

**Addendum to “Constraint on  $\bar{\rho}$ ,  $\bar{\eta}$  from  $B \rightarrow K^* \pi$ ”**

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A  $1\sigma$  range,  $20^\circ < \Phi_{3/2} < 115^\circ$ , defining the slope of a linear Cabibbo-Kobayashi-Maskawa relation,  $\bar{\eta} = \tan\Phi_{3/2}(\bar{\rho} - 0.24 \pm 0.03)$ , was obtained from  $B^0 \rightarrow K^* \pi$  amplitudes measured in two Dalitz plot analyses of  $B^0 \rightarrow K \pi \pi$ . A correction reported recently by the *BABAR* Collaboration in results for  $B^0 \rightarrow K^+ \pi^- \pi^0$  is shown to imply a somewhat narrower  $1\sigma$  range for the slope parameter,  $39^\circ < \Phi_{3/2} < 112^\circ$ .

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A Dalitz analysis of  $B^0 \rightarrow K^+ \pi^- \pi^0$  by the *BABAR* collaboration reported in Ref. [1] has been very recently corrected [2]. We had used the earlier uncorrected version of this analysis to obtain a Cabibbo-Kobayashi-Maskawa (CKM) constraint [3]. In this Addendum we recalculate the constraint using the corrected experimental results.

The following linear constraint between the Wolfenstein parameters [4]  $\bar{\rho}$  and  $\bar{\eta}$  was first derived in Ref. [5]:

$$\bar{\eta} = \tan\Phi_{3/2}(\bar{\rho} - 0.24 \pm 0.03). \quad (1)$$

$2\Phi_{3/2} \equiv \arg(A_{3/2}/\bar{A}_{3/2})$  is the relative phase between the amplitude for  $B^0 \rightarrow (K^* \pi)_{I=3/2}$  and its charge conjugate. This phase can be measured in Dalitz analyses of  $B^0 \rightarrow K^+ \pi^- \pi^0$  and  $B^0(t) \rightarrow K_S \pi^+ \pi^-$ . Two corresponding analyses, performed by the *BABAR* collaboration in Refs. [1,6], measured the magnitudes of amplitudes for  $B^0 \rightarrow K^{*+} \pi^-$ ,  $B^0 \rightarrow K^{*0} \pi^0$ , their charge conjugates, and three relative phases,

$$\begin{aligned} \phi &\equiv \arg\left(\frac{A(B^0 \rightarrow K^{*0} \pi^0)}{A(B^0 \rightarrow K^{*+} \pi^-)}\right), \\ \bar{\phi} &\equiv \arg\left(\frac{A(\bar{B}^0 \rightarrow \bar{K}^{*0} \pi^0)}{A(\bar{B}^0 \rightarrow \bar{K}^{*-} \pi^+)}\right), \\ \Delta\phi &\equiv \arg\left(\frac{A(B^0 \rightarrow K^{*+} \pi^-)}{A(\bar{B}^0 \rightarrow \bar{K}^{*-} \pi^+)}\right). \end{aligned} \quad (2)$$

In Ref. [3] we have used these measurements, including negative log-likelihood values for  $\phi$  and  $\bar{\phi}$  [1], to calculate a  $\chi^2$  dependence on  $\Phi_{3/2}$ . The log-likelihood values for  $\phi$  and  $\bar{\phi}$  have been recently corrected for a missing factor of 2 [2]. This affects the  $\chi^2$  dependence on  $\Phi_{3/2}$ . The corrected dependence is plotted in Fig. 1. The broken purple

curve corresponds to an unconstrained  $|\bar{A}_{3/2}/A_{3/2}|$ , while the solid blue curve is obtained by imposing the bounds  $0.8 < |\bar{A}_{3/2}/A_{3/2}| < 1.2$ , expected to hold in the standard model [3]. The latter curve defines a  $1\sigma$  range,

$$39^\circ < \Phi_{3/2} < 112^\circ. \quad (3)$$

Figure 2 shows the linear constraint (1) with the large range of slopes (3) overlaid on CKMfitter results following from [7,8]  $|V_{ub}|/|V_{cb}| = 0.086 \pm 0.009$ , obtained in semi-leptonic  $B$  decays, and values  $\beta = (21.5 \pm 1.0)^\circ$ ,  $\alpha = (88 \pm 6)^\circ$ , and  $\gamma = (53_{-18}^{+15} \pm 3 \pm 9)^\circ$  [9], obtained in  $B \rightarrow J/\psi K_S$ ,  $B \rightarrow \pi\pi$ ,  $\rho\rho$ ,  $\rho\pi$ , and  $B^+ \rightarrow D^{(*)} K^{(*)+}$ , respectively. The small theoretical error in the  $B \rightarrow K^* \pi$  constraint [ $\pm 0.03$  in Eq. (1)] is described by the difference between dark and light shaded regions in Fig. 2.

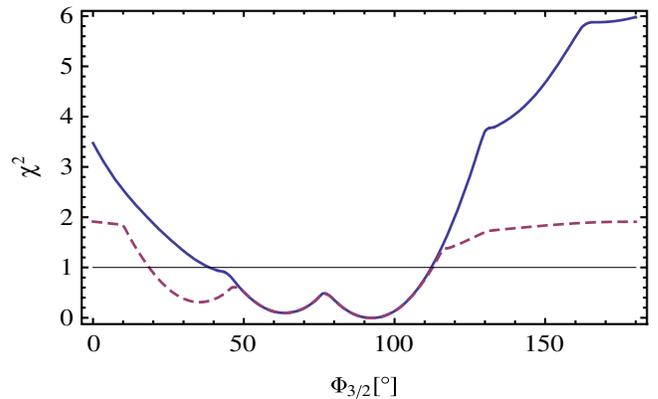


FIG. 1 (color online).  $\chi^2$  dependence on  $\Phi_{3/2}$  for unconstrained  $|R_{3/2}|$  (broken purple line) and for  $0.8 < |R_{3/2}| < 1.2$  (solid blue line). A black horizontal line at  $\chi^2 = 1$  defines  $1\sigma$  ranges for  $\Phi_{3/2}$ .

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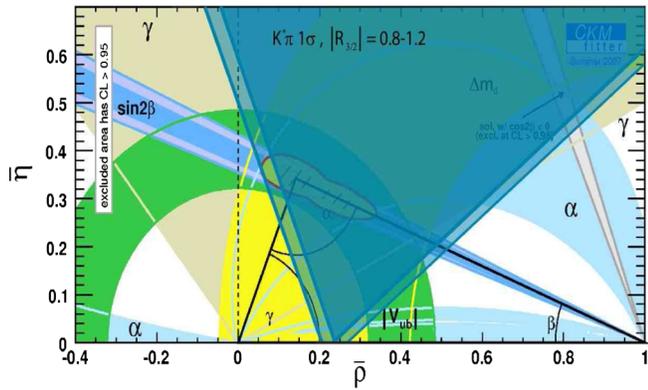


FIG. 2 (color online). Constraint in the  $\bar{\rho} - \bar{\eta}$  plane following from Eqs. (1) and (3). The dark shaded region marked  $K^* \pi 1\sigma$  corresponds to the experimental error on  $\Phi_{3/2}$  given by the  $1\sigma$  range (3), while the light shaded region includes also the error  $\pm 0.03$  in (1). Also shown are CKMfitter constraints obtained using  $|V_{ub}|/|V_{cb}|$ ,  $\beta$ ,  $\alpha$ ,  $\gamma$ , and  $\Delta m_d$  [8].

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