge theory for the local symmetry group of Minkowski spacetime: the Poincaré group [1,2]. Such a consideration led to the development of the Poincaré gauge theory (PGT) of gravity [3–8]. The PGT has *a priori* independent local rotation and translation potentials, which correspond to the metric-compatible connection and orthonormal coframe; their associated field strengths are the *curvature* and *torsion*. The spacetime then has generically a Riemann-Cartan geometry. Because of its gauge structure and geometric properties the PGT has been regarded as an attractive alternative to GR. The general theory includes as exceptional cases Einstein’s general relativity (GR) with *vanishing* torsion, the Einstein-Cartan theory with *nondynamic* torsion algebraically coupled to the intrinsic spin of the source, as well as the teleparallel theories wherein curvature vanishes and torsion represents the gravitational force (a sort of opposite to Riemannian geometry). Aside from these exceptions the generic PGT has, in addition to the metric familiar from Einstein’s GR, a connection with some independent dynamics. This additional dynamics is reflected in the torsion tensor.

There is a natural physical source for the torsion of spacetime, namely, spin 1/2 fermions. The effect is generally assumed to be small at ordinary densities, but could have a major influence at high densities (e.g., beyond 10^{48} gm/cm³), and thus it was expected to have important physical effects in the early universe [1]. Torsion cosmology investigations were initiated by Kopczyński [9]. Some early investigations attracted attention especially because they noted that torsion might prevent the (at that time newly recognized) singularities. However, this hope quickly faded. Indeed, it soon was argued that *nonlinear* torsion effects were more likely to produce stronger singularities [10].

The various PGT dynamic modes beyond those of the metric were first investigated via the linearized theory (for outstanding examples of such investigations see [4,11]). To this order the connection dynamics (which can be represented by the torsion tensor) decomposes into six modes with certain spins and parity: 2^\pm , 1^\pm , 0^\pm . Many possible combinations of well-behaved (carrying positive energy at speed $\leq c$, criterion often referred to as “no ghost, no tachyon”) propagating modes in the linear PGT theory were identified. They were classified into about a dozen separate cases, almost any combination of up to 3 dynamic modes is allowed. Some nice investigations of the PGT theory were also made using the Hamiltonian analysis [8,12,13], with findings consistent with the conclusions of the linearized investigation. Later, however, some potential problems were identified [14]. This prompted deeper investigations, which noted that effects due to nonlinearities in the constraints could be expected to render most of the aforementioned dynamic cases physically unacceptable [15]. A fundamental investigation identified two special cases, the so-called “scalar torsion” modes, which could

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be proved to be problem free, having a well posed initial value problem [16]. Subsequently, Hamiltonian investigations [17,18] supported the conclusion that these two dynamic “scalar torsion” modes may well be the only physically acceptable dynamic PGT torsion modes.

In one mode (referred to as the “pseudoscalar” because of its 0^- spin content) only the axial-vector torsion is dynamic. (As a consequence of the dynamic field equations it turns out to be dual to the gradient of a scalar field; however, it is not possible to treat this scalar field as the primary dynamical object without changing the nature of the theory [19].) Axial torsion is naturally driven by the intrinsic spin of fundamental fermions; in turn, it naturally interacts with such sources. Thus, for this mode one has some observational constraints [20]. Note that except in the early universe one does not expect large spin densities. Consequently, it is generally thought that axial torsion must be small and have small effects at the present time. This is one reason why we do not focus on this mode here.

The other good mode 0^+ , the so-called “scalar torsion” mode, has a certain type of dynamic vector torsion. (As a consequence of the dynamic equations given below in Sec. II B, it too turns out to be the gradient of a scalar field; this scalar field cannot, however, be regarded as a fundamental potential—for essentially the same reasons as those mentioned in connection with the “pseudoscalar” mode.) There is no known fundamental source that directly excites this mode. Conversely, this type of torsion does not interact in any direct obvious fashion with any familiar type of matter [21]. Hence, we do not have much in the way of constraints as to its magnitude. We could imagine it as having significant magnitude and yet not being dramatically noticed—except indirectly through the nonlinear equations. This mode, in particular, has also attracted our interest because of a conspicuous consequence of the nonlinear equations: in this case, there is a critical nonzero value for the affine scalar curvature.

Our theoretical PGT analysis thus led us to consider just two dynamic torsion modes. An obvious place where we might see some physical evidence for these modes is in cosmological models. The homogeneous and isotropic assumptions of cosmology greatly restrict the possible types of nonvanishing fields. Curiously, for torsion there are only two possibilities: 0^+ , i.e., vector torsion which, moreover, has only a time component (and is thus effectively the gradient of a time-dependent function), and axial torsion 0^- , which is effectively the dual of a vector with only a time component (and thus can be specified as the gradient of a time-dependent function). Hence, the homogeneous and isotropic cosmologies are *naturally* very suitable for the exploration of the physics of the dynamic PGT “scalar modes.”

Thus, cosmological models offer a situation where dynamic torsion may lead to observable effects. Here, we will not focus on the early universe, where one could surely

expect large effects (although their signature would have to be disentangled from other large effects), and instead inquire whether one can see traces of torsion effects today. In particular, we will here consider accounting for the outstanding present day mystery—the accelerated universe, in terms of an alternate gravity theory with an additional natural dynamic geometric quantity—torsion [22].

The observed accelerating expansion of the Universe suggested the existence of a kind of dark energy with a negative pressure. The idea of a dark energy is one of the greatest challenges for our current understanding of fundamental physics [23–25]. Among a number of possibilities to describe this dark energy component, the simplest may well be by means of a cosmological constant Λ . However, there are some reasons for dissatisfaction with this model. In particular, the so-called *cosmological constant problem* notes that the theoretically estimated value of the vacuum energy density is about 10^{120} times larger than the inferred cosmological constant. Moreover, the *coincidence* or *fine-tuning* problem notes that it is highly unlikely that we should be living in the relatively short era when the rapidly changing ratio of the material energy and the cosmological constant is nearly unity.

In the light of these problems there have been many interesting dynamical dark energy proposals. A popular idea is some unusual type of minimally coupled scalar field Φ (quintessence field), which has not yet reached its ground state and whose current dynamics is basically determined by its potential energy $V(\Phi)$. This idea has received much attention over the past few years and a considerable effort has been made in understanding the role of quintessence fields on the dynamics of the Universe (see, e.g., [26–28]). However, without a specific motivation from fundamental physics for the light scalar fields, these quintessence models can be constructed relatively arbitrarily. There is a lot of room for speculation.

Here, we consider another possibility for explaining the accelerating universe: dynamic scalar torsion. We explore the possibility that the dynamic PGT connection, reflected in dynamic PGT torsion, provides the accelerating force in the universe. As noted above, there are certain “scalar torsion” modes, which could have dynamical behavior. They could naturally provide the accelerating force in the universe. Here, we will show that the effect of torsion cannot only make the expansion rate oscillate, but also can force the universe to naturally have an accelerating expansion in some periods and a decelerating expansion at other times. Scalar torsion cosmology can avoid some of the problems that occur in other models.

A comprehensive survey of the PGT cosmological models was presented some time ago by Goenner and Müller-Hoissen [29]. Although that work only solved in detail a few particular cases, it developed the equations for all the PGT cases—including those for the particular model we consider here. However, that work was done prior to the

discovery of the accelerating universe, and torsion was thus imagined as playing a big role only at high densities in the early universe. More recently, investigators have begun to consider torsion as a possible cause of the accelerating universe (see, e.g., [30,31]), but the subject has not yet been explored in detail [32].

We have taken a first step in the exploration of the possible evolution of the Universe with the scalar torsion mode of the PGT. The main motivation is twofold: (1) to have a better understanding of the PGT, in particular, the possible physics of the dynamic “scalar torsion” modes; (2) to consider the prospects of accounting for the outstanding present day mystery—the accelerating universe—in terms of an alternate gravity theory, more particularly in terms of the PGT dynamic torsion. With the usual assumptions of isotropy and homogeneity in cosmology, we find that, under the model, the Universe will oscillate with generic choices of the parameters. The torsion field in the model plays the role of the imperceptible “dark energy.” With a certain range of parameter choices, it can account for the current status of the Universe, i.e., an accelerating expanding universe with a value of the Hubble constant, which is approximately the present one. These promising results should encourage further investigations of this model, with a detailed comparison of its predictions with the observational data.

The remainder of this work is organized as follows: We summarize the formulation of the PGT and the “scalar torsion” mode, and then translate the equations into a certain effective Riemannian form in Sec. II; the specialization of these relations to the form describing a cosmological model is presented in Sec. III. Then a preliminary analytical analysis aimed at revealing the behavior of the solutions is presented in Sec. IV. In Sec. V, we present the results of our numerical demonstrations for various choices of the parameters and the initial data. The implications of our findings are discussed in Sec. VI, and Sec. VII is a conclusion.

II. THE FIELD EQUATIONS

A. Poincaré gauge theory of gravitation

Our considerations in this work are entirely classical. The form of the gravity theory we wish to consider here, the PGT, was worked out some time ago on the basis of the fundamental principles of gauge theory and geometry [1–8]. In the PGT, there are two sets of local gauge potentials, the orthonormal frame field (tetrad) e_i^μ and the metric-compatible connection $\Gamma_{i\mu}^\nu$, which are associated with the translation and the Lorentz subgroups of the Poincaré gauge group, respectively. The field strengths associated with the frame and connection are the torsion

$$T_{ij}{}^\mu = 2(\partial_{[i}e_{j]}{}^\mu + \Gamma_{[i\nu}{}^\mu e_{j]}{}^\nu), \quad (1)$$

and the curvature

$$R_{ij\mu}{}^\nu = 2(\partial_{[i}\Gamma_{j]\mu}{}^\nu + \Gamma_{[i\sigma}{}^\nu\Gamma_{j]\mu}{}^\sigma), \quad (2)$$

which satisfy the Bianchi identities

$$\nabla_{[i}T_{jk]}{}^\mu \equiv R_{[ijk]}{}^\mu, \quad (3)$$

$$\nabla_{[i}R_{jk]}{}^{\mu\nu} \equiv 0. \quad (4)$$

From the frame one constructs some auxiliary quantities: the reciprocal frame $e^i{}_\mu$, which satisfies $e^i{}_\mu e_i{}^\nu = \delta_\mu{}^\nu$ and $e^i{}_\mu e_j{}^\mu = \delta_j{}^i$, and the metric $g_{ij} = e_i{}^\mu e_j{}^\nu \eta_{\mu\nu}$. Here, our conventions are as follows: the Greek indices are the local Lorentz indices; whereas the Latin indices are the coordinate indices. We use the metric signature $(-, +, +, +)$.

Following the standard paradigm, the conventional form of the PGT action, which is invariant under local Poincaré gauge transformations, is taken to have the form

$$A = \int d^4x e (L_G + L_M). \quad (5)$$

Here $e = \det(e_i{}^\mu)$, $eL_G(e_i{}^\mu, \partial_j e_i{}^\mu, \Gamma_{i\mu}{}^\nu, \partial_j \Gamma_{i\mu}{}^\nu) = eL_G(e_i{}^\mu, T_{ij}{}^\mu, R_{ij}{}^{\mu\nu})$ is the geometric gravity Lagrangian density and $eL_M(e, \Gamma, \psi, \partial\psi) = eL_M(e_i{}^\mu, \psi, D_i\psi)$ is the minimally coupled source Lagrangian density, where ψ represents all the matter and other interaction fields. We will not explicitly need the field equations for the non-geometric fields. Varying with respect to the geometric-gauge potentials gives the gravitational field equations. As explained in detail in the aforementioned references, they take the form

$$\nabla_j H_\mu{}^{ij} - E_\mu{}^i = \mathcal{T}_\mu{}^i, \quad (6)$$

$$\nabla_j H_{\mu\nu}{}^{ij} - E_{\mu\nu}{}^i = S_{\mu\nu}{}^i, \quad (7)$$

with the field momenta

$$H_\mu{}^{ij} := \frac{\partial eL_G}{\partial \partial_j e_i{}^\mu} = 2 \frac{\partial eL_G}{\partial T_{ji}{}^\mu}, \quad (8)$$

$$H_{\mu\nu}{}^{ij} := \frac{\partial eL_G}{\partial \partial_j \Gamma_{i\mu}{}^\nu} = 2 \frac{\partial eL_G}{\partial R_{ji}{}^{\mu\nu}}, \quad (9)$$

and

$$E_\mu{}^i := e^i{}_\mu eL_G - T_{\mu j}{}^\nu H_\nu{}^{ji} - R_{\mu j}{}^{\nu\sigma} H_{\nu\sigma}{}^{ji}, \quad (10)$$

$$E_{\mu\nu}{}^i := H_{[\nu\mu]}{}^i. \quad (11)$$

The source terms here

$$\mathcal{T}_\mu{}^i := \frac{\partial eL_M}{\partial e_i{}^\mu}, \quad (12)$$

$$S_{\mu\nu}{}^i := \frac{\partial eL_M}{\partial \Gamma_{i\mu}{}^\nu}, \quad (13)$$

are, respectively, the Noether energy-momentum and spin density currents, which (as a consequence of the assumed

minimal coupling) automatically satisfy suitable energy-momentum and angular momentum conservation laws.

The Lagrangian is chosen (as usual in gauge theories) to be at most of quadratic order in the field strengths, then the field momenta are linear in the field strengths

$$H_{\mu}{}^{ij} = \frac{e}{l^2} \sum_{k=1}^3 a_k T_{\mu}{}^{ji(k)} \quad (14)$$

$$H_{\mu\nu}{}^{ij} = -\frac{a_0 e}{l^2} e^i{}_{[\mu} e^j{}_{\nu]} + \frac{e}{\kappa} \sum_{k=1}^6 b_k R_{\mu\nu}{}^{ji(k)} \quad (15)$$

here, the three $T_{\mu}{}^{ji(k)}$ and the six $R_{\mu\nu}{}^{ji(k)}$ are the algebraically irreducible parts of the torsion and the curvature, respectively. The torsion, in particular, splits into the algebraically irreducible torsion vector, axial vector and tensor

$$\begin{aligned} T_i &= T_{ij}{}^j, & P_i &= \frac{1}{2} \epsilon_{ijkm} T^{jkm}, \\ Q_{ijk} &= T_{i(jk)} - \frac{1}{3} T_i g_{jk} + \frac{1}{3} g_{i(j} T_{k)}, \end{aligned} \quad (16)$$

which recompose to give

$$T_{ijk} = \frac{4}{3} Q_{[ij]k} + \frac{2}{3} T_{[i} g_{j]k} + \frac{1}{3} \epsilon_{ijkm} P^m. \quad (17)$$

The a_k and b_k in the Lagrangian are free coupling parameters. Because of the Bach-Lanczos identity only five of the six b_k 's are independent. a_0 is the coupling parameter of the scalar curvature $R := R_{\mu\nu}{}^{\nu\mu}$. Note that (because of the assumed quadratic Lagrangian, linear-in-field strength canonical momenta) one obtains, as in the standard physics paradigm, 2nd order equations for the potentials by varying the Lagrangian L_g of the PGT independently with respect to the frame and connection. It should be remarked that these PGT equations are quite different from the problematical 4th order type of equations obtained from Riemannian geometry based Lagrangians of the form $R + (R_{\dots})^2$ when varied with respect to the metric.

In the PGT, in addition to the dynamic metric represented by the translational gauge potential (the orthonormal frame), the rotational gauge potential (the connection) has some independent dynamics. As in other gauge theories, it is usually convenient to describe the dynamics of the connection (a noncovariant, gauge dependent potential) in terms of a tensorial field strength. In the PGT case, these modes can be described by the torsion tensor. As mentioned in the introduction, the various PGT dynamic torsion modes were first investigated via the linearized theory; it was shown that the torsion decomposes into six modes with certain spins and parity 2^{\pm} , 1^{\pm} , 0^{\pm} . Later investigations [15–18] concluded that effects due to nonlinearities in the constraints could be expected to render all of these cases physically unacceptable except for the two ‘‘scalar torsion’’ modes spin-0⁺ and spin-0⁻. These two dynamic

scalar torsion modes apparently are the only physically acceptable dynamic PGT torsion modes.

B. Simple spin-0⁺ mode

Here, we only investigate the *simple* spin-0⁺ case, i.e., choosing $a_2 = -2a_1$, $a_3 = -a_1/2$ and taking all the b_k 's to vanish except for $b_6 = b \neq 0$. (For a detailed analysis of this case please see [17].) Our gravitational Lagrangian density for this spin-0⁺ mode is then

$$\begin{aligned} L_g &= -\frac{a_0}{2} R + \frac{b}{24} R^2 + \frac{a_1}{8} (T_{\nu\sigma\mu} T^{\nu\sigma\mu} + 2T_{\nu\sigma\mu} T^{\mu\sigma\nu} \\ &\quad - 4T_{\mu} T^{\mu}), \end{aligned} \quad (18)$$

where $T_{\mu} := T_{\mu\nu}{}^{\nu}$. The Hamiltonian analysis showed that the number of degrees of freedom in L_g is three: the scalar torsion mode and two helicity states of the usual massless graviton (provided the scalar torsion mode is massive, i.e., $a_1 \neq a_0$). It is necessary to impose certain sign conditions on the parameters (see [4,8,11–13,17]):

$$a_1 > 0, \quad b > 0. \quad (19)$$

There is a simple argument that accounts for the signs of these two parameters. In order to have *least action* the kinetic energy contribution from any dynamic variable must be positive (for if such a term were negative the action would have no lower bound, since we could have an arbitrarily large time rate of change for a dynamic variable). Consider that b is the parameter associated with the quadratic scalar curvature term R^2 . With the help of Eq. (2), it can be seen that the scalar curvature includes some time derivatives of one of our basic dynamic fields, the connection components $R^2 = (e^i{}_{\nu} e^j{}_{\mu} R_{ij}{}^{\mu\nu}) = (2e^t{}_{\nu} e^j{}_{\mu} \dot{\Gamma}_j{}^{\mu\nu} + \dots)^2 = 4(e^t{}_{\nu} e^j{}_{\mu} \dot{\Gamma}_j{}^{\mu\nu})^2 + \dots \geq 0$. Hence, the coefficient of this term in the action should be positive. A similar argument based on (1), taking into account the chosen metric signature and the restricted form of the torsion in our model as a consequence of the field equations, Eq. (24) below, gives the sign of a_1 .

Varying L_g (18) with respect to the potentials $e_i{}^{\mu}$, $\Gamma_i{}^{\mu\nu}$ gives the specific second order field equations of the general form (6) and (7) for this mode. Assuming $S_{\mu\nu}{}^i = 0$ (i.e., the source spin current is negligible) we investigate first Eq. (7), obtaining for this mode the three decomposed equations

$$\nabla_{\mu} R = -\frac{2}{3} \left(R + \frac{6\mu}{b} \right) T_{\mu}, \quad (20)$$

$$0 = -\left(R + \frac{6\mu}{b} \right) P_{\mu}, \quad (21)$$

$$0 = -\left(R + \frac{6\mu}{b} \right) Q_{\mu\nu\sigma}, \quad (22)$$

where $\mu := a_1 - a_0$ is the effective mass of the linearized 0^+ mode. There is a special case with no dynamical scalar torsion if $R = -6\mu/b$, a constant. We do not treat this exceptional degenerate situation as an isolated case (it is considered below as a limit of the generic case). Assuming that $R + 6\mu/b \neq 0$ generically leads to

$$P_\mu = Q_{\mu\nu\sigma} = 0. \quad (23)$$

Using these two constraints gives the restricted form of the torsion

$$T_{ij}{}^\mu = \frac{2}{3}T_{[i}e_{j]}{}^\mu. \quad (24)$$

Substituting into Eq. (6) with our specific parameter choices gives the restricted field equation

$$\begin{aligned} \nabla_j H_\mu{}^{ij} - E_\mu{}^i &= e \left\{ \frac{2a_1}{3} [e^i{}_\nu \nabla_\mu T^\nu - e^i{}_\mu \bar{\nabla}_j T^j] \right. \\ &\quad \left. - e^i{}_\mu \left[-\frac{a_0}{2}R + \frac{b}{24}R^2 - \frac{a_1}{3}T_i T^i \right] \right. \\ &\quad \left. + R_\mu{}^i \left[\frac{b}{6}R - a_0 \right] \right\} = \mathcal{T}_\mu{}^i. \end{aligned} \quad (25)$$

Now, we have a complete set of the field equations, (20) and (25) along with (23). In [16], it was argued that this system had a well posed initial value problem. However that may be, the two main field equations are rather complicated. They really look nothing like the familiar, well-analyzed equations of GR. To help understand the significance of these new relations, and to use our previous experience, we will do a translation of (20) and (25) into a certain effective Riemannian form—transcribing from quantities expressed in terms of the orthonormal tetrad $e_j{}^\mu$ and connection $\Gamma_i{}^{\mu\nu}$ into the ones expressed in terms of the metric g_{jk} and torsion $T_{ij}{}^k$. Then we can compare the result with the more familiar field equations in GR.

C. Translation

As is well known, the PGT affine connection can be represented in the form

$$\Gamma_{ij}{}^k = \bar{\Gamma}_{ij}{}^k + \frac{1}{2}(T_{ij}{}^k + T^k{}_{ij} + T^k{}_{ji}), \quad (26)$$

where $\bar{\Gamma}_{ij}{}^k$ is the Levi-Civita connection,

$$\bar{\Gamma}_{ij}{}^k = \frac{1}{2}g^{km}(g_{mji} + g_{mi,j} - g_{ij,m}), \quad (27)$$

and $T_{ij}{}^k$ is the torsion. Accordingly, the affine Ricci curvature and scalar curvature can be represented as

$$\begin{aligned} R_{ij} &= \bar{R}_{ij} + \bar{\nabla}_j T_i + \frac{1}{2}(\bar{\nabla}_k - T_k)(T_{ji}{}^k + T^k{}_{ij} + T^k{}_{ji}) \\ &\quad + \frac{1}{4}(T_{kmi}T^{km}{}_j + 2T_{jkm}T^{mk}{}_i), \end{aligned} \quad (28)$$

$$R = \bar{R} + 2\bar{\nabla}_i T^i + \frac{1}{4}(T_{ijk}T^{ijk} + 2T_{ijk}T^{kji} - 4T_i T^i), \quad (29)$$

where \bar{R}_{ij} and \bar{R} are the Riemannian Ricci curvature and scalar curvature, respectively, and $\bar{\nabla}$ is the covariant derivative with the connection $\bar{\Gamma}_{ij}{}^k$.

For the case of interest here the torsion tensor has the restricted form (24). Consequently, the affine Ricci curvature and scalar curvature become

$$R_{ij} = \bar{R}_{ij} + \frac{1}{3}(2\bar{\nabla}_j T_i + g_{ij}\bar{\nabla}_k T^k) + \frac{2}{9}(T_i T_j - g_{ij}T_k T^k), \quad (30)$$

$$R = \bar{R} + 2\bar{\nabla}_i T^i - \frac{2}{3}T_i T^i. \quad (31)$$

Applying this translation *selectively* [33] in Eqs. (20) and (25) gives an alternate form of the field equations

$$\bar{\nabla}_i R + \frac{2}{3}\left(R + \frac{6\mu}{b}\right)T_i = 0, \quad (32)$$

$$\begin{aligned} a_0\left(\bar{R}_{ij} - \frac{1}{2}g_{ij}\bar{R}\right) - \frac{b}{6}R\left(R_{(ij)} - \frac{1}{4}g_{ij}R\right) - \frac{2\mu}{3}(\bar{\nabla}_{(i}T_{j)} \\ - g_{ij}\bar{\nabla}_k T^k) - \frac{\mu}{9}(2T_i T_j + g_{ij}T_k T^k) = -\mathcal{T}_{ij}, \end{aligned} \quad (33)$$

while contracting Eq. (33) with the help of Eq. (31) yields

$$a_1\bar{R} - \mu R = \mathcal{T}. \quad (34)$$

Note that the relation (33) can be rewritten into the form of Einstein's equation

$$a_0\left(\bar{R}_{ij} - \frac{1}{2}g_{ij}\bar{R}\right) = -\tau_{ij} := -(\mathcal{T}_{ij} + \tilde{\mathcal{T}}_{ij}), \quad (35)$$

where \mathcal{T}_{ij} is the source energy-momentum tensor and the contribution of the scalar torsion mode to the effective total energy-momentum tensor τ_{ij} is

$$\begin{aligned} \tilde{\mathcal{T}}_{ij} &= -\frac{2\mu}{3}(\bar{\nabla}_{(i}T_{j)} - g_{ij}\bar{\nabla}_k T^k) - \frac{\mu}{9}(2T_i T_j + g_{ij}T_k T^k) \\ &\quad - \frac{b_6}{6}R\left(R_{(ij)} - \frac{1}{4}g_{ij}R\right). \end{aligned} \quad (36)$$

However, it should be kept in mind that $\tilde{\mathcal{T}}_{ij}$ is only an effective quantity. Using this effective quantity allows us to use some of the insight we have obtained from our experience with GR. Thus, we can regard the contribution of $\tilde{\mathcal{T}}_{ij}$ to the right-hand side of (35) as something like that of an exotic field. This hybrid form is practical for our needs here, even though it is not really a proper fundamental physical description (one way to see this is to note that $\tilde{\mathcal{T}}_{ij}$ cannot be obtained as the Hilbert energy-momentum density of some effective source Lagrangian).

Equations (35) and (36) do allow us to appreciate some of the similarities and differences between this model and other accelerating universe models. However, to properly understand this model, one should consider the torsion dynamics geometrically rather than trying to regard it as

just another field in a Riemannian spacetime obeying Einstein's equations.

It is remarkable that the torsion vector—as a consequence of the field Eq. (32)—turns out to be the gradient of a scalar function. In fact, we can identify the function as $-(3/2)\ln(R + 6\mu/b)$. However, (especially given its geometric nature) we do not see any way to introduce this scalar potential directly into the Lagrangian as a fundamental field. Were that possible one could then directly compare features of our scalar torsion model with the various scalar field dark energy models. But as far as we can see, notwithstanding a few similarities, our model is really not much like those scalar field models.

III. FIELD EQUATIONS FOR TORSION COSMOLOGY

For cosmology, assuming homogeneous and isotropic leads to the Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right], \quad (37)$$

where $a(t)$ is the expansion factor, and k is the curvature index. Here, to see the effects we are interested in as well as match the observations, it is sufficient to consider only the simplest case: the flat universe with $k = 0$. This yields the nonvanishing (Riemannian) Ricci and scalar curvature

$$\bar{R}_i{}^i = 3\frac{\ddot{a}}{a} = 3(\dot{H} + H^2), \quad (38)$$

$$\bar{R}_r{}^r = \bar{R}_\theta{}^\theta = \bar{R}_\phi{}^\phi = \frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} = \dot{H} + 3H^2, \quad (39)$$

$$\bar{R} = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = 6(\dot{H} + 2H^2), \quad (40)$$

where $H := \dot{a}/a$. The torsion T_i should also be only time dependent, i.e., $T_i = T_i(t)$. So from (32) the spatial parts of T_i vanish. Letting $T_i(t) = \Phi(t)$ we have

$$\dot{R} = -\frac{2}{3}\left(R + \frac{6\mu}{b}\right)\Phi. \quad (41)$$

Integrating this equation leads to

$$\begin{aligned} R &= 6(\dot{H} + 2H^2 - H\Phi) - 2\dot{\Phi} + \frac{2}{3}\Phi^2 \\ &= -\frac{6\mu}{b} + \left(R(t_0) + \frac{6\mu}{b}\right) \exp\left(-\frac{2}{3} \int_{t_0}^t \Phi dt'\right). \end{aligned} \quad (42)$$

From the field equations we can finally give the necessary equations to integrate

$$\dot{a} = aH, \quad (43)$$

$$\dot{H} = \frac{\mu}{6a_1}R + \frac{1}{6a_1}\mathcal{T} - 2H^2, \quad (44)$$

$$\dot{\Phi} = -\frac{a_0}{2a_1}R + \frac{1}{2a_1}\mathcal{T} - 3H\Phi + \frac{1}{3}\Phi^2, \quad (45)$$

$$\dot{R} = -\frac{2}{3}\left(R + \frac{6\mu}{b}\right)\Phi, \quad (46)$$

where

$$\frac{b}{18}\left(R + \frac{6\mu}{b}\right)(3H - \Phi)^2 - \frac{b}{24}R^2 - 3a_1H^2 = \mathcal{T}_{ii} = \rho, \quad (47)$$

$$\mathcal{T} = g^{ij}\mathcal{T}_{ij} = 3p - \rho, \quad (48)$$

$$p = w\rho. \quad (49)$$

Here, we consider only the matter-dominated era, where the pressure p is negligible.

For the effective energy-momentum tensor contribution from the scalar torsion mode $\tilde{\mathcal{T}}_{ij}$, the explicit expression is

$$\tilde{\mathcal{T}}_i{}^i = -3\mu H^2 + \frac{b}{18}\left(R + \frac{6\mu}{b}\right)(3H - \Phi)^2 - \frac{b}{24}R^2, \quad (50)$$

$$\tilde{\mathcal{T}}_r{}^r = \tilde{\mathcal{T}}_\theta{}^\theta = \tilde{\mathcal{T}}_\phi{}^\phi = \frac{1}{3}[\mu(R - \bar{R}) - \tilde{\mathcal{T}}_i{}^i], \quad (51)$$

and the off-diagonal terms vanish.

We define $\rho_{\text{eff}} \equiv \rho + \rho_{\text{T}} = -3a_0H^2$, where $\rho_{\text{T}} \equiv \tilde{\mathcal{T}}_{ii}$ is the torsion-induced mass density. ρ_{eff} means the effective mass density which is deduced from GR. $p_{\text{T}} \equiv \tilde{\mathcal{T}}_r{}^r$ is an effective pressure due to contributions induced by the dynamic torsion.

IV. A PRELIMINARY ANALYSIS OF THE EQUATIONS

Equations. (43)–(46) are the main equations for the integrations to evolve the system. Regarding the parameters in the field equations, the Newtonian limit requires $a_0 \equiv -(8\pi G)^{-1}$. We take $a_1 > 0$ and $b > 0$ to satisfy the energy positivity requirement [17]. Moreover, the no tachyon condition for the scalar torsion is then also satisfied: $\mu = a_1 - a_0 > (8\pi G)^{-1} > 0$.

Before the detailed results are shown, we briefly analyze the equations to obtain some insight about their behavior. Let us first study the behavior of the affine scalar curvature R . The second derivative of R with respect to time can be obtained by operating a time derivative on Eq. (46) and using Eq. (45)

$$\ddot{R} = -\frac{2}{3}\dot{R}\Phi - \frac{2}{3}\left(R + \frac{6\mu}{b}\right)\dot{\Phi} \approx \frac{2a_0\mu}{a_1b}R, \quad (52)$$

here we assumed that all the variables, i.e., H , Φ , R are much smaller than the coefficient of the leading order term, i.e., $2a_0\mu/a_1b$. For \mathcal{T} , which appears on the right-hand side of (45), we know it consists at least of quadratic terms of H , R , and Φ from (47), so it should be smaller than the other variables. This shows that the coefficient of R on the right-hand side of (52) is negative $2a_0\mu/a_1b < 0$. From this analysis we find that the late-time behavior of R will be essentially oscillating with the period

$$T = 2\pi\sqrt{-\frac{a_1b}{2a_0\mu}}. \quad (53)$$

By a similar argument, it is easy to infer that Φ has the same periodical behavior.

Next, we direct our attention to the behavior of the expansion factor a . The acceleration of the expansion factor a can be obtained by combining Eqs. (34), (43), (44), (50), and (51)

$$\ddot{a} = \frac{\mu R + \mathcal{T}}{6a_1}a - \frac{\dot{a}^2}{a} = \frac{3p_T + \rho_{\text{eff}}}{6a_0}a. \quad (54)$$

From this the relation between the acceleration of the expansion \ddot{a} and the quantity $3p_T + \rho_{\text{eff}}$ can be clearly seen. Since $a_0 < 0$, it shows that $\ddot{a} > 0$ as long as $3p_T + \rho_{\text{eff}} < 0$, and vice versa. We will discuss this relation and its demonstration in the next section.

The period of a and H , if they exist, should be same as that of Φ and R . Because the variables are all highly coupled to each other to form an equation set, there should generically exist a common period in the solution. This point will be demonstrated in the later numerical analysis.

We need to look into the scaling features of this model before we can obtain the sort of results we seek on a cosmological scale. In terms of fundamental units we can scale the variables and the parameters as

$$\begin{aligned} t \rightarrow t/\ell, \quad a \rightarrow a, \quad H \rightarrow \ell H, \quad \Phi \rightarrow \ell\Phi, \\ R \rightarrow \ell^2 R, \quad a_0 \rightarrow \ell^2 a_0, \quad a_1 \rightarrow \ell^2 a_1, \quad (55) \\ \mu \rightarrow \ell^2 \mu, \quad b \rightarrow b, \end{aligned}$$

where $\ell \equiv \sqrt{8\pi G}$. So the variables and the scaled parameters a_0 , a_1 , and b all become dimensionless, and $a_0 = -1$. Furthermore, Eqs. (43)–(46) remain unchanged under such a scaling. However, as we are interested in the cosmological scale, it is practical to use another scaling to turn the numerical values of the scaled variables “gentler” (i.e., not stiff) from the numerical integration. In order to achieve this goal, let us introduce a dimensionless constant T_0 , which represents the magnitude of the Hubble time ($T_0 = H_0^{-1} \doteq 4.41504 \times 10^{17}$ seconds.) Then the scaling is

$$\begin{aligned} t \rightarrow T_0 t, \quad a \rightarrow a, \quad H \rightarrow H/T_0, \\ \Phi \rightarrow \Phi/T_0, \quad R \rightarrow R/T_0^2, \quad a_0 \rightarrow a_0, \quad (56) \\ a_1 \rightarrow a_1, \quad \mu \rightarrow \mu, \quad b \rightarrow T_0^2 b. \end{aligned}$$

With this scaling, all the field equations are kept unchanged while the period $T \rightarrow T_0 T$.

A. Parameter choice with constant scalar curvature

Equation (46) is of special interest among the field equations because of the existence of a constant value $6\mu/b$. It shows that the scalar affine curvature remains a constant $R = -6\mu/b$ forever as long as its initial data has this special critical value. It is tempting to see how the system evolves with $R = -6\mu/b$ initially.

As mentioned in Sec. II, the positivity of the kinetic energy in the Hamiltonian analysis of the spin-0⁺ case requires $b > 0$, $a_1 > 0$, thus $\mu > 0$, since $a_0 < 0$. With such an assumption, the scalar affine curvature should not have the value $R = -6\mu/b$ initially since this initial choice will require the matter density ρ to be negative from Eq. (47). Such a choice violates the assumption of energy positivity.

However, if we tentatively relax the parameter requirement for positive kinetic energy, i.e., allowing $a_1 = -\bar{a}_1 < 0$ such that $\mu = -m < 0$, the scenario will turn out to be quite intriguing. Under such a new parameter requirement, if we set initially the scalar affine curvature

$$R = -\frac{6\mu}{b} = \frac{6m}{b} > 0, \quad (57)$$

then R will remain at this constant value for all the time. From Eqs. (47) and (50), we can derive

$$-3a_0 H^2 = \rho + \rho_T > 0, \quad (58)$$

where

$$\rho = 3\bar{a}_1 H^2 - \frac{3m^2}{2b}, \quad (59)$$

$$\rho_T = \frac{3m^2}{2b} - 3mH^2. \quad (60)$$

Here, the matter density will be positive as long as the parameters are chosen suitably, such that $\bar{a}_1 H^2 - m^2/2b > 0$. And the more interesting point is that the torsion-induced mass density ρ_T could “act like a dark energy” if the suitable parameter values are chosen. We can simplify the field Eq. (44) to

$$\dot{H} = \frac{3m^2}{4\bar{a}_1 b} - \frac{3}{2}H^2, \quad (61)$$

and it leads to

$$\ddot{a} = \frac{1}{2}\left(\frac{3m^2}{2\bar{a}_1 b} - H^2\right)a. \quad (62)$$

Combining with Eq. (54), $3m^2/2\bar{a}_1 b > H^2$ as long as $3p_T + \rho_{\text{eff}} < 0$. By solving Eq. (61), the solution will show that

$$H \rightarrow \frac{m}{\sqrt{2\bar{a}_1 b}} \quad \text{for } t \rightarrow \infty. \quad (63)$$

By comparing this to a universe with a cosmological constant Λ where the Hubble function H approaches to $\sqrt{\Lambda/3}$ as $t \rightarrow \infty$, we can see how to choose suitable values such that the cosmological constant Λ and thus the dark energy can be mimicked in this torsion cosmological model with a constant affine scalar curvature. We will demonstrate numerically the behavior of this case in the next section.

V. NUMERICAL DEMONSTRATION

In this section, we would like to demonstrate two points: (1) The degenerate case $R = -6\mu/b$ with the relaxed parameter choice mentioned in Sec. , i.e., $a_1 < 0$ and $\mu < 0$ instead of the normal choice $a_1 > 0$ and $\mu > 0$. Although such a choice is against the positivity of kinetic energy, we would like to explore this scenario a little bit more, since it could mimic the cosmological constant and the other cosmological models with a negative kinetic energy [34]. We can see that the torsion in the system becomes kinetic instead of being dynamic, and the expansion is accelerating at late time; (2) In generic cases, i.e., $R + 6\mu/b \neq 0$, with the proper parameter choice (i.e., $a_1 > 0$ and $\mu > 0$), the torsion in the system is dynamic, and its functional pattern has a periodic feature, i.e., it could be accelerating for a while, and then be followed by a period of deceleration with the pattern repeating. With suitable adjustments of the parameters and the initial values of the fields involved, it is possible to change the period of the dynamic fields as well as their amplitudes. Furthermore, in the model, with some choices of the parameters and the initial values of the fields, it is possible to mimic the main apparent dynamic features of the Universe, i.e., the value of the Hubble function is the current Hubble constant in an accelerating universe after a period of time on the order of the Hubble time. In such a case, this model will describe an oscillating universe with a period on the order of magnitude of the Hubble time. This allows us to constrain the parameters and/or the value of the torsion field by comparing the observed data with the result from this model.

The 4th-order Runge-Kutta method is applied for the integration of the field Eqs. (43)–(46). The Universe is assumed to be matter-dominated, thus, $\mathcal{T} \approx -\rho$. The mass density ρ is determined from the fields via Eq. (47). The fields and the parameters are scaled with Eq. (55) and (56) to be dimensionless, and to achieve a realistic cosmology.

A. Case I: Constant R case

In this case, the initial values of the fields are as follows:

$$\begin{aligned} a(t_0) &= 50, & H(t_0) &= 1, \\ \Phi(t_0) &= 10, & R(t_0) &= \frac{6m}{b}, \end{aligned}$$

and the parameters are taken to be

$$a_0 = -1, \quad b = 10^{-4},$$

where t_0 is the initial time $t_0 = 1$, the present time of our universe. Under this setting,

$$\rho = -3a_0 H^2 + 3mH^2 - \frac{3m^2}{2b} = 3 + 3m - 1.5 \times 10^4 m^2. \quad (64)$$

In order that the mass density in the current universe is about $\rho \approx 30\%$, the parameter m is chosen as

$$m = 0.012.$$

This shows that $H \rightarrow m/\sqrt{2\bar{a}_1 b} \approx 0.84$. The detailed result is shown in Fig. 1, where the evolved values of H , \ddot{a} , Φ , and R are plotted as the (black) solid curve in different panels.

It is obvious in the bottom-right panel of Fig. 1 that the affine scalar curvature R remains constant, $6m/b$. The behavior of the torsion Φ can be understood through Eq. (45). Φ will increase (or decrease) until its value balances the right-hand side of Eq. (45); this mainly depends on the sign change of the term $3H\Phi$ provided $H > 0$ and $\mathcal{T} > 0$. With the current initial choice in this case, Φ decreases promptly at present until the balancing point is reached, as seen in the bottom-left panel of Fig. 1. However, Φ will not be a constant since the right-hand side of Eq. (45) still changes with time. The Hubble function H will always decrease, and approach to the fixed value $m/\sqrt{2\bar{a}_1 b} \approx 0.84$, as shown in the upper-left panel of Fig. 1, since the right-hand side of Eq. (61) is always negative. The acceleration of the expansion factor \ddot{a} is positive at late time, as seen in the upper-right panel of Fig. 1.

It is very interesting to see how the Universe evolves if the scalar affine curvature R has a tiny deviation from the constant value $6m/b$. Therefore, we chose the initial values of R as $R(t = 1) = 6m/b - 10^{-8}$ and $R(t = 1) = 6m/b + 10^{-8}$ and evolved the system while keeping all the other initial choices the same as in the $R = 6m/b$ case. The results are also plotted in Fig. 1. In Fig. 1, the (blue) dashed lines are for the $R(t = 1) = 6m/b - 10^{-8}$ case and the (red) dotted-dashed lines are for the $R(t = 1) = 6m/b + 10^{-8}$ case. The results show that once the scalar affine curvature R is smaller than $6m/b$ by a tiny amount, the

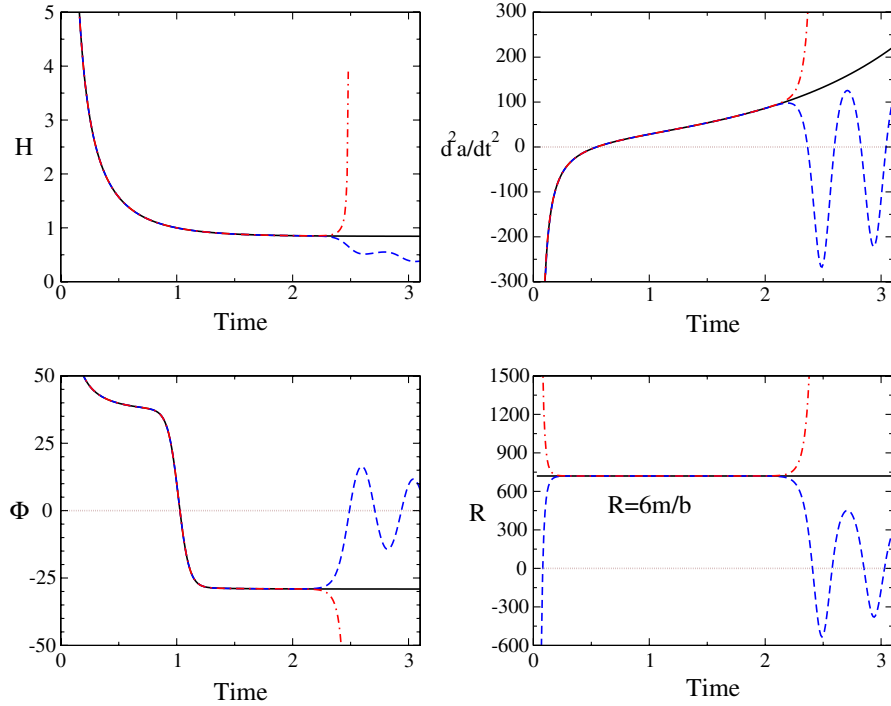


FIG. 1 (color online). Evolution of the Hubble function H , the 2nd time derivative of the expansion factor \ddot{a} , the temporal component of the torsion Φ , and the affine scalar curvature R as functions of time with the parameter choice and the initial data in Case I. The (black) solid lines represent the result of $R(t=0) = 6m/b$, the (blue) dashed lines represent the result of $R(t=1) = 6m/b - 10^{-8}$, and the (red) dotted-dashed lines represent the result of $R(t=1) = 6m/b + 10^{-8}$, while all the other initial choices are fixed.

values of R and thus the other fields will eventually return to a damped-oscillating mode. Therefore, the Universe will eventually approach a static condition. On the other hand, if the scalar affine curvature R is bigger than $6m/b$ by a tiny amount, the values of R and thus the other fields will rise unboundly. Either way, the affine curvature will never recover its constant value. Therefore, the constant curvature case represents an *unstable* universe with an effective cosmological constant or a negative-kinetic-energy field. This phenomenon demonstrates the inherent instability of a system with a negative kinetic energy. However, as long as the deviation of R from $6m/b$ is small enough, it would be difficult from Fig. 1 to predict the future of the Universe, since all the lines of these three cases are virtually overlapped together until a very late time. By careful fine-tuning we can arrange for a large variety of outcomes. This “chaotic” behavior well illustrates just how we can lose all physical predictability if we allow such *unphysical* parameter choices.

Although the above parameter choice has the virtue of explaining the accelerating expansion of the Universe and the cosmological constant, we cannot accept such a parameter choice here, since it violates the fundamental assumption of the positivity of the kinetic energy. Therefore, in the following cases, we will return to our normal *physical* assumption, i.e., $a_1 > 0$ and $\mu > 0$.

B. Case II: Oscillating acceleration of a

For this case, we take the initial values of the field to be

$$\begin{aligned} a(0) &= 10, & H(0) &= 5 \times 10^{-3}, \\ \Phi(0) &= 2 \times 10^{-4}, & R(0) &= -2 \times 10^{-3}, \end{aligned}$$

and the parameters are taken to be

$$\mu = 1.2, \quad b = 4,$$

The results plotted in Fig. 2 show that \ddot{a} , Φ , and R are damped periodic.

In particular, R has a periodic character as shown in the bottom-right panel of Fig. 2. According to Eq. (53), its period is $T = 2\pi\sqrt{-a_1 b / 2a_0 \mu} \approx 3.63$, which is close to the period of the variables shown in Fig. 2. The most interesting part is the behavior of \ddot{a} , which is periodic with the same period as Φ and R . As shown in the top-right panel of Fig. 2, \ddot{a} could be positive as well as being negative, and the pattern of its function is similar to the pattern of R . Therefore, the behavior of H is a declining baseline plus a damped oscillation, as shown in the top-left panel of Fig. 2.

On a broader viewpoint of the evolution of this system, \ddot{a} , Φ , and R will be slowly damped, and H will approach zero after a long time. The important feature of this case is

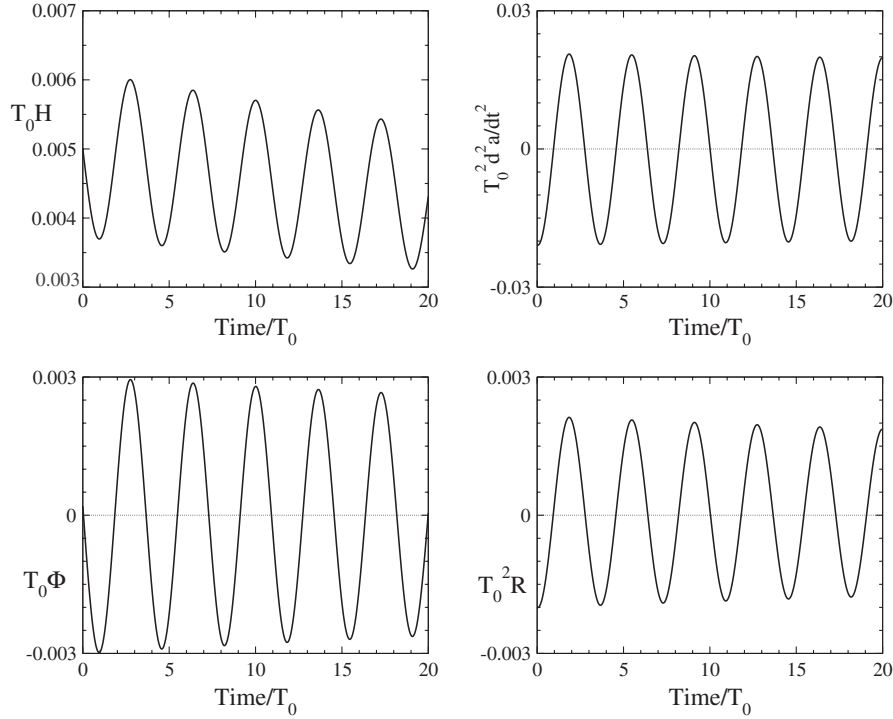


FIG. 2. Evolution of the Hubble function, the 2nd time derivative of the expansion factor, the temporal component of the torsion, and the affine scalar curvature as functions of time with the parameter choice and the initial data in Case II.

that the Universe could oscillate due to dynamic torsion. In such a scenario, the present day acceleration of the Universe is not so strange, \ddot{a} is oscillating, and it happens to be increasing at this time. Furthermore, the oscillation period would be determined mainly by the parameters and the initial values of the fields in this model. This encouraged us to try to find parameter values and initial conditions that more nearly resemble the current status of the Universe. Such a choice will be shown in the next case.

C. Case III: A presently accelerating universe

In this case, we would like to compare the numerical values of the torsion model with the observational data of the Universe. The initial data is set at the current time $t_0 = 1$, after scaling, instead of $t_0 = 0$. The parameters and initial conditions chosen are as follows:

$$\begin{aligned} a(t_0 = 1) &= 50, & H(t_0 = 1) &= 1, \\ \Phi(t_0 = 1) &= 1.4, \\ Y(t_0 = 1) &\equiv R(t_0 = 1) + \frac{6\mu}{b} = 6.2, \end{aligned}$$

and

$$\mu = 1.09, \quad b = 1.4.$$

Here, the initial data has been scaled according to Eqs. (55) and (56) such that the current value of the Hubble function

is unity. Therefore, we get realistic values in our universe: the Hubble constant at present $H(t_0 = 1)$ is

$$H = \frac{1}{4.41504 \times 10^{17}} \cdot \frac{1}{s} \approx 70 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}. \quad (65)$$

The results of the evolution with the parameters and initial conditions are plotted in Fig. 3. In the top-left panel the Hubble function H is damped oscillating at late time. In the top-right panel, it is obvious that \ddot{a} is damped and oscillating during the evolution and is positive at the current time $t \approx 1$, which means the expansion of the universe is currently accelerating. $\Phi(t)$ and $R(t)$ are also plotted in Fig. 3 to show the correlation of the evolution between these variables. We observe that the values of the variables $H(t)$, $\ddot{a}(t)$, $R(t)$, and $\Phi(t)$ become relatively high before $t/T_0 = 0.4$. However, this situation need not be taken too seriously, since it describes the earlier time of the universe, and our matter-dominated era assumption is not appropriate for such an early period of time.

In order to have a deeper understanding of the settings of this case, the matter density ρ , the effective mass density $\rho_{\text{eff}} = \rho + \rho_{\text{T}}$, and the quantity $3p_{\text{T}} + \rho_{\text{eff}}$ are plotted in the bottom panels of Fig. 3. The value of ρ , shown in the bottom-left panel, is decreasing at $t \approx 1$ as the Universe is expanding and is always positive, while the effective mass density ρ_{eff} , plotted in the same panel, shows an “oscillating” behavior around the curve of ρ . The oscillating behavior of ρ_{eff} comes from the contribution of the torsion-

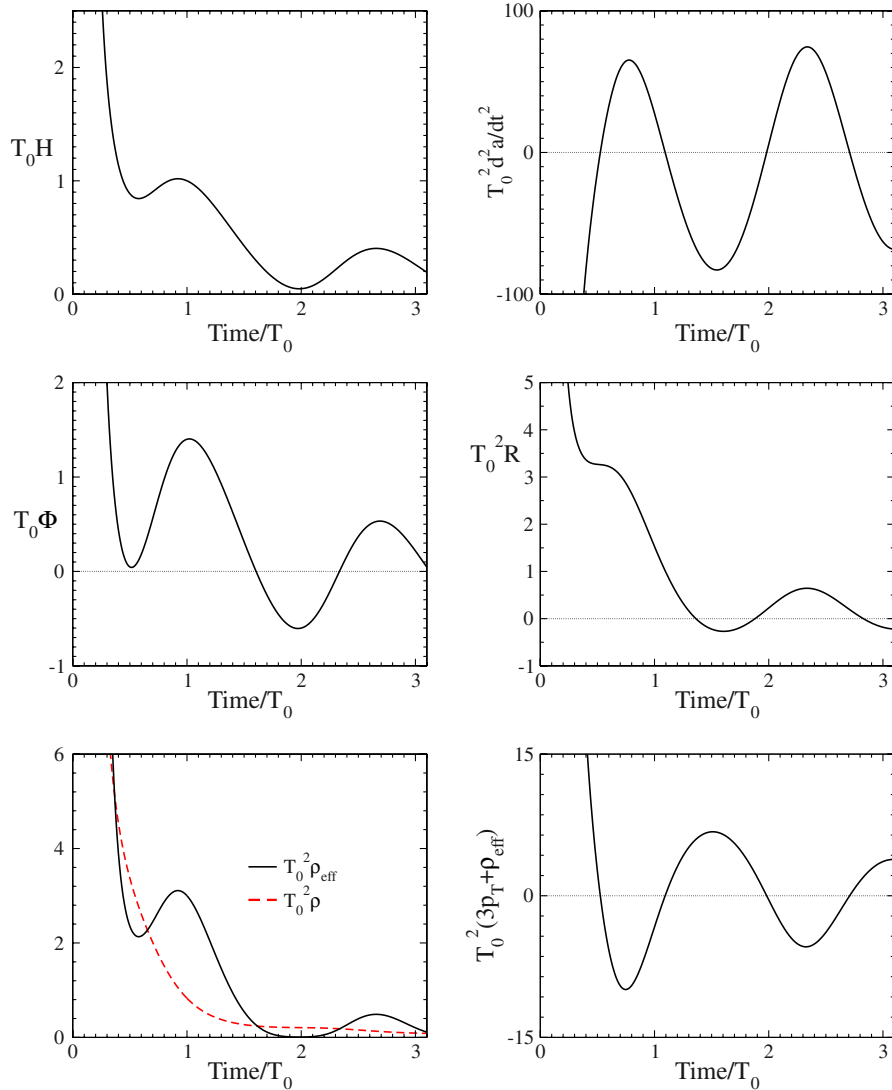


FIG. 3 (color online). Evolution of the Hubble function H , the 2nd time derivative of the expansion factor \ddot{a} , the temporal component of the torsion Φ , the affine scalar curvature R , the mass density ρ , the effective mass density $\rho_{\text{eff}} \equiv \rho + \rho_T$, and the quantity $3p_T + \rho_{\text{eff}}$ as functions of time with the parameter choice and the initial data in Case III.

induced mass density ρ_T and simply indicates that ρ_T is not positive definite in general. In fact, the value of ρ_T turns from negative to positive when the time is around $t \approx 0.7$. As to the quantity $3p_T + \rho_{\text{eff}}$, we can understand its importance for the evolution of the Universe through Eq. (54) in which the value of this quantity decides the status of the acceleration. We can see this much more clearly by checking the correlation between the curves of \ddot{a} and $3p_T + \rho_{\text{eff}}$ in Fig. 3. Also by comparing the two bottom panels of Fig. 3, it is obvious that the torsion-induced pressure p_T is negative when the Universe accelerates, and positive when the Universe decelerates.

In this case, the scaled value of $\rho(t=1) = 0.83$ and its physical value is $\rho(t=T_0) = 2.61 \times 10^{-30} \text{ g/cm}^3$. The Universe is supposed to be very close to the critical density, $\rho_c \equiv 3c^2 H^2 / 8\pi G = 9.47 \times 10^{-30} \text{ g/cm}^3$; we find the ra-

tio $\Omega_m \equiv \rho / \rho_c = 28\%$. In the standard Λ CDM model, $\Omega_m \sim 30\%$ with 5% baryonic matter and 25% dark matter. For our model $\Omega_T \equiv \rho_T / \rho_c = 72\%$ acts like the energy density of dark energy. Therefore, this torsion model is able to describe a presently accelerating expansion of the Universe with a proper amount of matter density. From the field equations we can see that the *effect* of the “dark energy” mainly comes from the nonlinearity of the field equation driven by the dynamic scalar torsion.

D. Other cases

We continue to look at two more cases, which are listed in Table I along with Case III, obtained by taking different values of the parameters and the initial conditions, along with physical values of the significant mass density ρ . We

TABLE I. Here the parameter a_0 is set to be 1 in all of the three cases; $H(1)$ means $H(t = \text{now})$, $\Phi(1)$ means $\Phi(t = \text{now})$, etc, under the scaling Eqs. (55) and (56).

Case	μ	b	$H(1)$	$\Phi(1)$	$Y(1)$	$a(1)$	$\ddot{a}(1)$	$\frac{\rho(1)}{10^{-30} \text{ g/cm}^3}$
III	1.09	1.4	1	1.4	6.2	50	27.59	2.61
IV	1.27	1.1	1	0.8	11.3	50	70.29	5.23
V	1.38	1.1	1	1.1	9.9	50	4.57	2.48

find that the results of the other two cases have a behavior qualitatively similar to that of Case III.

Now, we would like to compare our results with the supernovae data. Distance estimates from SN Ia light curves are derived from the luminosity distance

$$d_L \equiv \sqrt{\frac{L_{\text{int}}}{4\pi\mathcal{F}}} = cT_0 a(1)(1+z) \int_1^t \frac{dt}{a(t)}, \quad (66)$$

where L_{int} and \mathcal{F} are the intrinsic luminosity and observed flux of the SN, and the redshift $z \equiv a(1)/a(t) - 1$. Logarithmic measures of the flux (apparent magnitude, m) and luminosity (absolute magnitude, M) were used to derive the predicted distance modulus [35]

$$\mu = m - M = 5\log_{10}d_L + 25, \quad (67)$$

where m is the flux (apparent magnitude), M is the luminosity (absolute magnitude), and d_L in the formula should be in units of megaparsecs. We found the relations between the predicted distance modulus μ and the redshift z in the

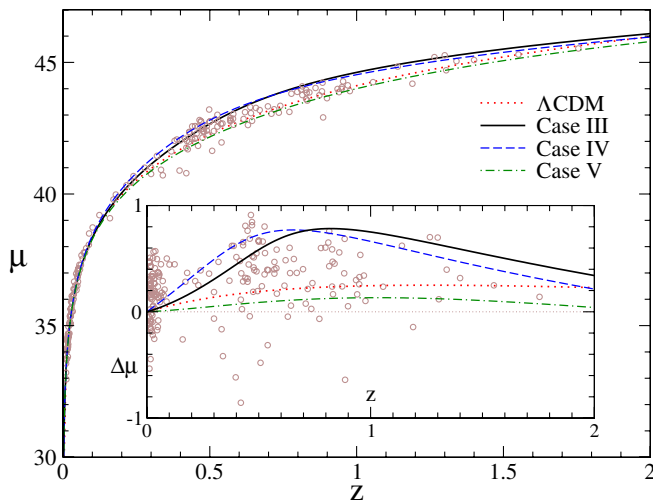


FIG. 4 (color online). Comparison of different torsion models and the standard Λ CDM model with the observational data via the relation between the distance modulus μ and the redshift z . The supernovae data points, plotted with (brown) circles, come from [36]. The result of standard Λ CDM model ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$) is plotted by the bold (red) dotted line. The results of Case III, IV, V are represented by the bold solid line, the (blue) dashed line, and the (green) dotted-dashed line, respectively. In the inset, the models and data are shown relative to an empty universe model ($\Omega = 0$).

three cases; they are plotted in Fig. 4. For comparison, we also plot the prediction of the Λ CDM model with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ by employing the following formula [36]

$$d_L = cT_0(1+z) \int_0^z \frac{dz}{\sqrt{(1+z)^2(1+\Omega_m z) - z(2+z)\Omega_\Lambda}}. \quad (68)$$

The astronomical observational data [36,37] are also plotted in Fig. 4 for comparison. The plots show that for small redshift z (e.g., $z < 1.9$) all three cases of the dynamical torsion models give an accelerating universe just like the Λ CDM model does. For larger z these cases might turn the Universe into a deceleration mode, which is consistent with the behavior of the various quantities shown in Fig. 3. We can see that Case V gives the closest curve behavior to the one from the Λ CDM model, although in Case V the matter density is only about 26% of the critical density. However, it was not meant to have a detailed comparison in this plot between our models with the Λ CDM model. Instead, in Fig. 4, we demonstrate the possibility of the scalar torsion field accounting for the effect of dark energy with a suitable set of parameters and initial data. This allows us to study the dark energy problem from a new and different angle.

VI. DISCUSSION

In this work we introduce into the evolution of a universe without a cosmological constant a certain dynamical PGT scalar torsion mode taken from our earlier work [17]. From the assumption of the homogeneity and isotropy of the Universe, only the temporal component of the torsion Φ will survive and affect the evolution of the Universe at late times. With the field Eqs. (43)–(46), we analyzed analytically and numerically the evolution of the system. We found that in generic cases, i.e., $R + 6\mu/b \neq 0$, with the proper parameter choice (i.e., $a_1 > 0$ and $\mu > 0$), the torsion Φ in the system is dynamic, and \ddot{a} , Φ , R tend to have a damped periodic behavior with the same period, while the behavior of H is a declining baseline plus a damped oscillation. With certain choices of the parameters of μ and b , and of the initial data of H , Φ , and R , like Cases III–V in the previous section, this model can describe an oscillating universe with an accelerating expansion at the present time.

Before we can give an adequate discussion of the viability of this model as an explanation of the accelerating universe, we should check whether this model can survive under the constraints of the theoretical and experimental tests.

There have been numerous investigations on the existence of torsion since this geometric quantity entered the realm of gravity (see [38–40] and the references therein). As mentioned above, this model has not only passed the important classical tests (“no ghosts” and “no tachyons”), it is also one of the two scalar torsion modes—the only PGT cases which are known to have a well posed initial value problem [17] and which may well be the only viable dynamic PGT torsion modes that can evade the nonlinear constraint problems. There have also been some laboratory tests in search of torsion [41,42]. The main idea among these experiments is the spin interaction between matter and torsion. The cosmological tests on torsion investigate the effect of torsion-induced spin flips of neutrinos in the early universe, which could alter the helium abundance and have other effects on the early nucleosynthesis [43,44]. However, Dirac fermions interact only with the totally antisymmetric pseudoscalar part of the torsion. Thus, these tests can only consider the pseudo-scalar mode (axial-vector torsion), not the scalar-mode torsion used in our model. The type of torsion used in our model does not interact directly with any known matter. Thus, these tests cannot really give a serious constraint on the amplitude of our scalar-mode torsion.

Among the models in which torsion is applied to the cosmological problem, Capozziello *et al.* [31,45] have done a serious study on replacing the role of the cosmological constant in the accelerating universe. With a totally antisymmetric torsion without dynamical evolution, their model is consistent with the observational data by tuning the amount of the torsion density, although this model cannot solve the coincidence problem. On the other hand, the oscillating universe models with a designed mechanism: an oscillating potential, an oscillating parameter of the equation of state, etc. [46–48] aim to solve the coincidence problem. Here, we found that our model takes some virtues from both kind of models, i.e., our model is capable of solving the coincidence problem of an accelerating universe with a dynamical scalar-mode torsion, which is *naturally* obtained from the geometry of the Riemann-Cartan spacetime, instead of from an exotic scalar field or a designed mechanism.

If we consider the spacetime as Riemannian instead of Riemann-Cartan, by absorbing the contribution of the torsion of this model into the stress-energy tensor on the right-hand side of the Einstein equation, then this contribution will act as a source of the Riemannian metric, effectively like an *exotic* fluid with its mass density ρ_T and pressure p_T varying with time (even though the time evolution of the torsion is not like that of a such a fluid). Moreover, the

effective fluid appears to have presently a negative pressure, and consequently a negative parameter in the effective equation of state, i.e., $\omega_T \equiv p_T/\rho_T$, which drives the Universe into accelerating expansion. Note that there is no constraint on the value of ω_T , which appears here, and its value could vary from time to time. It should be stressed that this is not a real physical fluid situation; the truth is that ω_T is nothing like “a torsion field equation of state,” it is just a proportionality factor between ρ_T and p_T , two expressions which effectively summarize the contribution of torsion acting as a source of the metric. The ratio ω_T is of interest only to help understand the acceleration of this model and to enable a limited comparison with other dark energy proposals.

One might be concerned about the value of the parameter b . Its value should be small enough to be consistent with the constraints on the effect of the quadratic order term R^2 on the large-scale structure of the Universe. The values of b we choose, i.e., $b/(a_0 T_0^2)$ in the conventional unit, are on the order of unity. These chosen values are bigger than the magnitude of a related parameter, estimated in [49]; however, one cannot expect that estimate to be applicable here—since in that work quadratic *Riemannian curvature* terms were considered (they lead to 4th order field equations) instead of the affine curvature terms we have used (which give 2nd order equations). As far as we know the parameter μ does not have too much constraint on it, except for its positivity as a mass parameter, since the baryonic matter will only interact with the scalar torsion indirectly by gravitation.

One may wonder how large must the torsion be in order to produce observable effects in the the present day universe, e.g., the observed acceleration? Conversely, how large can the torsion be without violating some observational constraint? The questions merit a detailed study. Here is a simple argument that indicates a magnitude. Let us compare the terms in the Lagrangian density and the field equations for the PGT scalar torsion model and the Einstein theory with a cosmological constant. Note that the presumed cosmological constant is “so small” that it has no noticeable effect in the laboratory, nor on the solar system scale, nor on the galactic scale. Nevertheless, it is large enough to have the dominant effect on the cosmological scale. Hence, we are led to infer that we should consider $a_1 T^2 \sim b R^2 \sim \Lambda \sim a_0 \rho \sim H^2$. With such a choice we can expect that torsion may be able to accelerate the Universe and yet not be conspicuous on smaller scales.

The 0^+ torsion mode in this model effectively gives a scalar field, yet this scalar field is, in fact, quite different from the various scalar field models of “exotic matter,” e.g., the *quintessence* models, in several significant ways: (i) torsion cosmology is derived naturally from a geometric gravitational theory, which is based on fundamental gauge principles, instead of on the hypothesis of the existence of a dark energy tailored to producing an explanation of an

accelerating universe; (ii) thus, there are only a couple of free parameters in torsion cosmology, instead of an *ad hoc* potential that can be rather arbitrarily chosen to fit the observations. Therefore, a torsion cosmological model should be more restrictive, and should be easier to be confirmed or falsified; (iii) based on its tensorial character, the coupling of torsion to the other fields is nothing like that which has ever been advocated for hypothetical scalar fields. Consequently, we see no way to simply replace the scalar-mode torsion with an effectively equivalent quintessence model. Thus, torsion cosmology and the quintessence models are characteristically different, even though there are some similarities.

Because of its intriguing behavior, we also turned our attention to a degenerate case $R = -6\mu/b$ in Case I with the relaxed parameter choice of $a_1 < 0$ and $\mu < 0$ instead of the normal choice $a_1 > 0$ and $\mu > 0$. Although such a choice is against the positivity of kinetic energy, we explored this scenario, since it could mimic the cosmological constant and the other cosmological models with a negative kinetic energy. Indeed, the result does show an accelerating universe at late time. However, our further numerical experiments also show that in such a case it describes a very unstable universe. A small perturbation from the constant curvature will cause a sudden change, which would only become apparent at some time in the future.

VII. CONCLUSION

In this work we considered the scalar torsion mode of the PGT on in a cosmological setting and proposed it as a viable model for explaining the current status of the Universe. Besides having a better understanding of the PGT, we study the prospects of accounting for the outstanding present day mystery—the accelerating universe—in terms of an alternate gravity theory, more particularly in terms of the PGT dynamic torsion. With the usual assumptions of isotropy and homogeneity in cosmology, we find that, under the model, the Universe will oscillate with generic choices of the parameters. The torsion field in the model could play the role of dark energy. With a certain range of parameter choices, it can account for the current status of the Universe, i.e., an accelerating expanding universe with a value of the Hubble constant, which is approximately the present one. Thus, we have considered the possibility that a certain geometric field, dynamic scalar torsion—which is naturally expected from spacetime gauge theory—could fully account for the accelerated universe.

The source of the torsion could come indirectly from the huge density of the particles with sufficient spin alignment in the early universe. This scalar mode of torsion could be

considered as a “phantom” field, at least in the matter-dominated epoch, since it will not interact directly with matter; it only interacts indirectly via gravitation. Then the dynamics of the scalar torsion mode could drive the Universe in an oscillating fashion with an accelerating expansion at present. It is quite remarkable that a gauge theory of dynamic geometry naturally presents us with such a phantom field. This natural geometric field could act like a dark energy.

However, there are also some points that need to be studied in much more detail before this model can more closely conform to reality. The model in Cases III–V of the previous section, suggests that the mass parameter of the torsion μ might be close to a_0 , and the parameter for the “kinetic” energy density of the torsion b may need to be as huge as T_0^2 to achieve an accelerating universe. The restricted window of the parameter choices, which allows a behavior like that of our universe, might render the model less favored, even though the matter in the Universe is not able to directly interact with the torsion. Meanwhile, the required choice of initial data and the values of the parameters may make this model unsuited to solving the fine-tuning problem.

These dark sides should not be able to diminish the possibility of the scalar mode of the torsion in this model playing a significant role in the evolution of the Universe. The model has only a few adjustable parameters, so scalar torsion may be easily falsified—as “dark energy.” If it turns out that the accelerated universe cannot be explained in this way—that something else has the dominant dark energy role—it would still be reasonable to expect that there may be some observable cosmological effects from dynamic scalar torsion. Also, here we only used one of the viable modes of torsion in PGT; thus, our model will be more general if it is extended to include all the viable PGT torsion modes. We believe that future investigations along this line should be open to these possibilities.

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