Strong decays of heavy-light mesons in a chiral quark model

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We carry out a systematic study of the heavy-light meson strong decays in a chiral quark model. For the S-wave vectors $[D^*(2007), D^{*+}(2010)]$, *P*-wave scalars $[D_0^*(2400), B_0^*(5730)]$, and tensors $[D_2^*(2460), D^{*}(2573)]$ we obtain results in good agreement with the experimental data. For the axial vectors $D_{s2}^*(2573)$, we obtain results in good agreement with the experimental data. For the axial vectors $D_{s2}(2420)$ and $D'(2430)$ a state mixing scheme between $1^{1}R_{s2}$ and $1^{3}R_{s2}$ is favored with a mixing angle $D_1(2420)$ and $D'_1(2430)$, a state mixing scheme between 1^1P_1 and 1^3P_1 is favored with a mixing angle $\phi \sim -(55 + 5)$ °, which is consistent with previous theoretical predictions. The same mixing scheme also $\phi \approx -(55 \pm 5)^\circ$, which is consistent with previous theoretical predictions. The same mixing scheme also
applies to D. (2460) and D. (2536) that accounts for the parrow width of the D. (2536) and its dominant applies to $D_{s1}(2460)$ and $D_{s1}(2536)$ that accounts for the narrow width of the $D_{s1}(2536)$ and its dominant decay into D^*K . For $B_1(5725)$ and $B'_1(5732)$, such a mixing explains well the decay width of the former
but leads to an even broader $B'(5732)$. Predictions for the strange bottom axial vectors are also made. For but leads to an even broader $B_1^1(5732)$. Predictions for the strange-bottom axial vectors are also made. For
the undetermined meson $D^*(2640)$, we find that they fit well in the radially excited state 2^3S , according the undetermined meson $D^*(2640)$, we find that they fit well in the radially excited state 2^3S_1 according to its decay mode. The newly observed $D_{sJ}^*(2860)$ strongly favors the D-wave excited state 1^3D_3 . For $D^*(2632)$ and $D^*(2690)$ we find they are difficult to fit in any D, excitations in that mass region if the $D_{sJ}^*(2632)$ and $D_{sJ}^*(2690)$, we find they are difficult to fit in any D_s excitations in that mass region, if the
experimental data are accurate. Theoretical predictions for decay modes of those upobserved states a experimental data are accurate. Theoretical predictions for decay modes of those unobserved states as multiplets of 2S and 1D waves are also presented, which should be useful for the further experimental search for those states.

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I. INTRODUCTION

During the past several years, significant progress has been made in the observation of the heavy-light mesons. In 2003, two new narrow charm-strange mesons $D_{sJ}^*(2317)$
and $D^*(2460)$ were observed by $BARR$ CLEO and Belle and $D_{sJ}^*(2460)$ were observed by *BABAR*, CLEO, and Belle
[1–5] Recently *BABAR* reported another two new charm-[1–[5](#page-13-0)]. Recently, *BABAR* reported another two new charmstrange mesons, i.e. $D_{sJ}^*(2860)$ with a width of (47 ± 17) MeV and $D^*(2690)$ with a width of $(112 +$ 17) MeV and $D_{sJ}^*(2690)$ with a width of (112 ± 43) MeV in the *DK* decay channel [6] Meanwhile Belle 43) MeV in the DK decay channel $[6]$. Meanwhile, Belle reported a new vector state $D_{sJ}^{*}(2708)$ with a width of $(108 + 23^{+36})$ MeV [7]. The $D^{*}(2690)$ and $D^{*}(2708)$ $(108 \pm 23^{+36}_{-31})$ MeV [\[7](#page-13-0)]. The $D_{sJ}^*(2690)$ and $D_{sJ}^*(2708)$
are believed to be the same state since their masses and are believed to be the same state since their masses and widths are consistent with each other. In the B meson sector two narrow states $B_1(5725)$ and $B_2^*(5740)$ were
reported by CDE [8], and are assigned as orbitally excited reported by CDF [[8\]](#page-13-0), and are assigned as orbitally excited B mesons. They were confirmed by the D0 Collaboration with slightly different masses [[9\]](#page-13-0). The CDF Collaboration also reported their strange analogues, $B_{s1}(5829)$ and $B_{s2}^*(5840)$, as orbitally excited B_s mesons [10]. The $B_s^*(5840)$ is also observed by the D0 Collaboration [11] $B_{s2}^*(5840)$ is also observed by the D0 Collaboration [11].
About the recent experimental status of the heavy-light About the recent experimental status of the heavy-light mesons, many reviews can be found in Refs. [12–18].

To understand the nature of the heavy-light mesons, especially the newly observed states, and to establish the heavy-light meson spectroscopy, a lot of effort has been put into both experiment and theory. For example, one can find recent discussions about the dynamics and decay properties of the heavy-light mesons given by Close and Swanson [19], Godfrey [[20](#page-13-0)], and other previous analyses in Refs. [[21–32\]](#page-13-0). For the new observed heavy-light mesons, such as $D_{sJ}^{*}(2860)$ and $D_{sJ}^{*}(2690)$, various attempts on the explanation of their nature have been made [33–43]. Many explanation of their nature have been made [\[33–43](#page-13-0)]. Many systematic studies are devoted to establishing the D, D_s, B , and B_s spectroscopies $[44–49]$ $[44–49]$ $[44–49]$, while some earlier work can be found in Refs. [[21,](#page-13-0)[50](#page-14-0)]. Recent reviews of the status of the theory study of the heavy-light mesons can be found in Refs. [\[51–58\]](#page-14-0).

On the one hand, the improved experimental measurements help clarify some old questions on the spectrum. On the other hand, they also raise some new ones which need further experimental and theoretical study [[59](#page-14-0),[60](#page-14-0)]. For instance, $D^*(2640)$ reported by DELPHI in $D^{*+}\pi^+\pi^-$
[61] as the first radial excited state still has not yet been [\[61\]](#page-14-0) as the first radial excited state still has not yet been confirmed by any other experiments. The spin-parity of the narrow $D_1(2420)$ also needs confirmation. The status of the broad $D_0^*(2400)$ is not clear at all: its measured mass and
width have uncertainties that are too large. For the D width have uncertainties that are too large. For the D_s spectroscopy, the low masses of the $D_{sJ}^{*}(2317)$ and $D^{*}(2460)$ still cannot be well explained by theory $D_{sJ}^*(2460)$ still cannot be well explained by theory;
whether they are exotic states is an open question whether they are exotic states is an open question. Theoretical predictions for the $D_{sJ}^{*}(2860)$ and $D_{sJ}^{*}(2690)$
are far from convergence. The narrow state $D^{*}(2632)$ seen are far from convergence. The narrow state $D_{sJ}^*(2632)$ seen
by the SELEX Collaboration [62] cannot be naturally by the SELEX Collaboration [\[62\]](#page-14-0) cannot be naturally explained by any existing theory. Nevertheless, since the flavor symmetry of the heavy-light mesons is badly broken, a mixture of states with the same J^P may occur. This will add further complexities into the meson spectrum, and further theoretical investigationis needed.

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In this work, we make a systematic study of the strong decays of heavy-light mesons in a chiral quark model. In the heavy-quark infinite mass limit, the flavor symmetry no longer exists in the heavy-light mesons, which allows us to describe the initial and final D, D_s, B , and B_s mesons in a nonrelativistic framework self-consistently. The meson decay will proceed through a single-quark transition by the emission of a pseudoscalar meson. An effective chiral Lagrangian is then introduced to account for the quarkmeson coupling. Since the quark-meson coupling is invariant under the chiral transformation, some of the lowenergy properties of QCD are retained. This approach is similar to that used in Refs. [[21,22](#page-13-0)], except that the two constants in the decay amplitudes of Refs. [\[21,22\]](#page-13-0) are replaced by two energy-dependent factors deduced from the chiral Lagrangian in our model.

The chiral quark model approach has been well developed and widely applied to meson photoproduction reactions [[63](#page-14-0)–[71](#page-14-0)]. Its recent extension to describe the process of πN scattering and investigate the strong decays of charmed baryons also turns out to be successful and inspiring [[72,73](#page-14-0)].

The paper is organized as follows. In the subsequent section, the heavy-light meson in the quark model is outlined. Then, the nonrelativistic quark-meson couplings are given in Sec. III. The decay amplitudes are deduced in Sec. IV. We present our calculations and discussions in Sec. V. Finally, a summary is given in Sec. VI.

II. MESON SPECTROSCOPY

A. Harmonic oscillator states

For a heavy-light $\overline{O}q$ system consisting of light quark 1 and heavy quark 2 with masses m_1 and m_2 , respectively, its eigenstates are conventionally generated by a harmonic oscillator potential

$$
\mathcal{H} = \frac{1}{2m_1} \mathbf{p}_1^2 + \frac{1}{2m_2} \mathbf{p}_2^2 + \frac{3}{2} K (\mathbf{r}_1 - \mathbf{r}_2)^2, \qquad (1)
$$

where vectors \mathbf{r}_i and \mathbf{p}_i are the coordinate and momentum for the *j*th quark in the meson rest frame, and K describes the oscillator potential strength which is independent of the flavor quantum number. One defines the Jacobi coordinates to eliminate the c.m. variables:

$$
\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \tag{2}
$$

$$
\mathbf{R}_{\text{c.m.}} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2}.
$$
 (3)

With the above relations (2) and (3) , the oscillator Hamiltonian ([1](#page-1-0)) is reduced to

$$
\mathcal{H} = \frac{P_{\text{c.m.}}^2}{2M} + \frac{1}{2\mu} \mathbf{p}^2 + \frac{3}{2}Kr^2,\tag{4}
$$

where

$$
\mathbf{p} = \mu \dot{\mathbf{r}}, \qquad \mathbf{P}_{\text{c.m.}} = M \dot{\mathbf{R}}_{\text{c.m.}}, \tag{5}
$$

with

$$
M = m_1 + m_2, \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}.
$$
 (6)

From Eqs. (2), (3), and (5), the coordinate \mathbf{r}_i can be expressed as functions of the Jacobi coordinate r,

$$
\mathbf{r}_1 = \mathbf{R}_{\text{c.m.}} + \frac{\mu}{m_1} \mathbf{r},\tag{7}
$$

$$
\mathbf{r}_2 = \mathbf{R}_{\text{c.m.}} - \frac{\mu}{m_2} \mathbf{r},\tag{8}
$$

and the momentum \mathbf{p}_i is given by

$$
\mathbf{p}_1 = \frac{m_1}{M} \mathbf{P}_{\text{c.m.}} + \mathbf{p},\tag{9}
$$

$$
\mathbf{p}_2 = \frac{m_2}{M} \mathbf{P}_{\text{c.m.}} - \mathbf{p}.\tag{10}
$$

Using standard notation, the principal quantum numbers of the oscillator are $N = (2n + l)$, the energy of a state is given by

$$
E_N = (N + \frac{3}{2})\omega,\tag{11}
$$

and the frequency of the oscillator is

$$
\omega = (3K/\mu)^{1/2}.\tag{12}
$$

In the quark model the useful oscillator parameter is defined by

$$
\alpha^2 = \mu \omega = \sqrt{\frac{2m_2}{m_1 + m_2}} \beta^2,
$$
 (13)

where β is the often used harmonic oscillator parameter with a universal value $\beta = 0.4$ GeV. Then, the wave function of an oscillator is given by

$$
\psi_{lm}^n = R_{nl} Y_{lm}.\tag{14}
$$

B. Spin wave functions

The usual spin wave functions are adopted. For the spin-0 state, it is

$$
\chi^0 = \frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow), \tag{15}
$$

and for the spin-1 states, the wave functions are

$$
\chi_1^1 = \text{1}, \qquad \chi_{-1}^1 = \text{1}, \qquad \chi_0^1 = \frac{1}{\sqrt{2}} (\text{11} + \text{11}).
$$
 (16)

We take the heavy-quark infinite mass limit as an approximation to construct the total wave function without flavor symmetry. All the wave functions up to $1D$ states are listed in Table [I.](#page-2-0)

TABLE I. The total wave function for the heavy-light mesons, denoted by $|n^{2S+1}L_J\rangle$. The Clebsch-Gordan series for
the spin and angular-momentum addition $|n^{2S+1}L_J\rangle =$ the spin and angular-momentum addition $\sum_{m+s_z=J_z} \langle Lm, SS_z | JJ_z \rangle \psi_{Lm}^n \chi_{S_z} \Phi$ has been omitted, where Φ is the flavor wave function.

$\ket{n^{2S+1}L_J}$	J^P	Wave function
1S_0	0-	$\psi^0_{00}\chi^0\Phi$
$3S_1$ $\mathbf{1}$	1^-	$\psi_{00}^0 \chi_{S_7}^1 \Phi$
$1^{-1}P_1$	1^+	$\psi_{1m}^0 \chi^0 \Phi$
$3P_0$	0^+	$\psi_{1m}^0 \chi_{S_7}^1 \Phi$
$1^{3}P_1$	1^+	$\psi_{1m}^0 \chi_{S_z}^1 \Phi$
$1^{3}P_{2}$	2^+	$\psi_{1m}^0 \chi_{S_z}^1 \Phi$
$2^{-1}S_0$	0^{-}	$\psi_{00}^1 \chi^0 \Phi$
$2^{3}S_1$	1^{-}	$\psi_{00}^1 \chi_{S_z}^1 \Phi$
$1^{-1}D_2$	2^{-}	$\psi_{2m}^0 \chi^0 \Phi$
3D_1	$1 -$	$\psi_{2m}^0 \chi_{S_z}^1 \Phi$
$3D_2$	2^{-}	$\psi_{2m}^0 \chi_{S_7}^1 \Phi$
3D_3	$3-$	$\psi_{2m}^0 \chi_{S_\tau}^1 \Phi$

III. THE QUARK-MESON COUPLINGS

In the chiral quark model, the low-energy quark-meson interactions are described by the effective Lagrangian [\[67,69\]](#page-14-0)

$$
\mathcal{L} = \bar{\psi} [\gamma_{\mu} (i\partial^{\mu} + V^{\mu} + \gamma_{5} A^{\mu}) - m] \psi + \cdots, \quad (17)
$$

where V^{μ} and A^{μ} correspond to vector and axial currents, respectively. They are given by

$$
V^{\mu} = \frac{1}{2} (\xi \partial^{\mu} \xi^{\dagger} + \xi^{\dagger} \partial^{\mu} \xi),
$$

\n
$$
A^{\mu} = \frac{1}{2i} (\xi \partial^{\mu} \xi^{\dagger} - \xi^{\dagger} \partial^{\mu} \xi),
$$
\n(18)

with $\xi = \exp(i\phi_m/f_m)$, where f_m is the meson decay constant. For the $SU(3)$ case, the pseudoscalar-meson octet ϕ_m can be expressed as

$$
\phi_m = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}} \eta \end{pmatrix}, (19)
$$

and the quark field ψ is given by

$$
\psi = \begin{pmatrix} \psi(u) \\ \psi(d) \\ \psi(s) \end{pmatrix} . \tag{20}
$$

From the leading order of the Lagrangian [see Eq. (17)], we obtain the standard quark-meson pseudovector coupling at tree level

$$
H_m = \sum_j \frac{1}{f_m} I_j \bar{\psi}_j \gamma^j_\mu \gamma^j_5 \psi_j \partial^\mu \phi_m, \tag{21}
$$

where ψ_i represents the *j*th quark field in a hadron, and I_i is the isospin operator to be given later.

In the quark model, the nonrelativistic form of Eq. (21) is written as [[67,69,72\]](#page-14-0)

$$
H_m^{nr} = \sum_j \left\{ \frac{\omega_m}{E_f + M_f} \boldsymbol{\sigma}_j \cdot \mathbf{P}_f + \frac{\omega_m}{E_i + M_i} \boldsymbol{\sigma}_j \cdot \mathbf{P}_i - \boldsymbol{\sigma}_j \cdot \mathbf{q} \right. \\ \left. + \frac{\omega_m}{2\mu_q} \boldsymbol{\sigma}_j \cdot \mathbf{p}'_j \right\} I_j \varphi_m,
$$
 (22)

where σ_i corresponds to the Pauli spin vector of the jth quark in a hadron. μ_q is a reduced mass given by $1/\mu_q =$ $1/m_j + 1/m'_j$, where m_j and m'_j stand for the masses of the
it due in the initial and final hadrons, respectively. For jth quark in the initial and final hadrons, respectively. For emitting a meson, we have $\varphi_m = \exp(-i \mathbf{q} \cdot \mathbf{r}_i)$, and for absorbing a meson we have $\varphi_m = \exp(i\mathbf{q} \cdot \mathbf{r}_j)$. In the above nonrelativistic expansions, $\mathbf{p}'_j = \mathbf{p}_j - \frac{\hat{m}_j}{M} \mathbf{P}_{c.m.}$ is the internal momentum for the jth quark in the initial meson rest frame. ω_m and **q** are the energy and threevector momentum of the light meson, respectively. The isospin operator I_i in Eq. (21) is expressed as

$$
I_{j} = \begin{cases} a_{j}^{\dagger}(u)a_{j}(s) & \text{for } K^{+}, \\ a_{j}^{\dagger}(s)a_{j}(u) & \text{for } K^{-}, \\ a_{j}^{\dagger}(d)a_{j}(s) & \text{for } K^{0}, \\ a_{j}^{\dagger}(u)a_{j}(d) & \text{for } \bar{K}^{0}, \\ a_{j}^{\dagger}(u)a_{j}(u) & \text{for } \pi^{-}, \\ a_{j}^{\dagger}(d)a_{j}(u) & \text{for } \pi^{+}, \\ \frac{1}{\sqrt{2}}[a_{j}^{\dagger}(u)a_{j}(u) - a_{j}^{\dagger}(d)a_{j}(d)] & \text{for } \pi^{0}, \\ \cos\theta\frac{1}{\sqrt{2}}[a_{j}^{\dagger}(u)a_{j}(u) + a_{j}^{\dagger}(d)a_{j}(d)] - \sin\theta a_{j}^{\dagger}(s)a_{j}(s) & \text{for } \eta, \end{cases}
$$
(23)

where $a_j^T(u, d, s)$ and $a_j(u, d, s)$ are the creation and anni-
bilation operators for the u, d and squarks. Generally θ hilation operators for the u , d , and s quarks. Generally, θ ranges from $\simeq 32^{\circ} \sim 43^{\circ}$ depending on the quadratic or line mass relation applied [\[74\]](#page-14-0). In our convention, $\theta = 45^{\circ}$

corresponds to the mixing scheme of Ref. [19]. We applied the same value in order to compare with Ref. [19]. However, we note in advance that within the commonly accepted range of θ , our results do not show great sensi-

TABLE II. The spin factors used in this work.

$g_{10}^z = \langle \chi^0 \sigma_{1z} \chi_0^1 \rangle = 1$
$g_{10}^+ = \langle \chi^0 \sigma_1^+ \chi_{-1}^1 \rangle = \sqrt{\frac{1}{2}}$
$g_{10}^- = \langle \chi^0 \sigma_1^- \chi_1^1 \rangle = -\sqrt{\frac{1}{2}}$
$g_{01}^z = \langle \chi_0^1 \sigma_{1z} \chi^0 \rangle = 1$
$g_{01}^+ = \langle \chi_1^1 \sigma_1^+ \chi^0 \rangle = -\sqrt{\frac{1}{2}}$
$g_{11}^z = \langle \chi_1^1 \sigma_{1z} \chi_1^1 \rangle = 1$
$g_{11}^+ = \langle \chi_1^1 \sigma_1^+ \chi_0^1 \rangle = \sqrt{\frac{1}{2}}$

tivities due to the relatively large uncertainties of the present experimental data.

IV. STRONG DECAYS

For a heavy-light meson $\overline{Q}q$, because the pseudoscalar mesons $\mathbb P$ only couple with the light quarks, the strong decay amplitudes for the process $\mathbb{M}_i \to \mathbb{M}_f \mathbb{P}$ can be written as

$$
\mathcal{M}(\mathbb{M}_i \to \mathbb{M}_f \mathbb{P}) = \langle \mathbb{M}_f | \{ \mathcal{G} \sigma_1 \cdot \mathbf{q} + h \sigma_1 \cdot \mathbf{p}'_1 \} \times I_1 e^{-i\mathbf{q} \cdot \mathbf{r}_1} | \mathbb{M}_i \rangle, \tag{24}
$$

with

$$
G = -\left(\frac{\omega_m}{E_f + M_f} + 1\right), \qquad h = \frac{\omega_m}{2\mu_q}.\tag{25}
$$

 \mathbb{M}_i and \mathbb{M}_f are the initial and final meson wave functions, and they are listed in Table [I](#page-2-0). In the initial meson rest frame the energies and momenta of the initial mesons \mathbb{M}_i are denoted by (E_i, \mathbf{P}_i) , while those of the final state mesons

TABLE III. The decay amplitudes for $|n^{2S+1}L_1\rangle \rightarrow |1^1S_0\rangle \mathbb{P}$. g_I is an isospin factor which is defined by $g_I = \langle \phi_{\Sigma} | I_1 | \phi_{\Lambda} \rangle$. In the Table III, IV, V, [VI,](#page-4-0) and [VII](#page-4-0), the overall factor $F(q') =$ $\exp(-\frac{q^2}{4\alpha^2})$, which plays the role of the decay form factor, is
emitted for simplicity, where $q' = (u/m)q$. In the tables, we omitted for simplicity, where $q' = (\mu/m_1)q$. In the tables, we have defined $\overline{\mathbf{R}} \equiv (\mathbf{G}q - \frac{1}{2}h\overline{q})$. Various spin factors used in this work are listed in Table II this work are listed in Table II.

Initial state	Amplitude
$1^3S_1(1^-)$	$g_{I}g_{10}^{z}R$
$1^1P_1(1^+)$	Forbidden
$1^3P_0(0^+)$	$i\frac{1}{\sqrt{6}}g_{I}g_{10}^{z}R\frac{q'}{\alpha}+i\frac{1}{\sqrt{6}}g_{I}(\sqrt{2}g_{10}^{+}+g_{10}^{z})h\alpha$
$1^3P_1(1^+)$	Forbidden
$1^3P_2(2^+)$	$i\frac{1}{\sqrt{3}}g_{I}g_{10}^{z}R\frac{q'}{\alpha}$
$2^1S_0(0^-)$	Forbidden
$2^3S_1(1^-)$	$\frac{1}{\sqrt{24}}g_{I}g_{10}^{z}\mathcal{R}\left(\frac{q'}{\alpha}\right)^{2}+\sqrt{\frac{1}{6}}g_{I}g_{10}^{z}hq'$
$1^1D_2(2^-)$	Forbidden
$1^3D_1(1^-)$	$\frac{1}{\sqrt{30}}g_{I}g_{10}^{z}\mathcal{R}\left(\frac{q'}{\alpha}\right)^{2}+\sqrt{\frac{3}{5}}g_{I}g_{10}^{+}hq'$
$1^3D_2(2^-)$	Forbidden
$1^3D_3(3^-)$	$-\frac{1}{\sqrt{20}}g_{I}g_{10}^{z}\mathcal{R}(\frac{q'}{\alpha})^{2}$

TABLE IV. The decay amplitudes for $|n^{2S+1}L_J\rangle \rightarrow |1^3S_1\rangle \mathbb{P}$.

$\ket{n^{2S+1}L_J}$	J_z	Amplitude
$1^1P_1(1^+)$	± 1	$ig_Ig_{01}^+h\alpha$
	θ	$-i\frac{1}{\sqrt{2}}g_{I}g_{01}^{z}\mathcal{R}\frac{q'}{\alpha}-i\frac{1}{\sqrt{2}}g_{I}g_{01}^{z}h\alpha$
$1^3P_0(0^+)$		Forbidden
$1^3P_1(1^+)$	±1	$i\frac{1}{2}g_{I}g_{11}^{z}\mathcal{R}\frac{q'}{\alpha}+i\frac{1}{2}g_{I}(g_{11}^{z}+\sqrt{2}g_{11}^{+})h\alpha$
	0	$\sqrt{2}g_Ig_{11}^+h\alpha$
$1^3P_2(2^+)$	$+1$	$-i\frac{1}{2}g_{I}g_{11}^{z}\mathcal{R}\frac{q'}{\alpha}$
	0	
$2^1S_0(0^-)$	Ω	$\frac{1}{\sqrt{24}}g_{I}g_{10}^{z}\mathcal{R}(\frac{q'}{\alpha})^{2}+\sqrt{\frac{1}{6}}g_{I}g_{10}^{z}hq'$
$2^3S_1(1^-)$	± 1	$\pm \{\frac{1}{\sqrt{24}}g_I g_{11}^z \mathcal{R}(\frac{q'}{\alpha})^2 F + \sqrt{\frac{1}{6}}g_I g_{11}^z h q'\}$
	Ω	
$1^1D_2(2^-)$	±1	$\frac{1}{\sqrt{2}}g_{01}^+g_I h q'$
	θ	$-\sqrt{\frac{1}{12}g_I g_{01}^z \mathcal{R}(\frac{q'}{\alpha})^2} - \sqrt{\frac{1}{3}g_I g_{01}^+ h q'}$
$1^3D_1(1^-)$	± 1	$\pm[\sqrt{\frac{1}{120}}g_Ig_{11}^z\mathcal{R}(\frac{q'}{\alpha})^2+\sqrt{\frac{5}{12}}g_Ig_{11}^+hq']$
	Ω	
$1^3D_2(2^-)$	± 1	$\sqrt{\frac{1}{24}}g_Ig_{10}^z \mathcal{R}(\frac{q'}{\alpha})^2 + \sqrt{\frac{3}{4}}g_Ig_{11}^+ hq'$
	θ	$g_{11}^{+}g_{I}hq'$
$1^3D_3(3^-)$	$+1$	$\pm \sqrt{\frac{1}{30}g_I g_{11}^z \mathcal{R}(\frac{q'}{\alpha})^2}$
	0	

 \mathbb{M}_f and the emitted pseudoscalar mesons $\mathbb P$ are denoted by (E_f, \mathbf{P}_f) and (ω_m, \mathbf{q}) . Note that $\mathbf{P}_i = 0$ and $\mathbf{P}_f = -\mathbf{q}$.

The form of Eq. (24) is similar to that in Refs. $[21,22]$, except that the factors G and h in this work have explicit dependence on the energies of final hadrons. In the calculations, we select $\mathbf{q} = q\hat{z}$; namely, the meson moves along the z axial. The spin factors can then be worked out, and we list them in Table II. Finally, we can work out the decay amplitudes for various processes, $\mathbb{M} \rightarrow |1^1S_0\rangle \mathbb{P}$, $\mathbb{M} \rightarrow$ $|1^3S_1\rangle \mathbb{P}$, $\mathbb{M} \rightarrow |1^3P_0\rangle \mathbb{P}$, $\mathbb{M} \rightarrow |1^1P_1\rangle \mathbb{P}$, and $\mathbb{M} \rightarrow$ $|1^3P_1\rangle$ P, which are listed in Tables III, IV, V, [VI](#page-4-0), and [VII](#page-4-0), respectively.

Some analytical features can be learned here. Table III shows that the decays of 1^1P_1 , 1^3P_1 , 2^1S_0 , 1^1D_2 , and 1^3D_2 into $|1^1S_0\rangle$ are forbidden by parity conservation. The

TABLE V. The decay amplitudes for $|n^{2S+1}L_J\rangle \rightarrow |1^3P_0\rangle \mathbb{P}$, where we have defined $W = Gq(-1 + \frac{q^2}{4\alpha^2})$, $S = h\alpha(1 - \frac{q^2}{2\alpha^2})$.

$ n^{2S+1}L_1\rangle$	J_z	Amplitude
$2^1S_0(0^-)$	θ	$i\frac{1}{3}g_{I}g_{01}^{z} \mathcal{W}_{\alpha}^{\underline{q}'}-i\frac{1}{3}g_{I}g_{01}^{z}h\alpha\mathcal{A}$
$2^3S_1(1^-)$		Forbidden
	0	$i\frac{\sqrt{2}}{3}g_{I}g_{01}^{z} \mathcal{W}\frac{q'}{\alpha} + i\frac{\sqrt{2}}{3}g_{I}g_{01}^{z}h\alpha\mathcal{A}$
$1^1D_2(2^-)$	$+1$	
	± 2	$-i\frac{\sqrt{2}}{3}g_{I}g_{01}^{+}h\alpha$
$1^3D_1(1^-)$		Forbidden
	0	$-i\frac{\sqrt{6}}{3}g_{11}^+g_I\mathcal{S}$
$1^3D_2(2^-)$	$+1$	
	$+2$	$-i\frac{2}{3}g_{11}^+g_Ih\alpha F - i\frac{\sqrt{2}}{6}g_{11}^zg_I\mathcal{S}$
$1^3D_3(3^-)$	$+2$	$-i\frac{\sqrt{2}}{3}g_{11}^+g_Ih\alpha F+i\frac{1}{3}g_{11}^zg_I\mathcal{S}$

$ n^{2S+1}L_{I}\rangle$	(J_z^f, J_z^i)	Amplitude
$2^1S_0(0^-)$		Forbidden
$2^3S_1(1^-)$ $\pm(1,-1)$		$-i\sqrt{\frac{2}{3}}g_Ig_{10}^+h\alpha(1+\frac{q^{\prime\prime}}{4\alpha^2})$
	(0,0)	$-i\frac{1}{\sqrt{3}}g_{I}g_{10}^{z}W\frac{q'}{\alpha}+i\frac{1}{\sqrt{3}}g_{I}g_{10}^{z}h\alpha A$
$1^1D_2(2^-)$		Forbidden
$1^3D_1(1^-)$ $\pm(1,-1)$		$-i\sqrt{\frac{3}{20}}g_Ig_{10}^zGq\frac{q'}{\alpha}-i\sqrt{\frac{1}{30}}g_Ig_{10}^+\mathcal{S}$
	(0,0)	$-i\sqrt{\frac{4}{15}}g_I g_{10}^z \mathcal{W}\frac{q'}{\alpha} + i\sqrt{\frac{4}{15}}g_I g_{10}^z h \alpha \mathcal{A}$
$1^3D_2(2^-)$	$\pm(1,-1)$	$-i\frac{1}{\sqrt{12}}g_Ig_{10}^z\mathcal{R}\frac{q'}{\alpha}+i\frac{1}{\sqrt{24}}g_Ig_{10}^+hq'\frac{q'}{\alpha}$
$1^3D_3(3^-)$ $\pm(1,-1)$		$-i\sqrt{\frac{1}{30}}(\sqrt{2}g_{10}^z-g_{10}^+)g_Ihq'\frac{q'}{\alpha}$
	(0,0)	$i\sqrt{\frac{1}{5}}g_I[g_{10}^z \mathcal{W}\frac{q'}{\alpha} - \sqrt{2}g_{10}^z h\alpha \mathcal{A} + 2g_{10}^+\mathcal{S}]$

TABLE VI. The decay amplitudes for $|n^{2S+1}L_1\rangle \rightarrow |1^1P_1\rangle \mathbb{P}$.

decay amplitudes for 1^3S_1 , 1^3P_2 , and $1^3D_3 \rightarrow |1^1S_0\rangle \mathbb{P}$ are proportional to $\mathcal R$ (i.e. proportional to q), $\mathcal R q'/\alpha$, and $\mathcal{R}(q/\alpha)^2$, respectively. This is crucial for understanding
the small branching ratios for $D^*(2007) \rightarrow D\pi$ as we will the small branching ratios for $D^*(2007) \to D\pi$ as we will see later see later.

In contrast, the decay amplitude for $1^3P_0 \rightarrow |1^1S_0\rangle \mathbb{P}$ has two terms. One is proportional to $\mathcal{R}q'/\alpha$, while the other is proportional to α . Similarly, the decay amplitudes for $2^{3}S_{1} \rightarrow |{}^{1}S_{0}\rangle$ and $2^{3}D_{1} \rightarrow |{}^{1}S_{0}\rangle$ also have two terms of which one is proportional to $\mathcal{R}(q'/\alpha)^2$, and the other to q' . This feature will have certain implications for their q' . This feature will have certain implications for their branching ratio rates into different $|1^1S_0\rangle \mathbb{P}$ states.

Table [IV](#page-3-0) shows that decays of 1^3P_0 into $|1^1S_0\rangle \mathbb{P}$ are forbidden. Among those three helicity amplitudes \mathcal{M}_{\pm} and \mathcal{M}_{\pm} the longitudinal one \mathcal{M}_{\pm} vanishes for 1^3P $\frac{3^3S}{2^3S}$ \mathcal{M}_0 , the longitudinal one \mathcal{M}_0 vanishes for 1^3P_2 , 2^3S_1 , $1³D₁$, and $1³D₃$ into $|1¹S₀\rangle$ \mathbb{P} .

From Tables [V,](#page-3-0) VI, and VII, we can see that the decays of $2^{3}S_{1}$ and $1^{3}D_{1}$ into $1^{3}P_{0}$) \mathbb{P} , $2^{1}S_{0}$ and $1^{1}D_{2}$ into

TABLE VII. The decay amplitudes for $|n^{2S+1}L_J\rangle \rightarrow |1^3P_1\rangle \mathbb{P}$.

$ n^{2S+1}L_1\rangle$	(J_z^f, J_z^i)	Amplitude
$2^1S_0(0^-)$		Forbidden
$2^3S_1(1^-)$	$\pm(1,-1)$	$-i\frac{1}{\sqrt{3}}g_{I}g_{11}^{+}h\alpha(1+\frac{q^{\prime2}}{4\alpha^2})$
	$\pm(1,1)$	$i\frac{1}{\sqrt{6}}g_Ig_{11}^z\mathcal{W}\frac{q'}{\alpha}-i\frac{1}{\sqrt{6}}g_Ig_{11}^zh\alpha\mathcal{A}$
	$\pm(0, 2)$	$\pm i g_{01}^{\dagger} h \alpha$
$1^1D_2(2^-)$	$\pm(1, 1)$	$\pm i \frac{1}{\sqrt{2}} g_{01}^+ \mathcal{S}$
	$\pm(1,-1)$	$\pm i\frac{1}{2}g_Ig_{01}^z(\mathcal{R}\frac{q'}{\alpha}+h\alpha)$
$1^3D_1(1^-)$	$\pm(1,-1)$	$-i\sqrt{\frac{1}{60}g_I g_{11}^+ h\alpha(1+\frac{q'^2}{2\alpha^2})}$
	$\pm(1, 1)$	$-i\sqrt{\frac{1}{30}g_I g_{11}^z}(\mathcal{W} \frac{q'}{\alpha} - h\alpha \mathcal{A} - \frac{3}{2}\mathcal{S})$
	$\pm(0, 2)$	$\pm i \frac{1}{\sqrt{12}} g_I g_{11}^z (R \frac{q'}{\alpha} + 3h\alpha)$
$1^3D_2(2^-)$	$\pm(1,-1)$	$\pm [-i\sqrt{\frac{1}{3}}g_Ig_{11}^z h\alpha - i\sqrt{\frac{1}{12}}g_{11}^+\mathcal{S}]$
	$\pm(1, 1)$	$\pm i\sqrt{\frac{1}{6}}g_Ig_{11}^z(\mathcal{W}\frac{q'}{\alpha}-h\alpha\mathcal{A}-\frac{1}{2}\mathcal{S})$
	$\pm(0, 2)$	$i\frac{1}{\sqrt{6}}g_{I}g_{11}^{z}\mathcal{R}\frac{q'}{\alpha}$
$1^3D_3(3^-)$	$\pm(1,-1)$	$-i\sqrt{\frac{1}{60}g_I g_{11}^+ h q' \frac{q'}{\alpha}}$
	$\pm(1,1)$	$-i\sqrt{\frac{2}{15}}g_Ig_{11}^z[\mathcal{W}\frac{q'}{\alpha}-h\alpha\mathcal{A}+\mathcal{S}]$

 $|1^1P_1\rangle \mathbb{P}$, and 2^1S_0 into $|1^1P_1\rangle \mathbb{P}$ are forbidden parity conservation. These selection rules are useful for the state classifications.

V. CALCULATIONS AND ANALYSIS

With the transition amplitudes, one can calculate the partial decay width with

$$
\Gamma = \left(\frac{\delta}{f_m}\right)^2 \frac{(E_f + M_f)|\mathbf{q}|}{4\pi M_i (2J_i + 1)} \sum_{J_{iz}, J_{fz}} |\mathcal{M}_{J_{iz}, J_{fz}}|^2, \tag{26}
$$

where J_{iz} and J_{fz} stand for the third components of the total angular momenta of the initial and final heavy-light mesons, respectively. δ as a global parameter accounts for the strength of the quark-meson couplings. In the heavy-light meson transitions, the flavor symmetry does not hold anymore. Treating the light pseudoscalar meson as a chiral field while treating the heavy-light mesons as constituting a quark system is an approximation. This will bring uncertainties to coupling vertices and form factors. Parameter δ is introduced to take into account such an effect. It has been determined in our previous study of the strong decays of the charmed baryons [[73](#page-14-0)]. Here, we fix its value as the same as that in Ref. [[73](#page-14-0)], i.e. $\delta = 0.557$.

In the calculation, the standard parameters in the quark model are adopted. For the u , d , s , c , and b constituent quark masses we set $m_u = m_d = 350$ MeV, $m_s =$ 550 MeV, $m_c = 1700$ MeV, and $m_b = 5100$ MeV, respectively. The decay constants for π , K, η , and D mesons, $f_{\pi} = 132$ MeV, $f_K = f_{\eta} = 160$ MeV, and $f_D = 226$ MeV are used. For the masses of all the heavy-light 226 MeV, are used. For the masses of all the heavy-light mesons the Particle Data Group (PDG) values are adopted in the calculations [[74](#page-14-0)].

To partly remedy the inadequacy of the nonrelativistic wave function as the relative momentum q increases, a commonly used Lorentz boost factor is introduced into the decay amplitudes [\[66,71,72\]](#page-14-0),

$$
\mathcal{M}(q) \to \gamma_f \mathcal{M}(\gamma_f q), \tag{27}
$$

where $\gamma_f = M_f/E_f$. In most decays, the three momenta carried by the final state mesons are relatively small, which means the nonrelativistic prescription is reasonable and corrections from the Lorentz boost are not drastic.

A. Strong decays of 1S states

Because of isospin violation, D^{*+} is about 3 MeV heavier than the neutral D^{*0} [[74](#page-14-0)]. This small difference leads to a kinematic forbiddance of $D^{*0} \to D^+ \pi^-$, while $D^{*+} \to D^0 \pi^+$ and $D^+ \pi^0$ are allowed but with a strong kinematic $D^0\pi^+$ and $D^+\pi^0$ are allowed, but with a strong kinematic suppression. Nevertheless, it shows in Table [III](#page-3-0) that the decay amplitudes of 1S states are proportional to the final state momentum q. For the decays of $D^{*0} \to D^0 \pi^0$,
 $D^{*+} \to D^0 \pi^+$ and $D^+ \pi^0$ of which the decay thresholds $D^{*+} \rightarrow D^0 \pi^+$, and $D^+ \pi^0$ of which the decay thresholds
are close to the D^* masses it leads to further dynamic are close to the D^* masses, it leads to further dynamic suppressions to the partial decay widths. As shown in

TABLE VIII. Predictions of the strong decay widths (in MeV) for the heavy-light mesons. For comparison, the experimental data and some other model predictions are listed.

	Notation	Channel	Γ (this work)	Γ [19]	Γ [20]	Γ [32]	Γ [30,31]	Γ [26]	$\Gamma_{\underline{\mathrm{exp}}}$
$D^{*}(2007)^{0}$	$(1^3S_1)1^-$	$D^0\pi^0$	58 keV	16 keV		39 keV			$<$ 2.1 MeV
$D^*(2010)^+$		$D^0\pi^+$	77 keV	25 keV		60 keV			64 ± 15 keV
		$D^+\,\pi^0$	35 keV	11 keV		27 keV			29 ± 7 keV
$D_0^*(2352)$	$(1^3P_0)0^+$	$D\pi$	248		277				261 ± 50
$D_0^*(2403)$			266	283					283 ± 58
$D_1(2420)$	$(1^1P_1)1^+$	$D^*\pi$	84						
	$(P_1)1^+$		21.6	$22\,$	25				25 ± 6
$D'_{1}(2430)$	$(1^3P_1)1^+$	$D^*\pi$	152						
	$(P'_1)1^+$		220	272	244				384 ± 117
$D_2^*(2460)^0$	$(1^3P_2)2^+$	$D\pi$	39	35	37		13.7		
		$D^*\pi$	19	$20\,$	18		6.1		
		$D\eta$	0.1	$0.08\,$					
		Total	59	55	55		$20\,$		43 ± 4
$D_{s1}(2536)$	$(1^1P_1)1^+$	$D^{\ast}K$	59						
	$(P_1)1^+$		0.35	$0.8\,$	0.34				< 2.3
$D_{s2}^*(2573)$	$(1^3P_2)2^+$	$\cal{D}K$	16	$27\,$	$20\,$				
		$D^{\ast}K$	$\mathbf{1}$	3.1	$\mathbf{1}$				
		$D_s \eta$	0.4	$0.2\,$					
		Total	17	30	21				15^{+5}_{-4}
$D_{sI}^*(2860)$	$(1^3D_3)3^-$	$\mathcal{D} \mathcal{K}$	$27\,$						
		$D^{\ast}K$	$11\,$						
		$D_s \eta$	$\overline{3}$						
		$D_s^*\eta$	0.3						
		DK^*	$0.4\,$						
		Total	42						48 ± 17
$B_0^*(5730)$	$({}^3P_0)0^+$	$B\pi$	272			141	250		
$B_1(5725)$	$(P_1)1^+$	$B^*\pi$	30			20			20 ± 12
	$(1^3P_1)1^+$	$B^*\pi$	153						
$B'_1(5732)$	$(P'_1)1^+$	$B^*\pi$	219			139	250		128 ± 18
$B_2^*(5740)$	$(1^3P_2)^2$ ⁺	$B\pi$	25			15	3.9		
		$B^*\pi$	22			14	3.4		
		Total	47			29	7.3	16 ± 6	22^{+7}_{-6}
$B_{s0}^*(5800)$	$(^3P_0)0^+$	$\cal{B}K$	227						
$B_{s1}(5830)$	$(P_1)1^+$	$B^{\ast}K$	$0.4 \sim 1$					3 ± 1	$\mathbf{1}$
$B'_{s1}(5830)$	$(P'_1)1^+$	$B^{\ast}K$	149						
$B_{s2}^*(5839)$	$(1^3P_2)2^+$	$\cal{B}K$	$\sqrt{2}$						
		$B^{\ast}K$	0.12						
		Total	$\sqrt{2}$					7 ± 3	$\mathbf{1}$

Table VIII, our calculations are in remarkable agreement with the experimental data. Since q is small, the form factor corrections from the quark model are negligibly small. One would expect that the ratio $\Gamma(D^0 \pi^+) / \Gamma(D^+ \pi^0) \approx 2$ is then dominated by the isospin
factor g_x , which agrees well with the prediction in factor g_1 , which agrees well with the prediction in Ref. [19].

B. Strong decays of 1P states

In the LS coupling scheme, there are four 1P states: 3P_0 ,
 3P_1 , 3P_2 , and 1P_1 . For 3P_0 , its transition to $|1{}^3S_1\rangle \mathbb{P}$ is forbidden. States of 1P_1 and 3P_2 can couple into $|1{}^3S_1\rangle$ forbidden. States of ¹ P_1 and ³ P_1 can couple into $\left[1^3S_1\right)\mathbb{P}$,
but not $\left[1^1S_1\right)\mathbb{P}$. In contrast ³ P_1 can be coupled to both but not $|1^1S_0\rangle$ P. In contrast, 3P_2 can be coupled to both $|1^3S_1\rangle$ P and $|1^1S_0\rangle$ P. In the decay amplitudes of 3P_0 , 1P_1 , and ${}^{3}P_{1}$, the term $h\alpha F$ dominates the partial decay widths, and usually their decay widths are much broader than that of ${}^{3}P_{2}$. Between the amplitudes of the ${}^{1}P_{1}$ and ${}^{3}P_{1}$ decays, we approximately have

$$
\mathcal{M}({}^{1}P_{1} \rightarrow |1^{3}S_{1}\rangle \mathbb{P})_{J_{z}} \simeq \frac{1}{\sqrt{2}} \mathcal{M}({}^{3}P_{1} \rightarrow |1^{3}S_{1}\rangle \mathbb{P})_{J_{z}}, \quad (28)
$$

since the term $\mathcal{R} \frac{q'}{\alpha} F$ is negligible when the decay channel threshold is close to the initial meson mass. As a consequence, the decay widths of the ${}^{1}P_1$ states are narrower than those of ${}^{3}P_{1}$.

1. 3P_0 states

 $D_0^*(2400)$ is listed by the PDG [[74](#page-14-0)] as a broad 3P_0 state.
mass values from Belle [75] and the FOCUS Its mass values from Belle [[75](#page-14-0)] and the FOCUS Collaboration [[76](#page-14-0)] are quite different though the FOCUS result is consistent with the potential quark model predic-tion of 2403 MeV [\[21\]](#page-13-0). In experiments, only the $D\pi$ channel is observed since the other channels are forbidden. The term $h\alpha F$ in the amplitude, which is in proportion to the oscillator parameter α , accounts for the broad decay width. By applying the PDG averaged mass 2352 MeV and the FOCUS value 2403 MeV, its partial decay widths into $D\pi$ are calculated and presented in Table [VIII.](#page-5-0) They are in good agreement with the data [\[74,76\]](#page-14-0).

In the B meson sector, B_0^* and B_{s0}^* , as the 3P_0 states, have not been confirmed in any experiments. The predicted mass for B_0^* and B_{s0}^* mesons is about 5730 MeV and 5800 MeV, respectively [\[47\]](#page-14-0). Their strong decays only open to $B\pi$ or BK. Applying the theory-predicted masses, we obtain broad decay widths for both states, i.e. $\Gamma(B_0^*)$ = we obtain broad decay widths for both states, i.e. $\Gamma(B_0) = 272$ MeV and $\Gamma(B_{s0}^*) = 227$ MeV, respectively. Our pre-
diction of $\Gamma(B^*) = 272$ MeV is compatible with the OCD diction of $\Gamma(B_0^*) = 272 \text{ MeV}$ is compatible with the QCD
sum rules prediction $\Gamma(B^*) \approx 250 \text{ MeV}$ [30]. Such broad sum rules prediction $\Gamma(B_0^*) \simeq 250$ MeV [\[30\]](#page-13-0). Such broad
widths may explain why they have not yet been identified widths may explain why they have not yet been identified in experiments.

2. 3P_2 states

In the PDG, the decay width of $D_2^*(2460)^0$ is $\Gamma = 43 \pm$
MeV and that of $D_2^*(2460)^{\pm}$ is $\Gamma = 29 \pm 5$ MeV. Since 4 MeV and that of $D_2^*(2460)^{\pm}$ is $\Gamma = 29 \pm 5$ MeV. Since
there is no obvious dynamic reason for such a significant there is no obvious dynamic reason for such a significant difference, it may simply be due to experimental uncertainties. Our prediction $\Gamma = 59$ MeV as a sum of the partial widths of $D\pi$, $D^*\pi$, and $D\eta$ is comparable with the data. Nevertheless, the partial width ratio

$$
R = \frac{\Gamma(D\pi)}{\Gamma(D^*\pi)} \approx 2.1\tag{29}
$$

obtained here is also in good agreement with the data $R \simeq$ 2.3 ± 0.6 [[74](#page-14-0)].
 $D^*(2573)$ is

 $D_{s2}^*(2573)$ is assigned to be a ³ P_2 state. Its total width is
= 15⁺⁵ and the width ratio between D^*K and DK is $\Gamma_{\text{exp}} = 15^{+5}$, and the width ratio between D^*K and DK is $R = \Gamma(D^*K)/\Gamma(DK) < 0.33$ [741. Our predictions for the $R = \Gamma(D*K)/\Gamma(DK)$ < 0.33 [[74](#page-14-0)]. Our predictions for the total width and ratio R are

$$
\Gamma = 17 \text{ MeV}, \tag{30}
$$

$$
R = \frac{\Gamma(D^*K)}{\Gamma(DK)} \approx 6\%,\tag{31}
$$

which are consistent with the data.

Notice that the width of D^*K , \sim 1 MeV, is 1 order of magnitude smaller than that of DK . Apart from the kinematic phase space suppression, its transition amplitude also suffers dynamic suppressions since it is proportional to $\mathcal{R}q'/\alpha$. This explains its absence in experiments. Although the decay channel $D\eta/D_s\eta$ is also opened for

 $D_2^*(2460)/D_{s2}^*(2573)$, its partial width is negligibly small, i.e. $\lt 1$ MeV i.e. <1 MeV.

In the B meson sector, a candidate of the ${}^{3}P_{2}$ state is from the CDF Collaboration with mass [\[8\]](#page-13-0)

$$
M(B_2^*) = 5740 \pm 2 \pm 1 \text{ MeV.}
$$
 (32)

The D0 Collaboration also observed the same state with slightly different masses, $M(B_2^*) = 5746.8 \pm 2.4 \pm 1.7$ MeV [9]. By assigning B^* as a ${}^{3}P$ state, the predicted 1.7 MeV [\[9](#page-13-0)]. By assigning B_2^* as a 3P_2 state, the predicted total width as a sum of $B\pi$ and $B^*\pi$ is

$$
\Gamma(B_2^*) \simeq 47 \text{ MeV},\tag{33}
$$

which is consistent with the CDF measurement $\Gamma(B_2^*)_{\text{exp}} \simeq$ which is consistent with the CDF measurement $\Gamma(B_2)_{exp} = 22^{-7}_{-6}$ MeV. It shows that these two partial widths of $B\pi$
and $R^*\pi$ are comparable, and the predicted width ratio is and $B^* \pi$ are comparable, and the predicted width ratio is

$$
R \equiv \frac{\Gamma(B^*\pi)}{\Gamma(B^*\pi) + \Gamma(B\pi)} = 0.47. \tag{34}
$$

This is also in good agreement with the recent D0 data $R =$ $0.475 \pm 0.095 \pm 0.069$ [[9\]](#page-13-0).
The CDE Collaboration

The CDF Collaboration also reported an observation of B_2^* 's strange analogue B_{s2}^* [10], of which the mass is

$$
M(B_{s2}^*) = 5840 \pm 1 \text{ MeV.}
$$
 (35)

With this mass, we obtain its partial decay widths, $\Gamma(B^*K) = 0.12$ MeV and $\Gamma(BK) = 2$ MeV, respectively. This gives its strong decay width and width ratio between B^*K and BK :

$$
\Gamma(B_{s2}^*) \simeq 2 \text{ MeV}, \qquad R \equiv \frac{\Gamma(B^*K)}{\Gamma(BK)} \simeq 6\%. \tag{36}
$$

The decay width is in good agreement with the data $\Gamma(B_{s2}^{*})_{\text{exp}} \sim 1$ MeV [\[77\]](#page-14-0). It also shows that the partial
width of the $B_{s}^{*}K$ channel is negligibly small, and will width of the B^*K channel is negligibly small, and will evade from observations in experiments. But a measurement of $\Gamma(BK)$ with improved statistics should be very interesting.

3. The mixed states

The $D_1(2420)$ and $D'_1(2430)$ listed by PDG [[74](#page-14-0)] corre-
and to a narrow and broad state, respectively. Their two spond to a narrow and broad state, respectively. Their two body pionic decays are only seen in $D^*\pi$. If they are pure P-wave states, they should be correspondent to ${}^{1}P_1$ and $3P_1$. The calculated decay widths by assigning them as $1P_1$ and ${}^{3}P_{1}$, are listed in Table [VIII](#page-5-0). It shows that for $D_{1}(2420)$ as a pure ${}^{1}P_1$ state, its decay width is overestimated by about an order, while for $D'_1(2430)$ as a pure 3P_1 state, its
decay width is underestimated by about a factor of 2 decay width is underestimated by about a factor of 2. Similarly large discrepancies are also found if one simply exchanges the assignments. Thus, the pure ${}^{1}P_1$ and ${}^{3}P_1$ scenario cannot explain the nature of $D_1(2420)$ and $D'_1(2430)$.
Since the

Since the heavy-light mesons are not charge conjugation eigenstates, state mixing between spin $S = 0$ and $S = 1$

states with the same J^P can occur. The physical states with $J^P = 1⁺$ would be given by

$$
|P_1'\rangle = +\cos(\phi)|^1 P_1\rangle + \sin(\phi)|^3 P_1\rangle,\tag{37}
$$

$$
|P_1\rangle = -\sin(\phi)|^1 P_1\rangle + \cos(\phi)|^3 P_1\rangle. \tag{38}
$$

Our present knowledge about the $D_1(2420)$ and $D'_1(2430)$
mixing is still limited. The determination of the mixing mixing is still limited. The determination of the mixing angle is correlated with quark potential, and masses of the states [[22\]](#page-13-0). An analysis by Ref. [19] suggests that a mixed state dominated by S-wave decay will have a broad width, and the D-wave-dominant decay will have a narrow one. By assuming that the heavy-quark spin-orbit interaction is positive, this leads to an assignment of $D'_1(2430)$ and $D_1(2430)$ as a mixed $|P' \rangle$ and $|P_1 \rangle$ respectively with a $D_1(2420)$ as a mixed $|P'_1\rangle$ and $|P_1\rangle$, respectively, with a negative mixing angle $\phi = -54.7^\circ$. However, this will negative mixing angle $\dot{\phi} = -54.7^{\circ}$. However, this will lead to the mass of D_1 being heavier than that of D'_1 , and the present experimental precision seems unable to rule out such a possibility [\[74\]](#page-14-0). An additional piece of information supporting such a scenario is that a positive spin-orbit interaction will lead to a heavier 2^+ state than 0^+ which indeed agrees with experiments [19].

In our calculation, we plot the pionic decay widths of the mixed states $|P_1\rangle$ and $|P_1\rangle$ as functions of ϕ in Fig. 1. By
looking for the best description of the experimental data looking for the best description of the experimental data, we determine the optimal mixing angle. It shows that with $\phi = -(55 \pm 5)^{\circ}, D_1(2420)$, as the $|P_1\rangle$ mixed state, has a narrow decay width of $\Gamma \approx 22$ MeV. This value agrees narrow decay width of $\Gamma \approx 22$ MeV. This value agrees well with the experimental data (see Table [VIII](#page-5-0)). Our prediction for the width of $D'_1(2430)$ as a $|P'_1\rangle$ broad state
is $\Gamma \approx 217$ MeV which also agrees with the data [741] is $\Gamma \approx 217$ MeV, which also agrees with the data [\[74\]](#page-14-0). Note that there are still large uncertainties with the

FIG. 1. The decay widths of $D_1(2420)$, $D'_{1}(2430)$, and $D_{s1}(2536)$ as functions of the mixing angle ϕ .

 $D_1^{\prime}(2430)$ measurements, and further experimental investigation is needed tigation is needed.

Such a mixing scenario may occur within the D_{s1} states, which leads to $D_{s1}(2460)$ and $D_{s1}(2536)$ as the mixed $|P_1\rangle$ which reads to $D_{s1}(2460)$ and $D_{s1}(2550)$ as the finxed $|I_1|$
and $|P_1|$, respectively. Note that $D_{s1}(2460)$ has a relatively
light mass which is below the D^*K threshold, and also light mass which is below the D^*K threshold, and also slightly below the DK threshold. Therefore, its strong decay is nearly forbidden, which makes it a narrow state. On the other hand, $D_{s1}(2536)$, as a $|P_1\rangle$ mixed state with the mixing angle $\phi = -(55 \pm 5)^\circ$, can give a decay width consistent with the data ($\Gamma < 2.3$ MeV) consistent with the data (Γ < 2.3 MeV)

$$
\Gamma(D_{s1}(2536)) \simeq 0.4 \sim 2.5 \text{ MeV.}
$$
 (39)

In contrast, if $D_{s1}(2536)$ is a pure ${}^{1}P_1$ state, its decay width will be 59 MeV, which is overestimated by a factor of 20.

We also derive the width ratio

$$
R = \frac{\Gamma(D^*(2007)^0 K^+)}{\Gamma(D^*(2010)^+ K^0)} \approx 1.2 \sim 1.7,
$$
 (40)

which is consistent with the experimental result, $R =$ 1.27 ± 0.27 . In Fig. $1(b)$, the change of the strong decay width $\Gamma(D_1(2536))$ in terms of the mixing angle ϕ is width $\Gamma(D_{s1}(2536))$ in terms of the mixing angle ϕ is presented by treating it as a mixed $|P_1\rangle$ state. It should be mentioned that the recent measurements of the angular decomposition of $D_{s1}(2536)^+ \rightarrow D^{*+}K_S^0$ indicate configuration mixings within $D_{s2}(2536)$ [78] ration mixings within $D_{s1}(2536)$ [[78](#page-14-0)].

In the B meson sector, two new narrow excited B_1 and B_{s1} mesons are recently reported by CDF, with masses

$$
M(B_1) = 5725 \pm 2 \pm 1 \text{ MeV}, \tag{41}
$$

$$
M(B_{s1}) = 5829 \pm 1 \text{ MeV.}
$$
 (42)

The D0 Collaboration also observed the same B_1 state with a slightly different mass, $M(B_1) = 5720 \pm 2.4 \pm 1.4$ MeV 1:4 MeV.

The narrowness of these two axial vector states makes them good candidates as the narrow heavy partners in the state mixing. With B_1 as a $|P_1\rangle$ state, its strong decay width to $B^*\pi$ is predicted to be

$$
\Gamma(B_1) \simeq 30 \text{ MeV.} \tag{43}
$$

With the strong decay widths for $B_2^* \rightarrow B\pi$ and $B^* \pi$
calculated we obtain the strong decay width ratio calculated, we obtain the strong decay width ratio

$$
R = \frac{\Gamma(B_1)}{\Gamma(B_1) + \Gamma(B_2^*)} = 0.34,
$$
 (44)

which is in good agreement with the recent D0 data $R =$ $0.477 \pm 0.069 \pm 0.062$ [[9\]](#page-13-0).
Note that $R^*(5732)$ in the

Note that $B_J^*(5732)$ in the PDG [[74](#page-14-0)] is a broad state with
= 128 + 18 MeV. The PDG averaged mass is $\Gamma_{\rm exp}$ = 128 \pm 18 MeV. The PDG averaged mass is
5608 + 8 MeV, which makes it lighter than *B*. (5725) 5698 \pm 8 MeV, which makes it lighter than B_1 (5725).
This makes it a natural candidate as the mixed light partner This makes it a natural candidate as the mixed light partner $|P'_1\rangle$, for which the predicted width is $\Gamma(B'_1) = 219$ MeV;
this result is compatible with the OCD sum rules prediction this result is compatible with the QCD sum rules prediction $\Gamma(B'_1) \simeq 250 \text{ MeV}$. As a test, we find that for $B_J^*(5732)$ as a

pure ${}^{3}P_1$ state its decay width is 153 MeV, which seems to agree well with the PDG suggested value. Whether $B_J^*(5732)$ is a mixed state $|P'_1\rangle$, a pure 3P_1 state, or another
configuration needs further improved experimental configuration needs further improved experimental measurement.

Similarly, for B_{s1} as a $|P_1\rangle$ state, its strong decay width and decay width ratio to the sum of B_{s1} and B_{2s}^* widths are

$$
\Gamma(B_{s1}) \simeq 0.4 \sim 1 \text{ MeV},\tag{45}
$$

$$
R = \frac{\Gamma(B_{s1})}{\Gamma(B_{s1}) + \Gamma(B_{2s}^*)} = 0.02 \sim 0.6.
$$
 (46)

The predicted width $\Gamma(B_{s1})$ agrees with the data $\Gamma(B_{s1})_{\text{exp}} \sim 1 \text{ MeV}$ [[77](#page-14-0)].

Since the mass of $|P'_1\rangle$ is slightly lower than that of $|P_1\rangle$,
a mass of R' (as a $|P'\rangle$ state) should be less than the mass of B'_{s1} (as a $|P'_1\rangle$ state) should be less than
5830 MeV If we assume the mass of B'_{s1} is around 5830 MeV. If we assume the mass of B_{s1} is around 5830 MeV, this gives a broad decay width to the B^*K channel

$$
\Gamma(B_{s1}') \simeq 149 \text{ MeV}.\tag{47}
$$

We should point out that the mass of B'_{s1} is most likely below the threshold of B^*K ; thus, the decay $B'_{s1} \to B^*K$ is kinematically forbidden. In this case the decay width of B' kinematically forbidden. In this case the decay width of B_{s1} will be very narrow. Its decay properties should be similar to those of $D_s(2460)$. The isospin violation decay $B_{s1}' \rightarrow$ $B_s^* \pi$ and radiative decay $B_{s1}' \rightarrow B_s^* \gamma$ will be the dominant
decay modes. A recent study of this scenario was given by decay modes. A recent study of this scenario was given by Wang with light-cone sum rules [[79](#page-14-0)].

C. Strong decays of 2S states

The radially excited heavy-light mesons are still not well established in experiments, although there are several candidates, such as $D^*(2640)^{\pm}$ [[61](#page-14-0)], $D^*_{sJ}(2632)$ [[62](#page-14-0)], and $D^*(2700)$ [6.7]. In theory, the radially excited D states $D_{sJ}^*(2700)$ [\[6,7](#page-13-0)]. In theory, the radially excited D states $2^{1}S$, and $2^{3}S$, were predicted to have masses \sim 2.58 and $2^{1}S_{0}$ and $2^{3}S_{1}$ were predicted to have masses \sim 2.58 and \sim 2.64 GeV, respectively [\[21\]](#page-13-0), while the radially excited D_s states 2^1S_0 and 2^3S_1 were \sim 2.6 and \sim 2.7 GeV, respectively $[21,36]$ $[21,36]$ $[21,36]$ $[21,36]$ $[21,36]$. In this section, we study the strong decays of these excited states into various channels. The mass uncertainties bring uncertainties into the predicted partial decay widths. Occasionally, some of the predicted partial widths exhibit sensitivities to the meson masses. Therefore, we present the strong decay widths of the D and D_s radially excited states as functions of their masses within a reasonable range as predicted by theory, and plot them along with their partial decay widths in Figs. 2 and [3](#page-9-0), respectively. For a given initial mass, by comparing the relative magnitudes among different partial widths from theoretical prediction and experimental measurement, one can extract additional information about the initial meson quantum numbers.

1. Radially excited D mesons

For a $2^{1}S_{0}$ state with a mass around 2.64 GeV, it can decay into $D^*\pi$, $D^*\eta$, D_s^*K , and $D_0^*(2400)\pi$. In Fig. 2, the partial widths and total strong decay width are shown for a partial widths and total strong decay width are shown for a mass range. In these channels, the $D^*\pi$ dominates the decays, the total decay width is \sim 14 MeV at $m(D(2¹S₀)) = 2.58$ GeV, and it shows a flat behavior. Note that the threshold of the D_s^*K channel is very close to 2.64 GeV. Some sensitivities to this open channel thus occur in a mass range around 2.6 GeV. It shows that the $D_s^* K$ width increases quickly with the masses and will compete against $D^*\eta$.

For the radially excited state $2³S₁$, its dominant decay channel is $D_1(2420)\pi$, while the other partial widths are
much smaller (see the lower part of the right panel of much smaller (see the lower part of the right panel of Fig. 2). Again, the $D^*\pi$ partial width appears insensitive to the initial D meson mass.

Comparing the decay patterns between $2^{1}S_0$ and $2^{3}S_1$ in Fig. 2 is useful for clarifying $D^*(2640)^{\pm}$. This state was
first seen by DELPHI in the $D^{*+}\pi^+\pi^-$ channel with a first seen by DELPHI in the $D^{*+}\pi^+\pi^-$ channel with a narrow width \leq 15 MeV [[61](#page-14-0)], but has not yet been confirmed by other experiments. If it is a genuine resonance, it will fit better into the 2^3S_1 state instead of 2^1S_0 due to its dominant decays into $D^{*+}\pi^+\pi^-$ which can occur via the main channel $D^*(2640)^+ \to D_1(2420)^0 \pi^+$. In contrast, the assignment to a 2¹ S, state will jmply a dominant decay assignment to a $2^{1}S_{0}$ state will imply a dominant decay channel to $D^*\pi$, which is not supported by the data. Although the predicted width of \sim 34 MeV overestimates

FIG. 2 (color online). The partial widths of $D(2¹S₀)$ and $D(2³S₁)$ as functions of the mass.

the data by nearly a factor of 2, it should be more urgent to establish it in experiments and have a more precise measurement of its partial decay widths to both $D^*\pi$ and $D^*\pi\pi$.

2. Radially excited D_s mesons

There are experimental signals for several excited D_s states, i.e. $D_{sJ}(2632)$ [[62](#page-14-0)], $D_{sJ}(2690)$, $D_{sJ}(2860)$ [\[6](#page-13-0)], and $D_{sJ}(2708)$ $D_{sJ}(2708)$ $D_{sJ}(2708)$ [7,[80](#page-14-0)], for which the spectroscopic classification is still unsettled. The $D_{sJ}(2690)$ and $D_{sJ}(2708)$ are likely to be the same state as they have similar masses and both are broad. We shall compare these experimental observations with our model predictions in order to learn more about their spectroscopic classifications.

 $D_{sJ}(2632)$ was reported by SELEX as a narrow state, i.e. Γ < 17 MeV, in $D_s \eta$ and DK channels [\[62\]](#page-14-0). The measured ratio of the partial widths is

$$
R = \frac{\Gamma(D^0 K^+)}{\Gamma(D_s \eta)} = 0.16 \pm 0.06. \tag{48}
$$

Its dominant decay into $D_s \eta$ makes it difficult to assign it into any simple $c\bar{q}$ scenario [\[81\]](#page-14-0). In particular, since a $2^{1}S_{0}$ state is forbidden to decay into $D_s \eta$ and DK, this rules out $D_{sJ}(2632)$ to be a radially excited 0^{-} .

As shown by Fig. 3, the decay of a $2³S₁$ state turns out to be dominated by D^*K and possibly DK, while its decay into $D_s \eta$ is rather small. Therefore, a simple 2^3S_1 cannot explain its decay pattern as well. Some more investigations

FIG. 3 (color online). The partial widths of $D_s(2^1S_0)$ and $D_s(2^3S_1)$ as functions of the mass.

of the nature of $D_{sJ}(2632)$ can be found in the literature, and here we restrict our attention to the output of our model calculations.

 $D_{sJ}^*(2860)$ $D_{sJ}^*(2860)$ $D_{sJ}^*(2860)$ and $D_{sJ}^*(2690)$ from BABAR [6] [or $D_{sJ}(2708)$ $D_{sJ}(2708)$ $D_{sJ}(2708)$ from Belle [7[,80\]](#page-14-0)] have widths

$$
\Gamma(D_{sJ}(2860)) = 48 \pm 17 \text{ MeV}, \tag{49}
$$

$$
\Gamma(D_{sJ}(2690)) = 112 \pm 43 \text{ MeV}, \tag{50}
$$

and both are observed in the DK channel, and no evidence is seen in D^*K and $D_s\eta$ modes. Compare these with Fig. 3, which shows that neither of them can easily fit in $2^{1}S_{0}$ or 2^3S_1 .

By fixing the masses of 2^1S_0 and 2^3S_1 states as sug-gested by the quark model [\[36\]](#page-13-0), i.e. $m(D_s(2^1S_0)) =$ 2.64 GeV and $m(D_s(2^3S_1)) = 2.71$ GeV, we obtain their strong decay widths

$$
\Gamma(D_s(2^1S_0)) \simeq 11 \text{ MeV},\tag{51}
$$

$$
\Gamma(D_s(2^3S_1)) \simeq 14 \text{ MeV},\tag{52}
$$

which turn out to be narrow. For $D_s(2^1S_0)$, the predicted dominant decay mode is D^*K , while the DK channel is forbidden. For $D_s(2^3S_1)$, there are two main decay channels, D^*K and DK , and they give a ratio of

$$
R = \frac{\Gamma(D^*K)}{\Gamma(DK)} \approx 2.6.
$$
 (53)

The $D_s \eta$ channel is also opened, but is negligibly small in comparison with DK and D^*K .

As $D_{sJ}^{*}(2860)$ has a relatively larger mass to fit in a D-wave state, we shall examine it with D-wave decays in the following subsection.

D. Strong decays of 1D states

Theoretical predicted masses of the D-wave excited D and D_s mesons are in a range of 2.8 \sim 2.9 GeV [\[21](#page-13-0)[,46\]](#page-14-0). To see the decay properties of those D-wave states, we plot their main decay partial widths in their possible mass region in Figs. [4](#page-10-0) and [5](#page-11-0) for D and D_s , respectively.

1. Excited D mesons

In Fig. [4,](#page-10-0) decays of four D-wave states are presented. It shows that the widths of $D(1^1D_2)$ and $D(1^3D_2)$ states are dominated by $D^*\pi$ decay, while the $D\pi/D\eta$ channels are forbidden. At \sim 2.8 GeV, they have very broad widths larger than 300 MeV. As D^* dominantly decays into $D\pi$, such broad widths imply that their dominant final states are $D\pi\pi$, and it might be difficult to identify them in experiments. This may explain why these states are still evading experimental observations.

For $D(1^3D_1)$, $D^*\pi$ is also the main decay channel, but
the a much smaller width. In contrast, $D\pi$ dominates the with a much smaller width. In contrast, $D\pi$ dominates the decay of $D(1^3D_3)$. With theory-suggested masses

FIG. 4 (color online). The partial widths of $D(1^1D_2)$, $D(1^3D_1)$, $D(1^3D_2)$, and $D(1^3D_3)$ as functions of the mass.

 $m(D(1^3D_1)) = 2.82$ and $m(D(1^3D_2)) = 2.83$ [\[21\]](#page-13-0), the total pionic decay widths for $D(1^3D_1)$ and $D(1^3D_2)$ are predicted to be

$$
\Gamma(D(1^3D_1)) \simeq 93 \text{ MeV}, \tag{54}
$$

$$
\Gamma(D(1^3D_3)) \simeq 130 \text{ MeV},\tag{55}
$$

and the predicted ratios between the $D^*\pi$ and $D\pi$ widths are

$$
R(D(1^{3}D_{1})) = \frac{\Gamma(D\pi)}{\Gamma(D^{*}\pi)} \approx 0.12,
$$
 (56)

$$
R(D(13D3)) \equiv \frac{\Gamma(D\pi)}{\Gamma(D^*\pi)} \simeq 1.7. \tag{57}
$$

For $D(1^3D_3)$, the dominance of $D\pi$ decay suggests that it is relatively more accessible in experiments is relatively more accessible in experiments.

2. Excited D_s mesons

For $D_s(1^1D_2)$ and $D_s(1^3D_2)$, three important partial widths of D^*K , DK^* , and $D_s^*\eta$ are presented in Fig. [5](#page-11-0), and both states are dominated by the D^*K decay. It is interesting to see that the mass of \sim 2.8 GeV is about the threshold for DK^* . Although this decay mode is negligible near threshold, it can become important in case the masses for these two D-wave states are above 2.8 GeV.

At 2.8 GeV, the total widths are dominated by the D^*K mode, which are

$$
\Gamma(D_s(1^1D_2)) \simeq 61 \text{ MeV},\tag{58}
$$

$$
\Gamma(D_s(1^3D_2)) \simeq 84 \text{ MeV}.
$$
 (59)

These results can guide a search for these two states around 2.8 GeV.

As shown in Fig. [5,](#page-11-0) D^*K and DK are two dominant decay modes for $D_s(1^3D_1)$ if its mass is around 2.8 GeV, and the predicted width is relatively narrow. Implications from such an assignment will be discussed in Sec. V E.

Compared with the $D_s(1^3D_1)$ decay, the dominant decay mode of the $D_s(1³D₃)$ state is DK around 2.8 GeV. With a higher mass the D^*K channel becomes increasingly important. This feature fits well the experimental observation for $D_{sJ}^{*}(2860)$, and makes it a possible assignment for this state state.

To be more specific, for $D_{sJ}^{*}(2860)$ as a $D_{s}(1^{3}D_{3})$ state,
predicted width is its predicted width is

$$
\Gamma(D_s(1^3D_3)) \simeq 41 \text{ MeV},\tag{60}
$$

and the dominant decay mode is DK. In comparison, the decays via DK^* and $D_s\eta$ are much less important (see Fig. [5](#page-11-0) and Table [VIII](#page-5-0)). The ratio of DK and D^*K is found to be

$$
R(D_s(1^3D_3)) = \frac{\Gamma(DK)}{\Gamma(D^*K)} \approx 2.3,
$$
 (61)

which is also consistent with the experiment $[6]$ $[6]$ $[6]$. This assignment agrees with results of Refs. [\[34,42,43\]](#page-13-0).

FIG. 5 (color online). The partial widths of $D_s(1^1D_2)$, $D_s(1^3D_1)$, $D_s(1^3D_2)$, and $D_s(1^3D_3)$ as functions of the mass. When we calculate the partial width of the DK^* channel, the D meson is looked at as the emitted pseudoscalar meson in the $SU(4)$ case [\[73\]](#page-14-0).

Some models also suggested that $D_{sJ}^*(2860)$ could be a
P, state [33,34,36], for which only decay mode DK and $2^{3}P_{0}$ state [\[33,34,36](#page-13-0)], for which only decay mode DK and $D_s \eta$ are allowed. In our model, a $2^3 P_0$ state leads to decay amplitude

$$
\mathcal{M}(2^3 P_0 \to |1^1 S_0 \rangle \mathbb{P})
$$

= $i\frac{1}{4} \sqrt{\frac{1}{15} g_I F} \frac{q'}{\alpha} \left[g_{10}^z G q \frac{q'^2}{\alpha^2} - \frac{1}{3} g_{10}^z h q' \left(1 + \frac{q'^2}{2\alpha^2} \right) - 2\sqrt{2} g_{10}^+ h q' \left(7 - \frac{q'^2}{\alpha^2} \right) \right]$ (62)

with which its partial decay width to DK is about $\Gamma =$ 184 MeV, and much broader than the experimental observation.

E. The $2^{3}S_{1}$ -1³D₁ mixing

In Ref. [\[36\]](#page-13-0), a mixing scheme between $2^{3}S_{1}$ and $1^{3}D_{1}$ was proposed as a solution for the relatively broad $D_{sJ}^*(2690)$, i.e.

$$
|D_{s1}^*(2690)\rangle = \sin(\phi)|1^3D_1\rangle + \cos(\phi)|2^3S_1\rangle,
$$

\n
$$
|D_{s1}^*(2810)\rangle = \cos(\phi)|1^3D_1\rangle - \sin(\phi)|2^3S_1\rangle,
$$
 (63)

where an orthogonal state $D_{s1}^{*}(2810)$ was also predicted.
The mixing angle was found to favor $\phi = -0.5$ radians The mixing angle was found to favor $\phi = -0.5$ radians, i.e. $\phi \approx -27$ °. According to such a mixing scheme, $D_{s1}^*(2810)$ will also be a broad state and dominated by a 1^3D configuration $1³D₁$ configuration.

Taking the mixing scheme of Eq. (63) , we plot the widths of $D_{sJ}^{*}(2690)$ and $D_{sJ}^{*}(2810)$ in terms of mixing
angle ϕ in Fig. 6. The figure shows that if we take the angle ϕ in Fig. 6. The figure shows that if we take the mixing angle $\phi \approx -27^{\circ}$ as predicted in Ref. [\[36\]](#page-13-0), the decay modes of $D_{sJ}^*(2690)$ are dominated by D^*K , which
disagrees with the experimental observation. Nevertheless disagrees with the experimental observation. Nevertheless, the predicted decay width of $D_{sJ}^{*}(2690)$, $\Gamma \sim 25$ MeV, is
underestimated by at least a factor of 2 compared with the underestimated by at least a factor of 2 compared with the data. If the 2S-1D mixing is small, e.g. $D_{sJ}^*(2690)$ is a pure

FIG. 6. The partial widths of low vector (A) and high vector (B) as functions of the mixing angle ϕ .

 $1³D₁$ state, the predicted decay width is $\Gamma \sim 42$ MeV, which is close to the lower limit of the data. However, the ratio $R = \Gamma(DK)/\Gamma(D^*K) \sim 0.8$ disagrees with the observation that the DK channel dominates the decay modes.

If we set $\phi \approx 30^{\circ}$, which implies that the sign of the spin-orbit splitting term is now negative to keep the correct mass ordering, the D^*K channel in the $D^*_{sJ}(2690)$ decay
will be completely suppressed. This is consistent with the will be completely suppressed. This is consistent with the observations except that the decay width $\Gamma \sim 15$ MeV is too small to compare with the data.

For the high vector $D_{sJ}^*(2810)$, with $\phi \approx \pm 30^\circ$, its width $\phi \approx \pm 40.60$ MeV and is dominated by the D^*K reads \sim 40–60 MeV and is dominated by the D^*K . Meanwhile, its branching ratio to DK is still sizeable.

Our test of the property of $D_{sJ}^{*}(2690)$ in the 2S-1D
xing scenario does not fit in the data very well. To clarify mixing scenario does not fit in the data very well. To clarify the nature of $D_{sJ}^*(2690)$, more accurate measurements of
its width and ratio $R = DK/D^*K$ and experimental reits width and ratio $R = DK/D^*K$, and experimental research for the accompanying $D_{sJ}^*(2810)$ in the DK and D^*K channels are needed D^*K channels, are needed.

VI. SUMMARY

In the chiral quark model framework, we systematically study the strong decays of heavy-light mesons in $\mathbb{M} \rightarrow$ $|1^1S_0\rangle \mathbb{P}, \mathbb{M} \rightarrow |1^3S_1\rangle \mathbb{P}, \mathbb{M} \rightarrow |1^3P_0\rangle \mathbb{P}, \mathbb{M} \rightarrow |1^1P_1\rangle \mathbb{P},$ and $\mathbb{M} \rightarrow |1^3P_1\rangle \mathbb{P}$. By adopting commonly used values for the constituent quark masses and pseudoscalar-meson decay constants, we make a full analysis of the strong decays of all the excited D^* , D_s^* , B^* , and B_s^* , and find that most available data can be consistently explained in this framework. We summarize our major results as follows.

A. Excited D mesons

The calculated partial decay widths for the $D^*(1^3S_1)$, $D_0^*(2400)$ as a 1^3P_0 state, and $D_2^*(2460)$ as a 1^3P_2 state are
in good agreement with the data in our model and support in good agreement with the data in our model and support their assignments as the low-lying excited D^* .

State mixing between the 1^1P_1 and 1^3P_1 is favored. With the mixing angle, $\phi \approx -(55 \pm 5)^{\circ}$, which is consistent with the prediction of Ref. [22] the narrow $D(2420)$ tent with the prediction of Ref. [\[22\]](#page-13-0), the narrow $D_1(2420)$ and broad $D'_1(2430)$ can be well explained as mixing states
between 1^1P , and 1^3P . Precise measurement of the mass between $1^{1}P_1$ and $1^{3}P_1$. Precise measurement of the mass of the broad D'_1 (2430) is needed.
Our result shows that assigning

Our result shows that assigning $D^*(2640)$ to be a radially excited $2^{3}S_{1}$ state can naturally explain its observation in the $D^{*+}\pi^+\pi^-$ final state, although the predicted width \sim 34 MeV still possesses some discrepancies with the data. A search for the $D^*(2640)$ in the $D_1(2420)\pi$ channel
is strongly recommended is strongly recommended.

Although the 2S and 1D excited D states are still not well established, we analyzed their partial decay widths in their possible mass regions, which should be useful for future experimental studies. The decay widths of 2S states turn out to be narrow, namely, at the order of a few tens of MeV. Our results show that $D(2¹S₀)$ is dominated by the $D^*\pi$ decay channel, while both $D^*\pi$ and $D_1(2420)\pi$ are
important for $D(2^3S_+)$. Both $D(1^1D_+)$ and $D(1^3D_+)$ have important for $D(2^3S_1)$. Both $D(1^1D_2)$ and $D(1^3D_2)$ have very broad widths ≥ 300 MeV, which may be difficult to identify in experiments. The decay widths of $D(1^3D_1)$ and $D(1^3D_3)$ are ≥ 100 MeV. The former has dominant decays into $D^*\pi$ and sizeable widths to $D\pi$ as well. For $D(1^3D_3)$, both $D\pi$ and $D^*\pi$ are important.

B. Excited D_s mesons

 $D_s(2460)$ and $D_{s1}(2536)$ are identified as partners due to mixing between ${}^{1}P_1$ and ${}^{3}P_1$. With the mixing angle $\phi \approx$ $-(55 \pm 5)^{\circ}$, our predictions for $D_{s1}(2536)$ are in good
agreement with the data. Note that D (2460) is below the agreement with the data. Note that $D_s(2460)$ is below the strong decay threshold.

 $D_{s2}^*(2573)$ is consistent with a 1^3P_2 state. Its partial
cay width to DK is dominant while to D^{*}K is very small decay width to DK is dominant while to D^*K is very small (\sim 1 MeV). This explains its absence in the D^{*}K channel [\[74\]](#page-14-0).

For those not-well-established states, by analyzing their decay properties, we find that $D_{sJ}^{*}(2860)$ strongly favors a
D $(13D_1)$ state, while the $D^{*}(2632)$ and $D^{*}(2690)$ cannot $D_s(1^3D_3)$ state, while the $D_{sJ}^*(2632)$ and $D_{sJ}^*(2690)$ cannot
fit in any pure D^* configurations fit in any pure D_s^* configurations.

For the unobserved 2S states, $D_s(2^1S_0)$ and $D_s(2^3S_1)$, their decay widths are predicted to be an order of \sim 15 MeV, and dominated by the D^{*}K decay mode.

For the unobserved 1D states, $D_s(1^1D_2)$, $D_s(1^3D_2)$, and $D_s(1^3D_1)$, their decay widths are of the order of 60 ~ 80 MeV at a mass of \sim 2.8 GeV. The $D_s(1^1D_2)$ and $D_s(1^3D_2)$ decay widths are dominated by the D^*K mode, while $D_s(1^3D_1)$ is dominated by the D^*K and DK together.

C. Excited B mesons

We also study the decay properties of the newly observed bottom states $B_1(5725)$, $B_2^*(5829)$. Our calculations strongly suggest that $B_1(5725)$ is a mixed $|P_1\rangle$ state and strongly suggest that $B_1(5725)$ is a mixed $|P_1\rangle$ state, and $B_2^*(5829)$ satisfies an assignment of 1^3P_2 .
The $B^*(5732)$ which was first repo

The $B_J^*(5732)$, which was first reported by the L3
pllaboration can be naturally explained as the broad Collaboration, can be naturally explained as the broad partner ($|P_1'\rangle$) of $B_1(5725)$ in the ${}^{1}P_1$ and ${}^{3}P_1$ mixing
scheme Its predicted width is $\Gamma(B') = 219$ MeV which scheme. Its predicted width is $\Gamma(B_1') = 219 \text{ MeV}$, which
is larger than the PDG suggested value $\Gamma =$ is larger than the PDG suggested value $\Gamma_{\text{exp}} =$ 128 ± 18 MeV. In contrast, as a pure 1^3P_1 state, its decay
width is $\Gamma(R') = 153$ MeV. Whether $R^*(5732)$ is a mixed width is $\Gamma(B_1^l) = 153$ MeV. Whether $B_2^*(5732)$ is a mixed
state $|B_1^l\rangle$ a pure 1^3P , state, or other configuration should state $|P_1\rangle$, a pure 1^3P_1 state, or other configuration should
be further studied be further studied.

The theoretical prediction of the mass for the $B_0^*(^3P_0)$
reson is about 5730 MeV Its decay into $R\pi$ has a broad The theoretical prediction of the mass for the $D_0(T_0)$
meson is about 5730 MeV. Its decay into $B\pi$ has a broad width $\Gamma(B_0^*) = 272$ MeV according to our prediction.

D. Excited B_s mesons

The two new narrow bottom-strange mesons $B_{s1}(5830)$ and $B_{s2}^*(5839)$ observed by CDF are likely the mixed state

 P_1 and the 1^3P_2 state, respectively, though their decay widths and ratios are not given. For $B_{s1}(5830)$ as a $|P_1\rangle$ state, its predicted width and decay ratio are $\Gamma(B_{s1}) \approx (0.4 \sim 1)$ MeV and $\Gamma(B_{s1})/(\Gamma(B_{s1}) + \Gamma(B_{2s}^*)) =$ $(0.4 \sim 1)$ MeV and $\Gamma(B_{s1})/(\Gamma(B_{s1}) + \Gamma(B_{2s}^*) = 0.02 \sim 0.6$. For $B_s(5839)$ as the ³ P_2 state, its decay width and width ratio predicted by us are $\Gamma(B_s^*) \sim 2$ MeV and and width ratio predicted by us are $\Gamma(B_{s2}^*) \simeq 2 \text{ MeV}$ and $\Gamma(B^*K)/\Gamma(RK) \sim 6\%$ $\Gamma(B^*K)/\Gamma(BK) \simeq 6\%.$

The theoretical prediction for the B_{s0}^* mass is about 5800 MeV. In our model it has a broad width $\Gamma(B_0^*)$ = 227 MeV. For B'_{s1} if its mass is above the threshold of B^*K , it will be a broad state with $\Gamma(B'_{s1}) \simeq 149$ MeV.
Otherwise it should be a narrow state Otherwise, it should be a narrow state.

It should be mentioned that uncertainties with quark model parameters can give rise to uncertainties with the theoretical results. A qualitative examination shows that such uncertainties can be as large as 10%–20%, which are a typical order for quark model approaches. Interestingly, systematics arising from such a simple prescription are useful for us to gain insights into the effective degrees of freedom inside those heavy-light mesons and the underlying dynamics for their strong decays.

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