Typicality derived

Don N. Page[*]

Institute for Theoretical Physics, Department of Physics, University of Alberta,
Room 238 CEB,
11322-89 Avenue, Edmonton, Alberta, Canada T6G 2G7[+], and
Asia Pacific Center for Theoretical Physics, Pohang 790-784, Korea

Hartle and Srednicki have suggested that standard quantum theory does not favor our typicality. Here an alternative version is proposed in which typicality is likely, *Eventual Quantum Mechanics*. This version allows one to calculate normalized probabilities for alternatives obeying what I call the *principle of observational discrimination*, that each possible complete observation or data set should uniquely distinguish one element from the set of alternatives.

I. INTRODUCTION

Hartle and Srednicki [1] use a type of probabilistic reasoning that includes standard quantum theory to argue that "it is perfectly possible (and not necessarily unlikely) for us to live in a universe in which we are not typical." However, this leads to their conclusion (iv): "Cosmological models that predict that at least one instance of our data exists (with probability one) somewhere in space-time are indistinguishable no matter how many other exact copies of these data exist." If one were forced to abide by that limitation, then [2] a huge variety of cosmological models giving sufficiently large universes would predict nearly unit probability for our data set and hence the same likelihoods. Thus observations would count for nothing in distinguishing between these theories, and much of cosmology would cease to be an observational science.

Hartle and Srednicki [1] note that a common kind of reasoning in cosmology starts from an assumption that some property of human observers is typical. They cite [3–13] as giving examples of this reasoning, which they question. They point out that this reasoning would not be valid in a version of quantum theory in which highly atypical observations are not highly unlikely. On the other hand, more recent arguments against the conclusions of Hartle and Srednicki have been given in [2,14–17]. For example, Bousso, Freivogel, and Yang argue [17] that "the Hartle-Srednicki prescription would put an end to experimental science. It would render all experiments pointless, because we could not reject any theory until we know how many other laboratories there are. Given the success of the scientific method thus far, we may conclude the Hartle-Srednicki prescription is inappropriate."

How can we rescue science from the dire conclusions Hartle and Srednicki draw from standard quantum theory and other similar types of probabilistic reasoning? Here I

argue that this can be done by reformulating quantum theory so that it gives normalizable probabilities for the alternatives of all possible distinct observations.

Here I shall define *standard quantum theory* to be any version of quantum theory in which observably distinct alternatives are restricted to orthogonal projection operators (with the probabilities of these alternatives being given by the expectation values of the corresponding orthogonal projection operators). Such a quantum theory may be suitable for quantum states in which there is no more than one copy of any observer (or set of communicating observers, or civilization, or human scientific information gathering and utilizing system, HSI [1], though here I shall henceforth just say "observer" for any of these possibilities). Then different possible observations by that observer presumably can be described by orthogonal projection operators. However, for cosmological quantum states for a universe sufficiently large that there is more than one copy of an observer that can jointly make distinct observations, these distinct observations need not correspond to orthogonal projection operators. Therefore, standard quantum theory is not able to assign normalizable probabilities to such sets of distinct alternative observations.

For example, suppose we consider the observation of how many heads occur when two coins are tossed in a certain recorded way. There are three possible distinct observations for the numbers of heads that occur in one tossing of two coins (0, 1, and 2). If only one set of two coins is tossed (e.g., by only one observer), then these distinct observations can be assigned orthogonal projection operators. If one has a quantum state in which it is definitely true that exactly one set of two coins is tossed, and each head is observed to land definitely heads or tails, then the expectation value of each of the three projection operators is interpreted in standard quantum theory to be the probability for that number of heads, and these probabilities are normalized to sum to unity.

However, if there is more than one tossing of two coins each (say by more than one copy of an observer), then

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[*]don@phys.ualberta.ca
[+]Permanent address.

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distinct observations for the numbers of heads do *not* correspond to orthogonal projection operators. For example, one can have N_1 heads in one of the tossings (say by one copy of the observer) and N_2 heads in a second tossing of two coins (say by another copy of the observer), and even if $N_1 \neq N_2$, these distinct observations do not correspond to orthogonal projection operators. In nonquantum language, one says that these two distinct observations are not mutually exclusive, since both can occur (one for each copy of the observer). If one calculates the expectation values of the projection operators corresponding to all the distinct three observations of the number of heads in a tossing, these expectation values will have a sum that is greater than unity.

For example, if the coin is fair, then for one tossing of two coins the probability of 0 heads is 1/4, of 1 head is 1/2, and of 2 heads is 1/4, which sum to unity. However, for two tossings of two coins each, the probability is $1 - (1 - 1/4)^2 = 7/16$ for the existence of an observation of 0 heads, $1 - (1 - 1/2)^2 = 3/4$ for the existence of an observation of 1 head, and $1 - (1 - 1/4)^2 = 7/16$ for the existence of an observation of 0 heads, which sum to 13/8, greater than unity. That is, the expectation value for the projection operator for at least one observation of 0 heads is 7/16, for at least one observation of 1 head is 3/4, and for at least one observation of 2 heads is 7/16. When one has in mind a view that encompasses both coin tossings, one says that these three possibilities for the number of heads observed in a tossing are not mutually exclusive, since, for example, there can be both an observation of 1 head (in one tossing by one observer) and of 2 heads (in the other tossing by the other copy of the observer). Therefore, the three projection operators are not orthogonal, and the sum of their expectation values can be greater than unity.

If one had observational access to both coin tossings, one could avoid this problem by taking a finer-grained set of projection operators, each the product of the projection operator onto a certain number of heads for a particular one of the tossings of two coins and of the projection operator onto a certain number of heads for the other one of the two tossings of two coins. Then one would get nine projection operators, one for each ordered pair of the number of heads for each of the two tossings. These nine projection operators are all orthogonal, and their expectation values will sum to unity if the quantum state gives no other possibilities (e.g., possibilities that not both coins are tossed twice or that not all coins each fall heads or tails).

This all works well in laboratory experiments in which one has observational access to all the relevant possibilities. However, in cosmology in which there may be experiments being made far away by distant copies of the observer, for which one does not have observational access by the copy here on earth, one cannot distinguish all of the alternatives corresponding to a full set of orthogonal pro-

jection operators. For example, in the coin-tossing experiment in which two tossings occur, one might only have access to the observation of the number of heads for one tossing, and one might not even be able to distinguish which tossing one is observing (e.g., the copies of the observer making the observation might not have any distinguishable data). Then one cannot construct from standard quantum theory projection operators which distinguish the distinct observations (whether 0, 1, or 2 heads) and which also are orthogonal. As given in the example above, if one uses projection operators onto the set of the three possible distinct observations, they are not orthogonal and can have expectation values whose sum exceeds unity.

When the probabilistic reasoning of Hartle and Srednicki is cast into the language of standard quantum theory, it uses the following technique to avoid the problem of the nonorthogonality of the set of projection operators for the distinct three observation possibilities: It uses the actual observation of the one observer to select the corresponding projection operator and its complementary projection operator (the identity operator minus the projection operator onto the actual observed result). These two projection operators certainly are orthogonal and sum to the unit operator, so in a quantum state which is normalized (which we shall always assume), the sum of the expectation values of these two projection operators is unity. However, these two orthogonal projection operators do not correspond to the results that are observationally distinguishable to any single copy of the observer.

For example, assume that there are two tossings of two fair coins, but that the observer of one of the two tossings cannot distinguish which of the two tossings he or she is observing. (The two tossings might be observed by two copies of a locally identical observer, very distantly separated in a huge spacetime so that the two copies cannot communicate with each other.) Suppose that for one of the tossings of two coins, the observer observes a total of one head. Hartle and Srednicki make the interpretation [1], "All we know is that there exists at least one such region containing our data." Therefore, they would calculate the probability for the existence of one head (out of two coins tossed per tossing) in either or both of the tossings, which for fair coins would be 3/4. This would be the expectation value of the projection operator onto the existence of one head in either or both of the two tossings of two coins each. The complementary probability would be 1/4, the expectation value of the complementary projection operator onto the nonexistence of exactly one head in either of the two tossings of two coins each.

However, this complementary projection operator cannot be tested by a single copy of the observer, since even if it finds that the number of heads in its tossing is not one, it cannot know whether or not the other distant tossing gets a result of just one head out of the two coins tossed. Therefore, although the one copy of the observer can confirm the existence of one head (if that is what it observes), it

cannot falsify the existence of one head (no matter what it observes).

On the other hand, if the observer wants a set of projection operators for which in principle it can confirm any one of them, it could use the three projection operators onto the existence of 0, 1, and 2 heads, respectively. However, these are not orthogonal, and for fair coins their expectation values sum to 13/8, greater than unity. Therefore, these three expectation values for the three projection operators whose positive results can be confirmed by the one observer cannot be interpreted as normalizable probabilities.

As I see it, this apparent consequence of standard quantum theory and of similar probabilistic reasoning that has been beautifully deduced by Hartle and Srednicki [1] seems to be a *reductio ad absurdum* of standard quantum theory and similar reasoning for cosmologies in which there are indistinguishable copies of observational situations. However, we shall see below that replacing standard quantum theory by Eventual Quantum Mechanics can rectify the situation.

II. GENERAL ANALYSIS OF STANDARD QUANTUM THEORY

We can generalize this discussion to the case in which there are m distinguishable possible observations (labeled by a subscript i that runs from 1 to m) in each of N observationally indistinguishable observational situations. (That is, which of the N situations being observed cannot be distinguished by the identical copies of the observer, but only the observation outcome itself.) For notational purposes, suppose each observational situation is labeled by a superscript index K that runs from 1 to N. (We might suppose that in principle K can be determined by some hypothetical superobserver, but not by the ordinary observer confined to a particular observational situation.)

Now suppose P_i^K denotes the projection operator in the entire quantum state space onto the ith observation in the Kth situation. This would be the tensor product of the local projection operator onto the ith observation in the Kth local observation situation and of the local identity operators in all the other local observation situations and in all other regions of spacetime. For fixed situation K, the different projection operators P_i^K for different values of i will be assumed to be orthogonal, $P_i^K P_j^K = 0$ for $i \neq j$, because for a fixed observational situation (fixed copy of the observer), the different possible observations are assumed to be mutually exclusive.

However, projection operators for different K's (different observation situations for different observers) will not be orthogonal, even if their i's are different: $P_i^K P_j^L \neq 0$ for $K \neq L$, even if $i \neq j$. In fact, if $\langle O \rangle$ denotes the expectation value of the operator O, then although $\langle P_i^K P_j^K \rangle = \delta_{ij} \langle P_j^K \rangle$ when both projection operators apply to the same situation K, when they apply to different situations $K \neq L$ (here assumed to be in separate local regions, with the quantum

state space a tensor product of the state spaces for each local region), one gets $\langle P_i^K P_j^L \rangle = \langle P_i^K \rangle \langle P_j^L \rangle$, the product of the expectation values for the individual projection operators, which need not be zero.

Because the observer within one observational situation (one copy of the observer) cannot observe which particular situation he or she is in and therefore has no access to the index K that is known only to the hypothetical superobserver, he or she has no justification for using any particular projection operator P_i^K associated with a particular K. However, casting the reasoning of Hartle and Srednicki [1] into quantum language, one can construct the projection operators $P_i = I - \prod_K (I - P_i^K)$ onto the existence of the observation i in at least one of the observational situations, where I is the identity operator for the full quantum state space, and where $\prod_K$ denotes the product over all K from 1 to N. If the observer does observe i, that would confirm the truth value of the corresponding projection operator P_i, but its complement, $I - P_i$, cannot be confirmed by any observation restricted to a single observational situation.

One can take $p_S(i) = \langle P_i \rangle$ to be the probability in standard quantum theory (denoted by the subscript S) that at least one observation of i occurs, and $p_S(\neg i) = \langle (I - P_i) \rangle = 1 - p_S(i)$ to be the probability that no observation of i occurs. However, since for $N \geq 1$ the different P_i's are not orthogonal, the sum of the $p_S(i)$'s generically will not be unity. One can follow Hartle and Srednicki [1] and say that one has normalizable probabilities $p_S(i)$ and $p_S(\neg i)$ for any particular i, but one can only test these if one uses the value of i actually observed. With some probability the existence of i can be confirmed by an observer within a single observational situation, but the negation of its existence, $\neg i$, cannot be confirmed at all. Because of the asymmetry between the confirmability of i and the nonconfirmability of $\neg i$, it seems inappropriate to use $p_S(i)$ as a likelihood in a Bayesian analysis.

We can also see, using an example modeled after that in [17], quantitatively how $p_S(i)$ can be much larger than the expectation value of P_i^K for any K and hence can be highly misleading to use as a likelihood in a Bayesian analysis. For example, take the case in which 20 coins are tossed by each of a billion widely separated observers ($N = 10^9$), and let the observational results be the sequence of 20 heads and tails ($m = 2^{20} = 1\,048\,576$ possibilities). Let $i = 1$ correspond to the sequence of all tails (0 heads in the tossing of 20 fair coins). If one hypothesizes that all the coins are fair, the expectation value of P_1^K for any K is $2^{-20} \approx 0.954 \times 10^{-6}$, less than one part in a million. However, $p_S(1) = \langle P_1 \rangle = \langle I - \prod_K (I - P_1^K) \rangle = 1 - (1 - 2^{-20})^{10^9} \approx 1 - 6.5 \times 10^{-415}$. If one used $p_S(1)$ as a likelihood in a Bayesian analysis, one might say that its value, very near unity, would tend to confirm the hypothesis that the coins are fair, whereas after getting 20 tails in a row (probability less than one in a million if there were only

one tossing of 20 fair coins), it would seem much more reasonable to interpret this as evidence against the fair-coin hypothesis.

So how can we modify standard quantum theory, which seems to exemplify the type of reasoning used by Hartle and Srednicki [1], to get more reasonable results, results in which one can get likelihoods that would not nearly all tend to unity if there were vastly many indistinguishable observational situations (identical copies of the observer)?

III. EVENTUAL QUANTUM MECHANICS

Let me now propose a version of quantum theory in which the probability of an observation within a particular observational situation does not depend on how many such situations there are if the quantum state restricted to each situation (e.g., its density matrix, after tracing over all other regions) is the same. Since the basic elements will be the events observable within an observational situation, I shall call this class of quantum theories *Eventual Quantum Mechanics*, or EQM.

To motivate what I am aiming for, let me propose that the alternatives for the observations within an observational situation should obey the following key principles for the set of alternatives:

(1) *Prior alternatives principle* (PAP)—The set of alternatives to be assigned likelihoods by theories T_i should be chosen prior to (or independent of) the observation O_j to be used to test the theories.

(2) *Principle of observational discrimination* (POD)—Each possible complete observation should uniquely distinguish one element from the set of alternatives.

(3) *Normalization principle*—The sum of the likelihoods each theory assigns to all of the alternatives in the chosen set should be unity,

$$\sum_j P(O_j|T_i) = 1. \qquad (3.1)$$

I am always assuming that the alternatives within any set to be considered are mutually exclusive and exhaustive (complete). For example, if the alternatives are observed data sets within some class, then each alternative data set must be different, and all data sets within the class must be included within that set of alternatives.

For pedagogical simplicity, assume initially that the universe does have N observational situations that are sufficiently indistinguishable that the observer within each one cannot distinguish which one is his or hers. (For example, the distinction might be only in terms of what the surroundings are at sufficiently great distances that the observer within the region does not have observational access to this information.) Suppose that in each situation there are m distinguishable observations, say given in the Kth situation by the m projection operators P_i^K for i running from 1 to m

(P_i^K each acting on the entire quantum state space, but trivially as the identity operator outside the Kth observation situation). Assume they are all orthogonal for each different i (but the same K), $P_i^K P_j^K = \delta_{ij} P_i^K$, and that they sum to the identity operator I for the entire quantum state space when summed over i, $\sum_i P_i^K = I$ for each fixed K.

Now construct the operator $R_i = \sum_K P_i^K$, the sum of the projection operators over all observational situations K but for the same observation i. Then define the probability of the observation i in Eventual Quantum Mechanics as the normalized expectation value of this sum of projection operators,

$$p_E(i) = \frac{\langle R_i \rangle}{\sum_j \langle R_j \rangle} = \frac{\left\langle \sum_K P_i^K \right\rangle}{\sum_j \left\langle \sum_K P_j^K \right\rangle} = \frac{1}{N} \sum_K \langle P_i^K \rangle. \qquad (3.2)$$

If one has only one observational situation, $N = 1$, as has been the usual implicit assumption in traditional formulations of quantum theory, then of course R_i is just the projection operator for the observation in that one situation, and one has the usual probability interpretation of standard quantum theory. Thus in that situation, Eventual Quantum Mechanics reduces to ordinary standard quantum theory.

It is also easy to see that, if all the N regions have the same quantum state (e.g., the same density matrix) and if the P_i^K's are all essentially the same, except for the specification of which region on which the specific P_i^K acts nontrivially, then $\langle P_i^K \rangle$ is the same for each K, and $\langle R_i \rangle$ is just N times this expectation value. Therefore, in this case $p_E(i)$ would be the same as $\langle P_i^K \rangle / \sum_j \langle P_j^K \rangle = \langle P_i^K \rangle$ for any K, the last equality being true because $\sum_j P_j^K = I$ and $\langle I \rangle = 1$ in a normalized quantum state. Thus in the case of N identical regions, Eventual Quantum Mechanics reduces to what ordinary standard quantum theory would predict for a single one of these regions. On the other hand, Eventual Quantum Mechanics does *not* reduce to what standard quantum theory predicts for many such regions, as has been nicely shown by Hartle and Srednicki [1], because the R_i are *not* projection operators that are used in standard quantum theory to give probabilities.

Where Eventual Quantum Mechanics would allow more general predictions than standard quantum theory applied to a single observational situation would be in cases in which the different regions (the different observational situations for different observers) have different density matrices. Then the EQM probabilities $p_E(i)$ would be the average of the expectation values of the projection operators P_i^K over the N regions, an average probability for the observation i in each of the N regions.

Moreover, one might further generalize Eventual Quantum Mechanics beyond the last expression of Eq. (3.2) to allow that the existence of each region, or the existence of the observer within each region, might itself have a quantum uncertainty and hence a probability less than unity.

This could be reflected in the normalization of the effective density matrix for each region and in the possibility that one might define the P_j^K more generally so that $\sum_j \langle P_j^K \rangle$ does not necessarily equal one for each region. Then the fundamental operator R_i, whose normalized expectation value gives the probability of the observation i, might not be simply a sum of projection operators, but perhaps a weighted sum of projection operators, where the weights could effectively give the probabilities of the different regions being realized, or of the existence of the observer within each of the different regions.

The weights for each region, or for the observer within each region, need not even be existence probabilities. For example, they might instead reflect how long each region lasts, or how long the observer lasts within each region.

So far as I can see, the main essential feature is that one have a positive operator R_i for each possible event or observation (or at least an operator R_i whose expectation value is positive for the actual quantum state of the cosmos, even if it need not be positive for all possible quantum states, though it might seem simpler just to require that each R_i be positive). It might be easiest to understand the cases in which each operator R_i is a sum of projection operators, or perhaps a weighted sum of easily understandable projection operators, but I do not see that such a requirement would be essential.

IV. MAKING EVENTUAL QUANTUM MECHANICS MORE STANDARD

One might try to interpret Eventual Quantum Mechanics in a way that appears more nearly like standard quantum theory. For example, first consider the case in which each region, and its observer, definitely exists with unit quantum probability. Then although the probability $p_E(i)$ given by Eq. (3.2) cannot in general be written as the expectation value of any natural projection operator for the problem, it could simply be written as the expectation value of the projection operator P_i^K for any particular choice of the region K *if* the quantum state were independent of the labeling of the different regions. That is, if one replaced the arbitrary density matrix ρ for the tensor product of the N regions and for whatever else exists outside them by the density matrix $\bar{\rho}$ that is the average of ρ over all $N!$ permutations of the regions, then $p_E(i)$ is simply the expectation value of any one P_i^K (arbitrary K) in the averaged state $\bar{\rho}$:

$$p_E(i) = \text{tr}(P_i^K \bar{\rho}). \tag{4.1}$$

However, even this conversion of $p_E(i)$ to an expectation value of a projection operator by changing the state to an averaged state fails to be true when the quantum probability for the existence of the observer is not unity for each region. If in that case one calculates by Eq. (4.1) each $p_E(i)$ and sums them over all possible observations i, the sum will not be normalized to unity but will be the average probability of the existence of the observer for each region.

To get a normalized set of probabilities $p_E(i)$ in that case, one should instead in the averaged quantum state $\bar{\rho}$ take each $p_E(i)$ to be the conditional probability for the observation i in any one of the regions (say K), given that there is an observer in that region. If P_O^K is the projection operator onto the existence of an observer in the region K, then instead of Eq. (4.1), one should use the conditional probability

$$p_E(i) = \frac{\text{tr}(P_O^K P_i^K P_O^K \bar{\rho})}{\text{tr}(P_O^K \bar{\rho})}. \tag{4.2}$$

One might go even further and define the observer-conditioned density matrix

$$\hat{\rho} = \frac{P_O^K \bar{\rho} P_O^K}{\text{tr}(P_O^K \bar{\rho})}, \tag{4.3}$$

so that then the conditional probability $p_E(i)$ can be written as the expectation value of any one of the projection operators P_i^K:

$$p_E(i) = \text{tr}(P_i^K \hat{\rho}). \tag{4.4}$$

However, being able to write $p_E(i)$ as the expectation value of a projection operator in Eventual Quantum Mechanics involves replacing an arbitrary quantum state ρ with the conditionalized averaged quantum state $\hat{\rho}$. If one wanted to stick with the original quantum state ρ, the probability $p_E(i)$ for the observation i (normalized out of all possible observations) cannot in general be written as the expectation value of any natural projection operator.

V. SENSIBLE QUANTUM MECHANICS

One subclass of Eventual Quantum Mechanics theories is those of Sensible Quantum Mechanics [18–23] or Mindless Sensationalism [24], in which the alternative events or data sets or observations are conscious perceptions. Roughly, each individual conscious perception is all that a conscious observer is aware of or consciously experiencing at once, what Lockwood [25] calls a "phenomenal perspective" or "maximal experience" or "conscious state," and what Bostrom [26] calls an "observer-moment." If this conscious perception is regarded as a observed data set, the data would be the content of that awareness. In this set of alternatives, each different possible conscious perception would be a member, and any two perceptions with different contents would be different observations.

In the case of a discrete set of conscious perceptions, a particular Sensible Quantum Mechanics theory assigns a probability to each conscious perception that is the expectation value of a corresponding positive "awareness operator." There is no requirement that these positive operators be orthogonal to each other or even that they be proportional to projection operators (though they might be approximately proportional to the integral over all of

spacetime and over the local Lorentz group of projection operators in local regions).

VI. CONCLUSIONS

Whether typicality is likely depends on the way likelihoods are calculated. The way likelihoods are calculated depends on the set of alternatives to be assigned likelihoods. The set of alternatives must be chosen even before one can do a Bayesian analysis, so one cannot compare different theories with different sets of alternatives.

Hartle and Srednicki [1] use a type of probabilistic reasoning that includes standard quantum theory to select alternatives that in the quantum case are given by orthogonal projection operators, but their alternatives are not distinguished by the possible observations in universes large enough to have many observational situations so similar that they are not distinguishable to the observers within them.

On the other hand, alternatives obeying the prior alternatives principle and the principle of observational discrimination have normalized likelihoods in which typicality is automatically favored in the likelihoods. Since this preference comes directly from the likelihoods normalized over all possible distinguishable observations or data sets, it is not and need not be introduced "through a suitable choice of priors" as Hartle and Srednicki [1] suggest. Instead, the prior probabilities for theories may be chosen to "favor theories that are simple, beautiful, precisely formulable mathematically, economical in their assumptions, comprehensive, unifying, explanatory, accessible to existing intuition, etc. etc.," as Hartle and Srednicki propose.

Cosmological theories obeying the prior alternatives principle, the principle of observational discrimination, and the normalization principle, but apparently not standard quantum theory for a very large universe, can in principle be tested by observations. It therefore seems quite reasonable to adopt these principles. Eventual Quantum Mechanics and its subclass of Sensible Quantum Mechanics are frameworks for quantum theories which do obey these principles and which would make typical observations more likely. That is, they enable typicality to be derived as likely.

Typicality by itself does not guarantee that the theory with the highest posterior probability will make us typical. However, typicality is favored in the likelihoods. One need not impose it separately, but in discussions in which one does not explicitly invoke the full Bayesian framework, assuming typicality may be a legitimate shortcut for selecting between different theories for our observations. We are unlikely to be highly atypical.

ACKNOWLEDGMENTS

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