Joint description of the e^+e^- annihilation into both four-pion channels

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The $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ reaction cross section as a function of the incident energy is calculated using a model that is an extension of our recently published model of the e^+e^- annihilation into four charged pions. The latter considered the intermediate states with the π, ρ, and a_1 mesons and fixed the mixing angle of the $a_1\rho\pi$ Lagrangian and other parameters by fitting the cross section data. Here we supplement the original intermediate states with those containing $\omega(782)$ and $h_1(1170)$, but keep unchanged the values of those parameters that enter both charged and mixed channel calculations. The inclusion of ω is vital for obtaining a good fit to the cross section data, while the intermediate states with h_1 further improve it. Finally, we merge our models of the $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ and $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ reactions and obtain a simultaneous good fit.

The electron-positron annihilation into four pions has been theoretically studied by several authors [1–5]. Assuming the one-photon approximation and vector meson dominance, this process goes via the $\rho(770)$ meson and its recurrences. If some conditions on the rho-decay amplitude into four pions are met [6], the cross section of the e^+e^- annihilation into four pions at invariant energy W can be expressed in terms of the decay width of a ρ meson with mass W into four pions [3,7]. Some of the models of the four-pion decay of the ρ meson [3,8–10] can thus be conveniently utilized when determining the excitation function of the e^+e^- annihilation into four pions.

In our recent work [7] we calculated the excitation function of the e^+e^- annihilation into four charged pions and compared it to the existing data. We confirmed the conclusion of several experimental and theoretical papers [2,4,5,11,12] that the axial-vector isovector resonance $a_1(1260)$ plays an important role. The new feature of our approach was that we did not take some *a priori* chosen Lagrangian of the $a_1\rho\pi$ interaction, but considered a two-component Lagrangian that contained two parameters: a mixing angle and an overall coupling constant. We varied the mixing angle in an effort to get the best fit to the data. The coupling constant was determined for each mixing angle from the given total width of the a_1 resonance. Besides the intermediate states with the a_1 resonance, we considered also those with only pions and ρ's as given in various theoretical schemes [3,8–10]. They influence the calculated cross section mainly in the rho mass region. However, the quality of the fit throughout the energy region covered by the *BABAR* experiment [13] has improved only slightly.

As far as the e^+e^- annihilation cross section data are concerned, the situation in the $\pi^+\pi^-\pi^0\pi^0$ sector is worse than in the $\pi^+\pi^-\pi^+\pi^-$ one. The data coming from various experimental groups did not agree with one another very well; see, e.g., Fig. 10 in [14]. It is good news that the latest published data by the SND Collaboration at BINP in Novosibirsk [15] agree well with the newest *BABAR* [14,16] and CMD-2 [14,17] data. Unfortunately, those data are still preliminary and publicly unavailable [18,19]. We can therefore use only the SND data, which cover the energy interval from 980 to 1380 MeV. The statistical errors are combined with the 8% systematic error [15] in quadrature.

We will first compare the cross section data to a simple pure-a_1 model, which is characterized by four Feynman diagrams depicted in Fig. 1. The model is an obvious modification of the model used in [7], which is defined by a two-component $a_1\rho\pi$ interaction Lagrangian

$$\mathcal{L}_{a_1\rho\pi} = \frac{g_{a_1\rho\pi}}{\sqrt{2}}(\mathcal{L}_1\cos\theta + \mathcal{L}_2\sin\theta) \qquad (1)$$

where

$$\mathcal{L}_1 = \mathbf{A}^\mu \cdot (\mathbf{V}_{\mu\nu} \times \partial^\nu\phi), \qquad (2)$$

$$\mathcal{L}_2 = \mathbf{V}_{\mu\nu} \cdot (\partial^\mu\mathbf{A}^\nu \times \phi), \qquad (3)$$

and $\mathbf{V}_{\mu\nu} = \partial_\mu\mathbf{V}_\nu - \partial_\nu\mathbf{V}_\mu$. The isovector composed of the ρ-meson field operators is denoted by $\mathbf{V}_\mu$; similar objects for π and a_1 are ϕ and $\mathbf{A}^\mu$, respectively. The sine of the mixing angle $\sin\theta = 0.4603(28)$ was determined in [7] by
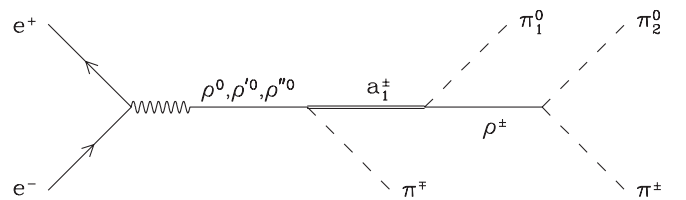

FIG. 1. Two Feynman diagrams of a pure-a_1 model of $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$. Two others can be obtained by exchanging π^0's.

fitting the $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ cross section data from the *BABAR* Collaboration [13] supplemented with the experimental value of the D/S ratio in the $a_1 \to \rho\pi$ decay [20]. The value of the coupling constant $g_{a_1\rho\pi}$ follows from $\sin\theta$ and the total width of the a_1 meson, which was chosen at 600 MeV.

Also defining our model is the form factor generated by the $\rho(770)$, $\rho' \equiv \rho(1450)$, and $\rho'' \equiv \rho(1700)$ resonances,

$$F(s) = F_\rho(s) + \delta F_{\rho'}(s) + \epsilon F_{\rho''}(s). \tag{4}$$

As far as the individual contributions on the right-hand side are concerned, we refer the reader to formulas in [7]. Here we only note that the complex parameters ϵ and δ as well as the masses and widths of the ρ' and ρ'' resonances hidden in $F_{\rho'}$ and $F_{\rho''}$ were determined by fitting the four-charged-pion *BABAR* data [13].

The last ingredient of our model is connected with the structure of the strongly interacting particles. Each vertex is usually modified by a strong form factor to soften the interaction. In [7], we used a simplified approach. We merged all form factors to one, effective, strong form factor of the Kokoski-Isgur [21] type, which multiplies the total annihilation amplitude

$$F_{KI}(s) = \exp\left\{-\frac{s - s_0}{48\beta^2}\right\}, \tag{5}$$

where $s_0 = 16m_\pi^2$. The value of $\beta = 0.3695(98)$ GeV follows from the fit to the *BABAR* data [13]. The same form factor is also used here.

When calculating the $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ excitation curve in the pure-a_1 model, we keep all the parameters at values determined in [7]. The result $\chi^2 = 2076$ for 35 data points is disastrous. A poor result for the pure-a_1 model clearly signifies that an additional contribution to the amplitude of the electron-positron annihilation into two charged and two neutral pions is needed. The intermediate states with the ω meson, considered already by Renard in 1969 [1] and later by other authors [3–5,9,11], are an obvious choice.

We will pursue two different ways of including the intermediate states with the ω meson. First, we adopt the approach of Eidelman, Silagadze, and Kuraev (ESK) [9], who used the anomalous part of the chiral Lagrangian [22–24] to describe the $\omega\rho\pi$ and $\omega3\pi$ vertices. The Feynman diagrams are shown in Figs. 2 and 3. The second approach (PL) [25] is more phenomenological. It does not consider, like [3,11], the $\omega3\pi$ contact term. The Lagrangian

$$L_{\omega\rho\pi} = G_{\omega\rho\pi}\epsilon_{\mu\nu\rho\sigma}(\partial^\mu\omega^\nu)(\phi \cdot \partial^\rho V^\sigma) \tag{6}$$

has the same form as in [3,9,11]. But now, the coupling constant $G_{\omega\rho\pi}$ is determined from the $\omega \to 3\pi$ decay width, taking into account the value of the $\rho \to \pi\pi$ coupling constant g_ρ as it follows from the $\rho \to \pi\pi$ decay width, $g_\rho^2 = 35.77 \pm 0.24$. We get $G_{\omega\rho\pi}^2 = (216.2 \pm 3.0)$ GeV^{-2}. The value of $G_{\omega\rho\pi}$ is higher than the corre-
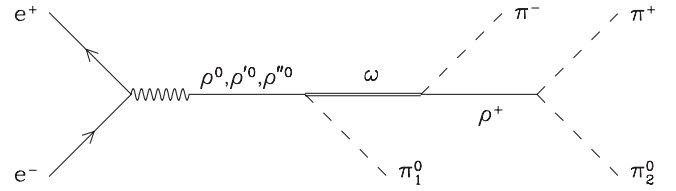

FIG. 2. The generic Feynman diagram describing the ω contribution to $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$. The other five diagrams are obtained by obvious modifications.

sponding quantity in [9] by only about 2.6%, so the main difference between the two approaches lies in the diagrams with the $\omega3\pi$ contact terms. To check the soundness of our approach, we performed two tests.

First, we calculated the width of the radiative decay $\omega \to \pi^0\gamma$ assuming the strength of the $\rho^0\gamma$ coupling, as it follows from the normalization of the pion form factor,

$$\mathcal{L}_{\gamma\rho^0} = \frac{em_\rho^2}{g_\rho}A^\mu\rho_\mu^0. \tag{7}$$

The calculated branching fraction of $(9.48 \pm 0.28)\%$ differs a little from the current experimental value $B(\omega \to \pi^0\gamma) = (8.90^{+0.27}_{-0.23})\%$ [26].

Second, we determined the strength of the $\omega\gamma$ coupling from the $\omega \to e^+e^-$ decay width and used it in calculating the rate of the $\pi^0 \to \gamma\gamma$ decay. The result, expressed in terms of the π^0 mean lifetime, $\tau = (7.7 \pm 0.4) \times 10^{-17}$ s agrees well with the experimental value of $(8.4 \pm 0.6) \times 10^{-17}$ s.

What remains unsettled is a possible transfer-momentum-squared (t) dependence of the $\rho\gamma$ coupling. In fact, the experimental width of the $\rho^0 \to e^+e^-$ decay, where $t = m_\rho^2$, requires about 20% stronger coupling than that indicated in (7). The latter gives good results for the $t = 0$ processes $\omega \to \pi^0\gamma$ and $\pi^0 \to \gamma\gamma$. Our derivation [7] of the $e^+e^- \to 4\pi$ cross section formula used the standard $\gamma\rho^0$ coupling (7) and assumed that all the t dependence is absorbed in the form factor. We use the same approach here.

Now, we can also vary the form-factor parameters δ and ϵ, as the structure of the intermediate states is different from the pure-a_1 model. We will distinguish them from the $\pi^+\pi^-\pi^+\pi^-$ case by primes. There is no free parameter
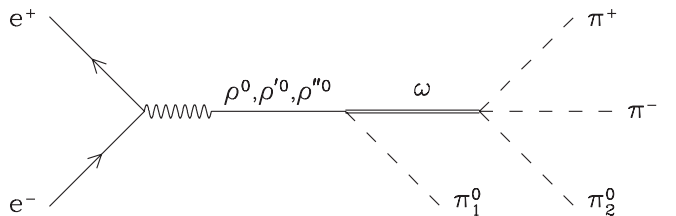

FIG. 3. One of the two Feynman diagrams with the contact $\omega3\pi$ term. Another is obtained by exchanging π^0's.

connected with the $\omega\pi$ intermediate states. The results are shown in Table I.

In an effort to further improve the agreement of our model with data, we include the intermediate states with the isoscalar axial-vector meson $h_1(1170)$. The corresponding Feynman diagrams can be obtained from those in Fig. 2 by replacing ω with h_1. The interaction Lagrangian is again assumed in a two-component form similar to (1) but respecting the isoscalar character of $h_1(1170)$,

$$\mathcal{L}_{h_1\rho\pi} = \frac{g_{h_1\rho\pi}}{\sqrt{3}}(\mathcal{L}_a \cos\eta + \mathcal{L}_b \sin\eta), \qquad (8)$$

where

$$\mathcal{L}_a = h^\mu \cdot (\mathbf{V}_{\mu\nu} \cdot \partial^\nu \phi), \qquad (9)$$

$$\mathcal{L}_b = -\partial^\mu h^\nu \cdot (\mathbf{V}_{\mu\nu} \cdot \phi). \qquad (10)$$

In the following, the sine of the mixing angle η will be varied to achieve the best possible description of the cross section data. For each $\sin\eta$ the coupling constant will be determined from the condition that the total width of the $h_1(1170)$, calculated as $\Gamma(h_1 \to \rho\pi)$, should be equal to 360 MeV. While fitting the $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ data the mixing angle of the $a_1\rho\pi$ Lagrangian (1) is kept fixed at the value determined in the $\pi^+\pi^-\pi^+\pi^-$ case, as it represents a universal process-independent parameter. The results of the fit are shown in Table II for both approaches to the intermediate states with ω. It is clear that the inclusion of the $h_1\pi$ intermediate states greatly improves the confidence level of the model with ω described by the ESK scheme. The confidence level of the model utilizing the PL scheme for ω also rises, but because it was already high in the $a_1 + \omega$ model, the inclusion of the intermediate states with h_1 is not necessary. The excitation curves are compared to data in Fig. 4.

Following the idea that a simultaneous fit to more processes may lead to a more precise value of the mixing angle of the $a_1\rho\pi$ Lagrangian, we are now going to perform a joint fit to the $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ and $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ cross section data. We merge the pure-a_1 model of our previous paper [7] with the $a_1 + \omega + h_1$ model described here. For the $\omega\pi$ intermediate states we

TABLE I. Fitting the $a_1 + \omega$ model to the $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ cross section data (35 data points).

Approach to ω	ESK [9]	PL [25]
χ^2/NDF	2.19	0.82
CL (%)	0.01	74.9
Re δ'	−0.52(25)	−0.36(27)
Im δ'	−1.27(25)	−1.05(32)
Re ϵ'	−0.79(34)	−0.71(37)
Im ϵ'	0.953(97)	0.628(79)

TABLE II. Fitting the $a_1 + \omega + h_1$ model to the $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ cross section data (35 data points).

Approach to ω	ESK [9]	PL [25]
χ^2/NDF	1.18	0.80
CL (%)	22.4	77.8
$\sin\eta$	0.3434(36)	0.3433(46)
Re δ'	0.092(62)	0.102(74)
Im δ'	0.028(22)	0.035(23)
Re ϵ'	0.022(71)	0.028(86)
Im ϵ'	−0.030(58)	−0.049(65)

will use the PL version, which describes the data better than ESK. Simultaneous handling of both four-pion channels enables us to use a more correct description of the $\rho(770)$ part of the electromagnetic form factor [4], namely,

$$F_\rho(s) = \frac{M_\rho^2(0)}{M_\rho^2(s) - s - im_\rho\tilde{\Gamma}_\rho(s)}, \qquad (11)$$

where $M_\rho(s)$ is the running mass of the ρ meson calculated in [27]. The total decay width of the ρ^0 meson,

$$\tilde{\Gamma}_\rho(s) = \Gamma_\rho(s) + \Gamma_{\pi^+\pi^-\pi^+\pi^-}(s) + \Gamma_{\pi^+\pi^-\pi^0\pi^0}(s), \qquad (12)$$

now includes not only the contribution of several two- and three-body decay channels $\Gamma_\rho(s)$ given in [27], but also the contributions from both four-pion decay modes. Similarly to [7] we also consider the masses and widths of ρ' and ρ'' as free parameters. They have the same values in both channels, as well as the $a_1\rho\pi$ Lagrangian mixing parameter $\sin\theta$ and the parameter β of the strong form factor (5).
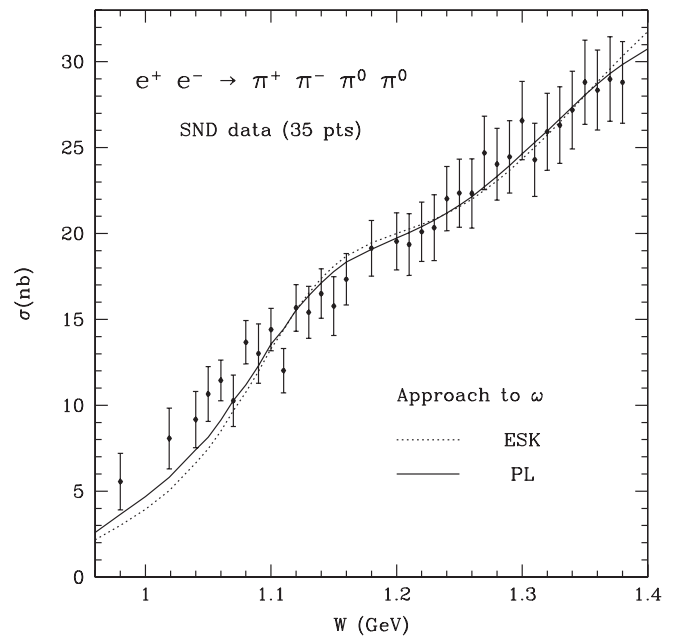

FIG. 4. Comparison of the $a_1 + \omega + h_1$ model with the $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ data for two approaches to ω.

Besides those six common parameters, there are two sets of parameters specific for each of the two four-pion annihilation channels. The charged-pion-channel set contains four real parameters entering the electromagnetic form factor (4). The four form-factor parameters in the mixed-pion channel are distinguished from those in the $\pi^+\pi^-\pi^+\pi^-$ channel by primes. The fifth parameter in the mixed-pion channel is the $h_1\rho\pi$ Lagrangian mixing parameter $\sin\eta$. All together, this makes 15 real free parameters.

In Table III we present the optimized values of all free parameters. The corresponding confidence level is 28.4%. If we compare them with those obtained in individual models (Table II here and Table VI in [7]), we can say that they are in good agreement. Only the error domains of $m_{\rho'}$, $m_{\rho''}$, and δ do not overlap, but the disagreement is very small. The excitation curves of individual reactions do not visually differ from those obtained when fitting the two models separately and are not shown.

Unfortunately, our goal to narrow the interval of the $a_1\rho\pi$ Lagrangian mixing parameter and thus make the calculations of the dilepton and photon production from hadron gas more reliable (see the analysis in [28]) has not been reached. The uncertainty of $\sin\theta$ is larger than that obtained in [7]. We hope that the situation will improve when more precise cross section data in the mixed-pion channel are available. The identification of the essential contributions to the e^+e^- annihilation into four pions is

important for the reliable assessment of the dilepton production by the four-pion annihilation in heavy ion collisions [29]. A new result of our work is the mixing angle of the $h_1\rho\pi$ Lagrangian $\sin\eta \approx 0.34$. It may help in investigating the role of the $h_1(1170)$ resonance in thermal production of dileptons and photons from hadron gas, which has been ignored so far.

TABLE III. Joint fit of the pure-a_1 model of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ and the $a_1 + \omega(PL) + h_1$ model of $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ to a combined set of the cross section data and the D/S ratio (180 data points).

Quantity	Value	Quantity	Value
χ^2/NDF	1.06	Re δ'	0.088(73)
$m_{\rho'}$ (GeV)	1.383(16)	Im δ'	0.024(22)
$\Gamma_{\rho'}$ (GeV)	0.551(21)	Re ϵ'	0.025(81)
$m_{\rho''}$ (GeV)	1.883(18)	Im ϵ'	$-0.011(74)$
$\Gamma_{\rho''}$ (GeV)	0.237(35)	Re δ	0.171(14)
$\sin\theta$	0.4662(52)	Im δ	0.045(29)
β (GeV)	0.3617(70)	Re ϵ	0.0002(14)
$\sin\eta$	0.336(11)	Im ϵ	$-0.0050(12)$

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