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# Recollapsing quantum cosmologies and the question of entropy

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Recollapsing homogeneous and isotropic models present one of the key ingredients for cyclic scenarios. This is considered here within a quantum cosmological framework in the presence of a free scalar field with, in turn, a negative cosmological constant and spatial curvature. Effective equations shed light on the quantum dynamics around a recollapsing phase and the evolution of state parameters such as fluctuations and correlations through such a turn around. In the models considered here, the squeezing of an initial state is found to be strictly monotonic in time during the expansion, turn around, and contraction phases. The presence of such monotonicity is of potential importance in relation to a long-standing debate concerning the (a)symmetry between the expanding and contracting phases in a recollapsing universe. Furthermore, together with recent analogous results concerning a bounce, one can extend this monotonicity throughout an entire cycle. This provides a strong motivation for employing the degree of squeezing as an alternative measure of (quantum) entropy. It may also serve as a new concept of emergent time described by a variable without classical analog. The evolution of the squeezing in emergent oscillating scenarios can in principle provide constraints on the viability of such models.

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## I. INTRODUCTION

Classical cosmology has shown enormous progress over the recent years. Despite this, a number of fundamental questions remain. Central among these is the fact that within the classical general relativistic framework, the initial state of the universe is singular, which would result in the breakdown of the laws of physics. To obtain a satisfactory scenario with a nonsingular initial state, one often looks to quantum gravity and quantum cosmology. In fact, with a loop quantization one can generically resolve the big bang singularity in cosmological and other models [1,2]. In the simplest cases, a bounce results, which keeps the volume nonzero and the universe away from the classical singularity reached otherwise at the big bang. The possibility of a contracting phase (or several phases) before the hot big bang has recently been invoked in a number of cosmological scenarios, including several models proposed as alternatives to standard inflation, such as, for example, pre-big bang [3] and the ekpyrotic/cyclic scenarios [4–6]. The assumed nature of such phases, however, has so far been mostly rather *ad hoc*, without a satisfactory treatment of the classical singularity. The presence of such phase(s) raises important questions, including their nature and their relation to the present phase of the universe. This in turn relates to fundamental questions such as, among others, cosmological entropy and the arrow of time.

Now, given that the big bang was a high-energy, strong-curvature regime, the understanding of the pre- and post-bounce phases would require a full control of dynamical evolution of the quantum state through such a bounce. Moving through a bounce, a wave packet can spread and deform significantly, implying that the universe before the bounce could, for all we know, have been in a state very different from what we see now. Thus, to understand the cosmological dynamics through such bounces, all aspects of a quantum space-time are essential, including its fluctuations and higher moments.

In loop quantum cosmology, solvable models with controlled state properties exist if the matter source is a free, massless scalar. This has been analyzed numerically [7–10] and analytically [11,12]. More general models can be treated by means of effective equations [13,14], as they are also employed here for the recollapse. Note that the concept of effective equations is much more general than simply providing correction terms to classical equations. With a complete set of consistent effective equations one can, in fact, derive dynamical properties such as expectation values, fluctuations, correlations, or higher moments for full quantum states. As we will see below, state properties can be studied directly by using effective equations, which provide an economical and representation-independent approximation scheme of the evolution of states. (For another discussion of effective equations especially in quantum cosmology, see [15].)

If one combines the quantum bounce with a classical recollapse, cyclic models ensue. Such oscillatory models, according to which the universe undergoes many (and possibly an infinite number of) bounces, have been employed in order to construct nonsingular emergent models which can set the initial conditions for a successful phase of inflation. Since such a universe can pass through many cycles, and hence many high energy, strong-curvature regimes, this could result in even more severe changes of its state compared to a single bounce. We should note that oscillatory models have a long history in cosmology at least since the studies by Tolman in the 1930's [16,17]—albeit within a classical setting. Interestingly, Tolman also considered the question of cosmological entropy for these models, claiming that the entropy during the expanding phase should be slightly lower than during the subsequent collapsing phase. In these studies entropy refers to that of the content of the universe [16,17] and ignores contributions from (quantum) gravity.

There are, however, important problems with these models, including the lack of treatment of singularities and the uncorroborated assumption that the bounces themselves leave the entropy of the universe unchanged. The consideration of oscillatory models within a quantum cosmological framework, on the other hand, not only allows singularities to be avoided, but also introduces many more quantum degrees of freedom, thus allowing the question of entropy to be considered in a different light.

This is the setting we consider in this paper. We will analyze the recollapse in detail, which is a semiclassical regime but, crucially, still described in terms of a quantum state. We especially focus on the evolution of state parameters through the recollapse, which provides insights to the question of what their generic change may be. In particular, we are interested in how strongly fluctuations of a generic state respect time-reversal symmetry for time reflections around the recollapse point. If fluctuations are symmetric in this sense, there is not much change between the pre- and post-recollapse phases. A violation of the symmetry, on the other hand, would provide a measure for the generic change of the quantum state in the recollapse phase. The analysis is thus complementary to what has already been studied for the bounce [18,19]: Can the quantum state after the recollapse be very different from what it was before? Especially in the presence of many cycles, this question is important for understanding the viability of oscillatory cosmological models over epochs long compared to the life time of individual cycles.

For technical reasons, we shall take a free massless scalar as the matter source in all models considered in detail here. However, we shall also demonstrate the robustness of our claims under the inclusion of potentials. The free scalar has the advantage that it can be used as a global internal time parameter and thus gives rise to true Hamiltonian, rather than constrained, evolution. Any nonconstant potential or even a mass term would spoil this feature. (Here we refer to the classical situation. We will later encounter and entertain the possibility of genuine quantum variables as a measure for time even in situations where no obvious classical clock may exist.) Moreover, in the absence of a cosmological constant and for flat, isotropic space, this matter content provides an exactly solvable model even after quantization (loop or otherwise) [11]. Thus, there are no dynamical quantum corrections whatsoever in this case; the system is harmonic and presents the simplest and most controlled model of quantum cosmology. (There may, however, be quantum geometry corrections of kinematical type which give rise to a bounce in loop quantum cosmology. But they turn out not to spoil the dynamical solvability [11].) However, this exact model does not allow a recollapse, and we therefore have to add extra ingredients and with them nontrivial quantum corrections. Nevertheless, the resulting systems will be manageable and provide key contributions for highly controlled cyclic models. While there is no scalar potential in the main part of the paper, we verify that in fact our results remain robust in the presence of general nonzero potentials. Moreover, our analysis provides a starting point to analyze equations in the presence of a potential perturbatively. For the bounce, such equations are developed in [13,14], which in some cases even allow conclusions valid to all orders in the potential and in quantum moments [20]. Since our main question is about limitations to the symmetry of fluctuations around cosmological turning points, a highly controlled model is reliable as any limitation there would only grow if the model becomes more complicated. (See also [18,19] in this context.) In Sec. III C we will comment in more detail on possible effects of a potential.

## II. RECOLLAPSING MODELS

We shall confine ourselves to isotropic and homogeneous settings. There are two different ways to achieve a recollapsing cosmological model: by including a negative cosmological constant or by allowing positive spatial curvature. We shall first describe the general scheme of our analysis and then specialize to these two cases. In deriving our central result, namely, the monotonic increase of squeezing during a single recollapse phase, we employ effective equations based on a Wheeler-DeWitt quantization, rather than loop quantization. These two schemes are close to each other in the regime of interest here. Only at the end of this paper will we use results of loop quantum cosmology in the application to a cyclic scenario where a recollapse phase is joined to a bounce phase. In anticipation of this we choose the specific form of our basic variables with motivation from loop quantum cosmology, as explained below, although this does not affect the main results.

### A. Prescription

In the presence of a cosmological constant and a free massless scalar field, the Friedmann equation takes the form

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{4\pi G}{3}\frac{p_\phi^2}{a^6} + \Lambda, \tag{1}$$

where $p_\phi$ is the momentum corresponding to the homogeneous scalar field $\phi$ and can be written as

$$\begin{aligned} p_\phi &= \pm a^2\sqrt{\frac{3}{4\pi G}(\dot{a}^2 + k - \Lambda a^2)} \\ &= \pm 2\sqrt{\frac{8\pi G}{3}}(1-x)V\sqrt{P^2 + kf_0^2\left(\frac{8\pi G(1-x)f_0V}{3}\right)^{2x/(1-x)} - \Lambda f_0^2\left(\frac{8\pi G(1-x)f_0V}{3}\right)^{(1+2x)/(1-x)}} \end{aligned} \tag{2}$$

in terms of canonical gravitational variables

$$V = \frac{3a^{2-2x}}{8\pi G(1-x)f_0} \quad \text{and} \quad P = -f_0 a^{2x}\dot{a} \tag{3}$$

with $\{V, P\} = 1$. The introduction of this pair of canonical variables and the parameters $f_0$ and $x$ is motivated by loop quantum cosmology and deserves further explanation: While $f_0$ will not be of much consequence in what follows, we keep it for general reference. It has dimensions such that $P$ becomes dimensionless; for $x = -1/2$, for instance, it has the dimension of length and for $x = 0$ it is itself dimensionless. Its significance lies in the fact that it determines a fundamental scale for the loop quantization which becomes relevant at the bounce. Moreover, the fundamental length can depend on the evolution of the universe, and thus $a$, if the underlying discreteness of quantum gravity is being refined during evolution. This possibility is taken into account by the parameter $x$, which makes the momentum $P$ depend differently on $a$ for different choices of $x$. How precisely these parameters arise has been discussed, e.g., in [21,22]. The dynamical behavior of loop quantum cosmology is sensitive to their values, but in this paper we will mainly analyze recollapses where effects of the loop quantization are not expected to play large roles. We will nevertheless see that it is of interest to keep all possibilities, especially of $x$. For all choices of $f_0$ and $x$, the variables used here are canonically related to each other, and we emphasize that the choice of the values of the parameters does not qualitatively change our results. Nevertheless, some quantitative aspects can change, and also equations of motion may be easier to solve for some $x$ than others.

Physically, different values of $x$ correspond to different ways in which an inhomogeneous discrete quantum state can be refined during its evolution on microscopic levels [21]. For $x = 0$ the variable $P$ corresponds to an underlying state which has a constant number of lattice sites as the universe expands, while for $x = -1/2$ the state has a constant geometrical size at each lattice site and thus requires new sites to be generated during expansion. A precise value of $x$ could, in principle, be determined if one could derive a reduced Hamiltonian of an isotropic model from a full, inhomogeneous Hamiltonian (such as those introduced in [23]). Since this is not yet available, we have to keep the value of $x$ free and look instead for possible phenomenological constraints.

Classical solutions as functions of $\phi$ are readily determined from the Hamiltonian $H \propto p_\phi$ and its canonical equations of motion in terms of $\phi$, which will be presented below. Such equations of motion determine the relational dependence of, e.g., $V(\phi)$ through the Hamiltonian equation of motion $dV/d\phi = \{V, H\}$. Our main interest, however, is in possible effects which may result from the behavior of quantum states. In particular, a quantum system has not only expectation values as free variables, which could be associated with the classical variables $(V, P)$, but also fluctuations, correlations, and higher moments. Dynamically, all these variables couple in a general quantum system. These coupled equations of motion can be derived from the usual commutator relations such as $d\langle\hat{V}\rangle/d\phi = -i\hbar^{-1}\langle[\hat{V}, \hat{H}]\rangle$ or, more compactly, from a quantum Hamiltonian $H_Q := \langle\hat{H}\rangle$. (For details we refer to [24,25] or, in the context of cosmological models, [12,19].) Here, the expectation value is computed in a state with a general set of moments. As is well known, for a general classical Hamiltonian $H(V, P)$ we have $\langle H(\hat{V}, \hat{P})\rangle \neq H(\langle\hat{V}\rangle, \langle\hat{P}\rangle)$ where the difference amounts to quantum corrections to the classical dynamics. These corrections depend, e.g., on quantum fluctuations or, more generally, on moments

$$G^{a,b} = \langle(\hat{V} - \langle\hat{V}\rangle)^a(\hat{P} - \langle\hat{P}\rangle)^b\rangle_{\text{Weyl}}, \tag{4}$$

of the state used for the expectation values. (In the definition of moments, we assume the basic operators to be totally symmetric or Weyl ordered as indicated by the subscript.) Upon writing $H_Q = \langle\hat{H}\rangle$ in terms of expectation values and the moments, we obtain the complete quantum Hamiltonian. This in turn generates the Hamiltonian equations of motion for [26] $V := \langle\hat{V}\rangle$, $P := \langle\hat{P}\rangle$ as well as all the moments $G^{a,b}$. (As before, the equations of motion are given by $df/d\phi = \{f, H_Q\}$ where $\{V, P\} = 1$ and for $G^{a,b}$ the Poisson brackets follow from expectation values of commutators divided by $i\hbar$.)

This is the basis for the derivation of effective equations which may provide good approximations in regimes where the infinite set of all moments can be truncated to finitely many variables. In the following we shall only consider the second order moments which, for better clarity, we denote as

$$G^{PP} = G^{0,2} = \langle \hat{P}^2 \rangle - P^2 \tag{5}$$

$$G^{VP} = G^{1,1} = \tfrac{1}{2}\langle \hat{V}\hat{P} + \hat{P}\hat{V} \rangle - VP \tag{6}$$

$$G^{VV} = G^{2,0} = \langle \hat{V}^2 \rangle - V^2. \tag{7}$$

Their Poisson brackets can be then derived as in

$$\begin{aligned}\{G^{VV}, G^{PP}\} &= \{\langle \hat{V}^2 \rangle - V^2, \langle \hat{P}^2 \rangle - P^2\} \\ &= \frac{1}{i\hbar}\langle [\hat{V}^2, \hat{P}^2] \rangle - \frac{2P}{i\hbar}\langle [\hat{V}^2, \hat{P}] \rangle - \frac{2V}{i\hbar}\langle [\hat{V}, \hat{P}^2] \rangle \\ &\quad + \frac{4VP}{i\hbar}\langle [\hat{V}, \hat{P}] \rangle \\ &= 2\langle \hat{V}\hat{P} + \hat{P}\hat{V} \rangle - 4VP = 4G^{VP}. \end{aligned} \tag{8}$$

Similarly,

$$\{G^{VV}, G^{VP}\} = 2G^{VV} \quad \text{and} \quad \{G^{VP}, G^{PP}\} = 2G^{PP}. \tag{9}$$

Such Poisson brackets, when used in $dG^{a,b}/d\phi = \{G^{a,b}, H_Q\}$, determine the evolution of the quantum variables of a state. This demonstrates how effective equations are able to go well beyond simple corrections to classical equations, which will be made ample use of in this article.

### B. Negative cosmological constant

For $\Lambda < 0$, $k = 0$, our system has the classical Hamiltonian

$$H = (1 - x) \times V\sqrt{P^2 + |\Lambda| f_0^2 (8\pi\gamma G(1 - x) f_0 V/3)^{(1+2x)/(1-x)}}, \tag{10}$$

for $\phi$-evolution, i.e. $p_\phi = 2\gamma\sqrt{8\pi G/3}H$ (a specific sign has been chosen here for the square root; the other choice simply amounts to replacing $\phi$ with $-\phi$). The factor in $p_\phi$ can be eliminated by redefining $\phi$. Evolution is analyzed best for $x = -1/2$, in which case

$$H = \frac{3}{2}V\sqrt{P^2 + |\Lambda| f_0^2}, \tag{11}$$

is linear in $V$. The corresponding quantum Hamiltonian, including moments of second order, is

$$H_Q = \frac{3}{2}V\sqrt{P^2 + |\Lambda| f_0^2} + \frac{3}{4}|\Lambda| f_0^2 \frac{V}{(P^2 + |\Lambda| f_0^2)^{3/2}} G^{PP} + \frac{3}{2}\frac{P}{\sqrt{P^2 + |\Lambda| f_0^2}} G^{VP}, \tag{12}$$

which includes the quantum moments $G^{PP}$, $G^{VP}$ in correction terms. Higher moments are ignored here, and $G^{VV}$ does not occur thanks to the linearity of $H$ in $V$. (For $\Lambda = 0$ we have the solvable free system, in which no coupling terms between expectation values and moments arise [11].) The quantum Hamiltonian determines the Hamiltonian equations of motion

$$\frac{dV}{d\phi} = \frac{3}{2}\frac{VP}{\sqrt{P^2 + |\Lambda| f_0^2}} - \frac{9}{4}|\Lambda| f_0^2 \frac{VP}{(P^2 + |\Lambda| f_0^2)^{5/2}} G^{PP} + \frac{3}{2}|\Lambda| f_0^2 \frac{G^{VP}}{(P^2 + |\Lambda| f_0^2)^{3/2}} \tag{13}$$

$$\frac{dP}{d\phi} = -\frac{3}{2}\sqrt{P^2 + |\Lambda| f_0^2} - \frac{3}{4}|\Lambda| f_0^2 \frac{G^{PP}}{(P^2 + |\Lambda| f_0^2)^{3/2}}. \tag{14}$$

Quantum fluctuations appear here in coupling terms and are themselves dynamical, subject to equations of motion

$$\frac{dG^{PP}}{d\phi} = -3\frac{P}{\sqrt{P^2 + |\Lambda| f_0^2}} G^{PP} \tag{15}$$

$$\frac{dG^{VP}}{d\phi} = \frac{3}{2}|\Lambda| f_0^2 \frac{V}{(P^2 + |\Lambda| f_0^2)^{3/2}} G^{PP} \tag{16}$$

$$\frac{dG^{VV}}{d\phi} = 3|\Lambda| f_0^2 \frac{V}{(P^2 + |\Lambda| f_0^2)^{3/2}} G^{VP} + 3\frac{P}{\sqrt{P^2 + |\Lambda| f_0^2}} G^{VV}. \tag{17}$$

These equations satisfy

$$\frac{d}{d\phi}(G^{VV}G^{PP} - (G^{VP})^2) = 0$$

such that a state initially saturating the (generalized) uncertainty relation

$$G^{VV}G^{PP} - (G^{VP})^2 \geq \frac{\hbar^2}{4} \tag{18}$$

will keep saturating it. Such a state would be considered a dynamical coherent state whose properties can be analyzed by our equations. In what follows, however, we will not restrict states to be on the saturation surface although they certainly must satisfy the uncertainty relation.

If we first ignore all moments and their quantum back-reaction, we find the classical solutions

$$P_{\text{classical}}(\phi) = P_0 \cosh(3(\phi - \phi_0)/2) + \sqrt{P_0^2 - |\Lambda| f_0^2}\sinh(3(\phi - \phi_0)/2) \tag{19}$$

$$V_{\rm classical}(\phi) = V_0 \frac{\sqrt{P_0^2 + |\Lambda| f_0^2}}{-P_0 \sinh(3(\phi - \phi_0)/2) + \sqrt{P_0^2 + |\Lambda| f_0^2} \cosh(3(\phi - \phi_0)/2)}. \tag{20}$$

The volume has a turning point, and we can simplify expressions without loss of generality by choosing our initial values there, i.e. $P_0 = P(\phi_0) = 0$ and shift $\phi$ such that $\phi_0 = 0$. Then, we have simply

$$P_{\rm classical}(\phi) = -\sqrt{|\Lambda|} f_0 \sinh(3\phi/2) \tag{21}$$

$$V_{\rm classical}(\phi) = \frac{V_0}{\cosh(3\phi/2)}. \tag{22}$$

These solutions describe the recollapse of a universe with a past and a future singularity. Analytical solutions of equations amended by quantum geometry effects, where the singularities are replaced by bounces and thus provide cyclic solutions, have been derived e.g. in [27]. However, quantum backreaction effects, which complicate the analysis, were not included in the equations used there.

In a next step, we can solve the equations of motion (15)–(17) approximately by assuming the classical solutions for $P$ and $V$. Thus, we are still ignoring quantum backreaction effects at this stage, which if present would imply that the moments back-react by the coupling terms in (13) and (14) and change the classical solutions. For small fluctuations, this will be a good approximation, and solutions obtained for the moments will allow us to check self-consistently for how long in $\phi$ it will remain valid.

It is then easy to solve for $G^{PP}$, to give

$$G^{PP}(\phi) = G_0^{PP} \cosh^2(3\phi/2), \tag{23}$$

which shows that $G^{PP}$ is inversely proportional to the volume squared, and which in turn allows to solve for

$$G^{VP}(\phi) = G_0^{VP} + \frac{V_0 G_0^{PP}}{\sqrt{|\Lambda|} f_0} \frac{\sinh(3\phi/2)}{\cosh(3\phi/2)}. \tag{24}$$

With this, one can finally solve for

$$G^{VV} = \frac{G_0^{VV} + 2\frac{V_0 G_0^{VP}}{\sqrt{|\Lambda|} f_0} \tanh(3\phi/2) + \frac{V_0^2 G_0^{PP}}{|\Lambda| f_0^2} \tanh^2(3\phi/2)}{\cosh^2(3\phi/2)}. \tag{25}$$

With quantum backreaction to second order in moments, i.e. solving the full Eqs. (13)–(17) without starting with the classical solutions, the equations are more highly coupled. One can derive some solutions by dividing (14) by (15), thus providing a differential equation for $P(G^{PP})$:

$$\frac{dP}{dG^{PP}} = \frac{P^2 + |\Lambda| f_0^2}{2PG^{PP}} + \frac{1}{4} \frac{|\Lambda| f_0^2}{P(P^2 + |\Lambda| f_0^2)}. \tag{26}$$

This can be written in a simpler form thus

$$\frac{d(P^2 + |\Lambda| f_0^2)}{d \log G^{PP}} = P^2 + |\Lambda| f_0^2 + \frac{1}{2} \frac{\Lambda}{P^2 + |\Lambda| f_0^2} G^{PP}, \tag{27}$$

whose solution yields

$$P = \sqrt{-|\Lambda| f_0^2 + \sqrt{c(G^{PP})^2 - |\Lambda| G^{PP}}} \tag{28}$$

such that

$$\sqrt{P^2 + |\Lambda| f_0^2} = \left(c(G^{PP})^2 - |\Lambda| f_0^2 G^{PP}\right)^{1/4}, \tag{29}$$

with a constant of integration $c$.

### C. Positive spatial curvature

With $\Lambda = 0$ but $k = 1$, the system is simplest to solve for $x = 0$, which makes it again linear in $V$. The quantum Hamiltonian is then the same as before, (12), with $\Lambda$ replaced by $-1$ (and a missing factor of $3/2$ arising from $1 - x$ in the Hamiltonian, which simply rescales $\phi$). We can thus immediately take over the solutions already found. For other values of $x$, the equations are more highly coupled and do not allow simple solutions. Nevertheless, we can use the solutions already provided to find information also about these systems by simply replacing $(V, P)$ in the $x = 0$-solutions by

$$\tilde{V} := \frac{1}{G f_0} ((1 - x) G f_0 V)^{1/(1-x)} \tag{30}$$

$$\tilde{P} := \frac{P}{((1 - x) G f_0 V)^{x/(1-x)}} = P/(G f_0 \tilde{V})^x. \tag{31}$$

(We have chosen the factors of $G$ and $f_0$ such that $\tilde{P}$ has the same dimensions as $f_0$, which will be useful later.) This has to be done also in the moments, i.e. we will obtain their solutions not for $G^{VV}$, say, but for $G^{\tilde{V}\tilde{V}}$. These are not directly the fluctuations of our basic variables for $x \neq 0$ but they still give important information about the spreading and other properties of states. For instance, we will determine the correlation $G^{\tilde{V}\tilde{P}}$ instead of $G^{VP}$. Both parameters contain equally interesting information about squeezing and the symmetry of fluctuations around the recollapse. In particular, if $G^{\tilde{V}\tilde{V}}$ is not symmetric around the recollapse, then nor will be $G^{VV}$.

## III. IMPLICATIONS

Several conclusions can be drawn from the solutions found to the given order.

### A. Volume ratio between recollapse and high curvature regimes

Our solutions correspond to state parameters in a Wheeler-DeWitt quantization because we use elementary variables $(V, P)$ which are assumed to be quantized to well-defined operators. Those operators, together with the Hamiltonian, then determine the dynamics. The latter have not been written explicitly here, but they are the central ingredient to Hamiltonian equations of motion via the Poisson brackets of quantum variables such as (8) and (9).

The Wheeler-DeWitt quantization does not easily solve the singularity problem. For models without quantum backreaction effects, i.e. spatially flat models sourced by a free massless scalar, $\langle\hat{V}\rangle$ simply follows the classical trajectory into the singularity. On the other hand, in general models such as those considered here, there are quantum backreaction effects which one may expect to become stronger as the solution for $V$ approaches zero—the classical singularity. This could stop $V$ altogether, or delay its approach to zero sufficiently strongly such that zero would not be reached in a finite amount of proper time (but possibly still finite in $\phi$). However, this is difficult to analyze if all moments are required, and unlikely to result in a generic resolution of singularities.

A loop quantization does provide a natural solution of the singularity problem in isotropic models, but it requires one to use a different set of basic variables. (At a basic level, singularities in homogeneous and spherically symmetric models have been shown to be absent by allowing general wave functions to be extended through classical singularities [28–31]. More specific examples for bouncing wave packets are derived in [7,11]. For a discussion and comparison of results concerning singularities see [2].) While $V$ would still be represented as an operator in the quantization, the curvature (or connection) component $P$ is not. Instead, loop quantum gravity is based on a quantum representation in which only holonomies of the Ashtekar connection are represented, in this way providing the kinematical structures for a well-defined, background independent quantization of full gravity [32–34]. In the cosmological models studied here, this means that it is not $P$ which is part of the elementary algebra but $\exp(i\mu P)$, for arbitrary real $\mu$. (Note it is $P$ which enters here, rather than $\tilde{P}$ of (30) because $x$ represents the freedom in the refinement of a discrete underlying state and thus determines the form of holonomies in a reduced isotropic setting [21]. This is, in fact, the main reason why we allow for different values of $x$.) Using the exponential instead of an expression linear in $P$ changes the basic algebra as well as the Hamiltonian, in particular, at large $P$. In a flat, isotropic model with a free scalar field, the classical singularity is then resolved and replaced by a bounce.

To study the oscillating models we need to consider a combination of bounces and recollapses which is more complicated because of the structure of required quantum evolution equations. Nevertheless, one can study cyclic solutions by patching together bounce and recollapse phases. For small curvatures, we can use the equations and the corresponding solutions provided in this paper to an excellent approximation, even for a model of loop quantum cosmology. However, we can use this only when $P$ is not too large and have to cut off our solutions at the latest when $|P| \sim 1$. (At this point, the precise value of $f_0$ would set the corresponding scale for $\dot{a}$.) This leaves only a finite range of sizes for the universe between this high curvature regime and the recollapse. The high curvature regimes can also be described by effective equations, which are in fact precise without quantum backreaction, but require a different set of basic variables [11].

For a negative cosmological constant, we have the ratio $V_0/V_{|P|=1} = \sqrt{1 + 1/|\Lambda| f_0^2}$. Thus, for a small cosmological constant compared to $f_0^{-2}$, the ratio is huge. Since $f_0$ arises from quantum gravity and has the dimension of length in this case which is based on $x = -1/2$, $f_0$ should take a value near the Planck length. Thus, $|\Lambda|$ must only be small compared to a Planckian value which can safely be assumed to be the case. For the closed model with $x = 0$, on the other hand, we have $V_0/V_{|P|=1} = \sqrt{1 + 1/f_0^2}$ with a dimensionless $f_0$. In this case, there are no strong reasons to expect quantum gravity to provide a value of $f_0$ small compared to one (without reference to a second scale larger than the Planck length, which should not appear in the basic variables $V$ and $P$ where $f_0$ enters). This is certainly not enough for a macroscopic universe which has to grow large out of the high curvature regime. For this reason we have to use other values for $x$ in this case: then, $|P| = 1$ is reached at much smaller values for $\tilde{P}$ as provided by our solutions. Although the qualitative behavior is unchanged compared to other $x$, changes in $x$ have an important quantitative implication (which was first emphasized in [10]). For example for $x = -1/2$, the high curvature regime starts at

$$\cosh(\phi) \sim \frac{1}{6}\left(108C + 12\sqrt{-12 + 81C^2}\right)^{1/3} + \frac{2}{(108C + 12\sqrt{-12 + 81C^2})^{1/3}}$$

where $C = (Gf_0V_0)^{2/3}/f_0^2$. For large $V_0$, this is approximately $\cosh(\phi) \approx V_0^{2/9}$ (or, for general $x \neq 0$, $\cosh(\phi) \approx V_0^{-x/(1-x)^2}$). Thus, the ratio $V_0/V_{|P|=1} \approx V_0^{-x/(1-x)^2}$ is no longer constant and grows with $V_0$ for negative $x$. For $x = -1/2$, the ratio is given by $V_0^{2/9}$ which is large enough for large $V_0$, leaving ample room for a growing universe.

### B. Quantum backreaction effects

From our solutions we can determine whether quantum backreaction effects are strong around the recollapse. As one can easily see, there are no possible divergences in the equations of motion (13) and (14) which would enhance the coupling terms. Quantum backreaction effects can only be strong if the quantum variables are large, which can be avoided at least for some time by choosing a semiclassical initial state. Thus, the equations to the order provided here are reliable to a high degree and can be used to determine the state properties around the recollapse. In particular, our equations of motion and solutions for quantum variables themselves can be used to see how long the approximation remains valid.

### C. Evolution of the spread

Of particular interest is whether fluctuations depend strongly on $\phi$ or remain nearly constant during the evolution. If they change rapidly, the behavior of neighboring cycles would be noticeably different from each other because the state would have changed significantly. In scenarios with a large or an infinite number of cycles, large differences should even be generic between widely separated cycles.

As we have seen, $G^{PP}$ is always proportional to the inverse volume squared when quantum backreaction effects can be ignored. Thus, curvature fluctuations must be symmetric around the recollapse and do not change significantly: At any volume after the recollapse we have the same $G^{PP}$ as at the same volume before. For the other quantum variables, however, the situation is different. Ignoring products of quantum variables, we can rewrite (17) approximately as

$$\frac{d}{d\phi}\left(\frac{G^{VV}}{V}\right) = 3|\Lambda|f_0^2\frac{G^{VP}}{(P^2+|\Lambda|f_0^2)^{3/2}}, \tag{32}$$

for a negative cosmological constant with $x = -1/2$ or

$$\frac{d}{d\phi}\left(\frac{G^{VV}}{V}\right) = \frac{2G^{VP}}{(P^2+f_0^2)^{3/2}}, \tag{33}$$

for a closed model with $x = 0$ and $\Lambda = 0$. This shows that $G^{VV}$ would be a function only of $V$, and thus symmetric around the recollapse, if $G^{VP} = 0$, i.e. the state is unsqueezed. One may assume this as an initial condition, but $G^{VP}$ itself is dynamical and subject to the evolution equation (16). Its time derivative cannot be zero since, thanks to the uncertainty relation, $G^{PP}$ is nonzero unless volume fluctuations diverge. Even an initially unsqueezed state will become squeezed after some time, and thus also affect the volume fluctuations.

Even if $G^{VV}/V$ is not constant, $G^{VV}$ may be symmetric around the recollapse but behave differently with respect to $V$. In fact, (25) shows that $G^{VV}$ is symmetric around the recollapse if $G_0^{VP}$, i.e. the correlation at the recollapse, vanishes even though $G^{VP}$ would become nonzero away from the recollapse. But since this happens only under the special condition of $G_0^{VP} = 0$, it could generically be satisfied only in one cycle of an oscillatory universe.

From (24), we can estimate the change in squeezing per recollapse by

$$\lim_{\phi\to\infty} G^{VP}(\phi) - \lim_{\phi\to-\infty} G^{VP}(\phi) = \frac{2V_0G_0^{PP}}{\sqrt{|\Lambda|}f_0} \tag{34}$$

as an upper bound. The change may be small for small fluctuations $G_0^{PP}$, but is enlarged by a factor of $V_0$ (as well as $1/\sqrt{|\Lambda|}f_0$ in the presence of $\Lambda < 0$, which is large given that $|\Lambda|f_0^2$ is small; if the recollapse is triggered by positive spatial curvature, we have the same formula with $\Lambda$ set to $-1$). In a large universe, this change can be quite significant. Note that in (34) we have used $\phi \to \pm\infty$, and thus a range which includes the high curvature regimes where the equations have to be amended by effects of the loop quantization and the specific solution would change. We can take this into account by reducing the range of $\phi$; however, this does not change the result but only affects the numerical factor in the change of squeezing. There is thus a significant change during the classical recollapse, irrespective of how the high curvature regime is dealt with. For instance, we have

$$G^{VP}|_{\sinh(3\phi/2)=1} = G_0^{VP} + \frac{V_0G_0^{PP}}{\sqrt{2|\Lambda|}f_0}, \tag{35}$$

whose numerical coefficient is different, but which still carries the large factor of $V_0$. In fact, the tanh- behavior of $G^{VP}$ demonstrates that the greatest change in correlations occurs near the recollapse.

To quantify the production of squeezing during recollapse phases, it may be helpful to transform the solution for $G^{VP}(\phi)$ to proper time rather than using the relational formulation with respect to $\phi$. The relation between proper time $\tau$ and $\phi$ can in general be complicated, but can easily be obtained for $x = -1/2$ by integrating

$$\frac{d\phi}{d\tau} = \frac{p_\phi}{V} = \frac{p_\phi}{V_0}\cosh(3\phi/2)$$

to obtain

$$\phi(\tau) = \tfrac{2}{3}\mathrm{arsinh}(\tan(3p_\phi(\tau-\tau_1)/2V_0)).$$

Without loss of generality, we chose $\phi$ to vanish at $\tau_1$, which may be different from the recollapse time $\tau_0$. The whole range $-\infty < \phi < \infty$ corresponds to a finite proper time interval $-V_0\pi/3p_\phi < \tau - \tau_1 < V_0\pi/3p_\phi$. This highlights the fact that we are not including effects of the loop quantization, such that the endpoints of the $\phi$-range, where the volume vanishes, correspond to future and past singularities a finite proper time away.

Inserting this in the solution (24) for $G^{VP}$, we obtain

$$G^{VP}(\tau) - G^{VP}(\tau_0) = \frac{V_0 G_0^{VP}}{\sqrt{|\Lambda|} f_0} \sin(3p_\phi(\tau - \tau_1)/V_0), \tag{36}$$

which shows the growth of squeezing in proper time during each recollapse (which is in fact monotonic in the given range of $\tau$).

Starting with an initially unsqueezed state it may seem that for many cycles the state remains almost unchanged from cycle to cycle. Its volume fluctuations may always seem to attain nearly the same size at the same volume. However, this is so only because of the special initial state chosen, from which squeezing builds up slowly. For small $G^{VP}$, (32) and (33), respectively, we show that $G^{VP}/V$ is nearly constant in both cases considered. The change in volume fluctuations before compared to after the recollapse seems insignificant from cycle to cycle but becomes noticeable over many cycles. Moreover, if the initial state had already had some squeezing, volume fluctuations relative to volume would change much more rapidly. In this way, the choice of initial state can strongly influence the long-term behavior.

In a cyclic model, it is especially important to ask what significance one should attribute to the choice of initial state. Is it to be posed in "our" cycle, and if not, how many cycles ago? If we could have observational input on properties of the state, we could certainly pose an initial condition in our cycle and see how the state evolves to or from there. However, state properties are hardly under control, and this possibility remains elusive. We thus have to pose initial conditions many cycles ago based on some general principle of emergence, but we never know how many. Thus, even though we know that an initially unsqueezed state builds up squeezing only slowly, this does not say much about the present state if we do not know how many cycles ago the state was unsqueezed.

An interesting question is whether in a cyclic model one generically expects to have a finite or an infinite number of past cycles. The problem with the finite case is that it does not resolve the origin question. In the emergent scenarios [35–39], as well as some other such models, the universe is assumed to have undergone an infinite number of past cycles so as to remove the question of the origin. In that case any given cycle would have an infinite number of precursors and generically we therefore have to expect the current state to be squeezed. (We will argue in the next subsection that bounces do not affect the qualitative behavior of the squeezing, especially its monotonicity). The question then is how the squeezing in a generic cycle is determined. If each cycle produces the same amount of squeezing, a generic cycle would have infinitely squeezed states, which could not be semiclassical. However, as (34) shows, the amount of new squeezing per cycle depends on the recollapse volume $V_0$ of that cycle. For growing cycles, as in the emergent scenario, the change in squeezing is initially small and approaches zero for cycles in the infinitely distant past. Depending on the precise scenario, the sum of all squeezing contributions may converge, such that a finite value results for a generic cycle. Whether this is the case and what this precise value could be depends on which concrete model one is using, and we will not follow this route here. It is, however, interesting that this in principle allows one to restrict the possibilities for emergent scenarios by the amount of squeezing they would predict.

Another interesting and related question is that in the emergent models the eventual nonuniformity of cycles is produced by a nonconstant potential. (In initial regions where the potential is flat, the universe would just periodically oscillate around the center point; the eventual asymmetric emergence is induced by a nontrivial change in the underlying potential.) This raises the question of what happens to these models when treated quantum mechanically. Taking the case of a negative cosmological constant, corresponding to a constant negative potential, as a guide suggests that, even though in the flat regions of the potential there is a classical symmetry expressed by the exact periodicity in the dynamics, we nevertheless acquire a quantum mechanical asymmetry due to the evolution in squeezing. A more complicated question is what happens in the regions where there is already a classical asymmetry induced by the nonflat potential.

To be specific, let us look at the closed model with $x = 0$, while including a scalar potential $W(\phi)$. In this case, $\phi$ will no longer serve as a global internal time, but it is still a good indicator of local internal time in phases where $\phi$ is monotonic (i.e. outside zeros of $p_\phi$). In this way, we can still draw conclusions for the behavior of quantum variables near a recollapse. In this case we have the Hamiltonian

$$H = V\sqrt{P^2 + f_0^2 - 8\pi\gamma G W(\phi) f_0^3 V/3}, \tag{37}$$

and a corresponding quantum Hamiltonian

$$\begin{aligned} H_Q = {}& V\sqrt{P^2 + f_0^2 - 8\pi\gamma G W(\phi) f_0^3 V/3} \\ &+ \frac{1}{2}\frac{V(f_0^2 - 8\pi\gamma G W(\phi) f_0^3 V/3)}{(P^2 + f_0^2 - 8\pi\gamma G W(\phi) f_0^3 V/3)^{3/2}} G^{PP} \\ &+ \frac{P(P^2 + f_0^2 - 4\pi\gamma G W(\phi) f_0^3 V/3)}{(P^2 + f_0^2 - 8\pi\gamma G W(\phi) f_0^3 V/3)^{3/2}} G^{VP} \\ &- \frac{4\pi\gamma G W(\phi) f_0^3 (P^2 + f_0^2 - 2\pi\gamma G W(\phi) f_0^3 V)/3}{(P^2 + f_0^2 - 8\pi\gamma G W(\phi) f_0^3 V/3)^{3/2}} \\ &\times G^{VV}, \end{aligned} \tag{38}$$

expanded to moments of second order. In contrast to the previous cases, this includes not only the quantum moments $G^{PP}$, $G^{VP}$ but also $G^{VV}$ in correction terms. The quantum Hamiltonian then determines equations of motion, which for $G^{VP}$ results in

$$\frac{dG^{VP}}{d\phi} = \frac{V(f_0^2 - 8\pi\gamma GW(\phi)f_0^3V/3)}{(P^2 + f_0^2 - 8\pi\gamma GW(\phi)f_0^3V/3)^{3/2}}G^{PP} + \frac{8\pi\gamma GW(\phi)f_0^3(P^2 + f_0^2 - 2\pi\gamma GW(\phi)f_0^3V)/3}{(P^2 + f_0^2 - 8\pi\gamma GW(\phi)f_0^3V/3)^{3/2}} \times G^{VV}. \quad (39)$$

Since $P^2 + f_0^2 - 8\pi\gamma GW(\phi)f_0^3V/3$ is required to be positive and $P$ is small near a recollapse, the sign of this expression remains unchanged compared to the free model. Thus, inclusion of a potential does not change our monotonicity result. Notice that we have not assumed the potential to be small since the analysis involves only an expansion in moments rather than in $W(\phi)$. The rate of change of correlations depends on the value of the potential, but it has a definite sign: $G^{VP}$ is either growing or decreasing during a recollapse phase. We should note that the rate of change of $G^{VP}$ is according to Eq. (39) defined with respect to $\phi$, which would seem to indicate that it should change sign as $\phi$ goes through a turning point. This is, however, not the case since for a given potential at a turning point of $\phi$ the sign of $p_\phi$, which is our quantum Hamiltonian $H_Q$, also changes. Thus, at such a turning point there is also a sign flip in the equation of motion, which ensures that the rate of change of $G^{VP}$ remains globally monotonic even though in this case $\phi$ is no longer a global monotonic time variable. A varying potential will affect the rate by which $G^{VP}$ changes, and thus lead to different absolute changes in squeezing before and after the recollapse. But correlations will always change, and thus our qualitative discussion remains unaltered in this case.

In the cyclic models with many cycles one can only draw conclusions from the consideration of generic rather than special initial states. Thus one needs to consider the consequences of generic initially squeezed states, rather than special unsqueezed initial states. While a state may have been uncorrelated at some time, we cannot know how many cycles ago this may have been, or after how many cycles it may be so in the future. For statements relevant to a single cycle, which are the only ones with a chance of being observable, it is not legitimate to use special initial states which are known to change between cycles. In fact as can be seen from Eqs. (32) or (33), there is no strong bound on the change of volume fluctuations relative to volume from one cycle to the next without a sharp limit on correlations. Quantum properties of the collapse phase can thus differ from those of the expansion phase. As (25) shows, the time-asymmetric term has a single factor of $V_0$, while the last term is multiplied with $V_0^2$. One can thus expect that the asymmetry is not pronounced strongly for a universe of large recollapse size $V_0$, but the precise behavior depends also on the moments. Then, the last term containing $V_0^2$ is suppressed by a factor of $G_0^{PP}$ which must be small near the recollapse where $P = 0$; see

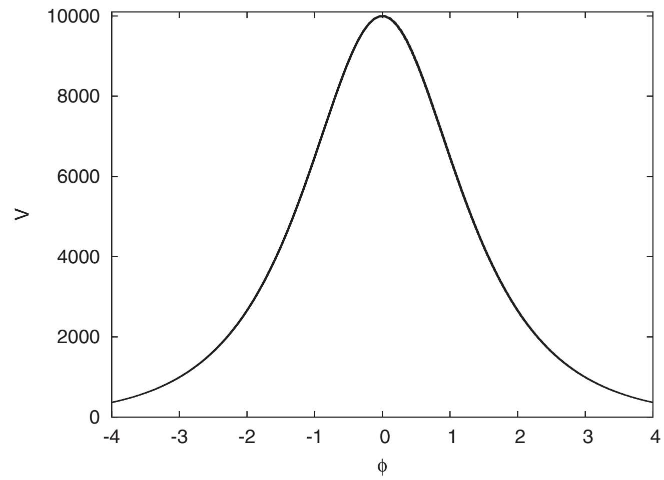


FIG. 1. An example of a recollapsing universe which grows to large volume. Plotted are the volume expectation value $\langle\hat{V}\rangle(\phi)$ as well as the fluctuations around it. While the detailed behavior of the fluctuations cannot be discerned from this total plot, the asymmetry around the recollapse is clearly visible in the zoom shown in Fig. 2.

Fig. 1 for a numerical example. Moreover, over several cycles the change in quantum properties will add up.

Correlations in a semiclassical state are bounded, and so $G^{VP}$ is restricted but may certainly vary. And as long as it can easily be nonzero and affect the behavior of single cycles, it must be taken into account in cyclic models with many cycles. Moreover, in addition to the recollapse

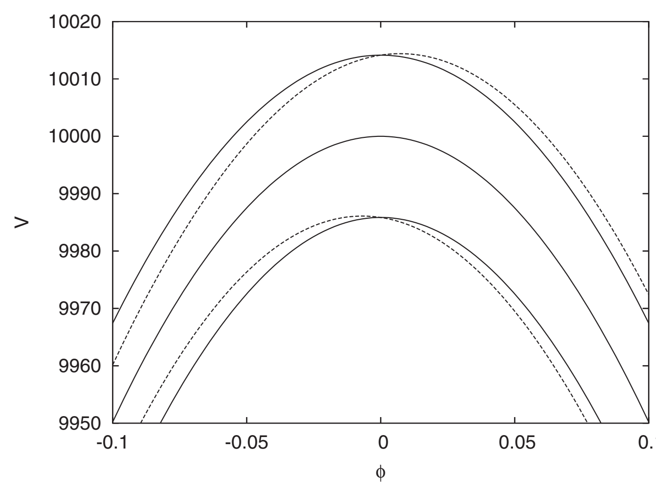


FIG. 2. The recollapse phase of Fig. 1 in more detail. The central line is the volume expectation value, and solid lines around it illustrate the spread $\Delta V$ of a state which is symmetric around the bounce. Dashed lines show how asymmetric the volume fluctuations can be if the state is correlated at the recollapse. Initial conditions are set at the recollapse, where in units with $\hbar = 0.2$ we have $V_0 = 10^4$, $G_0^{VV} = 200$, $G_0^{PP} = 10^{-4}$, and $G_0^{VP} = 0$ for the solid lines and $G_0^{VP} = 0.1$ for the dashed lines. The latter state thus saturates the generalized uncertainty relation (18).

phases discussed here, squeezing has a similar influence on the asymmetry of fluctuations around the bounce [12]. This shows that the different cycles of a universe can indeed be very different from each other, even though they are connected by deterministic evolution of an underlying state. The generic behavior of quantum properties is much more subtle than the assumption of unsqueezed states would suggest. Current knowledge is insufficient to determine what came before, or what will come after.

### D. Entropy

A central question in cosmology is how to successfully define a notion of cosmological entropy, and a number of attempts have been made in this direction. This is in turn hoped to provide a notion of cosmological arrow of time. The notion of entropy is connected to that of information associated with the degrees of freedom considered. In addition to the usual thermodynamic entropy which is normally associated with matter/energy degrees of freedom of the constituent components of the universe [17], possible notions of entropy associated with the geometrical [40] and gravitational [41] degrees of freedom of the universe have also been put forward.

Motivated by the thermodynamical notion of entropy and the associated second law of thermodynamics a necessary, but not sufficient, condition that has often been required of general notions of entropy is that of monotonicity in time. An important step has therefore often been to look for variables defined in terms of the underlying dynamics that evolve monotonically. In addition to notions of entropy associated with classical degrees of freedom, one would also expect entropic measures associated with the quantum mechanical degrees of freedom. An immediate question that any such measure needs to answer concerns the nature of its relationship with the thermodynamic measure of entropy. In particular an important question in the case of recollapsing/oscillating cosmological models is: do (or should) the expanding and recollapsing evolutionary phases possess oscillating or monotonic entropies? Furthermore, how should the entropy associated with different cycles evolve in oscillating models?

This question has in fact been the subject of a long standing and intense debate concerning the relation between the so-called thermodynamical and cosmological arrows of time [40,42]. The question is whether the observed asymmetric (monotonic) thermodynamical time arrow in the current expanding phase of the universe has a counterpart in cosmology, particularly in a recollapsing universe. A number of studies have been made in this connection [43–46]. Given the absence of a dynamical explanation for the observed asymmetry in the universe, most such studies assume that the observed thermodynamic arrow of time must arise from the boundary conditions of the universe [45,46].

Our results above seem to indicate that the degree of the squeeze of the quantum gravity state may provide a notion of entropy purely associated with quantum degrees of freedom. To the best of our knowledge, this is a new possibility not considered before. (Relating entropy to the squeezing of a matter state, however, has been considered in the context of particle production; see e.g. [47–54].) As can be seen from (16) and (39), the squeezing of a state is strictly monotonic in time during expansion, recollapse, and contraction of a cycle in the models considered. This demonstrates that even in isotropic models, which include the microscopic dynamics only in a highly averaged form, quantum aspects prevent one from viewing a collapsing universe simply as a time-reverse of its expansion. The quantum theory's arrow of time cannot reverse at the recollapse.

Unfortunately, it is difficult to follow its evolution through a bounce because this phase can only be described in a different set of basic variables [$J = V\exp(iP)$ for $P$] which make the equations solvable. For classical variables, these are easily translated into each other. But the transformation is nonlinear, such that moments transform in a highly complicated way. In any case, the change in squeezing is nevertheless generic because it is unlikely that the bounce will restore fluctuations to precisely the value of the preceding cycle. Moreover, one can roughly estimate the squeezing as it evolves through the bounce. In the bounce phase, only operators such as $\hat{J} := \hat{V}\exp(i\hat{P})$ exist and give rise to a solvable evolution. Moments between $V$ and $J$ can thus be computed exactly [12], but it is difficult to transform between the $V$-$J$ and $V$-$P$ moments. However, the bounce happens near $P \approx \pi/2$, and with $\delta P := P - \pi/2$ we have, up to reordering, $\mathrm{Re}J = \langle\hat{V}\cos(\hat{P})\rangle = \langle\hat{V}\cos(\pi/2 + \delta\hat{P})\rangle = -\langle\hat{V}\sin\delta\hat{P}\rangle \sim -\langle\hat{V}\delta\hat{P}\rangle = -\langle\hat{V}(\hat{P} - \pi/2)\rangle$. Thus, $\mathrm{Re}J + VP - \frac{\pi}{2}V \sim -G^{VP}$ provides an estimate for the $V$-$P$ squeezing as it evolves through the bounce in terms of expectation values. Since expectation values are symmetric around the bounce in the absence of a potential, not much additional squeezing is generated around the bounces.

Most of the squeezing is thus generated in the recollapse phases, which resembles recent results for cyclic models with bounces based on the Hagedorn phase of string theory [55]. In the present context with a quantum measure for entropy in the form of squeezing, this may seem counterintuitive given that the recollapse is a much more classical phase than the bounce. However, the production of correlations is not so much a matter of quantum versus semiclassical behavior but rather of the dynamics in a given regime. A state may remain semiclassical to an excellent degree, and yet receive a significant amount of squeezing. Whether or not this happens depends on the equation of motion for $G^{VP}$, or the underlying Hamiltonian. The analysis presented in this article unambiguously shows the production of correlations in a recollapse phase even though it is semiclassical. Although our qualitative estimates for the bounce phases are difficult to make precise, the monotonic behavior of correlations at small curvature appears to be an interesting and reliable property.

The precise amount of squeezing depends on initial conditions. If all moments could initially be zero, they would remain so and no squeezing would develop. However, this initial condition is impossible because the moments are subject to the uncertainty relation (18). Thus, unless the volume uncertainty diverges, $G^{PP}$ cannot be zero in (16) and an initially unsqueezed state inevitably develops squeezing over time which can grow large over many cycles. It is thus quantum uncertainty, together with the specific dynamics of the system, which prevents the existence of perfectly symmetric states.

There is a sense in which small squeezing presents a special state with a distinguished discrete symmetry. Under time reversal, we map $\phi \mapsto -\phi$, $P \mapsto -P$, and $G^{VP} \mapsto -G^{VP}$ while the other variables remain unchanged. Thus, a time reversible solution would have vanishing squeezing which one may view as a special state analogous to low entropy. As (25) shows, this is obtained for vanishing correlations at the recollapse. However, since $G^{VP}$ would generically be nonzero at a recollapse, especially in a cyclic model, there is no solution which is exactly time reversible. Again, it is the uncertainty relation as an additional condition, which eliminates those initial values which would correspond to time-reversal solutions.

## IV. CONCLUSIONS

We have studied the evolution of recollapsing models within an isotropic and homogeneous quantum cosmological framework in presence of a scalar field. To allow a recollapse we consider, in turn, a negative cosmological constant as well as a positive curvature model. We derive the resulting quantum evolution equations to second order in moments of a state and study their effects on the recollapsing dynamics of the universe, i.e. the expanding, turn around and contracting phases. These effective equations allow us to observe that state properties generically change during the recollapse, making quantum fluctuations in the expansion and contraction phases different. At large volumes as they are realized at a recollapse, the change is not as noticeable as it can be for states travelling through a bounce [18,19], but it is significant especially in a cyclic model with several recollapse phases. As in the case of the bounce, the asymmetry of fluctuations is controlled by quantum correlations which have often been ignored in previous studies.

The specific equations analyzed here thus allow us to identify correlations as a quantum measure for the change of fluctuations. More precisely, we find that the squeezing of an initial state is strictly monotonic in time throughout these three phases for the models considered. Importantly, we have shown this finding to be robust under the inclusion of a matter potential. Combining these results with the corresponding ones concerning a bounce in loop quantum cosmology we have shown that squeezing of an initial state evolves monotonically throughout a whole cycle. The absence of perfectly symmetric states is a combined consequence of the specific dynamics of the quantum system together with the presence of quantum uncertainty.

Such monotonicity is of potential importance in two regards. First, it sheds new light on a long standing intensive debate concerning the (a)symmetry between the expanding and contracting phases in a recollapsing universe. As shown here, the contracting phase cannot be a time reverse of the expanding phase. Second, it motivates the adoption of the degree of squeezing as an alternative measure of (quantum) entropy.

Qualitatively, we also consider the evolution of the squeezing of an initial state in emergent nonsingular oscillating universes in which the universe is assumed to have undergone a large (possibly infinite) number of past cycles. We argue that the consideration of the amount of squeezing in the universe can in principle provide some constraints on the viability of such emergent models. In any case, given that a generic cycle does have nonvanishing correlations, squeezings of the quantum gravity state must be taken into account in order to draw reliable conclusions about cyclic models.

## ACKNOWLEDGMENTS

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