Unparticle effects on unitarity constraints from Higgs boson scattering

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We study the effects of two-body Higgs boson scattering by exchanging unparticles. The unparticle contribution can change the standard model prediction for two-body Higgs boson scattering partial wave amplitude significantly leading to modification of the unitarity constraint on the standard model Higgs boson mass. For unparticle dimension $d_{\mathcal{U}}$ between 1 and 2, the unitarity constraint on the Higgs boson mass can be larger than that in the standard model. Information on unparticle interaction can also be obtained.

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Since the seminal work of Georgi on unparticle physics [1] last year, the study of unparticle effects has drawn a lot of attention [1–12]. The concept of the unparticle [1] stems from the observation that in certain high energy theories with a nontrivial infrared fixed point at some scale $\Lambda_{\mathcal{U}}$ may develop a scale-invariant degree of freedom below the scale. The kinematics is determined by its scaling dimension $d_{\mathcal{U}}$ under scale transformations. The unparticle must interact with standard model (SM) particles to be physically relevant. Even though at present the detailed dynamics of how the unparticle interacts with SM particles is not known, these interactions can be well described in effective field theory. In this approach the interactions are parametrized in the following way [1]

$$\lambda\Lambda_{\mathcal{U}}^{4-d_{\rm SM}-d_{\mathcal{U}}}O_{\rm SM}O_{\mathcal{U}}, \tag{1}$$

where $O_{\rm SM}$ is composed of the SM fields and $O_{\mathcal{U}}$ is an unparticle operator.

There has been a burst of activity on various aspects of unparticle physics from phenomenology to theoretical issues [1–12]. Some of the major tasks of phenomenological study are to search for new signals and effects in various physical processes and to determine (constrain) the unparticle scale and also the unparticle dimension $d_{\mathcal{U}}$. In this work we study unparticle interaction effects on unitarity constraints from two-body Higgs boson scattering using partial wave analysis. We find that the unparticle contribution to the scattering partial wave amplitude can be significant which affects the unitarity constraint on the Higgs boson mass. For $d_{\mathcal{U}}$ between 1 and 2, the unparticle contribution can relax the upper bound for the Higgs boson mass.

Partial wave analysis of scattering processes is one of the often used methods to constrain unknown parameters in a theory. The unitarity constraint on the Higgs boson mass from two-body Higgs boson scattering in the SM [13], and constraint on the extra dimension scale [14] are some of the interesting examples. A scattering amplitude $\mathcal{M}$ for a given process can be decomposed into the partial wave amplitude according to angular momentum $\vec{J}$ as

$$\mathcal{M} = \frac{1}{k}\sum a_J(2J+1)P_J(\cos\theta). \tag{2}$$

The unitarity condition is referred to as the condition that the magnitude for each of the partial wave amplitudes $|a_J|$ should not be too large. There are many discussions on how to implement the unitarity condition to constrain new physics [15]. We will use a weak condition $|a_J| < 1$ for $J = 0$ and work with the tree-level amplitude to show how interesting constraints on unparticle interactions and the Higgs boson mass can be obtained.

Potentially large contributions to the two-body Higgs boson scattering may come from the following lowest dimension operator involving a scalar unparticle and SM Higgs field [11,12]

$$O_{hh} = \lambda_{hh}\Lambda_{\mathcal{U}}^{2-d_{\mathcal{U}}}H^\dagger H O_{\mathcal{U}}, \tag{3}$$

where $H = (h^+, (v + h + iI)/\sqrt{2})$ is the SM Higgs doublet. The h^+ and I are the fields "eaten" by W and Z, and h is a physical Higgs. The parameter λ_{hh} is real.

There are s, t, and u channel contributions from the above effective operator to the two-body Higgs boson scattering amplitude as shown in Fig. 1. We obtain the scattering amplitude as

$$\mathcal{M}^{un}(hh\to hh) = (\lambda_{hh}\Lambda_{\mathcal{U}}^{2-d_{\mathcal{U}}})^2\frac{A_{d_{\mathcal{U}}}}{2\sin(\pi d_{\mathcal{U}})} \times\left[\frac{1}{(-s)^{d_{\mathcal{U}}}}+\frac{1}{(-t)^{d_{\mathcal{U}}}}+\frac{1}{(-u)^{d_{\mathcal{U}}}}\right]. \tag{4}$$

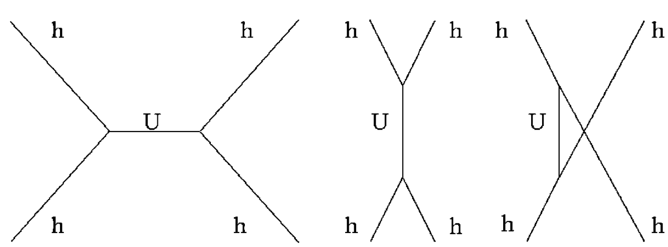

FIG. 1. Feynman diagrams for two-body Higgs boson scattering by exchanging an unparticle in s, t, and u channels.

In obtaining the above expression, we have used the scalar unparticle propagator $(iA_{d_{\mathcal{U}}}/2\sin(\pi d_{\mathcal{U}}))(1/(-p^2)^{2-d_{\mathcal{U}}})$. The factor $A_{d_{\mathcal{U}}}$ is normalized as $A_{d_{\mathcal{U}}} = (16\pi^{5/2}/(2\pi)^{2d_{\mathcal{U}}})\Gamma(d_{\mathcal{U}}+1/2)/(\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}}))$ following Ref. [1].

Using the above scattering amplitude, the $J = 0$ component in the partial wave expansion $a_0^{\rm un}$ can be easily obtained

$$\begin{aligned} a_0^{\rm un} &= \frac{1}{16\pi}\left(\frac{4\vec{p}^2}{s}\right)^{1/2}\frac{1}{s-4m_h^2}\int_{-(s-4m_h^2)}^{0}\mathcal{M}^{\rm un}dt \\ &= \frac{1}{16\pi}\lambda_{hh}^2\left(\frac{\sqrt{s}}{\Lambda_{\mathcal{U}}}\right)^{2d_{\mathcal{U}}-4}\frac{A_{d_{\mathcal{U}}}}{2\sin(\pi d_{\mathcal{U}})}\sqrt{1-\frac{4m_h^2}{s}} \\ &\quad\times\left[e^{-i\pi d_{\mathcal{U}}}+\frac{2}{d_{\mathcal{U}}-1}\left(1-\frac{4m_h^2}{s}\right)^{d_{\mathcal{U}}-2}\right], \end{aligned} \tag{5}$$

where $\vec{p} = \sqrt{1-4m_h^2/s}$ is the Higgs boson momentum in the center-of-mass frame.

As for any other processes involving the unparticle propagator, there is a $\sin(\pi d_{\mathcal{U}})$ factor in the denominator which has poles at integer $d_{\mathcal{U}}$ and makes $a_0^{\rm un}$ to diverge. In the first term in Eq. (5), the pole at $d_{\mathcal{U}} = 1$ is canceled by a zero in $A_{\mathcal{U}}$. However, the second term will blow off. Therefore, integer numbers are forbidden. Also the factor $A_{d_{\mathcal{U}}}$ decreases quickly as $d_{\mathcal{U}}$ increases, therefore for large $d_{\mathcal{U}}$ the unparticle contribution is suppressed.

For a complete analysis, one also needs to include the SM contribution where the $J = 0$ partial wave amplitude is given by [13]

$$\begin{aligned} a_0^{\rm SM} &= \frac{G_F m_h^2}{8\sqrt{2}\pi}\sqrt{1-\frac{4m_h^2}{s}}\left[3+\frac{9m_h^2}{s-m_h^2}\right. \\ &\quad\left.-\frac{18m_h^2}{s-4m_h^2}\ln\left(\frac{s}{m_h^2}-3\right)\right]. \end{aligned} \tag{6}$$

With this contribution included, the weak unitarity condition becomes

$$|a_0^T| = |a_0^{\rm SM}+a_0^{\rm un}| < 1. \tag{7}$$

There is an imaginary part from unparticle contribution to a_0 due to the s-channel unparticle exchange in Fig. 1 with $\mathrm{Im}a_0^{\rm un} = -(1/32\pi)r^{2d_{\mathcal{U}}-4}A_{d_{\mathcal{U}}}\sqrt{1-4m_h^2/s}$. Here $r = (\lambda_{hh})^{1/(d_{\mathcal{U}}-2)}\sqrt{s}/\Lambda_{\mathcal{U}}$. Since the SM contribution is real, the unitarity condition requires $|\mathrm{Im}a_0| < 1$. One can, in principle, obtain a constraint on the parameter r as a function of the unparticle dimension. We have analyzed this and found that the constraints are weak. The combined effects of real and imaginary parts can provide more interesting information which we study in the following.

In the limiting case of $s \gg 4m_h^2$, the weak unitarity condition is simply given by

$$\left| r^{2d_{\mathcal{U}}-4}\frac{A_{d_{\mathcal{U}}}}{32\pi\sin(\pi d_{\mathcal{U}})}\left[e^{-i\pi d_{\mathcal{U}}}+\frac{2}{d_{\mathcal{U}}-1}\right]+\frac{3G_F m_h^2}{8\sqrt{2}\pi}\right| < 1. \tag{8}$$

Since the strength of the unparticle contribution to the partial wave amplitude is a function of r, the unitarity condition may provide information about r. We plot, in Fig. 2, r as a function of $d_{\mathcal{U}}$ for several representative Higgs boson masses for $d_{\mathcal{U}}$ in the range between 1 and 2. Note that the unitarity bound gives a lower bound for r because $2d_{\mathcal{U}} - 4 < 0$. This reflects the fact that the interaction of the Higgs boson with the unparticle defined by operator O_{hh} does not decouple in the limit where $\Lambda_{\mathcal{U}}$ goes to infinity. Since $r \sim \sqrt{s}/\Lambda_{\mathcal{U}}$, naively, for $d_{\mathcal{U}}$ smaller than 2, small s is ruled out. However, one must keep in mind that $s > 4m_h^2$ must be satisfied, s smaller than $4m_h^2$ is not constrained by the unitarity condition. For $d > 2$, the unitarity bound gives an upper bound for r. In this case, in the large $\Lambda_{\mathcal{U}}$ limit, the interaction of the Higgs boson and unparticle decouples.

If the unparticle scale $\Lambda_{\mathcal{U}}$ is known from some theoretical considerations, one can use the weak unitarity condition to constrain the energy scale $\sqrt{s}$ with which one can reliably (satisfying the weak unitarity condition) use the operator O_{hh} for calculations. We have carried out a study keeping the $4m_h^2/s$ term in the expression for a_0^T. For $d_{\mathcal{U}}$ larger than 2, the unitarity condition enables one to obtain an upper bound for s since the leading s dependence is $s^{d_{\mathcal{U}}-2}$. s cannot be too large in order not to violate the unitarity condition, but numerically it is way above 10 TeV or any near future collider energies, such as LHC and ILC. For $d_{\mathcal{U}}$ between 1 and 2, the unitarity condition puts a low bound for s. Since the leading scale $\Lambda_{\mathcal{U}}$ and s dependence of a_0^T is $(s/\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}}-2}$, the unitarity condition gives a lower bound for s. A smaller $\Lambda_{\mathcal{U}}$ corresponds to a larger s. Numerically we find that for lower values of $\Lambda_{\mathcal{U}}$ (less than 1 TeV), s larger than the threshold is all allowed.

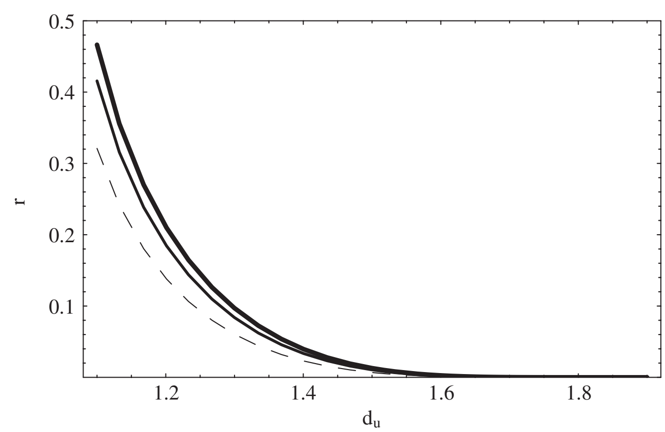

FIG. 2. Lower bound on r as a function of $d_{\mathcal{U}}$ with different Higgs masses, 115 (solid line), 500 (lighter solid line), 1000 (dashed line) GeV in the limit $s \gg 4m_h^2$.

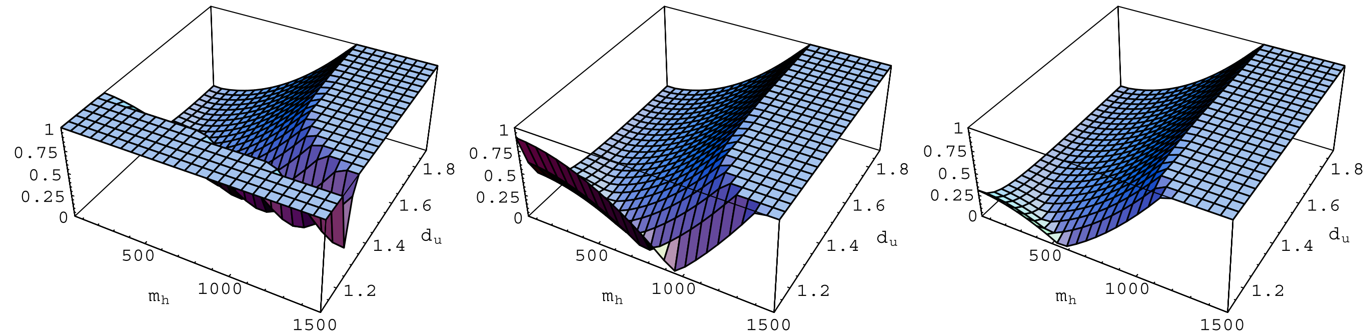

FIG. 3 (color online). $|a_t^0|$ (vertical axis) as a function of Higgs mass(in GeV) and $d_{\mathcal{U}}$ with $r = 0.1, 0.5$, and 1.

But for larger $\Lambda_{\mathcal{U}}$, for example, 10 TeV, there are regions with $d_{\mathcal{U}}$ close to 1 that violate the unitarity condition for s above the threshold. Note also that near the threshold, the second term in Eq. (5) becomes very large and therefore $|a_0^T|$, if $d_{\mathcal{U}}$ is smaller than 1.5. One should not use the value for s too close to the threshold if $d_{\mathcal{U}}$ is less than 1.5.

We find that the weak unitarity condition is satisfied for s significantly larger than the threshold of producing two Higgs bosons for $d_{\mathcal{U}}$ between 1 and 2.

We now discuss unparticle effects on unitarity constraint on the Higgs boson mass m_h. Without unparticle contribution, in the limit $s \gg 4m_h^2$, the weak unitarity condition implies that the Higgs boson mass must be smaller than $8\sqrt{2}\pi/3G_F = 1010$ GeV. With unparticle contributions, the constraint on the Higgs mass can be modified dramatically since the real part of the unparticle contribution can have either signs relative to the SM contribution depending on the unparticle dimension $d_{\mathcal{U}}$. For example, for $d_{\mathcal{U}}$ between 1 and 2, the real part of a^{un} is negative making the constraint on Higgs mass looser compared to the one for the SM. For $d_{\mathcal{U}}$ between 2 and 3, $\mathrm{Re}(a^{\mathrm{un}})$ is positive, the constraint on the Higgs mass becomes tighter. Since for large $d_{\mathcal{U}}$ there is a suppression from $A_{d_{\mathcal{U}}}$, the constraint on relevant parameters are weak. We will concentrate on $d_{\mathcal{U}}$ between 1 and 2.

In Fig. 3, we show $|a_0^T|$ as functions of the Higgs boson mass m_h and the unparticle dimension $d_{\mathcal{U}}$ for several finite values of r in the limit $s \gg 4m_h^2$. With a low value for r, the allowed region in m_h and $d_{\mathcal{U}}$ space is more restrictive than those for larger r. This is because that for smaller r, a_0^{un} becomes larger as $d_{\mathcal{U}}$ decreases. To satisfy the unitarity constraint, a large cancellation from the SM contribution is needed and results in a larger Higgs boson mass. Fixing $|a_0^T| = 1$, one can solve an upper bound for the Higgs mass m_h as a function of $d_{\mathcal{U}}$ and r. In Fig. 4 we show this upper bound. We see more clearly that for smaller r and $d_{\mathcal{U}}$, a much larger Higgs boson mass compared with the SM unitarity bound is allowed. When $d_{\mathcal{U}}$ and r become larger, the unparticle effects decreases. The unitarity bound on the Higgs boson mass quickly, from above, reaches the SM one.

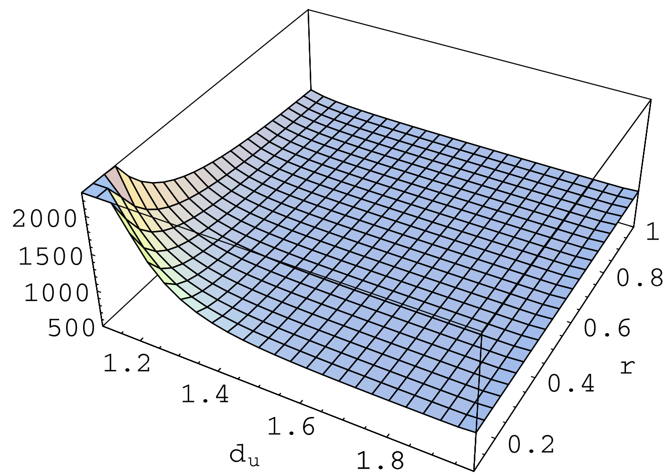

FIG. 4 (color online). The upper bound of Higgs mass m_h in GeV (vertical axis) as a function of $d_{\mathcal{U}}$ and r.

In summary, we have studied unparticle effects on the unitarity constraints from the two-body Higgs boson scattering process. We find that the unparticle contribution to the scattering partial wave amplitude can be significant which affects the unitarity constraint on the Higgs boson mass. For $d_{\mathcal{U}}$ between 1 and 2, the unparticle contribution can relax the upper bound for the Higgs boson mass. For $d_{\mathcal{U}}$ smaller than 1.3 and r smaller than 0.4, the allowed Higgs boson mass can be much larger than that in the SM.

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