

**Fermion masses and mixings in grand unified theories with noncanonical  $U(1)_Y$** Ilia Gogoladze,<sup>1,\*</sup> Chin-Aik Lee,<sup>1,+</sup> Tianjun Li,<sup>2,3,‡</sup> and Qaisar Shafi<sup>1,§</sup><sup>1</sup>*Bartol Research Institute, Department of Physics and Astronomy, University of Delaware, Newark, Delaware 19716, USA*<sup>2</sup>*George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, Texas 77843, USA*<sup>3</sup>*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, People's Republic of China*

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We discuss fermion masses and mixings in models derived from orbifold grand unified theories such that gauge coupling unification is achieved without low-energy supersymmetry by utilizing a non-canonical  $U(1)_Y$ . A gauged  $U(1)_X$  flavor symmetry plays an essential role, and the Green-Schwarz mechanism is invoked in anomaly cancellations. Models containing vectorlike particles with masses close to  $M_{\text{GUT}}$  are also discussed.

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**I. INTRODUCTION**

In some recent papers [1,2], it has been shown that unification of the standard model (SM) gauge couplings can be realized without invoking low-energy supersymmetry (SUSY). The unification scale turns out to be close to  $M_{\text{GUT}} \sim 4 \times 10^{16}$  GeV. Such models are realized from supersymmetric  $SU(N)$  gauge theories in higher dimensions compactified on suitable orbifolds, and showing that the normalization of  $U(1)_Y$  can be different from the standard value of 5/3. One well-known example utilizes a value of 4/3 for the hypercharge normalization, with SUSY broken at  $M_{\text{GUT}}$ . An important extension of these ideas implements gauge and Yukawa coupling unification at  $M_{\text{GUT}}$  [2]. For instance, with gauge-top quark Yukawa coupling unification and with SUSY broken at  $M_{\text{GUT}}$ , the SM Higgs boson mass turns out to be  $135 \pm 6$  GeV [2]. Somewhat larger values for the Higgs mass  $144 \pm 4$  GeV are found with gauge-bottom quark Yukawa coupling unification [2]. Examples based on split SUSY have also been discussed [3]. For a discussion of models with low-energy SUSY and noncanonical normalization of  $U(1)_Y$ , see [4].

Our main goal here is to understand the SM fermion masses and mixings in this framework by employing a flavor  $U(1)_X$  symmetry via the Froggatt-Nielsen (FN) mechanism [5]. The  $U(1)_X$  symmetry is gauged and has anomalies that are cancelled by the Green-Schwarz (GS) mechanism [6]. This avoids potential problems that may arise if a global  $U(1)_X$  symmetry is employed to implement the FN mechanism. Because the  $U(1)_X$  anomaly cancellations depend on the normalization of  $U(1)_Y$ , and our models exhibit SUSY at  $M_{\text{GUT}}$ , some care is required to realize a consistent framework for the SM-fermion masses and mixings in our setup. For viable fermion textures, we follow closely the discussions presented in Refs. [7–9].

One of the two scenarios includes vectorlike particles with masses close to  $M_{\text{GUT}}$ . Moreover, because SUSY is broken around the grand unified theory (GUT) scale, we do have the gauge hierarchy problem in our models. How to solve the gauge hierarchy problem is an interesting question but is out of the scope of our present work. One possible solution is the following: we embed our models into the string/M theory with flux compactifications, and then we might have many metastable string/M theory vacua, which form the string landscape [10,11]. Interestingly, the gauge hierarchy problem might be solved in the string landscape if there exist quite a few SUSY breaking hidden sectors [10,11].

The paper is organized as follows. In Sec. II, we briefly review the FN mechanism realized with an anomalous  $U(1)_X$ . The simplest models are discussed in Sec. III, while Sec. IV contains models with additional vectorlike particles. Our conclusions are summarized in Sec. V.

**II. FN MECHANISM AND YUKAWA TEXTURES**

The SM fermion masses and mixings can be explained elegantly via the FN mechanism [5], where an additional flavor dependent global  $U(1)_X$  symmetry is introduced. To stabilize this mechanism against quantum gravity corrections, we consider an anomalous gauged  $U(1)_X$  symmetry. In a weakly coupled heterotic string theory, there exists an anomalous  $U(1)_X$  gauge symmetry, where the corresponding anomalies are cancelled by the GS mechanism [6]. For completeness, let us briefly review gauge coupling unification and anomaly cancellations in weakly coupled heterotic string model building.

The generic four-dimensional Lagrangian in the Einstein metric from four-dimensional string theories can be written as

$$\mathcal{L} \supset \int d^4x \int d^2\theta S \left\{ \frac{k_C}{2} \text{Tr}_C [W_C^\alpha W_{C\alpha}] + \frac{k_W}{2} \text{Tr}_W [W_W^\alpha W_{W\alpha}] + \frac{k_Y}{4} W_Y^\alpha W_{Y\alpha} + \frac{k_X}{4} W_X^\alpha W_{X\alpha} + \frac{k_{XY}}{2} W_Y^\alpha W_{X\alpha} \right\} + \text{H.c.}, \quad (1)$$

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where we neglect the overall constant here. The last term is the gauge kinetic mixing term between  $U(1)_Y$  and  $U(1)_X$ , which is important for the anomaly cancellations in the following discussions.  $k_C, k_W, k_Y, k_X$ , and  $k_{XY}$  are the Kac-Moody levels for the gauge symmetries  $SU(3)_C, SU(2)_L, U(1)_Y, U(1)_X$ , and the  $U(1)_Y \times U(1)_X$  gauge kinetic mixing term, respectively.  $k_C$  and  $k_W$  are positive integers,  $k_Y$  and  $k_X$  are positive rational numbers, and  $k_{XY}$  is a rational number. Also, the generators for the non-Abelian gauge groups are normalized to 1/2 for the fundamental representation. With  $g_S^{-2}$  denoting the vacuum expectation value (VEV) of the real part of  $S$ , the gauge coupling unification [12,13] is given by

$$k_C g_C^2 = k_W g_W^2 = k_Y g_Y^2 = k_X g_X^2 = k_{XY} g_{XY}^2 = g_S^2, \quad (2)$$

where  $g_C, g_W, g_Y, g_X$ , and  $g_{XY}$  are the gauge couplings for  $SU(3)_C, SU(2)_L, U(1)_Y, U(1)_X$  gauge symmetries, and the  $U(1)_Y \times U(1)_X$  gauge kinetic mixing term, respectively. In addition, there exist the following terms for the imaginary part of  $S$  ( $\text{Im}(S)$ ) and the gauge field strengths

$$\begin{aligned} \mathcal{L} \supset \int d^4x \text{Im}(S) & \left\{ \frac{k_C}{2} \text{Tr}_C[F_C \wedge F_C] + \frac{k_W}{2} \text{Tr}_W[F_W \wedge F_W] \right. \\ & \left. + \frac{k_Y}{4} F_Y \wedge F_Y + \frac{k_X}{4} F_X \wedge F_X + \frac{k_{XY}}{2} F_X \wedge F_Y \right\} \\ & + \text{H.c.} \end{aligned} \quad (3)$$

The Neveu-Schwarz (NS-NS) twoform field  $B_{\mu\nu}$  in four dimensions can couple to the  $F_X$  as follows:

$$\mathcal{L} \supset \int d^4x c_0 B \wedge F_X, \quad (4)$$

where  $c_0$  is a constant. Because  $B_{\mu\nu}$  and  $\text{Im}(S)$  are dual to each other in four dimensions, we obtain that Eqs. (3) and (4) can contribute to the gauge anomalies after we contract  $B_{\mu\nu}$  and  $\text{Im}(S)$ . Therefore, the total anomaly cancellations give us the following constraints: [12,13]

$$\begin{aligned} \frac{\mathcal{A}_{CCX}}{k_C} = \frac{\mathcal{A}_{WWX}}{k_W} = \frac{\mathcal{A}_{YYX}}{k_Y} = \frac{\mathcal{A}_{XXX}}{3k_X} = \frac{\mathcal{A}_{YXX}}{k_{YX}} \\ = \frac{\mathcal{A}_{GGX}}{24}, \end{aligned} \quad (5)$$

where  $\mathcal{A}_{CCX}, \mathcal{A}_{WWX}, \mathcal{A}_{YYX}, \mathcal{A}_{XXX}$ , and  $\mathcal{A}_{YXX}$  are the gauge anomalies from the chiral fermions, and  $\mathcal{A}_{GGX}$  is the gauge-gravity mixed anomaly from the chiral fermions. The last equality is required to cancel the gauge-gravity mixed anomaly.

We follow the standard notation for the SM left-handed quark doublets, right-handed up-type quarks, right-handed down-type quarks, left-handed lepton doublets, right-handed neutrinos, and right-handed leptons by denoting them as  $q_i, u_i^c, d_i^c, l_i, \nu_i^c$ , and  $e_i^c$ , respectively. To reproduce the observed neutrino masses and mixings, we need at least two, or even three right-handed neutrinos for the seesaw

mechanism [14]. In the former case, one of the left-handed neutrinos will remain massless, which is consistent with the known data. To be concrete, we will consider three right-handed neutrinos in this paper. Moreover, since we consider GUT-scale SUSY breaking, there is one pair of Higgs doublets  $H_u$  and  $H_d$  close to the string scale.

To break the  $U(1)_X$  gauge symmetry, we introduce a flavon field  $A$  with  $U(1)_X$  charge  $-1$ . To preserve SUSY close to the string scale,  $A$  can acquire a VEV so that the  $U(1)_X$  D flatness can be realized. It was shown [7,8] that

$$0.171 \leq \epsilon \equiv \frac{\langle A \rangle}{M_{\text{Pl}}} \leq 0.221, \quad (6)$$

where  $M_{\text{Pl}}$  is the reduced Planck scale. Interestingly,  $\epsilon$  is about the size of the Cabibbo angle. Note that the  $X$  charges of the SM fermions and the Higgs fields are denoted by appropriate subscripts.

With SUSY broken around  $M_{\text{GUT}}$ , the SM fermion Yukawa couplings arising from the holomorphic superpotential at the string scale are given by

$$\begin{aligned} -\mathcal{L} = & y_{ij}^U \left( \frac{A}{M_{\text{Pl}}} \right)^{XYU_{ij}} q_i u_j^c H_u + y_{ij}^D \left( \frac{A}{M_{\text{Pl}}} \right)^{XYD_{ij}} q_i d_j^c H_d \\ & + y_{ij}^E \left( \frac{A}{M_{\text{Pl}}} \right)^{XYE_{ij}} l_i e_j^c H_d + y_{ij}^N \left( \frac{A}{M_{\text{Pl}}} \right)^{XYN_{ij}} l_i \nu_j^c H_u, \end{aligned} \quad (7)$$

where  $y_{ij}^U, y_{ij}^D, y_{ij}^E$ , and  $y_{ij}^N$  are order-one Yukawa couplings, and  $XYU_{ij}, XYD_{ij}, XYE_{ij}$ , and  $XYN_{ij}$  are non-negative integers

$$XYU_{ij} = X_{q_i} + X_{u_j^c} + X_{H_u}, \quad (8)$$

$$XYD_{ij} = X_{q_i} + X_{d_j^c} + X_{H_d},$$

$$XYE_{ij} = X_{l_i} + X_{e_j^c} + X_{H_d}, \quad (9)$$

$$XYN_{ij} = X_{l_i} + X_{\nu_j^c} + X_{H_u}.$$

Our goal is to generate Yukawa textures that can explain the SM fermion masses and mixings. The quark textures  $A, B$ , and  $C$  in Table I, with  $\epsilon \approx 0.2$ , as copied from [7–9], can reproduce the SM quark Yukawa couplings and the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix.

And the following lepton textures can reproduce the neutrino masses and Pontecorvo-Maki-Nakagata-Saki (PMNS) neutrino mixing matrix

$$Y^E \sim \epsilon^c \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^1 \\ \epsilon^3 & \epsilon^2 & \epsilon^0 \\ \epsilon^3 & \epsilon^2 & \epsilon^0 \end{pmatrix}, \quad (10)$$

$$M_{LL} \sim \frac{\langle H_u \rangle^2}{M_s} \epsilon^{-5} \begin{pmatrix} \epsilon^2 & \epsilon^1 & \epsilon^1 \\ \epsilon^1 & \epsilon^0 & \epsilon^0 \\ \epsilon^1 & \epsilon^0 & \epsilon^0 \end{pmatrix}, \quad (11)$$

TABLE I. The three quark textures.

Yukawa	A	B	C
$Y^U$	$\begin{pmatrix} \epsilon^8 & \epsilon^5 & \epsilon^3 \\ \epsilon^7 & \epsilon^4 & \epsilon^2 \\ \epsilon^5 & \epsilon^2 & \epsilon^0 \end{pmatrix}$	$\begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & \epsilon^0 \end{pmatrix}$	$\begin{pmatrix} \epsilon^8 & \epsilon^4 & \epsilon^2 \\ \epsilon^8 & \epsilon^4 & \epsilon^2 \\ \epsilon^6 & \epsilon^2 & \epsilon^0 \end{pmatrix}$
$Y^D$	$\epsilon^c \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^1 & \epsilon^0 & \epsilon^0 \end{pmatrix}$	$\epsilon^c \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^4 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon^0 & \epsilon^0 & \epsilon^0 \end{pmatrix}$	$\epsilon^c \begin{pmatrix} \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon^0 & \epsilon^0 \end{pmatrix}$

where  $c$  is either 0, 1, 2 or 3, and  $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle$  satisfies  $\epsilon^c \sim \epsilon^3 \tan\beta$ . This neutrino texture requires some amount of fine-tuning as it generically predicts

$$\sin\theta_{12} \sim \epsilon, \quad \Delta m_{12}^2 \sim \Delta m_{23}^2. \quad (12)$$

Interestingly, with  $\epsilon$  as large as 0.2, the amount of fine-tuning needed is not that huge, and this is shown in the computer simulations of [7–9] with random values for the coefficients. Although the authors considered the supersymmetric models in Refs. [7–9], we still use their GUT-scale Yukawa textures. The reason is that at the GUT-scale, the CKM quark mixing matrices are roughly the same, and the differences for the quark and lepton masses are generically smaller than about 20% for the SM and minimal supersymmetric standard model (MSSM) [15].

To obtain the desired textures, the  $X$  charge assignments for the SM fermions and Higgs fields satisfy the following equations:

$$X_{H_u} + X_{q_3} + X_{u_3^c} = 0, \quad (13a)$$

$$X_{q_2} - X_{q_3} = 2, \quad (13b)$$

$$X_{q_1} - X_{q_3} = 3, 4, 2, \quad (13c)$$

$$X_{u_2^c} - X_{u_3^c} = 2, \quad (13d)$$

$$X_{u_1^c} - X_{u_3^c} = 5, 4, 6, \quad (13e)$$

$$X_{H_d} + X_{q_3} + X_{d_3^c} = c, \quad (13f)$$

$$X_{d_2^c} - X_{d_3^c} = 0, \quad (13g)$$

$$X_{d_1^c} - X_{d_3^c} = 1, 0, 2, \quad (13h)$$

$$X_{H_d} + X_{l_3} + X_{e_3^c} = c, \quad (13i)$$

$$X_{l_2} - X_{l_3} = 0, \quad (13j)$$

$$X_{l_1} - X_{l_2} = 1. \quad (13k)$$

Finally, regardless of whether we use texture  $A$ ,  $B$ , or  $C$ , the following equations always hold:

$$3X_{H_u} + \sum_i [X_{q_i} + X_{u_i^c}] = 12, \quad (14a)$$

$$3X_{H_d} + \sum_i [X_{q_i} + X_{d_i^c}] = 6 + 3c, \quad (14b)$$

$$3X_{H_d} + \sum_i [X_{l_i} + X_{e_i^c}] = 6 + 3c. \quad (14c)$$

### III. MODELS WITHOUT EXTRA VECTORLIKE PARTICLES

In this paper, we assume that  $k_C = k_W = 1$  because the semirealistic string models have so far only been constructed at the Kac-Moody level one for the non-Abelian gauge factors. Because of the GUT-scale SUSY breaking, we only have the SM as the effective theory below  $M_{\text{GUT}}$ . It has been shown [1] that gauge coupling unification can be achieved if we choose

$$k_Y = \frac{4}{3}, \quad k_C = k_W = 1. \quad (15)$$

Thus, we shall work with  $k_C = k_W = 1$ .

Before we calculate the gauge anomalies, let us explain our conventions. We define the anomalous contributions of the chiral fermions as follows:

$$\mathcal{A}_{ABC} = \frac{1}{2} \text{Tr}_{\text{matter}} [T_A T_B T_C] \quad (16)$$

for Abelian symmetries  $A$ ,  $B$ , and  $C$ , and

$$\mathcal{A}_{AAC} \frac{\text{Tr}_A [T_A^a T_A^b]}{2} = \frac{1}{2} \text{Tr}_{\text{matter}} [T_A^a T_A^b T_C] \quad (17)$$

for a non-Abelian symmetry  $A$  and an Abelian symmetry  $C$ , where  $\text{Tr}_A$  is the trace over a fundamental representation of  $A$ . More specifically, the anomalies are given as follows:

$$\mathcal{A}_{CCX} = \sum_i [2X_{q_i} + X_{u_i^c} + X_{d_i^c}], \quad (18a)$$

$$\mathcal{A}_{WWX} = \sum_i [3X_{q_i} + X_{l_i}] + X_{H_u} + X_{H_d}, \quad (18b)$$

$$\begin{aligned} \mathcal{A}_{YYX} = & \sum_i \left[ \frac{1}{3} X_{q_i} + \frac{8}{3} X_{u_i^c} + \frac{2}{3} X_{d_i^c} + X_{l_i} + 2X_{e_i^c} \right] \\ & + X_{H_u} + X_{H_d}, \end{aligned} \quad (18c)$$

$$\begin{aligned} \mathcal{A}_{YXX} = & \sum_i [X_{q_i}^2 - 2X_{u_i^c}^2 + X_{d_i^c}^2 - X_{l_i}^2 + X_{e_i^c}^2] \\ & + X_{H_u}^2 - X_{H_d}^2, \end{aligned} \quad (18d)$$

$$\begin{aligned} \mathcal{A}_{XXX} = & \sum_i \left[ 3X_{q_i}^3 + \frac{3}{2} X_{u_i^c}^3 + \frac{3}{2} X_{d_i^c}^3 + X_{l_i}^3 + \frac{1}{2} X_{e_i^c}^3 + \frac{1}{2} X_{\nu_i^c}^3 \right] \\ & + X_{H_u}^3 - X_{H_d}^3 - \frac{1}{2} + \dots, \end{aligned} \quad (18e)$$

$$\mathcal{A}_{GGX} = \sum_{\phi} X_{\phi}, \quad (18f)$$

where  $\mathcal{A}_{XXX}$  and  $\mathcal{A}_{GGX}$  also receive contributions from the flavinos and possibly other superfields as well.

The gauge and gauge-gravity mixed anomalies can be cancelled by the GS mechanism, as discussed in the previous section. We will not consider  $\mathcal{A}_{XXX}$  and  $\mathcal{A}_{GGX}$ , since these anomalies can always be cancelled by introducing additional SM singlet superfields, which are charged under  $U(1)_X$ . These extra superfields can be decoupled from the SM after  $U(1)_X$  breaking, which is why we may neglect them. However, we would like to point out that to explain the SM fermion masses and mixings via the FN mechanism in  $U(1)_X$  models, the anomalies  $\mathcal{A}_{XXX}$  and  $\mathcal{A}_{GGX}$  are typically on the order of a thousand ( $\mathcal{O}(1000)$ )! To cancel these anomalies, we have to introduce literally hundreds of SM singlets with suitable  $U(1)_X$  charges. It remains to be seen whether such  $U(1)_X$  models can arise from string theory.

We will now combine Eq. (5) with above equations to solve for the  $X$  charges. First, we can derive the following equations:

$$\begin{aligned}\mathcal{A}_{CCX} &= -3(X_{H_u} + X_{H_d}) + \left\{ 3X_{H_u} + \sum_i [X_{q_i} + X_{u_i^c}] \right\} \\ &\quad + \left\{ 3X_{H_d} + \sum_i [X_{q_i} + X_{d_i^c}] \right\} \\ &= -3(X_{H_u} + X_{H_d}) + 18 + 3c,\end{aligned}\quad (19)$$

$$\begin{aligned}\left(k_Y - \frac{5}{3}\right)\mathcal{A}_{CCX} &= \mathcal{A}_{WWX} + \mathcal{A}_{YYX} - \frac{8}{3}\mathcal{A}_{CCX} \\ &= 2(X_{H_u} + X_{H_d}) + \sum_i [-2X_{q_i} - 2X_{d_i^c} \\ &\quad + 2X_{l_i} + 2X_{e_i^c}] \\ &= 2(X_{H_u} + X_{H_d}) + \left\{ 3X_{H_d} + \sum_i [X_{l_i} + X_{e_i^c}] \right\} \\ &\quad - \left\{ 3X_{H_d} + \sum_i [X_{q_i} + X_{d_i^c}] \right\} \\ &= 2(X_{H_u} + X_{H_d}),\end{aligned}\quad (20)$$

$$\begin{aligned}\left(k_Y - \frac{5}{3}\right)\{-3(X_{H_u} + X_{H_d}) + 18 + 3c\} \\ = 2(X_{H_u} + X_{H_d}), \\ X_{H_u} + X_{H_d} = \frac{\left(k_Y - \frac{5}{3}\right)(6 + c)}{k_Y - 1},\end{aligned}\quad (21)$$

$$\mathcal{A}_{WWX} = \mathcal{A}_{CCX},$$

$$X_{H_u} + X_{H_d} + \sum_i [3X_{q_i} + X_{l_i}] = -3(X_{H_u} + X_{H_d}) + 18 + 3c,$$

$$\sum_i [3X_{q_i} + X_{l_i}] = \frac{\left(-k_Y + \frac{11}{3}\right)(6 + c)}{k_Y - 1},\quad (22)$$

$$\begin{aligned}\mathcal{A}_{CCX} &= -3(X_{H_u} + X_{H_d}) + 18 + 3c \\ &= -3 \frac{\left(k_Y - \frac{5}{3}\right)(6 + c)}{k_Y - 1} + 18 + 3c \\ &= 2 \frac{6 + c}{k_Y - 1}.\end{aligned}\quad (23)$$

Equation (23) tells us that unless  $k_Y = 1$ , which is inconsistent with gauge coupling unification, we will have contributions to the gauge anomalies from the chiral fermions. This explains why the GS mechanism is necessary in the first place.

Assuming that all the neutrino Dirac and Majorana matrices are holomorphic due to SUSY, the  $ia$  entry of the Dirac coupling behaves as

$$Y_{Diracia} \sim \mathcal{O}(\epsilon^{X_{H_u} + X_{l_i} + X_{\nu_a^c}}),\quad (24)$$

while the  $ab$  entry of the Majorana coupling goes as

$$M_{RRab} \sim M_s \mathcal{O}(\epsilon^{X_{\nu_a^c} + X_{\nu_b^c}}).\quad (25)$$

From the seesaw relation [14]

$$M_{LL} = \langle H_u \rangle^2 Y_D M_{RR}^{-1} Y_D^T,\quad (26)$$

we find

$$\begin{aligned}M_{LLij} &\sim \frac{\langle H_u \rangle^2}{M_s^{-1}} \max_{ab} [\mathcal{O}(\epsilon^{X_{H_u} + X_{l_i} + X_{\nu_a^c}}) \mathcal{O}(\epsilon^{-X_{\nu_a^c} - X_{\nu_b^c}}) \\ &\quad \times \mathcal{O}(\epsilon^{X_{H_u} + X_{l_j} + X_{\nu_b^c}})] \\ &\sim \frac{\langle H_u \rangle^2}{M_s^{-1}} \mathcal{O}(\epsilon^{2X_{H_u} + X_{l_i} + X_{l_j}}).\end{aligned}\quad (27)$$

Combining this with the neutrino mass texture ansatz in Eq. (11), we obtain

$$2X_{H_u} + 2X_{l_1} = -3,\quad (28a)$$

$$2X_{H_u} + 2X_{l_2} = -5,\quad (28b)$$

$$2X_{H_u} + 2X_{l_3} = -5.\quad (28c)$$

The only constraints on the  $X$  charges of the right-handed neutrinos coming from the previous analysis are

$$X_{H_u} + X_{l_i} + X_{\nu_a^c} = \text{nonnegative integer},\quad (29a)$$

$$X_{\nu_a^c} + X_{\nu_b^c} = \text{nonnegative integer}\quad (29b)$$

for all  $i$ ,  $a$ , and  $b$ . This tells us that the  $X$  charges of  $\nu^c$  are all odd multiples of a half and that they are all at least  $5/2$ . As the right-handed neutrinos are SM singlets, they do not contribute to  $\mathcal{A}_{CCX}$ ,  $\mathcal{A}_{WWX}$ ,  $\mathcal{A}_{YYX}$ , or  $\mathcal{A}_{YXX}$ . We will only need to know their actual  $X$  charges if we wish to compute  $\mathcal{A}_{XXX}$  and  $\mathcal{A}_{GGX}$ .

### A. Embedded matter parity

In accordance with [7–9], we will assume that matter parity is embedded in the  $X$  charges, although this is not necessary in our models. The even matter parity superfields have integral  $X$  charges, and the odd matter parity superfields have  $X$  charges, which are odd multiples of  $1/2$ . The flavon and the SUSY breaking sector all have even matter parity. This simplifies our analysis appreciably, because it prevents Higgsino-lepton and axino-flavino-right-handed neutrino mixings. Matter parity also protects the zero VEV of the right-handed sneutrinos.<sup>1</sup> Without it, we will have to come up with more convoluted explanations for each of these inconvenient details. Embedding matter parity within  $U(1)_X$  in this manner circumvents problems with the non-conservation of global symmetries in quantum gravity. By making all even matter superfields have  $X$  charges that are even, and all odd matter superfields have  $X$  charges that are odd after being multiplied by  $2m$  will also give rise to matter parity for a positive integer  $m$ . This is because we can identify matter parity with the parity of the  $X$  charge multiplied by  $2m$ . Equation (28) tells us that  $m$  has to be odd. One consequence of our previous requirement is that  $X_{H_u} + X_{H_d}$  has to be an integral multiple of  $\frac{1}{m}$ , and according to Eq. (21), this can only happen if  $\frac{m(k_Y - \frac{5}{3})(6+c)}{k_Y - 1}$  is integral.<sup>2</sup> In addition, Eq. (22) tells us that

$$-(X_{H_u} + X_{H_d}) = 1 \pmod{3}. \quad (30)$$

Refs. [7–9] did not have to deal with this additional constraint even though they assumed that  $k_Y = \frac{5}{3}$  and  $X_{H_u} + X_{H_d} = 0$ , because they did not always insist upon the texture given in Eq. (10). We present some possible values of  $k_Y$  between 1 and  $5/3$  in Table II, where the previous two conditions are satisfied.

With matter parity and a huge  $M_{3/2}$ , the lightest SUSY particle (LSP) is both stable and extremely heavy. This may also give us SUSY dark matter. However, the LSP would overclose the Universe if they were in thermal equilibrium. Thus, if the LSP does contribute to the dark matter density, it must be produced nonthermally.

### B. Solution

We now have enough information to arrive at the  $X$  charge assignments in Table III with  $k_Y = \frac{4}{3}$  and  $c = 1$ .

Other solutions are obtained by adding arbitrary non-negative integers to the right-handed neutrinos'  $X$  charges and adding some multiples of the corresponding particles' hypercharges to their  $X$  charges. Only the above solutions are physical, while the other solutions only amount to a

<sup>1</sup>A nonzero VEV that is not too tiny will lead to appreciable Higgsino-lepton mixings.

<sup>2</sup>This constraint will definitely have to be modified if we add additional superfields, which are charged under the SM as in Sec. IV.

TABLE II. Permissible values for  $k_Y$  which are consistent with matter parity.

$c$	$k_Y$
0	$\frac{11}{7}, \frac{7}{5}, \frac{17}{13}, \frac{5}{4}, \frac{23}{19}, \frac{13}{11}, \frac{29}{25}, \frac{8}{7}, \dots$
1	$\frac{19}{47}, \frac{4}{4}, \frac{65}{37}, \frac{83}{46}, \frac{101}{87}, \dots$
2	$\frac{43}{13}, \frac{33}{61}, \frac{51}{35}, \frac{79}{19}, \frac{11}{97}, \frac{83}{45}, \dots$
3	$\frac{27}{9}, \frac{45}{27}, \frac{63}{9}, \frac{81}{45}, \dots$
	$\frac{8}{5}, \frac{19}{13}, \frac{11}{8}, \frac{25}{19}, \frac{14}{11}, \frac{31}{25}, \frac{17}{14}, \frac{37}{31}, \dots$

redefinition of  $X$  as hypercharge is itself another gauge symmetry.

Let us note that in order to have  $A_{YXX} = 0$ , we will find irrational  $U(1)_X$  charges for the SM fermions and Higgs fields, since we have a quadratic equation for  $U(1)_X$  charges.  $A_{YXX} = 0$  can be realized for rational  $U(1)_X$  charges of the SM fermions and Higgs fields if we introduce extra vectorlike particles.

### C. Supersymmetry breaking

Let us assume that a chiral superfield  $Z$  is responsible for the GUT-scale SUSY breaking and that the soft SUSY breaking corrections to the other superfields arise from their direct couplings to  $Z$ . This means that the VEV of  $F$  component of  $Z$  has to be nonzero  $\langle F_Z \rangle \sim M_{\text{Pl}} M_{3/2}$ , where  $M_{3/2}$  is comparable to the GUT scale. While  $Z$  is neutral under the SM gauge group, it may or may not be neutral under  $U(1)_X$ . If  $Z$  is neutral under  $U(1)_X$ , it could be identified with the dilaton field  $S$ .

To obtain the SM as the effective theory below the GUT scale, one linear combination of  $H_u$  and  $H_d^*$  has mass around the weak scale, while the orthogonal linear combination of  $H_u$  and  $H_d^*$ , gauginos, sfermions, and Higgsinos should have masses around the GUT scale. Thus, we must make sure that these masses can be generated at the correct scales.

If  $Z$  is neutral under  $U(1)_X$ , we have the following Kähler potential

$$\begin{aligned} & \int d^4x d^2\theta d^2\bar{\theta} \left( \frac{\bar{Z}Z}{M_{\text{Pl}}^2} (H_u \bar{H}_u + H_d \bar{H}_d) + \frac{\bar{Z}}{M_{\text{Pl}}} \right. \\ & \times \left( \frac{\bar{A}}{M_{\text{Pl}}} \right)^{-X_{H_u} - X_{H_d}} H_u H_d + \frac{\bar{Z}Z}{M_{\text{Pl}}^2} \\ & \left. \times \left( \frac{\bar{A}}{M_{\text{Pl}}} \right)^{-X_{H_u} - X_{H_d}} H_u H_d \right) + \text{H.c.}, \quad (31) \end{aligned}$$

where we do not display the order one coefficients, the third term gives us the  $\mu$  term, and the rest of the terms give us the scalar Higgs masses. Then we obtain the following “bare” scalar Higgs mass matrix:

$$\begin{array}{cc} & H_u^\dagger & H_d \\ H_u & \mathcal{O}(M_{3/2}^2) & \mathcal{O}(\epsilon^{-X_{H_u} - X_{H_d}} M_{3/2}^2) \\ H_d^\dagger & \mathcal{O}(\epsilon^{-X_{H_u} - X_{H_d}} M_{3/2}^2) & \mathcal{O}(M_{3/2}^2) \end{array}. \quad (32)$$

TABLE III. The  $X$  charge assignments for  $k_Y = \frac{4}{3}$  and  $c = 1$  in models without additional vectorlike particles.

	$H_u$	$H_d$	$Q_1$	$Q_2$	$Q_3$	$U_1^c$	$U_2^c$	$U_3^c$	$D_1^c$	$D_2^c$	$D_3^c$	$L_1$	$L_2$	$L_3$	$E_1^c$	$E_2^c$	$E_3^c$	$N_1^c$	$N_2^c$	$(N_3^c)$	$\mathcal{A}_{CCX}$	$\mathcal{A}_{YXX}$	$k_{YX}$
A	-6	-1	$\frac{11}{2}$	$\frac{9}{2}$	$\frac{5}{2}$	$\frac{17}{2}$	$\frac{11}{2}$	$\frac{7}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{9}{2}$	$\frac{7}{2}$	$\frac{7}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	$\geq \frac{5}{2}$	$\geq \frac{5}{2}$	$\geq \frac{5}{2}$	42	$\frac{101}{2}$	$\frac{101}{84}$
B	-5	-1	$\frac{11}{2}$	$\frac{9}{2}$	$\frac{5}{2}$	$\frac{13}{2}$	$\frac{9}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{7}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	$\geq \frac{5}{2}$	$\geq \frac{5}{2}$	$\geq \frac{5}{2}$	42	$\frac{165}{2}$	$\frac{55}{28}$
C	-7	0	$\frac{9}{2}$	$\frac{9}{2}$	$\frac{5}{2}$	$\frac{21}{2}$	$\frac{13}{2}$	$\frac{9}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$	$\frac{11}{2}$	$\frac{9}{2}$	$\frac{9}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{7}{2}$	$\geq \frac{5}{2}$	$\geq \frac{5}{2}$	$\geq \frac{5}{2}$	42	$\frac{85}{2}$	$\frac{85}{84}$

Note that for  $-X_{H_u} - X_{H_d} = 7$ , we cannot have one linear combination of  $H_u$  and  $H_d^*$  with mass around the weak scale.

To solve this problem, we assume that the  $U(1)_X$  charge for  $Z$  is equal to  $X_{H_u} + X_{H_d}$ . We also assume that  $\langle F_S \rangle \sim M_{\text{Pl}} M_{3/2}$ . Then the Kähler potential is given by

$$\int d^4x d^2\theta d^2\bar{\theta} \left( \left( \frac{\bar{Z}Z}{M_{\text{Pl}}^2} + \frac{\bar{S}S}{M_{\text{Pl}}^2} \right) (H_u \bar{H}_u + H_d \bar{H}_d) + \frac{\bar{Z}}{M_{\text{Pl}}} H_u H_d + \frac{\bar{Z}S}{M_{\text{Pl}}^2} H_u H_d \right) + \text{H.c.} \quad (33)$$

And the scalar Higgs mass matrix becomes

$$\begin{array}{ccc} & H_u^\dagger & H_d \\ H_u & \mathcal{O}(M_{3/2}^2) & \mathcal{O}(M_{3/2}^2) \\ H_d^\dagger & \mathcal{O}(M_{3/2}^2) & \mathcal{O}(M_{3/2}^2) \end{array} \quad (34)$$

Thus, we can fine-tune the SUSY breaking soft Higgs masses so that one linear combination of  $H_u$  and  $H_d^*$  has mass around the weak scale. We emphasize that the fine-tuning of the SUSY breaking soft Higgs masses has to be very precise and is a manifestation of the gauge hierarchy problem. In addition, we can show that the nonholomorphic contributions to the SM Yukawa couplings can be neglected.

The gaugino masses can be generated via Eq. (1), while the squark, slepton, and the sneutrino masses can be generated via the Kähler potential

$$\int d^4x d^2\theta d^2\bar{\theta} \left( \frac{\bar{Z}Z}{M_{\text{Pl}}^2} + \frac{\bar{S}S}{M_{\text{Pl}}^2} \right) \bar{\phi} \phi + \text{H.c.}, \quad (35)$$

where  $\phi$  denotes the SM fermion superfields.

#### IV. MODELS WITH ADDITIONAL SM VECTOR SUPERFIELDS

If there is a single superfield  $Z$  that breaks SUSY and is neutral under  $U(1)_X$ , to obtain one linear combination of  $H_u$  and  $H_d^*$  with mass around the weak scale, we find that

$$X_{H_u} + X_{H_d} = 2. \quad (36)$$

The soft SUSY breaking mass-squared terms for  $H_u H_d$  are generated from the superpotential term  $Z A^2 H_u H_d / M_{\text{Pl}}^2$ .

As shown above, to cancel anomalies via the GS mechanism, we now have to introduce additional superfields, which are vectorlike under the SM but not under  $U(1)_X$ . In this case, we can also have the rational solutions with  $A_{YXX} = 0$ . However, the price for adding these new superfields is that Eq. (18) will have to be modified and all of the equations that follow from them, as well as the charge assignments of Table III in general. To give a concrete example, let us introduce the new superfields  $H'_u$  and  $H'_d$  with the SM charges  $(1, 2)_1$  and  $(1, 2)_{-1}$ , respectively. To prevent Yukawa couplings with the SM matter superfields, we may set their  $X$  charges to some nonintegral value. The modified anomalies are now

$$\mathcal{A}_{CCX} = \sum_i [2X_{q_i} + X_{u_i^c} + X_{d_i^c}], \quad (37a)$$

$$\begin{aligned} \mathcal{A}_{WWX} &= (X_{H_u} + X_{H_d}) + (X_{H'_u} + X_{H'_d}) \\ &+ \sum_i [3X_{q_i} + X_{l_i}], \end{aligned} \quad (37b)$$

$$\begin{aligned} \mathcal{A}_{YYX} &= (X_{H_u} + X_{H_d}) + (X_{H'_u} + X_{H'_d}) + \sum_i \left[ \frac{1}{3} X_{q_i} + \frac{8}{3} X_{u_i^c} \right. \\ &\left. + \frac{2}{3} X_{d_i^c} + X_{l_i} + 2X_{e_i^c} \right], \end{aligned} \quad (37c)$$

$$\begin{aligned} \mathcal{A}_{YXX} &= X_{H_u}^2 - X_{H_d}^2 + X_{H'_u}^2 - X_{H'_d}^2 \\ &+ \sum_i [X_{q_i}^2 - 2X_{u_i^c}^2 + X_{d_i^c}^2 - X_{l_i}^2 + X_{e_i^c}^2], \end{aligned} \quad (37d)$$

$$\left( k_Y - \frac{5}{3} \right) \mathcal{A}_{CCX} = \mathcal{A}_{WWX} + \mathcal{A}_{YYX} - \frac{8}{3} \mathcal{A}_{CCX},$$

$$\begin{aligned} \left( k_Y - \frac{5}{3} \right) \{ -3(X_{H_u} + X_{H_d}) + 18 + 3c \} \\ = 2(X_{H_u} + X_{H_d}) + 2(X_{H'_u} + X_{H'_d}), \\ X_{H'_u} + X_{H'_d} = -\frac{3k_Y - 3}{2} (X_{H_u} + X_{H_d}) \\ + \frac{3k_Y - 5}{2} (6 + c). \end{aligned} \quad (38)$$

If  $1 \leq k_Y < 5/3$  and  $X_{H_u} + X_{H_d}$  is non-negative, then  $X_{H'_u} + X_{H'_d}$  is negative. If we want the scalar  $H'$  and its superpartner to get a mass from the flavon VEV, we would also want  $X_{H'_u} + X_{H'_d}$  to be integral. Since  $Z$  is neutral under  $U(1)_X$ , the  $H'$ -ino mass comes from the Kähler

term  $Z^\dagger A^{\dagger -X_{H'_u} - X_{H'_d}} H'_u H'_d$ . This is of order  $M_{3/2} \epsilon^{-X_{H'_u} - X_{H'_d}}$ , which is low enough ( $\sim \epsilon^4 M_{3/2}$ ) to modify somewhat the gauge coupling unification. In essence, what we have done is to shift the problem from the Higgs to the Higgs'. So, we have

$$\begin{aligned} \mathcal{A}_{WWX} &= \mathcal{A}_{CCX}, \\ \sum_i [3X_{q_i} + X_{l_i}] &= \frac{3k_Y - 11}{2} (X_{H_u} + X_{H_d}) \\ &\quad - \frac{3k_Y - 11}{2} (6 + c). \end{aligned} \quad (39)$$

We present the solutions with  $\mathcal{A}_{YXX} = 0$  to these equations in Table IV.

On the other hand, if we arrange to add full “ $SU(5)$ ” multiplet superfield pairs and make their masses of order  $\epsilon^4 M_{3/2}$ , the GUT scale is almost the same, but the unified gauge coupling becomes strong. Let us call the additional superfields  $XL$ ,  $XD^c$ ,  $\overline{XL}$ , and  $\overline{XD^c}$  with the SM quantum numbers  $(1, 2)_{-1}$ ,  $(\overline{3}, 1)_{2/3}$ ,  $(1, 2)_1$ , and  $(3, 1)_{-2/3}$ , respectively. The anomalies are now

$$\mathcal{A}_{CCX} = (X_{XD^c} + X_{\overline{XD^c}}) + \sum_i [2X_{q_i} + X_{u_i^c} + X_{d_i^c}], \quad (40a)$$

$$\begin{aligned} \mathcal{A}_{WWX} &= (X_{H_u} + X_{H_d}) + (X_{XL} + X_{\overline{XL}}) \\ &\quad + \sum_i [3X_{q_i} + X_{l_i}], \end{aligned} \quad (40b)$$

$$\begin{aligned} \mathcal{A}_{YYX} &= (X_{H_u} + X_{H_d}) + (X_{XL} + X_{\overline{XL}}) + \frac{2}{3} (X_{XD^c} + X_{\overline{XD^c}}) \\ &\quad + \sum_i \left[ \frac{1}{3} X_{q_i} + \frac{8}{3} X_{u_i^c} + \frac{2}{3} X_{d_i^c} + X_{l_i} + 2X_{e_i^c} \right], \end{aligned} \quad (40c)$$

$$\begin{aligned} \mathcal{A}_{YXX} &= X_{H_u}^2 - X_{H_d}^2 - X_{XL}^2 + X_{\overline{XL}}^2 + X_{XD^c}^2 - X_{\overline{XD^c}}^2 \\ &\quad + \sum_i [X_{q_i}^2 - 2X_{u_i^c}^2 + X_{d_i^c}^2 - X_{l_i}^2 + X_{e_i^c}^2], \end{aligned} \quad (40d)$$

$$\mathcal{A}_{CCX} = (X_{XD^c} + X_{\overline{XD^c}}) - 3(X_{H_u} + X_{H_d}) + 18 + 3c, \quad (41)$$

$$\begin{aligned} &\left( k_Y - \frac{5}{3} \right) [(X_{XD^c} + X_{\overline{XD^c}}) - 3(X_{H_u} + X_{H_d}) + 18 + 3c] \\ &= 2(X_{H_u} + X_{H_d}) + 2(X_{XL} + X_{\overline{XL}}) - \frac{8}{3} (X_{XD^c} + X_{\overline{XD^c}}), \end{aligned} \quad (42)$$

TABLE IV. The  $X$  charge assignments for  $k_Y = 4/3$ ,  $\mathcal{A}_{YXX} = 0$ , and  $c = 0$  and  $2$  in models with  $H'_u$  and  $H'_d$ .

Field	A		B		C	
	$c = 0$	$c = 2$	$c = 0$	$c = 2$	$c = 0$	$c = 2$
$H_u$	$3a - \frac{11}{6}$	$3a - \frac{25}{6}$	$3a - \frac{5}{6}$	$3a - \frac{19}{6}$	$3a - \frac{17}{6}$	$3a - \frac{31}{6}$
$H_d$	$-3a + \frac{23}{6}$	$-3a + \frac{37}{6}$	$-3a + \frac{17}{6}$	$-3a + \frac{31}{6}$	$-3a + \frac{29}{6}$	$-3a + \frac{43}{6}$
$H'_u$	$\frac{169}{72} - \frac{19}{3}a + 8a^2$	$\frac{121}{90} - \frac{107}{15}a + \frac{32}{5}a^2$	$\frac{97}{72} - \frac{1}{3}a + 8a^2$	$-\frac{41}{90} - \frac{7}{3}a + \frac{32}{5}a^2$	$\frac{421}{72} - \frac{37}{3}a + 8a^2 + 8a^2$	$\frac{463}{90} - \frac{179}{15}a + \frac{32}{5}a^2$
$H'_d$	$-\frac{457}{72} + \frac{19}{3}a - 8a^2$	$-\frac{571}{90} + \frac{107}{15}a - \frac{32}{5}a^2$	$-\frac{385}{72} + \frac{1}{3}a - 8a^2$	$-\frac{409}{90} + \frac{7}{3}a - \frac{32}{5}a^2$	$-\frac{709}{72} + \frac{37}{3}a - 8a^2$	$-\frac{913}{90} + \frac{179}{15}a - \frac{32}{5}a^2$
$q_1$	$a + 3$	$a + 3$	$a + 4$	$a + 4$	$a + 2$	$a + 2$
$q_2$	$a + 2$	$a + 2$	$a + 2$	$a + 2$	$a + 2$	$a + 2$
$q_3$	$a$	$a$	$a$	$a$	$a$	$a$
$u_1^c$	$-4a + \frac{41}{6}$	$-4a + \frac{55}{6}$	$-4a + \frac{29}{6}$	$-4a + \frac{43}{6}$	$-4a + \frac{53}{6}$	$-4a + \frac{67}{6}$
$u_2^c$	$-4a + \frac{23}{6}$	$-4a + \frac{37}{6}$	$-4a + \frac{17}{6}$	$-4a + \frac{31}{6}$	$-4a + \frac{29}{6}$	$-4a + \frac{43}{6}$
$u_3^c$	$-4a + \frac{11}{6}$	$-4a + \frac{25}{6}$	$-4a + \frac{5}{6}$	$-4a + \frac{19}{6}$	$-4a + \frac{17}{6}$	$-4a + \frac{31}{6}$
$d_1^c$	$2a - \frac{17}{6}$	$2a - \frac{19}{6}$	$2a - \frac{17}{6}$	$2a - \frac{19}{6}$	$2a - \frac{17}{6}$	$2a - \frac{19}{6}$
$d_2^c$	$2a - \frac{23}{6}$	$2a - \frac{25}{6}$	$2a - \frac{17}{6}$	$2a - \frac{19}{6}$	$2a - \frac{29}{6}$	$2a - \frac{31}{6}$
$d_3^c$	$2a - \frac{23}{6}$	$2a - \frac{25}{6}$	$2a - \frac{17}{6}$	$2a - \frac{19}{6}$	$2a - \frac{29}{6}$	$2a - \frac{31}{6}$
$l_1$	$-3a + \frac{1}{3}$	$-3a + \frac{8}{3}$	$-3a - \frac{2}{3}$	$-3a + \frac{5}{3}$	$-3a + \frac{4}{3}$	$-3a + \frac{11}{3}$
$l_2$	$-3a - \frac{2}{3}$	$-3a + \frac{5}{3}$	$-3a - \frac{5}{3}$	$-3a + \frac{2}{3}$	$-3a + \frac{1}{3}$	$-3a + \frac{8}{3}$
$l_3$	$-3a - \frac{2}{3}$	$-3a + \frac{5}{3}$	$-3a - \frac{2}{3}$	$-3a + \frac{2}{3}$	$-3a + \frac{1}{3}$	$-3a + \frac{8}{3}$
$e_1^c$	$6a - \frac{1}{6}$	$6a - \frac{17}{6}$	$6a + \frac{11}{6}$	$6a - \frac{5}{6}$	$6a - \frac{13}{6}$	$6a - \frac{29}{6}$
$e_2^c$	$6a - \frac{7}{6}$	$6a - \frac{23}{6}$	$6a + \frac{5}{6}$	$6a - \frac{11}{6}$	$6a - \frac{19}{6}$	$6a - \frac{35}{6}$
$e_3^c$	$6a - \frac{19}{6}$	$6a - \frac{25}{6}$	$6a - \frac{7}{6}$	$6a - \frac{23}{6}$	$6a - \frac{31}{6}$	$6a - \frac{47}{6}$
$\nu_1^c$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$
$\nu_2^c$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$
$\nu_3^c$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$	$\cong \frac{5}{2}$

TABLE V. The  $X$  charge assignments for  $k_Y = 4/3$  in the models with  $XL$ ,  $\overline{XL}$ ,  $XD^c$ , and  $\overline{XD}^c$ .

	$A$	$B$	$C$
$H_u$	$1 - c$	$2 - c$	$-c$
$H_d$	$1 + c$	$c$	$d$
$XL$	$d$	$d$	$d$
$\overline{XL}$	$24 + 3c - d$	$24 + 3c - d$	$24 + 3c - d$
$XD^c$	$e$	$e$	$e$
$\overline{XD}^c$	$24 + 3c - e$	$24 + 3c - e$	$24 + 3c - e$
$Q_1$	$\frac{7}{2}$	$\frac{9}{2}$	$\frac{5}{2}$
$Q_2$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$
$Q_3$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$U_1^c$	$\frac{7}{2} + c$	$\frac{3}{2} + c$	$\frac{11}{2} + c$
$U_2^c$	$\frac{1}{2} + c$	$-\frac{1}{2} + c$	$\frac{3}{2} + c$
$U_3^c$	$-\frac{3}{2} + c$	$-\frac{5}{2} + c$	$-\frac{1}{2} + c$
$D_1^c$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$D_2^c$	$-\frac{3}{2}$	$-\frac{1}{2}$	$-\frac{5}{2}$
$D_3^c$	$-\frac{3}{2}$	$-\frac{1}{2}$	$-\frac{5}{2}$
$L_1$	$-\frac{5}{2} + c$	$-\frac{7}{2} + c$	$-\frac{3}{2} + c$
$L_2$	$-\frac{7}{2} + c$	$-\frac{9}{2} + c$	$-\frac{5}{2} + c$
$L_3$	$-\frac{7}{2} + c$	$-\frac{9}{2} + c$	$-\frac{5}{2} + c$
$E_1^c$	$\frac{11}{2} - c$	$\frac{15}{2} - c$	$\frac{7}{2} - c$
$E_2^c$	$\frac{9}{2} - c$	$\frac{13}{2} - c$	$\frac{5}{2} - c$
$E_3^c$	$\frac{9}{2} - c$	$\frac{9}{2} - c$	$\frac{1}{2} - c$
$N_1^c$	$\geq \frac{5}{2}$	$\geq \frac{5}{2}$	$\geq \frac{5}{2}$
$N_2^c$	$\geq \frac{5}{2}$	$\geq \frac{5}{2}$	$\geq \frac{5}{2}$
$(N_3^c)$	$\geq \frac{5}{2}$	$\geq \frac{5}{2}$	$\geq \frac{5}{2}$
$\mathcal{A}_{CCX}$	$36 + 6c$	$36 + 6c$	$36 + 6c$

$$\begin{aligned} \mathcal{A}_{WWX} &= \mathcal{A}_{CCX}, \\ \sum_i [3X_{q_i} + X_{l_i}] &= -4(X_{H_u} + X_{H_d}) - (X_{XL} + X_{\overline{XL}}) \\ &\quad + (X_{XD^c} + X_{\overline{XD}^c}) + 18 + 3c. \end{aligned} \quad (43)$$

To ensure that the fermionic superpartners of  $XL$  and  $XD^c$  have masses of the same order of magnitude, we require that

$$X_{XL} + X_{\overline{XL}} = X_{XD^c} + X_{\overline{XD}^c}, \quad (44)$$

$$\begin{aligned} X_{XL} + X_{\overline{XL}} &= X_{XD^c} + X_{\overline{XD}^c} \\ &= -\frac{3k_Y - 5}{k_Y - 1}(6 + c) + 3(X_{H_u} + X_{H_d}). \end{aligned} \quad (45)$$

The sum is actually positive for the range of values of  $k_Y$  and  $X_{H_u} + X_{H_d}$  that we are interested in. This makes

a significant difference as the superpotential coupling  $A^{X_{XL} + X_{\overline{XL}}} XL \overline{XL}$  and  $A^{X_{XL} + X_{\overline{XL}}} XD^c \overline{XD}^c$  are actually allowed provided that the sum is an integer. Unfortunately, the suppression factor is way too large, as the sum is typically between 24 and 33. We list the concrete  $X$  charge assignments in Table V.

## V. DISCUSSION AND CONCLUSIONS

Gauge coupling unification is a powerful tool for constructing realistic models. We have considered the SM fermion masses and mixings in theories obtainable from higher dimensional models, in which the SM gauge couplings unify without invoking the low energy SUSY at scales of order  $10^3$  GeV. Because any global symmetry could, in principle, be broken by nonrenormalizable terms induced by quantum gravity effects, we considered an anomalous gauged flavor  $U(1)_X$  symmetry, which plays an essential role in our analysis. Although low energy SUSY is absent in our case, the latter does play an essential role at  $M_{\text{GUT}}$ .

We analyzed three kinds of models. All of these models can be made compatible not only with the observed SM fermion mass hierarchies and mixings by employing the FN mechanism, but also with the  $U(1)_X$  anomaly cancellations via the GS mechanism. And matter parity can be embedded as well. In the first class of models, we do not introduce vectorlike particles, and the anomaly  $A_{YXX}$  is not zero, otherwise the SM fermions and Higgs fields will have irrational  $U(1)_X$  charges. To fine-tune the SUSY breaking soft masses for the Higgs doublet fields so that one linear combination of  $H_u$  and  $H_d^*$  has weak-scale mass, we need to introduce a SUSY breaking chiral superfield with nonzero  $U(1)_X$  charge. Interestingly, the nonholomorphic contributions to the SM Yukawa couplings can be neglected. In the second and third class of models, we introduce new vectorlike particles. In these models, the anomaly  $A_{YXX}$  can be made zero for rational  $U(1)_X$  charges of the SM fermions and Higgs fields. Also, we can have one light scalar Higgs doublet via fine-tuning without introducing a SUSY breaking chiral superfield with nonzero  $U(1)_X$  charge. Finally, in class three models, gauge coupling unification can be preserved, while it is somewhat modified in models in the second class.

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