Fermion mass hierarchy and proton stability from nonanomalous $U(1)_F$ in supersymmetric $SU(5)$

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We present a realistic supersymmetric $SU(5)$ model combined with a nonanomalous $U(1)_F$ symmetry. We find a set of $U(1)_F$ charges which automatically lead to the realistic mass hierarchy and mixing patterns for quarks, leptons, and neutrinos. All gauge anomalies, including the $[U(1)_F]^3$ anomaly, are canceled in our model without invoking the Green-Schwarz mechanism or having exotic fields. Proton decay mediated by dimension-5 operators is automatically suppressed in our model, because the scale set by the largest right-handed neutrino mass is much less than the GUT scale.

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I. INTRODUCTION

The fermion mass hierarchy and mixings are some of the least understood aspects of the standard model (SM). Many attempts have been made to understand the large disparity among the masses and mixing angles [1]. One approach is the Froggatt-Nielsen (FN) mechanism [2], where an $U(1)_F$ family symmetry is introduced under which the SM fermions are charged. The $U(1)_F$ symmetry is broken by the vacuum expectation value of a SM-singlet scalar field ϕ whose $U(1)_F$ charge, without loss of generality, is normalized to be -1. Fermion masses are generated by the operators

$$Y_{ij}\left(\frac{\phi}{\Lambda}\right)^{|q_i+q_j+q_H|}\bar{\Psi}_i\Psi_j H, \tag{1}$$

if $(q_i + q_j + q_H)$ is a positive integer, and ϕ is replaced by $\phi^\dagger$ if $(q_i + q_j + q_H)$is a negative integer. Here i, j are generation indices, and q_i, q_j, and q_H are, respectively, the $U(1)_F$ charges of the fermions $\bar{\Psi}_i$, Ψ_j and the SM Higgs doublet H. The parameter Λ is the cutoff scale of the $U(1)_F$ symmetry. Upon breaking the $U(1)_F$ symmetry, the effective Yukawa couplings can be written as

$$Y_{ij}^{\text{eff}} = Y_{ij}\lambda^{|q_i+q_j+q_H|}, \tag{2}$$

where $\lambda = \langle\phi\rangle/\Lambda$ if $(q_i + q_j + q_H)$ is a positive integer, and $\lambda = \langle\phi^\dagger\rangle/\Lambda$ if $(q_i + q_j + q_H)$ is a negative integer. By having appropriate $U(1)_F$ charges for various fermions, the realistic masses and mixing patterns can be accommodated, with λ being smaller than unity and $Y_{ij} \sim \mathcal{O}(1)$. If the model is supersymmetric, which is the case in our model, the couplings to the $\phi^\dagger$ are not allowed, since the superpotential must be holomorphic. However, in order to ensure D-flatness, a $\bar{\phi}$ field, which carries charge $+1$, must be introduced, with a vacuum expectation value close to that of ϕ. In this case, $\lambda = \langle\bar{\phi}\rangle/\Lambda$ if $(q_i + q_j + q_H)$ is a negative integer. Note that as a result the contributions of ϕ, $\bar{\phi}$ to gauge anomalies cancel.

Models based on a global $U(1)_F$ symmetry have been constructed before (see references in [1]). However, as any global symmetry is broken by quantum gravity effects, one inevitably has to promote the $U(1)_F$ symmetry to be a gauge symmetry. In this case, the $U(1)_F$ charges of the fields are constrained by the anomaly cancellation conditions. Most attempts [3–5] so far have focused on an *anomalous* $U(1)_F$ symmetry, in which the mixed anomalies are canceled by the Green-Schwarz mechanism [6], while the $[U(1)_F]^3$ anomaly is not addressed. The $[U(1)_F]^3$ anomaly can be canceled by introducing exotic matter fields charged under $U(1)_F$ [7]. However, these models often involve a rather large number of exotic particles whose role is merely to cancel the anomalies [8].

Here we pursue an alternative scenario [9] where the theory is anomaly-free, without invoking the Green-Schwarz mechanism or exotic particles other than the right-handed neutrinos, which are required for neutrino masses. (We do have the FN pair of SM singlets ϕ, $\bar{\phi}$ described above, and we will find it necessary to introduce another such oppositely charged pair, but these do not contribute to gauge anomalies.) We propose a SUSY $SU(5)$ model, combined with a *nonanomalous* $U(1)_F$, in the presence of right-handed neutrinos. We find a set of $U(1)_F$ charges that satisfy *all* the anomaly cancellation conditions, including the $[U(1)_F]^3$ anomaly. Note that these charges have to be rational numbers in order for the model to be embedded into a simple group in the UV completed theory; it is thus highly nontrivial for these solutions to exist. We show that these $U(1)_F$ charges give rise to realistic quark and charged lepton masses and mixing patterns.

The charges we find also lead to a FN mixing pattern for the Dirac neutrino Yukawa matrix, Y_ν; however, to accommodate the right-handed neutrino masses required for the seesaw mechanism, we need to introduce a second pair of

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SM-singlet fields $(\chi, \bar{\chi})$ with charges $\mp\frac{5}{9}$. This leads to a realistic texture for the light neutrino mass matrix, and if we assume that the observed atmospheric neutrino (mass)^2 difference sets the scale of the heaviest neutrino mass, it follows that $\langle\chi\rangle, \langle\bar{\chi}\rangle \sim 10^{11}$ GeV.

Furthermore, we then find that the dimension-4 R-parity violating operators $\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_k$, are forbidden, and moreover the dimension-5 operators which mediate proton decay are automatically suppressed without additional assumptions or fine-tuning of parameters, because they arise from higher dimensional operators in the effective field theory involving powers of χ, $\bar{\chi}$. This solves a major problem with the usual minimal SUSY $SU(5)$ GUT theories, and allows our model to be viable.

II. THE MODEL

In $SU(5)$, the three generations of matter fields are unified into $\bar{\mathbf{5}}_i$ and $\mathbf{10}_i$ representations, where $i = 1, 2, 3$ is the generation index. Under $U(1)_F$, $\bar{\mathbf{5}}_i$ and $\mathbf{10}_i$ have charges q_{f_i} and q_{t_i}, respectively. We also introduce right-handed neutrinos, the number of which is a free parameter. If the type-I seesaw [10] is the mechanism that gives rise to light neutrino masses, we need at least two right-handed neutrinos to accommodate the current neutrino oscillation data; we will take three right-handed neutrinos, N_i, which are $SU(5)$ singlets and carry $U(1)_F$ charges, q_{n_i}.

To generate realistic fermion mass hierarchy utilizing the FN mechanism and to cancel the gauge anomalies, it turns out that two conjugate pairs of $\mathbf{5}$ and $\bar{\mathbf{5}}$ Higgses are required [this will be clear once the $U(1)_F$ charges are presented], which we denote as $\mathbf{5}_{H_1}$, $\bar{\mathbf{5}}_{H_1}$, $\mathbf{5}_{H_2}$, and $\bar{\mathbf{5}}_{H_2}$. The $U(1)_F$ charges of $\mathbf{5}_{H_1}$ and $\mathbf{5}_{H_2}$ are q_{H_1} and q_{H_2}, respectively. In addition, we need a $\mathbf{24}$-dim Higgs to break $SU(5)$ to the SM gauge group. We take this Higgs to be neutral under the $U(1)_F$ symmetry. The $U(1)_F$ symmetry is broken spontaneously by the vacuum expectation values of the $SU(5)$ singlets, ϕ and $\bar{\phi}$, whose $U(1)_F$ charges are normalized to -1 and $+1$, respectively. Note that $\langle\phi\rangle - \langle\bar{\phi}\rangle \ll \Lambda$, as required by D-flatness.

There are three anomaly cancellation conditions that have to be satisfied: the $[SU(5)]^2U(1)_F$, gravitation-$U(1)_F$ and $[U(1)_F]^3$ anomalies. Since the $\mathbf{24}$-dim Higgs is neutral under the $U(1)_F$ symmetry, it does not contribute to these anomalies. As the $\mathbf{5}$-dim Higgses and ϕ all appear in conjugate pairs, they do not contribute either. Therefore, only the matter fields, $\bar{\mathbf{5}}_i$, $\mathbf{10}_i$, and right-handed neutrinos N_i, contribute to the anomalies. To cancel the anomalies, their $U(1)_F$ charges must satisfy

$$\frac{1}{2}\sum_i q_{f_i} + \frac{3}{2}\sum_i q_{t_i} = 0, \tag{3}$$

$$5\sum_i q_{fi} + 10\sum_i q_{t_i} + \sum_i q_{n_i} = 0, \tag{4}$$

$$5\sum_i q_{fi}^3 + 10\sum_i q_{t_i}^3 + \sum_i q_{n_i}^3 = 0. \tag{5}$$

Following [9], we parametrize the charges as

$$q_{t_1} = -\tfrac{1}{3}q_{f_1} - 2a, \tag{6}$$

$$q_{t_2} = -\tfrac{1}{3}q_{f_2} + a + a', \tag{7}$$

$$q_{t_3} = -\tfrac{1}{3}q_{f_3} + a - a', \tag{8}$$

and

$$q_{n_1} = -\tfrac{5}{3}q_{f_1} - 2b, \tag{9}$$

$$q_{n_2} = -\tfrac{5}{3}q_{f_2} + b + b', \tag{10}$$

$$q_{n_3} = -\tfrac{5}{3}q_{f_3} + b - b'. \tag{11}$$

With this parametrization, the conditions (3) and (4) are satisfied automatically. The values of q_{f_i}, a, a', b, and b' are constrained by the cubic equation (5), as well as the observed fermion masses and mixing patterns.

III. FERMION MASSES AND MIXINGS

The up-type quark mass matrix is given by the Yukawa coupling,

$$\lambda^{|q_{t_i}+q_{t_j}+q_{H_1}|}\mathbf{10}_i\mathbf{10}_j\mathbf{5}_{H_1}, \tag{12}$$

where $\lambda = \langle\phi\rangle/\Lambda$. We will take the expansion parameter to be the Cabibbo angle, $\lambda \sim 0.22$. Note that if the sum of the charges $(q_{t_i} + q_{t_j} + q_{H_1})$ is noninteger for some i and j, that particular Yukawa coupling is forbidden.

In general, there are similar operators involving $\mathbf{5}_{H_2}$ which contribute to the up-type quark masses and thus must be included. As we will show later, due to the $U(1)_F$ charge of the $\mathbf{5}_{H_2}$, these operators are suppressed because the sum of charges $(q_{t_i} + q_{t_j} + q_{H_2})$ is noninteger for all i and j. It is thus sufficient to consider only the operators given in Eq. (12). In this paper, we restrict ourselves to the case with $(q_{t_i} + q_{t_j} + q_{H_1}) > 0$. The exponents that determine the quark mass matrix elements U_{ij} are then

$$\begin{pmatrix} |2q_{t_1} + q_{H_1}| & |q_{t_1} + q_{t_2} + q_{H_1}| & |q_{t_1} + q_{t_3} + q_{H_1}| \\ & |2q_{t_2} + q_{H_1}| & |q_{t_2} + q_{t_3} + q_{H_1}| \\ & & |2q_{t_3} + q_{H_1}| \end{pmatrix} \tag{13}$$

with $U_{ij} = U_{ji}$.

As the top quark mass is large, it is natural to assume that it is unsuppressed by the expansion parameter. We therefore demand $2q_{t_3} + q_{H_1} = 0$. We also assume $q_{f_2} = q_{f_3}$, which is motivated by the large atmospheric neutrino mixing. With these assumptions, the parameters

$$a' = 1, \qquad -\tfrac{1}{3}(q_{f_1} - q_{f_2}) - 3a = 2, \tag{14}$$

gives the up-type quark Yukawa couplings

$$Y_u \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \tag{15}$$

yielding a realistic up-type quark mass hierarchy [11].

Down-type quark masses are generated by the Yukawa couplings,

$$\lambda^{|q_{t_i}+q_{f_j}-q_{H_2}|}\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_{H_2}. \tag{16}$$

All couplings to the $\bar{\mathbf{5}}_{H_1}$ are highly suppressed, because the corresponding sums of the $U(1)_F$ charges are noninteger, as we shall see when we present the solutions for the charges.

Let us assume that the b-quark mass is generated at the renormalizable level. (In fact we have also examined the cases when the b-mass Yukawa is suppressed by a factor of λ^{α_b} with $\alpha_b = 1, 2, 3$, but without finding a solution more elegant than the one we present here). The exponents of the elements in the down-type quark mass matrix are then

$$\begin{pmatrix} |-9a-3| & 3 & 3 \\ |-9a-4| & 2 & 2 \\ |-9a-6| & 0 & 0 \end{pmatrix}. \tag{17}$$

The observed mass hierarchy among the down-type quarks can be obtained with $a = -\frac{7}{9}$ and $q_{f_1} - q_{f_2} = 1$ when we take $(q_{t_i} + q_{f_j} - q_{H_2}) > 0$. The Yukawa couplings for the down-type quarks and the charged leptons are

$$Y_d \sim Y_e^T \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}. \tag{18}$$

The Yukawa matrices Y_u and Y_d also give rise to realistic Cabibbo-Kobayashi-Maskawa matrix elements [11]. In addition, the Georgi-Jarlskog relations [12] for the first and second generations of down-type quarks and charged leptons can be obtained by introducing a **45**-dim Higgs (accompanied by a $\overline{\mathbf{45}}$ to maintain anomaly cancellation). The following discussion does not depend on this.

Let us return to the anomaly cancellation conditions. The cubic equation in terms of the three free parameters b, b', and q_{f_2} reduces to a linear one and it is given by

$$q_{f_2} = -\frac{4550 + 2430b^2 + 729b^3 - 81b(-25 + 9b'^2)}{45(124 + 90b + 81b^2 + 27b'^2)}. \tag{19}$$

Thus, for any rational values of b and b', there always exists a solution for q_{f_2}. The simplest set of solutions we found correspond to $b = -37/18$ and $b' = 3/2$. The corresponding $U(1)_F$ charges for all the fields in the model are shown in Table I. It is remarkable that such a simple solution to all the anomaly cancellation constraints exists. We note that there are degenerate solutions. With $b = -5/9$ and $b' = 3$, we again get $q_{f_2} = -1/2$, but now $q_{n_1} = 5/18$ and $q_{n_2} = 59/18$. These charges lead to the same effective neutrino mass matrix, m_ν, as given in Eq. (23) below.

We now consider the neutrino sector. The neutrino Dirac mass matrix is generated by the Yukawa couplings

$$\lambda^{|q_{f_i}+q_{n_j}+q_{H_1}|}\bar{\mathbf{5}}_i N_j \mathbf{5}_{H_1}. \tag{20}$$

As in the charged fermion sector, we will find that all couplings to the other Higgs (in this case $\mathbf{5}_{H_2}$) are highly suppressed. With the charge assignment given in Table I, the neutrino Dirac Yukawa matrix is given by

$$Y_\nu \sim \begin{pmatrix} \lambda^7 & \lambda^4 & \lambda \\ \lambda^6 & \lambda^3 & 1 \\ \lambda^6 & \lambda^3 & 1 \end{pmatrix}. \tag{21}$$

In order to use the type-I seesaw mechanism to generate the effective neutrino masses, we must introduce a second pair of SM singlets χ, $\bar{\chi}$ with charges $\mp 5/9$. Then the Majorana mass matrix for the right-handed neutrinos is

$$M_{RR} \sim \begin{pmatrix} \lambda^6 & \lambda^3 & 1 \\ \lambda^3 & 1 & \lambda^3 \\ 1 & \lambda^3 & \lambda^6 \end{pmatrix}\langle\chi\rangle. \tag{22}$$

The effective light neutrino mass matrix, after implementing the seesaw mechanism, is

$$m_\nu \sim Y_\nu M_{RR}^{-1} Y_\nu^T v^2 \sim \begin{pmatrix} \lambda^8 & \lambda^7 & \lambda^7 \\ \lambda^7 & \lambda^6 & \lambda^6 \\ \lambda^7 & \lambda^6 & \lambda^6 \end{pmatrix}\frac{v^2}{\langle\chi\rangle}, \tag{23}$$

where $v = \langle H_1 \rangle$. If we assume $v \sim 240$ GeV and that the largest neutrino $(\text{mass})^2$ is around 2×10^{-3} eV^2, we find $\langle\chi\rangle \sim 10^{11}$ GeV. The textures given in Eqs. (15), (18), and (23) have been shown to give successful fermion masses and mixings [11], including those in the neutrino sector [13].

IV. PROTON DECAY

The usual minimal SUSY $SU(5)$ GUT model suffers from the problem of having rapid proton decay due to

TABLE I. $U(1)_F$ charges of different fields.

Field	$\bar{\mathbf{5}}_1$	$\bar{\mathbf{5}}_2$	$\bar{\mathbf{5}}_3$	$\mathbf{10}_1$	$\mathbf{10}_2$	$\mathbf{10}_3$	N_1	N_2	N_3	$\mathbf{5}_{H1}$	$\mathbf{5}_{H2}$
$U(1)_F$ charge	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{25}{18}$	$\frac{7}{18}$	$-\frac{29}{18}$	$\frac{59}{18}$	$\frac{5}{18}$	$-\frac{49}{18}$	$\frac{29}{9}$	$-\frac{19}{9}$

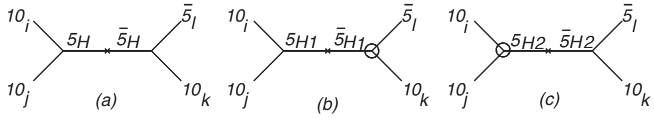

FIG. 1. Feynman diagrams of dimension-5 operators that lead to proton decay mediated by color triplet Higgsinos. (a) Dimension-5 proton decay operators in the usual minimal SUSY $SU(5)$ model [23]. (b), (c) In the presence of the $U(1)_F$, these operators are absent because the Yukawa couplings that are circled are highly suppressed.

the dimension-5 operators mediated by colored triplet Higgsinos, if the masses of the SUSY particles are ~ 1 TeV [14].

In our model the R-parity violating operators $\lambda_{ijk}\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_k$ are forbidden by our $U(1)_F$ charge assignments, and moreover cannot be generated via higher dimensional terms involving powers of ϕ, $\bar{\phi}$ and/or χ, $\bar{\chi}$. (In fact with right-handed neutrinos we also have the possibility of dimension-4 R-parity violating operators of the form $N_iN_jN_k$ and $N_i\mathbf{5}_{H_{1,2}}\bar{\mathbf{5}}_{H_{1,2}}$; these are similarly forbidden).

Let us now consider the dimension-5 operators $\kappa_{ijkl}\mathbf{10}_i\mathbf{10}_j\mathbf{10}_k\bar{5}_l$. In the usual SUSY GUT theory, one needs to tune the parameters κ_{1121}, κ_{1122} to be smaller than $10^{-8}/M_{Pl}$ [15]. These operators are also forbidden here by $U(1)_F$ conservation, but can be generated from higher-dimension operators involving ϕ, $\bar{\phi}$ and χ, $\bar{\chi}$. For example the operator $\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{5}_1$ has $U(1)_F$ charge $11/3$, which can be generated from the operator

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{5}_1\left(\frac{\phi^2\chi^3}{\Lambda^6}\right) \tag{24}$$

which for $\Lambda \sim M_{\rm GUT}$ is very suppressed. All the κ_{ijkl} operators are suppressed by $(\langle\chi\rangle/\Lambda)^3$.

In the standard $SU(5)$ treatment, these operators are generated by color triplet Higgs exchange. In our model, however, we have two conjugate pairs of Higgses, $(\mathbf{5}_{H_1} \oplus \bar{\mathbf{5}}_{H_1})$ and $(\mathbf{5}_{H_2} \oplus \bar{\mathbf{5}}_{H_2})$. With the $U(1)_F$ charge assignment given in Table I, the couplings $\mathbf{10}_i\mathbf{10}_j\mathbf{5}_{H_2}$ and $\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_{H_1}$ are also suppressed by $(\langle\chi\rangle/\Lambda)^3$ for any i and j, because the sums of the $U(1)_F$ charges of the fields involved in each of these operators are fractions of the form $2/3$, $-1/3$, $7/3$, etc. The mass terms that mix $\mathbf{5}_{H_1} - \bar{\mathbf{5}}_{H_2}$ and $\mathbf{5}_{H_2} - \bar{\mathbf{5}}_{H_1}$ are similarly suppressed by a factor of $(\langle\chi\rangle/\Lambda)^3\lambda^7$. In Fig. 1 we contrast the situations in the standard $SU(5)$ treatment and in our model.

Therefore, all dangerous dimension-5 operators that could lead to fast proton decay are absent in this model.[1]

We note that a similar mechanism to suppress dimension-5 proton decay operators has been discussed in, for example, [4,17]. For more recent work utilizing a discrete symmetry, see [18]. In these models, unlike in our case, the cubic anomaly cancellation condition was not imposed to constrain the charges.

[1]Of course it is possible to envisage models where dimension-5 proton decay contributions are permitted which are nevertheless consistent with observations [16]; however, it seems to us more attractive if they are forbidden in a natural way.

V. CONCLUSION

We have constructed a realistic model based on SUSY $SU(5) \times U(1)_F$, which is free *all gauge anomalies*. It is quite remarkable that we are able to find such a simple solution for the charges that achieves this. Realistic fermion masses and mixing angles are generated upon breaking of the $U(1)_F$ symmetry. We find that three right-handed neutrinos are required in this model in order to cancel the gauge anomalies, in addition to generating neutrino masses. Most interestingly, all dimension-5 operators that could lead to proton decay are automatically suppressed. The model therefore possesses all the successes of grand unification, while still being consistent with the limits from nonobservation of proton decay.

We have not discussed the supersymmetry-breaking sector of the theory, nor the origin of the low energy Higgs potential. One might, for example, consider anomaly mediated supersymmetry breaking; particularly since then introduction of an anomaly-free $U(1)$ has been advocated as leading to a solution of the tachyonic slepton problem. However, for this to work all the lepton doublets and the charged lepton singlets must have the same sign of the $U(1)$ charge, so it is not compatible with the structure of our model. In a non-GUT context, a viable marriage of FN textures with anomaly mediation and a $U(1)_F$ was described in Ref. [19] [although in that analysis, unlike here, exotic SM singlets are again required to cancel the $U(1)_F$ cubic and gravitational anomalies].

Since our quark and lepton mass matrices arise from Yukawa couplings to $\mathbf{5}_{H_1}$ and $\bar{\mathbf{5}}_{H_2}$, we need the light Higgs doublets H_u and H_d to come primarily from these representations. Note that as indicated above the $\mathbf{5}_{H_1} - \bar{\mathbf{5}}_{H_2}$ mass term is suppressed; however, the $\mathbf{5}_{H_1} - \bar{\mathbf{5}}_{H_1}$ and $\mathbf{5}_{H_2} - \bar{\mathbf{5}}_{H_2}$ mass terms are allowed. One way to obtain light Higgs doublets would be to introduce couplings $\mathbf{5}_{H_1}\mathbf{24}\,\bar{\mathbf{5}}_{H_1}$ and $\mathbf{5}_{H_2}\mathbf{24}\,\bar{\mathbf{5}}_{H_2}$ tuned as in the original supersymmetric $SU(5)$ model so as to leave two pairs of light Higgs doublets and heavy Higgs triplets. Having two pairs of Higgs doublets would mean that there would have to be quite large threshold corrections in order to maintain gauge unification, unless one arranged to have one light pair of color triplets. (This possibility has been considered recently in E_6-based models [20].) We hope to return elsewhere to a full construction of the Higgs sector of the theory.

It would be interesting to see if this model can be realized in a more direct way, for example, in a string theory model of intersecting branes. It may be possible to implement the mechanism of [21], in which the connection between leptogenesis and low energy leptonic CP violation can be established [22]. It would be interesting to investigate this further.

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