Weak-interaction corrections to hadronic top-quark pair production: Contributions from quark-gluon and $b\bar{b}$ induced reactions

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As an addendum to our previous evaluation of the weak-interaction corrections to hadronic top-quark pair production [W. Bernreuther, M. Fücker, and Z. G. Si, Phys. Rev. D **74**, 113005 (2006).], we determine the leading weak-interaction contributions due to the subprocesses $b\bar{b} \to t\bar{t}$ and $gq(\bar{q}) \to t\bar{t}q(\bar{q})$. For several distributions in $t\bar{t}$ production at the CERN LHC, we find that these contributions are non-negligible as compared to the weak corrections from the other partonic subprocesses.

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The physics of top quarks at the Tevatron and at the upcoming CERN Large Hadron Collider (LHC) offers the unique possibility to explore the interactions of the heaviest known fundamental particle. At the LHC one expects to also investigate with some precision the so-far unknown high-energy regime, i.e., single top-quark and top-antitop-quark $t\bar{t}$ events with transverse momenta and/or pair-invariant masses in the TeV range. The analysis and interpretation of such events will require, in particular, precise standard model (SM) predictions. In this context the electroweak corrections to hadronic $t\bar{t}$ production were recently determined: the $\mathcal{O}(\alpha_s^2\alpha)$ contributions of W, Z and Higgs boson exchange to quark-antiquark annihilation $q\bar{q} \to t\bar{t}$ [1,2] and to gluon fusion $gg \to t\bar{t}$ [3–5], extending earlier work of[1] [11], and the photonic corrections to hadronic top-quark pair production [12].

In this addendum to [3] we analyze a further set of weak-interaction corrections which we found to have some impact on a few kinematic distributions: (i) the contributions of order α^2 and $\alpha_s\alpha$ to

$$b\bar{b} \to t\bar{t}, \tag{1}$$

and (ii) the $\mathcal{O}(\alpha_s\alpha^2)$ and $\mathcal{O}(\alpha_s^2\alpha)$ contributions to the reactions

$$gq(\bar{q}) \to t\bar{t}q(\bar{q}) \qquad (q = u, d, s, c, b). \tag{2}$$

We employ here the so-called 5-flavor scheme [13], where the (anti)proton is considered to contain also b and $\bar{b}$ quarks in its partonic sea. Thus the reaction (1) is a leading-order (LO) process in this scheme, while (2), $q = b$, is a next-to-leading-order (NLO) QCD correction to (1). The $\mathcal{O}(\alpha_s^2\alpha)$ corrections to the processes (2) were calculated already in [3], which we include here for completeness. For several top-quark observables—in particular, for the $t\bar{t}$ cross section—the contributions (i) and (ii) are insignificant. However, here we show that for the pair-invariant mass distribution and for the top-quark helicity asymmetry, which are among the key observables in the toolkit for the search of new physics in $t\bar{t}$ events, these corrections do matter if one aims at predictions with a precision at the percent level.

The amplitude of (1) receives, in the Born approximation and putting $m_b = 0$, the following contributions: (a) t-channel W boson exchange $b\bar{b} \to^W t\bar{t}$, (b) s-channel photon and Z boson exchanges $b\bar{b} \to^{\gamma,Z} t\bar{t}$, and (c) s-channel gluon exchange $b\bar{b} \to^g t\bar{t}$. The t-channel W boson exchange contribution (a) is not suppressed by a small Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix element, in contrast to the corresponding t-channel amplitudes $b\bar{d}$, $b\bar{s} \to t\bar{t}$ + c.c. channels.

The lowest-order, weak-interaction-induced contributions to the squared transition matrix element $|\mathcal{M}(b\bar{b} \to t\bar{t})|^2$ are of order α^2 and $\alpha_s\alpha$; the latter arises from the interference of amplitudes (a) and (c).

The dominant part of $|\mathcal{M}(b\bar{b} \to t\bar{t})|^2$ is due to W exchange (a), as can be understood from inspecting the various terms in the limit of large parton center-of-mass energy $\sqrt{\hat{s}} \gg 2m_t$. It has the following properties: First, it is positive while the weak-interaction corrections to gg, $q\bar{q} \to t\bar{t}(q \neq b)$ are negative in most of the kinematic range of $\hat{s}$. Second, t-channel W exchange produces top quarks mostly in the forward region. Thus one expects these contributions to be relevant only for relatively small transverse momentum p_T of the (anti)top quark. For the distribution of the pair-invariant mass $M_{t\bar{t}} = \sqrt{(p_t + p_{\bar{t}})^2}$, no such conclusion can be drawn. Third, t-channel W exchange generates left-handed top quarks and right-handed antitop quarks. In the high-energy regime $M_{t\bar{t}} \gg m_t$, where the top quarks behave more and more like massless quarks, the (anti)top quarks due to (a) have, therefore, (positive) negative helicity.

The Feynman diagrams for the reactions (2) are shown in Fig. 1 to leading order in the weak and strong interactions. The exchange of the SM Higgs boson is numerically insignificant and therefore not taken into account.

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[1]Supersymmetric corrections to $t\bar{t}$ production were determined in [6–10].

Diagrams (b1)–(b4) in Fig. 1 with W-boson exchange are relevant only for b quarks in the initial state. For $q = s, d$ the corresponding amplitudes are suppressed by small CKM mixing matrix elements ($|V_{td}| \sim 7 \times 10^{-3}$ and $|V_{ts}| \sim 3.5 \times 10^{-2}$ [14]). Here we compute the $\mathcal{O}(\alpha_s \alpha^2)$ contributions to the squared matrix elements [Figs. 1(a) for $q \neq b$ and Figs. 1(a) and 1(b) for $q = b$]. The terms corresponding to the squares of (a2) and (b3), their interference, and the interference between (b3) and (c2) have initial-state collinear singularities which we removed within the standard $\overline{\text{MS}}$ factorization scheme. These terms are therefore expected to exhibit some sensitivity to variations of the factorization scale μ_F. For completeness, we also take into account in the numerical evaluations below the weak-interaction corrections of $\mathcal{O}(\alpha_s^2 \alpha)$ [i.e., the interferences of Figs. 1(a) and 1(c) for $q \neq b$ and Figs. 1(a)–1(c) for $q = b$] which were computed in [3].

The qualitative discussion in the previous paragraphs is corroborated by the numerical evaluation of corrections (i) and (ii). As far as the contributions of these terms to the hadronic $t\bar{t}$ production cross sections at the Tevatron and at the LHC are concerned, they are below the percent level and are, like the electroweak contributions from the other partonic subprocesses [2,3,12], smaller than the uncertainties of the present QCD predictions. Next we analyze three distributions relevant for top physics: the transverse momentum distribution, the pair-invariant mass distribution, and the helicity asymmetry. At the Tevatron (i.e., for $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV) b-quark induced $t\bar{t}$ production plays no role, and the weak-interaction induced contributions to these distributions from (1) and (2) are completely negligible. However, they matter for the LHC, i.e., for $pp \to t\bar{t}X$ at $\sqrt{s} = 14$ TeV.

Let us now discuss the p_T and $M_{t\bar{t}}$ distribution and the top-quark helicity asymmetry for the LHC. We compare corrections (i) and (ii) with the weak-interaction induced contributions of order α^2 and $\alpha_s^2 \alpha$ due to the subprocesses $q\bar{q} \to t\bar{t}(q \neq b)$ and $gg \to t\bar{t}$ [1,3], which we denote by corrections (iii) in the following. These depend on the unknown SM Higgs boson mass, for which we use the values $m_H = 120$ GeV and 200 GeV. In most of the results shown below, corrections (i), (ii), and (iii) will be normalized to the respective distributions $d\sigma_{\text{LO}}$ obtained in lowest-order QCD from $q\bar{q}$, $gg \to t\bar{t}$. In the case of the parity-violating helicity asymmetry, (i), (ii), and (iii) are normalized to $d\sigma_{\text{LO}}/dM_{t\bar{t}}$. As in [3] we use $m_t = 172.7$ GeV, $\alpha_s(2m_t) = 0.1$, and $\alpha(2m_t) = 1/126.3$. The LO QCD terms and the contributions of (i) and (iii) to the distributions are evaluated with the LO parton distribution functions (PDF) CTEQ6.L1, while for the computation of the contributions from (ii), which depend on the factorization scale, the set CTEQ6.1M [15] is used. The scale μ_F is varied between $m_t/2 \leq \mu_F \leq 2m_t$. Dependence on the renormalization scale μ_R enters only via the $\overline{\text{MS}}$ coupling α_s. The ratio of corrections (iii) and $d\sigma_{\text{LO}}$ is practically independent of α_s, while the corresponding ratios involving (i) and (ii) vary weakly with μ_R.

Figure 2(a) shows the various weak-interaction contributions to the transverse momentum distribution of the top quark at the LHC, normalized to $d\sigma_{\text{LO}}/dp_T$. The hatched areas depict the range of values when $\mu \equiv \mu_F = \mu_R$ is varied between $m_t/2$ and $2m_t$. Figure 2(b) displays the ratio of the sum of the weak corrections (i), (ii), and (iii) and the LO QCD contribution. In order to exhibit the significance of corrections (i) and (ii) relative to (iii), we have plotted in Fig. 2(c) the p_T-distribution ratio $d\sigma_{\text{weak}}^{(\text{i+ii})}/d\sigma_{\text{weak}}^{(\text{iii})}$. Figure 2(a) shows that the weak correction (i) to the p_T distribution of the top quark is positive, as expected. As Fig. 2(c) shows, contributions (i) and (ii) make up, for p_T around 100 GeV, a significant part of the weak-interaction contributions to the p_T spectrum. The ratio $d\sigma_{\text{weak}}^{(\text{i+ii})}/d\sigma_{\text{weak}}^{(\text{iii})}$ decreases in magnitude with increasing p_T, but is still $\sim -10\%$ for $p_T \sim 1$ TeV. Figure 2(b) shows the significance of the sum of the weak corrections to the p_T distribution.

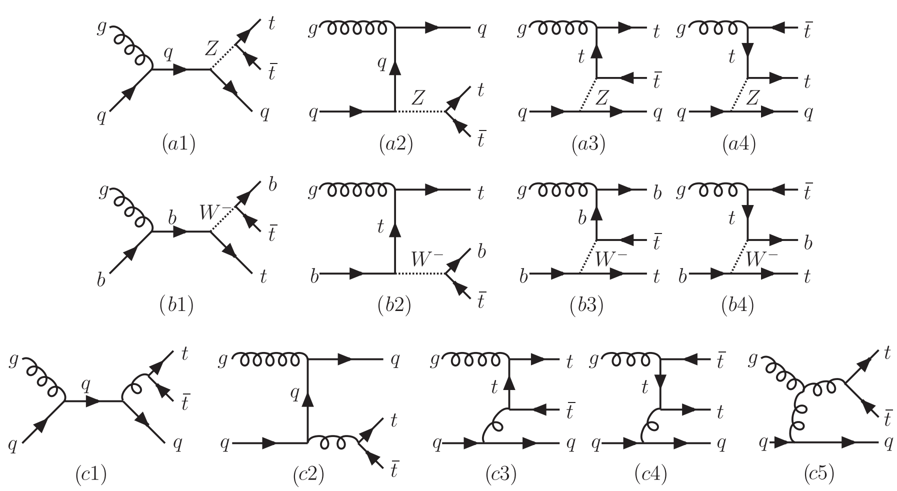

FIG. 1. Feynman diagrams for $gq(\bar{q}) \to t\bar{t}q(\bar{q})$ to leading order in the weak and strong interactions.

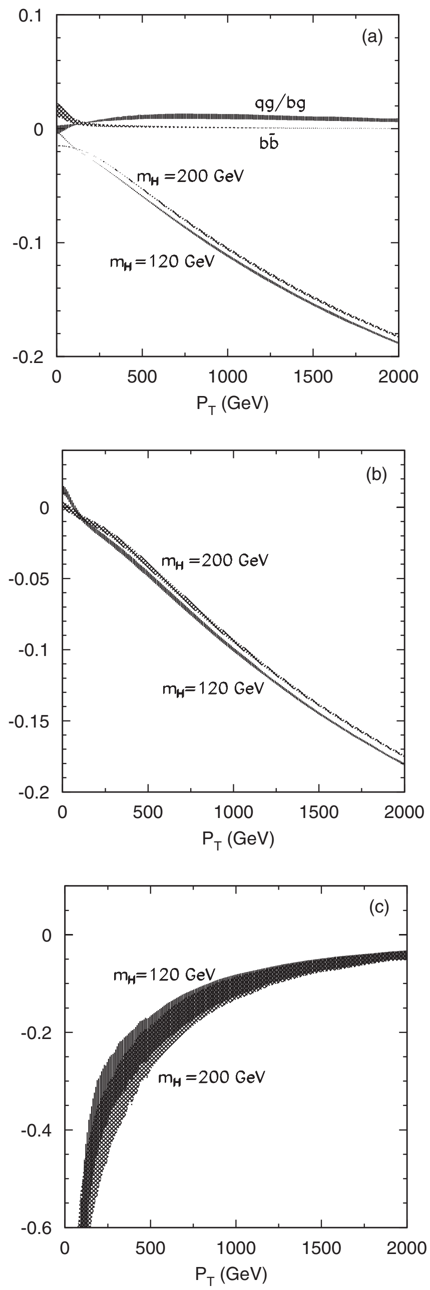

FIG. 2. (a) Ratios $(d\sigma_{\rm weak}/dp_T)/(d\sigma_{\rm LO}/dp_T)$ where $d\sigma_{\rm weak}$ are the weak-interaction corrections (i), (ii), and (iii) to the reactions (1) and (2), and $q\bar{q}$, $gg \to t\bar{t}(q \neq b)$, respectively. (b) Sum of the ratios shown in (a) for two different values of m_H. (c) The p_T-distribution ratio $d\sigma^{\rm (i+ii)}_{\rm weak}/d\sigma^{\rm (iii)}_{\rm weak}$ for $m_H = 120$ GeV (vertically hatched area) and $m_H = 200$ GeV (cross-hatched area).

In Figs. 3(a)–3(c) the analogous ratios are displayed for the $M_{t\bar{t}}$ distribution. The weak-interaction corrections (i) and (ii) are both positive and show a considerable scale uncertainty. As the weak corrections (iii) to the $M_{t\bar{t}}$ distribution are negative, contributions (i) and (ii) considerably reduce the magnitude of the sum of the weak-interaction corrections, as shown in Figs. 3(b) and 3(c).

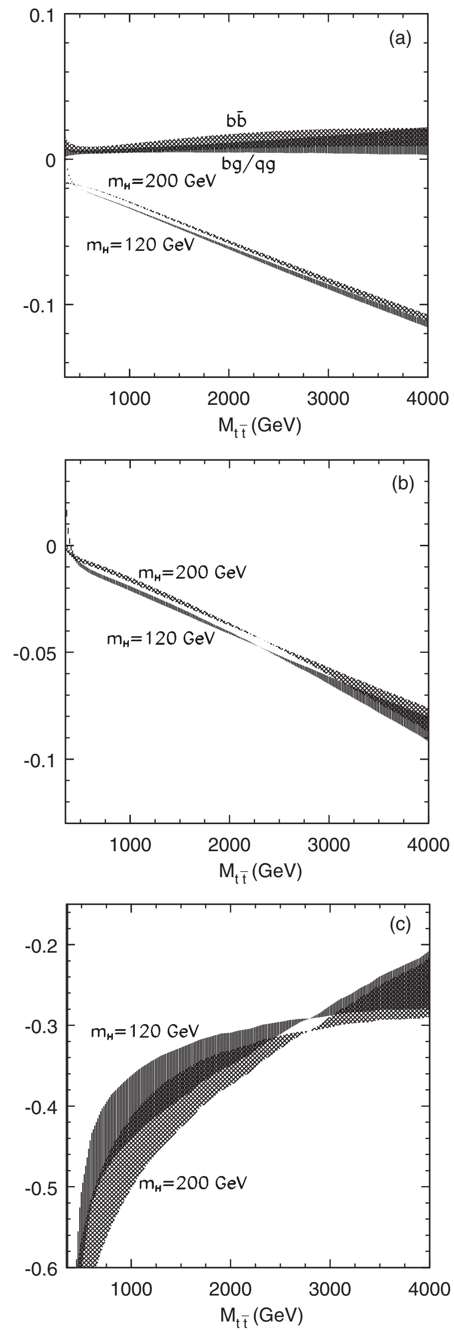

FIG. 3. Same as Figs. 2(a)–2(c), but for $(d\sigma_{\rm weak}/dM_{t\bar{t}})/(d\sigma_{\rm LO}/dM_{t\bar{t}})$.

Finally, we consider the parity-violating helicity asymmetry for t quarks defined by

$$\Delta_{\rm hel} = \frac{Z_{\rm hel}}{d\sigma_{\rm LO}/dM_{t\bar{t}}}, \qquad Z_{\rm hel} = \frac{d\sigma_+}{dM_{t\bar{t}}} - \frac{d\sigma_-}{dM_{t\bar{t}}}. \tag{3}$$

The subscripts $\pm$ in (3) refer to a t quark with positive/negative helicity while the helicity states of $\bar{t}$ are summed. (In [3] a different normalization was chosen for $\Delta_{\rm hel}$.) Figure 4(a) displays the weak-interaction induced contributions (i), (ii), and (iii) to $\Delta_{\rm hel}$. [As the SM Yukawa coupling is parity-conserving, (iii) does not depend on

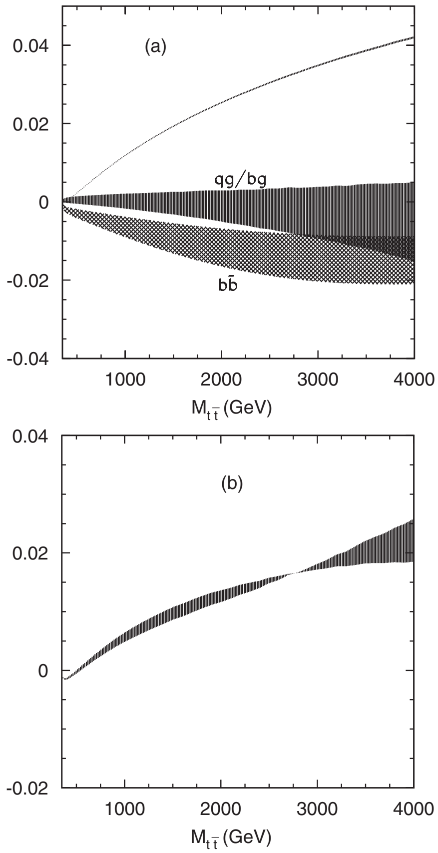

FIG. 4. (a) Contribution of the various partonic subprocesses to $\Delta_{\rm hel}$: initial states $q\bar{q}(q \neq b)$ and gg (thin line), qg and $\bar{q}g(q = u, \ldots, b)$ (vertically hatched area), and $b\bar{b}$ (cross-hatched area). (b) Sum of the three contributions shown in (a).

m_H.] Each correction (i) and (ii) shows a considerable scale dependence which, however, cancels to a large extent in the sum of the two contributions—cf. Fig. 4(b). Corrections (i) and (ii) reduce the contribution (iii) to $\Delta_{\rm hel}$ by about 50%. The t-quark helicity asymmetry in the SM is then $\Delta_{\rm hel} \lesssim 2\%$ for $M_{t\bar{t}} \lesssim 4$ TeV. Such a small effect will hardly be measurable at the LHC. Nevertheless, as emphasized in [3], this observable is an ideal experimental sensor for tracing possible new parity-violating interactions in $t\bar{t}$ production; thus $\Delta_{\rm hel}$ should be computed as precisely as possible within the SM.

If one takes into account only $t\bar{t}$ events with $p_T \geq p_{T\,\rm min}$, corrections (i), (ii) will not change significantly, as long as $p_{T\,\rm min}$ is not too large. Choosing, for instance, $p_{T\,\rm min} =$ 30 GeV does not lead to a significant change of the results shown in Figs. 2–4. Eventually, the weak corrections to the distributions discussed here should be evaluated [16] in conjunction with the known NLO QCD corrections, for which NLO PDF, in particular a NLO b-quark PDF, are to be used. (For recent updates of this PDF, see [17,18]). The NLO b-quark PDF enhances the b-quark induced weak contribution to the $M_{t\bar{t}}$ distribution and to $\Delta_{\rm hel}$ at large $M_{t\bar{t}}$.

In conclusion we have determined for hadronic $t\bar{t}$ production the leading weak-interaction corrections due to the subprocesses $b\bar{b} \to t\bar{t}$ and $gq(\bar{q}) \to t\bar{t}q(\bar{q})$. For the LHC we find that in the case of the pair-invariant mass distribution and of the helicity asymmetry, these contributions are non-negligible as compared to the weak corrections from $q\bar{q}, gg \to t\bar{t}$. As these distributions are key observables for investigating the interactions of top quarks in the high-energy regime, these corrections should be taken into account when it comes to precision analyses of future $t\bar{t}$ events at the LHC.

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