

Role of “intrinsic charm” in semileptonic B -meson decaysC. Breidenbach,^{1,*} T. Feldmann,^{1,+} T. Mannel,^{1,2,‡} and S. Turczyk^{1,§}¹*Theoretische Physik 1, Fachbereich Physik, Universität Siegen, D-57068 Siegen, Germany*^{||}²*CERN, Department of Physics, Theory Unit, CH-1211 Geneva 23, Switzerland*

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We discuss the role of so-called “intrinsic-charm” operators in semileptonic B -meson decays, which appear first at order $1/m_b^3$ in the heavy quark expansion. We show by explicit calculation that—at scales $\mu \leq m_c$ —the contributions from “intrinsic-charm” effects can be absorbed into short-distance coefficient functions multiplying, for instance, the Darwin term. Then, the only remnant of “intrinsic charm” are logarithms of the form $\ln(m_c^2/m_b^2)$, which can be resummed by using renormalization-group techniques. As long as the dynamics at the charm-quark scale is perturbative, $\alpha_s(m_c) \ll 1$, this implies that no additional nonperturbative matrix elements aside from the Darwin and the spin-orbit term have to be introduced at order $1/m_b^3$. Hence, no sources for additional hadronic uncertainties have to be taken into account. Similar arguments may be made for higher orders in the $1/m_b$ expansion.

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I. INTRODUCTION

The heavy quark expansion (HQE) has turned out to be a valuable tool for precision calculations of heavy hadron decays [1–4]. In particular, due to the HQE for semileptonic decays, where the $b \rightarrow c$ transition is described in the framework of a standard local operator product expansion (OPE), the relative uncertainty in the Cabbibo-Kobayashi-Maskawa matrix element $|V_{cb}|$ could be reduced to a level below 2% [5–8].

The expansion in inverse powers of the heavy quark mass m_b can be set up for both, the lepton-energy spectrum as well as for the total decay rate. The nonperturbative input, entering the theoretical description, is given by forward matrix elements of local operators in the OPE. The leading term represents the partonic rate and does not contain any unknown hadronic matrix element. The perturbative corrections to the partonic rate have been calculated to order α_s^2 , recently [9,10]. Terms of order $1/m_b$ vanish due to heavy quark symmetries. At order $1/m_b^2$, two hadronic parameters μ_π^2 and μ_G^2 appear, which can be interpreted as the kinetic energy and the chromomagnetic moment of the heavy quark inside the heavy hadron. The short-distance contribution to the coefficient of μ_π^2 is known to order α_s [11], while that of μ_G^2 is known at tree level. The dimension-six operators at order $1/m_b^3$ define two additional parameters, which correspond to the Darwin term ρ_D^3 and the spin-orbit term ρ_{LS}^3 , known from the usual nonrelativistic reduction of the Dirac equation. The coefficients at that order are only known at tree

level, so far. The terms at order $1/m_b^4$ have also been classified, and introduce five new hadronic parameters [12].

It has also been pointed out that at order $1/m_b^3$ a dimension-six operator appears, whose matrix element could be interpreted as the “intrinsic-charm” content of the B meson [13,14]. An order-of-magnitude estimate for the effect has been given in [13], and the additional uncertainty from the poor knowledge of these matrix elements has been included in the error budget for $|V_{cb}|$ [15]. However, as we are going to show in this paper, the inclusion of an “intrinsic-charm” contribution requires a proper definition of the short-distance functions appearing in the lepton-energy spectrum, since the “intrinsic-charm” operators and, for instance, the Darwin-term mix under renormalization. As long as the strong dynamics at the charm-mass scale is treated perturbatively, the effect of “intrinsic charm” can entirely be absorbed into short-distance coefficients defined at a low hadronic input scale, and the nonanalytic dependence on the charm-quark mass can be resummed by standard renormalization-group techniques, extending the results in [16]. In this case, no additional hadronic uncertainty due to “intrinsic charm” has to be included. On the other hand, treating the charm-quark as nonperturbative, the hadronic matrix elements of intrinsic-charm operators would remain as unknown parameters. In this case, however, the charm-quark dependent terms in the standard expressions for the lepton-energy spectrum and the total rate have to be modified accordingly, in order to avoid double counting.

In this paper, we are going to present a systematic study of how “intrinsic-charm” effects will enter the theoretical expressions for the lepton-energy spectrum, depending on the treatment of the charm-quark mass scale, with particular emphasis on the mixing of the “intrinsic-charm” operators into the Darwin term.

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II. CALCULATION OF THE CHARM CONTRIBUTION

Starting point for the calculation of inclusive rates within the OPE is the hadronic tensor $W_{\mu\nu}$ as it appears in the differential rate for $b \rightarrow c \ell \bar{\nu}_\ell$ transitions,

$$d\Gamma = 16\pi G_F^2 |V_{cb}|^2 W_{\mu\nu} L^{\mu\nu} d\phi. \quad (1)$$

Here $d\phi$ denotes the invariant phase space for the lepton-neutrino pair, and the leptonic tensor is given by

$$L^{\mu\nu} = 2(p_e^\mu p_{\nu_e}^\nu + p_e^\nu p_{\nu_e}^\mu - g^{\mu\nu} p_e \cdot p_{\nu_e} - i\epsilon^{\mu\nu\alpha\beta} p_{e\alpha} p_{\nu_e\beta}), \quad (2)$$

where $\epsilon^{0123} = -\epsilon_{0123} = +1$. Using translational invariance, the hadronic tensor may be cast into the form

$$2M_B W_{\mu\nu} = \int d^4x e^{i(m_b v - q)x} \times \langle \bar{B}(p) | \bar{b}_v(x) \gamma_\nu P_L c(x) \bar{c}(0) \gamma_\mu P_L b_v(0) | \bar{B}(p) \rangle, \quad (3)$$

where $P_L = (1 - \gamma_5)/2$ projects onto left-handed fields, $v^\mu = p^\mu/M_B$ is the velocity of the decaying \bar{B} meson, and $b_v(x)$ denotes the heavy b -quark field with the phase $e^{-im_b v \cdot x}$ factored out. Performing the OPE for this matrix element, the product of the two $b \rightarrow c$ currents is matched onto a set of local operators at scales μ of the order of the b -quark mass m_b . Now, as far as the charm-quark mass is concerned, one may take different points of view [17]:

(1) One may assume that $m_b \sim m_c \gg \Lambda_{\text{QCD}}$, which means that the short-distance matching coefficients and the phase-space integrals are functions of the fixed ratio $\rho = m_c^2/m_b^2$. In other words, one integrates out (hard) quantum fluctuations with virtualities of order $m_{b,c}^2$ and is left with light degrees of freedom: light quarks and gluons, together with the quasistatic b -quark field in HQET. In a standard renormalization scheme like the modified minimal subtraction scheme ($\overline{\text{MS}}$), operators with charm fields do not appear at scales $\mu < m_c$. More precisely, such operators would correspond to quasistatic charm quarks, which cannot contribute to the considered matrix elements, $\langle \bar{B} | \bar{b}_v \dots c_{\text{static}} \bar{c}_{\text{static}} \dots b_v | \bar{B} \rangle \equiv 0$, because of energy conservation, $m_b + 2m_c + \Delta E_{\text{soft}} > m_B$. This is in fact the point of view that is usually considered in the precision determination of $|V_{cb}|$.

(2) One may consider the power counting $m_b \gg m_c \gg \Lambda_{\text{QCD}}$, and integrate out hard b -quark fluctuations at a different scale than the charm quark. In this case, for the first matching at the high scale $\mu_h \sim m_b$ one still has to keep the charm quark dynamical, and the corresponding ‘‘intrinsic-charm’’ operators appear in the OPE [18]. The renormalization group for these operators can be used to scale down to the semihard scale $\mu_{sh} \sim m_c$, where the charm quark is finally integrated out. As before, the ‘‘intrinsic-charm’’ operators then match onto local opera-

tors built from light fields. Obviously, the main difference compared to case 1 is, that the logarithmic terms $\ln(m_c/m_b)$ can be resummed into short-distance coefficient functions [16], while the analytic terms should be expanded in powers of $m_c/m_b \sim \sqrt{\Lambda_{\text{QCD}}/m_b} \sim 0.3$.

(3) Finally, one may assume that $m_b \gg m_c \gtrsim \Lambda_{\text{QCD}}$. In this case, one cannot integrate out the charm-quark effects perturbatively, and is thus left with genuine intrinsic-charm operators, whose hadronic matrix elements have to be defined at a sufficiently high scale μ_0 , satisfying $m_b \geq \mu_0 \gg m_c$. Notice that the matrix elements of the intrinsic-charm operators contain the nonanalytic dependence on the charm-quark mass m_c , and consequently the partonic phase-space integration for the calculation of various moments of the differential rate has to be modified accordingly, in order to avoid double counting.

In the following we shall discuss the different cases in turn.

A. $m_b \sim m_c \gg \Lambda_{\text{QCD}}$

As explained above, when integrating out both, the hard b -quark fluctuations and the charm quark, at a common scale $\mu \sim m_b$, we are left with operators built from soft fields, only. Thus the only matrix elements appearing at order $1/m_b^3$ are the Darwin term ρ_D^3 and the spin-orbit term ρ_{LS}^3 defined by (we use the convention of [12], but omit the hat over $\hat{\rho}_D^3, \hat{\rho}_{LS}^3$)

$$\begin{aligned} 2M_B \rho_D^3 &= \langle \bar{B}(p) | \bar{b}_v(iD_\mu)(i\nu D)(iD^\mu)b_v | \bar{B}(p) \rangle, \\ 2M_B \rho_{LS}^3 &= \langle \bar{B}(p) | \bar{b}_v(iD_\mu)(i\nu D)(iD_\nu)(-i\sigma^{\mu\nu})b_v | \bar{B}(p) \rangle. \end{aligned} \quad (4)$$

In the charged-lepton-energy spectrum one obtains (among others) a contribution of the form

$$\frac{d\Gamma}{dy} \Big|_{\rho_D^3} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \frac{\rho_D^3}{m_b^3} \left\{ -\frac{8\theta(1-y-\rho)}{1-y} + \dots \right\}. \quad (5)$$

For later use, we have only quoted the most singular term in the limit $y = 2E_\ell/m_b \rightarrow 1$, and $\rho = m_c^2/m_b^2 \rightarrow 0$ (the full expressions are provided in Appendix A 2). Upon integration it yields a logarithmically enhanced contribution to the total rate

$$\Gamma \Big|_{\rho_D^3} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \frac{\rho_D^3}{m_b^3} \{8 \ln \rho + \dots\}, \quad (6)$$

where the ellipses denote the contributions from the sub-leading terms in (5) which are of order $\rho \ln \rho$. Similarly, we identify the leading terms in the moments (see, Appendix A 2)

$$\langle (y - y_0)^n \rangle \Big|_{\rho_b^3} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \frac{\rho_D^3}{m_b^3} \{8(1 - y_0)^n \ln \rho + \dots\}. \quad (7)$$

Note that for $m_b \sim m_c$ the logarithm is actually of order one and represents a regular contribution to the matching coefficient (and therefore the remaining terms in curly brackets enter on the same level). Also, the phase-space boundary for y is $y < 1 - \rho$ which is away from $y = 1$ by an amount of order one.

A similar logarithmically enhanced term also appears in the partonic rate,

$$\Gamma \Big|_{\text{partonic}} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \{1 - 12\rho^2 \ln \rho + \dots\}, \quad (8)$$

and in the related moment,

$$\langle 1 - y \rangle \Big|_{\text{partonic}} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \{6\rho^2 \ln \rho + \dots\}. \quad (9)$$

In contrast to the Darwin-term contribution, the logarithmic term vanishes in the limit $\rho \rightarrow 0$. Nevertheless, as has been shown in [16], such “phase-space logs” can be resummed into short-distance coefficients, as we are going to discuss in the following.

B. $m_b \gg m_c \gg \Lambda_{\text{QCD}}$

When we integrate out the b quark first at a scale $\mu_h \sim m_b$ and still keep the charm quark dynamical, we have to take into account operators with explicit charm quarks until those are finally integrated out at the semihard scale $\mu_{\text{sh}} \sim m_c$. In addition to the dimension-five and dimension-six operators, defining μ_π^2 , μ_G^2 and ρ_D^3 , ρ_{LS}^3 , one thus finds (at tree level) matrix elements of the local “intrinsic-charm” (IC) operators

$$\begin{aligned} 2M_B W_{\mu\nu}^{\text{IC}} &= (2\pi)^4 \delta^4(q - m_b v) \\ &\times \langle \bar{B}(p) | (\bar{b}_v \gamma_\nu P_L c) (\bar{c} \gamma_\mu P_L b_v) | \bar{B}(p) \rangle \\ &+ (2\pi)^4 \left(\frac{\partial}{\partial q_\alpha} \delta^4(q - m_b v) \right) \\ &\times \langle \bar{B}(p) | (i \partial_\alpha \bar{b}_v \gamma_\nu P_L c) (\bar{c} \gamma_\mu P_L b_v) | \bar{B}(p) \rangle \\ &+ \dots, \end{aligned} \quad (10)$$

which can be interpreted as the probability to find semihard (i.e., off shell) charm quarks inside the heavy \bar{B} meson.

Notice, that the power counting for the semihard charm fields $[c] = (m_c)^{3/2}$ is now different from the ones for soft HQET fields $[b_v] = \Lambda^{3/2}$, and therefore it may be convenient to use a notation as in [19], where the “intrinsic-charm” operators in the first line of (10) are suppressed by $\lambda^3 \equiv (m_c/m_b)^3$, the ones in the second line by λ^4 , the kinetic and chromomagnetic operators by $\lambda^4 \equiv (\Lambda/m_b)^2$, and the Darwin and spin-orbit term by λ^6 . Because of chiral symmetry, only the λ^4 “intrinsic-charm” operators

contribute to the partonic rate for $b \rightarrow c \ell \nu$, related to the $\rho^2 \ln \rho$ term in (8). Additional soft gluon couplings to semihard charm quarks are further suppressed, and this will give rise to the λ^6 suppressed terms $\rho_D^3 \ln \rho$ in (6), descending from the λ^3 “intrinsic-charm” operators.

Let us consider first the matrix elements of the operator in the first line of (10). They may be decomposed in terms of two hadronic parameters, $T_1(\mu)$ and $T_2(\mu)$,

$$\begin{aligned} (4\pi)^2 \langle \bar{B}(p) | \bar{b}_v \gamma_\nu P_L c \bar{c} \gamma_\mu P_L b_v | \bar{B}(p) \rangle \\ = 2M_B (T_1(\mu) g_{\mu\nu} + T_2(\mu) v_\mu v_\nu). \end{aligned} \quad (11)$$

The contribution to the rate of the matrix element of the local “intrinsic-charm” operators is concentrated at small hadronic mass m_X and in the endpoint of the lepton-energy spectrum. Performing the tree-level matching at $\mu = m_b$, we have

$$\begin{aligned} \frac{d^2 \Gamma^{\text{IC}}}{dm_X^2 dy} &= \delta(m_X^2) \delta(1 - y) \Gamma^{\text{IC}} \quad \text{and} \\ \frac{d \Gamma^{\text{IC}}}{dy} &= \delta(1 - y) \Gamma^{\text{IC}}, \end{aligned} \quad (12)$$

with

$$\Gamma^{\text{IC}} = - \frac{G_F^2 m_b^5}{24\pi^3} |V_{cb}|^2 \frac{3T_1(m_b)}{m_b^3}. \quad (13)$$

On the other hand, the calculation of the matching coefficients for the contribution of ρ_D^3 and ρ_{LS}^3 to the total rate now has to be performed in the limit $m_c \ll m_b$. Notice, that the naive limit $\rho \rightarrow 0$ in (5) would give ill-defined expressions. In particular, the integral over

$$dy \frac{\theta(1 - y)}{1 - y}$$

would be infrared divergent in the lepton-energy endpoint. As we will see, the new IR divergence in the phase-space integration, appearing in the limit $\rho \rightarrow 0$, is related to the UV renormalization of the “intrinsic-charm” operators (11). Defining the hadronic parameters $T_{1,2}(\mu)$ in the $\overline{\text{MS}}$ scheme, we also have to perform the phase-space integral in $D = 4 - 2\epsilon$ dimensions. As a result, the contribution of the Darwin term to the total rate is regularized by plus distributions,

$$\frac{\theta(1 - y)}{1 - y} \rightarrow \left[\frac{\theta(1 - y)}{1 - y} \right]_+ - \delta(1 - y) \ln \left(\frac{\mu^2}{m_b^2} \right), \quad (14)$$

which exactly subtracts the effects of semihard charm quarks, that would otherwise be double counted when adding (13) to the decay rate.

The final expression for the combined contributions of the Darwin term and the “intrinsic-charm” operators to the lepton-energy spectrum at order $1/m_b^3$ can be written as

$$\frac{d\Gamma^{(3)}}{dy} \Big|_{\rho_D^3+\text{IC}} = \frac{G_F^2 m_b^5}{24\pi^3} |V_{cb}|^2 \times \left\{ \frac{C_{\rho_D}(y, \mu) \rho_D^3(\mu)}{m_b^3} + \frac{C_{T_1}(y, \mu) T_1(\mu)}{m_b^3} \right\}, \quad (15)$$

which should be used for $m_c \leq \mu \leq m_b$. The matching conditions for the short-distance coefficient functions—including the limit $\rho \rightarrow 0$ for the subleading terms in (5) as given in the Appendix—are given by

$$C_{\rho_D}(y, m_b) = - \left[\frac{y^2(9-5y+2y^2)\theta(1-y)}{6(1-y)} \right]_+ + \frac{17}{12} \delta(y-1) + \frac{5}{24} \delta'(y-1) - \frac{1}{72} \delta''(y-1) + \mathcal{O}(\alpha_s), \quad (16)$$

$$C_{T_1}(y, m_b) = -3\delta(y-1) + \mathcal{O}(\alpha_s),$$

$$C_{T_2}(y, m_b) = \mathcal{O}(\alpha_s).$$

In Appendix B 1, we derive the leading terms in the anomalous-dimension matrix that describe the mixing of the “intrinsic-charm” operators $\{T_1(\mu), T_2(\mu)\}$ into the Darwin term $\rho_D^3(\mu)$, see, also Fig. 1(b),

$$\frac{d}{d \ln \mu} \begin{pmatrix} \rho_D^3 \\ T_1 \\ T_2 \end{pmatrix} = - \left\{ \begin{pmatrix} 0 & 0 & 0 \\ -2/3 & 0 & 0 \\ 4/3 & 0 & 0 \end{pmatrix} + \mathcal{O}(\alpha_s) \right\} \begin{pmatrix} \rho_D^3 \\ T_1 \\ T_2 \end{pmatrix}. \quad (17)$$

Neglecting the $\mathcal{O}(\alpha_s)$ contributions to the anomalous-dimension matrix, we only determine the leading-logarithmic terms [20], which are generated by the renormalization-group equation for the short-distance coefficients

$$C_{T_i}(y, \mu) \simeq C_{T_i}(y, m_b),$$

$$C_{\rho_D}(y, \mu) \simeq C_{\rho_D}(y, m_b) - \frac{1}{3} \ln \frac{\mu^2}{m_b^2} \left(C_{T_1}(y, m_b) - 2C_{T_2}(y, m_b) \right). \quad (18)$$

Now, integrating out the semihard charm quarks at $\mu_{\text{sh}} = m_c$, is equivalent to setting

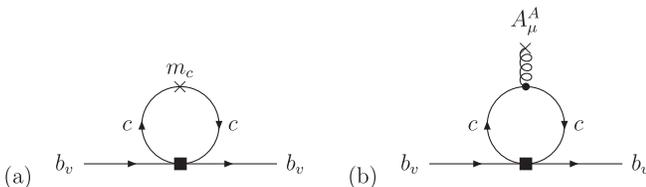


FIG. 1. Leading diagrams determining the mixing of four-quark into two-quark operators.

$$T_i(\mu \leq m_c) = 0. \quad (19)$$

In this case, the expression for the lepton-energy spectrum (15) simplifies to

$$\frac{d\Gamma^{(3)}}{dy} \Big|_{\rho_D^3+\text{IC}} = \frac{G_F^2 m_b^5}{24\pi^3} |V_{cb}|^2 \frac{C_{\rho_D}(y, m_c) \rho_D^3(m_c)}{m_b^3}, \quad (20)$$

and the information on “intrinsic charm”, i.e., the non-analytic dependence on the charm-quark mass, has been completely absorbed into the short-distance function $C_{\rho_D}(y, m_c)$. This can be made explicit by inserting the leading-order matching conditions (16) for $C_{T_i}(y, m_b)$, which results in

$$C_{\rho_D}(y, m_c) \simeq C_{\rho_D}(y, m_b) + \ln \frac{m_c^2}{m_b^2} \delta(y-1). \quad (21)$$

In this way (20) reproduces the logarithmic term in the lepton-energy moments in (7) as well as the finite terms [given by the limit $\rho \rightarrow 0$ of Eq. (A6) in Appendix A 2].

Similar considerations can be made for the $\rho^2 \ln \rho$ term in the partonic rate. We decompose the matrix elements of the operators in the second line of (10) as

$$(4\pi)^2 \langle \bar{B}(p) | (i\partial_\alpha \bar{b}_\nu \gamma_\nu P_L c) (\bar{c} \gamma_\mu P_L b_\nu) | \bar{B}(p) \rangle = 2M_B \left(T_3(\mu) g_{\mu\nu} v_\alpha + T_4(\mu) g_{\mu\alpha} v_\nu + T_5(\mu) g_{\nu\alpha} v_\mu + T_6(\mu) v_\mu v_\nu v_\alpha - T_7(\mu) i\epsilon_{\mu\nu\alpha\beta} v^\beta \right). \quad (22)$$

(Notice that in unpolarized observables, only the sum $T_4(\mu) + T_5(\mu)$ appears.) Generalizing the results for the total rate in [16] to the lepton-energy spectrum, and concentrating again on the leading-logarithmic terms, we find

$$\frac{d\Gamma}{dy} \Big|_{\text{partonic+IC}} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left\{ C_0(y, \mu) + \rho C_1(y, \mu) + \rho^2 C_2(y, \mu) + \frac{\sum_{i=3}^7 C_{T_i}(y, \mu) T_i(\mu)}{m_b^4} \right\}, \quad (23)$$

with

$$C_0(y, m_b) = (6y^2 - 4y^3)\theta(1-y) + \mathcal{O}(\alpha_s),$$

$$C_1(y, m_b) = -6y^2\theta(1-y) - 6\delta(y-1) + \mathcal{O}(\alpha_s),$$

$$C_2(y, m_b) = \left[\frac{12\theta(1-y)}{1-y} \right]_+ - \left[\frac{6\theta(1-y)}{(1-y)^2} \right]_{++} - 6\theta(1-y) + 6\delta(y-1) + 3\delta'(y-1) + \mathcal{O}(\alpha_s), \quad (24)$$

and

$$\begin{aligned}
 C_{T_3}(y, m_b) &= -24\delta'(y-1) + 48\delta(y-1) + \mathcal{O}(\alpha_s), \\
 C_{T_4, T_5}(y, m_b) &= -24\delta(y-1) + \mathcal{O}(\alpha_s), \\
 C_{T_6}(y, m_b) &= \mathcal{O}(\alpha_s), \\
 C_{T_7}(y, m_b) &= 24\delta'(y-1) + \mathcal{O}(\alpha_s).
 \end{aligned} \tag{25}$$

Again, the “intrinsic-charm” operators T_{3-7} mix into the two-particle operator $m_c^4 \bar{b} \nu_\mu \gamma^\mu b$ (see Appendix B 2), and consequently, the coefficient $C_2(y, \mu)$ evolves as

$$\begin{aligned}
 C_2(y, m_c) &\simeq C_2(y, m_b) - \frac{1}{8} \ln \frac{\mu^2}{m_b^2} (C_{T_3}(y, m_b) - C_{T_4}(y, m_b) \\
 &\quad - C_{T_5}(y, m_b) - C_{T_7}(y, m_b)).
 \end{aligned} \tag{26}$$

Inserting the leading-order matching conditions, one has

$$\begin{aligned}
 &-\frac{1}{8} (C_{T_3}(y, m_b) - C_{T_4}(y, m_b) - C_{T_5}(y, m_b) - C_{T_7}(y, m_b)) \\
 &= 6\delta'(y-1) - 12\delta(y-1),
 \end{aligned} \tag{27}$$

and one reproduces the logarithmic terms $-12\rho^2 \ln \rho$ in Γ_{part} and $6\rho^2 \ln \rho$ in $\langle 1-y \rangle_{\text{part}}$, respectively, see, (8) and (9).

C. $m_b \gg m_c \sim \Lambda_{\text{QCD}}$

If we consider the dynamics at the charm-quark mass scale to be in the nonperturbative regime, we cannot exploit the condition (19) and are left with the general formula for the leptonic-energy spectrum in (15), which should be evaluated at a scale μ_0 that satisfies $m_c \ll \mu_0 \leq m_b$. Moreover, we have to take seriously the new power counting which implies that terms of order ρ^2 now count as $(\Lambda/m_b)^4$ and should be neglected to the order that we are considering, we are thus left with

$$\begin{aligned}
 \frac{d\Gamma}{dy} \Big|_{\text{partonic}} &= \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \\
 &\quad \times \left\{ C_0(y, \mu_0) + \rho C_1(y, \mu_0) + \mathcal{O}(\rho^2) \right\},
 \end{aligned} \tag{28}$$

$$\begin{aligned}
 \frac{d\Gamma^{(3)}}{dy} \Big|_{\rho_D^3 + \text{IC}} &= \frac{G_F^2 m_b^5}{24\pi^3} |V_{cb}|^2 \\
 &\quad \times \left\{ \frac{C_{\rho_D}(y, \mu_0) \rho_D^3(\mu_0)}{m_b^3} + \frac{C_{T_1}(y, \mu_0) T_1(\mu_0)}{m_b^3} \right\},
 \end{aligned} \tag{29}$$

together with the contributions to the lepton-energy spectrum from μ_π^2 , μ_G^2 , and ρ_{LS}^3 (see, e.g., [12]), where the limit $\rho \rightarrow 0$ to the considered order ($1/m_b^3$) is trivial.

In that order, the genuinely intrinsic-charm contribution comes together with the Darwin term, only. In particular, to leading-logarithmic accuracy (18), the contributions to the total rate, and the moments $\langle y \rangle$ and $\langle y^2 \rangle$ can be obtained as

$$\Gamma^{(3)} \Big|_{\rho_D^3 + \text{IC}} = \frac{G_F^2 m_b^5}{24\pi^3} |V_{cb}|^2 \left\{ X(\mu_0) + \frac{\rho_D^3(\mu_0)}{m_b^3} \left[\frac{17}{12} \right] \right\}, \tag{30}$$

$$\langle y \rangle \Big|_{\rho_D^3 + \text{IC}} = \frac{G_F^2 m_b^5}{24\pi^3} |V_{cb}|^2 \left\{ X(\mu_0) + \frac{\rho_D^3(\mu_0)}{m_b^3} \left[\frac{47}{30} \right] \right\}, \tag{31}$$

$$\langle y^2 \rangle \Big|_{\rho_D^3 + \text{IC}} = \frac{G_F^2 m_b^5}{24\pi^3} |V_{cb}|^2 \left\{ X(\mu_0) + \frac{\rho_D^3(\mu_0)}{m_b^3} \left[\frac{287}{180} \right] \right\}, \tag{32}$$

where we defined the parameter combination

$$X(\mu_0) = -\frac{3T_1(\mu_0)}{m_b^3} + \ln \frac{\mu_0^2}{m_b^2} \frac{\rho_D^3(\mu_0)}{m_b^3}. \tag{33}$$

Considering a sizeable value for $T_1(\mu_0)$ at small hadronic scales (in contrast to the perturbative situation considered in the previous subsection), and taking into account that the ρ_D^3 contribution in $X(\mu_0)$ is formally enhanced by $\ln \mu_0^2/m_b^2$, we may ignore the (small) differences between the individual moments induced by the numbers in square brackets in (30)–(32), to first approximation. Therefore, even in this genuine intrinsic-charm scenario, the inclusion of a large nonperturbative intrinsic-charm effect, basically amounts to treating the Darwin term ρ_D^3 for the effective parameter X . In any case, one may consider the limit $m_c \sim \Lambda_{\text{QCD}}$ rather academic, and would prefer the scenario with semihard charm quarks as in the previous subsection for the discussion of “intrinsic-charm” effects in inclusive semileptonic B decays.

We should also mention that (28) and (29) provide the appropriate formulas for the massless limit, relevant to $b \rightarrow u\ell\nu$ decays, after appropriate changes $V_{cb} \rightarrow V_{ub}$ and reinterpretation of the intrinsic-charm operators as so-called weak annihilation operators [21,22]. Notice that the (local) annihilation operators enter at order $1/m_b^3$ in the standard OPE, whereas their nonlocal counterparts, necessary to describe the shape-function region, already enter at (relative) order Λ/m_b [23–25].

III. CONCLUSION

We have shown how the “intrinsic-charm” contribution in semileptonic B -meson decays is related to the renormalization of subleading operators (like $m_c^4 \bar{b}_\nu b_\nu$ and the Darwin term) appearing in the operator product expansion for the lepton-energy spectrum and the total rate. We have distinguished three different cases which correspond to different power counting for the charm-quark mass. In the first case, one assumes $m_b \sim m_c$, i.e., the charm quark is already integrated out at the hard scale, set by the large b -quark mass in the OPE. Consequently, all dependence on the charm-quark dynamics is already encoded in the matching conditions for the hard coefficient functions, and no “intrinsic-charm” operators should be introduced

below the hard scale. The only remnant of “intrinsic charm” is the nonanalytic dependence of the coefficient functions on the ratio $\rho = m_c^2/m_b^2$.

Another viable scenario treats the charm-quark mass as intermediate between the hard and the soft scale in the OPE, $m_b \gg m_c \gg \Lambda_{\text{QCD}}$. In that case, four-quark operators including soft b -quark fields and semihard charm quarks have to be included in the OPE. At the same time, in order to avoid double counting, the semihard region has to be subtracted from phase-space integrals by a suitable regularization of the decay spectra in the limit $m_c \ll m_b$. We have shown by explicit calculation how the mixing between the “intrinsic-charm” operators and the Darwin term generates the logarithmically enhanced terms entering the OPE at order $1/m_b^3$. Similarly, extending the results of [16], we could reproduce terms of order $\rho^2 \ln \rho$ in the partonic rate. After integrating out the charm quark at the semihard scale, the moments of the lepton-energy spectrum can be entirely described in terms of the standard hadronic input parameters, whereas—again—the complete charm-quark dependence enters via (eventually renormalization-group improved) short-distance coefficients, multiplying, for instance, the Darwin term.

A somewhat more exotic approach would treat the charm quark as light, i.e., of order Λ_{QCD} . Only in this case genuine intrinsic-charm (i.e., nonperturbative) effects have to be taken into account. Still, we have found that on the level of a few lepton-energy moments, the experimental data basically constrains a particular combination of the intrinsic-charm contribution and the Darwin term, such that to order $1/m_b^3$ the number of independent hadronic parameters effectively remains the same.

The main conclusion to be drawn is that, as long as the strong dynamics at the charm-quark scale can be treated perturbatively, “intrinsic-charm” effects do not induce an

additional source of hadronic uncertainties at the level of $1/m_b^3$ power corrections, apart from the usually considered Darwin and spin-orbit terms. The same will be true for higher orders in the $1/m_b$ expansion as classified in [12]. The issue of whether to resum logarithms $\ln(m_c^2/m_b^2)$ by introducing the above two-step matching procedure, or sticking to the standard one-step matching has to be decided by considering radiative corrections to the $1/m_b^3$ expressions which is beyond the scope of this work (see also [10]).

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APPENDIX A: LEPTON-ENERGY SPECTRUM AND MOMENTS

1. Partonic rate

The complete expression for the partonic contribution to the lepton-energy spectrum with $m_b \sim m_c \geq \mu$ is given by [12]

$$\begin{aligned} \frac{d\Gamma}{dy} \Big|_{\text{partonic}} &= \frac{G_F^2 m_b^5}{192 \pi^3} |V_{cb}|^2 \left\{ -\frac{4\rho^3}{(y-1)^3} - \frac{6(\rho^3 + \rho^2)}{(y-1)^2} \right. \\ &\quad \left. - \frac{12\rho^2}{y-1} - 4y^3 - 6(\rho-1)y^2 + 2(\rho-3)\rho^2 \right\} \\ &\quad \times \theta(1-y-\rho). \end{aligned} \quad (\text{A1})$$

From this one can obtain closed expressions for the $(1-y)^n$ moments,

$$\begin{aligned} \langle (1-y)^n \rangle_{\text{partonic}} &= \frac{G_F^2 m_b^5}{192 \pi^3} |V_{cb}|^2 \left\{ -\frac{12(\rho^n - 1)\rho^2}{n} - \frac{4(\rho^n - \rho^2)\rho}{n-2} - \frac{12(\rho^{n+2} - 1)\rho}{n+2} + \frac{6(\rho^2 + \rho)(\rho^n - \rho)}{n-1} \right. \\ &\quad \left. - \frac{2(\rho^3 - 3\rho^2 - 3\rho + 1)(\rho^{n+1} - 1)}{n+1} + \frac{6(\rho + 1)(\rho^{n+3} - 1)}{n+3} - \frac{4(\rho^{n+4} - 1)}{n+4} \right\}. \end{aligned} \quad (\text{A2})$$

Arbitrary moments can be derived via

$$\langle (y-y_0)^n \rangle = \sum_{k=0}^n \binom{n}{k} (1-y_0)^{n-k} (-1)^k \langle (1-y)^k \rangle. \quad (\text{A3})$$

Expanding (A2) in the small parameter ρ , the logarithmically enhanced terms at order ρ^2 appear only in the total rate and the first moment

$$\begin{aligned} \langle (1-y)^n \rangle_{\text{partonic}} &= \frac{G_F^2 m_b^5}{192 \pi^3} |V_{cb}|^2 (12\delta^{n0} - 6\delta^{n1}) \rho^2 \ln \rho \\ &\quad + \text{analytic/higher-order terms in } \rho. \end{aligned} \quad (\text{A4})$$

2. Darwin-term contribution

The full contribution related to the Darwin term in the lepton-energy spectrum for the case $m_b \sim m_c \geq \mu$ is given by [12]

$$\begin{aligned}
 \frac{d\Gamma^{(3)}}{dy} \Big|_{\rho_b^3} &= \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \frac{\rho_D^3}{m_b^3} \left[\left(\frac{40\rho^3}{3(y-1)^6} + \frac{8\rho^2(3\rho+1)}{(y-1)^5} + \frac{6\rho^2(3\rho+1)}{(y-1)^4} + \frac{16\rho(2\rho^2-\rho-1)}{3(y-1)^3} - \frac{28\rho}{3(y-1)^2} + \frac{8}{y-1} \right. \right. \\
 &\quad \left. \left. + \frac{2}{3}(5\rho^3 - 5\rho^2 + 10\rho + 22) + \frac{8}{3}(\rho+3)(y-1) + 4(y-1)^2 + \frac{8}{3}(y-1)^3 \right) \theta(1-y-\rho) \right. \\
 &\quad \left. - \left(\frac{2(\rho-1)^4(\rho+1)^2}{3\rho^2} \right) \delta(1-y-\rho) \right]. \tag{A5}
 \end{aligned}$$

From this one can obtain closed expressions for the moments,

$$\begin{aligned}
 \langle (1-y)^n \rangle_{\rho_b^3} &= \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \frac{\rho_D^3}{m_b^3} \left[\frac{8(\rho^n-1)}{n} - \frac{2}{3}(\rho-1)^4(\rho+1)^2\rho^{n-2} + \frac{28(\rho^n-\rho)}{3(n-1)} \right. \\
 &\quad - \frac{2(5\rho^3-5\rho^2+10\rho+22)(\rho^{n+1}-1)}{3(n+1)} + \frac{8(\rho+3)(\rho^{n+2}-1)}{3(n+2)} - \frac{4(\rho^{n+3}-1)}{n+3} + \frac{8(\rho^{n+4}-1)}{3(n+4)} \\
 &\quad \left. - \frac{16(2\rho^2-\rho-1)(\rho^2-\rho^n)}{3(n-2)\rho} + \frac{6(3\rho+1)(\rho^3-\rho^n)}{(n-3)\rho} - \frac{8(3\rho+1)(\rho^4-\rho^n)}{(n-4)\rho^2} + \frac{40(\rho^5-\rho^n)}{3(n-5)\rho^2} \right]. \tag{A6}
 \end{aligned}$$

Taking the limit $\rho \rightarrow 0$ in (A6), the logarithmically enhanced terms appear only in the total rate

$$\langle (1-y)^n \rangle_{\rho_b^3} = \frac{G_F^2 m_b^5}{24\pi^3} |V_{cb}|^2 \frac{\rho_D^3}{m_b^3} (\delta^{n0} \ln \rho + \mathcal{O}(\rho \ln \rho)). \tag{A7}$$

APPENDIX B: OPERATOR MIXING

1. Dimension-six

In the following we briefly sketch the derivation of the elements of the anomalous-dimension matrix that govern the mixing of the four-quark (“intrinsic-charm”) operators into the Darwin term. For simplicity, we do not construct the complete set of independent operators that would be needed to describe the full one-loop anomalous-dimension matrix, but rather focus on the effect of the charm-loop diagram in Fig. 1(b). For this purpose it is sufficient to consider the two operator structures which enter the hadronic tensor at tree level (10),

$$\begin{aligned}
 2M_B T_1(\mu) &= \frac{(4\pi)^2}{3} (\langle \bar{B}(p) | \bar{b}_v \gamma_\mu P_L c \bar{c} \gamma^\mu P_L b_v | \bar{B}(p) \rangle \\
 &\quad - \langle \bar{B}(p) | \bar{b}_v \not{P}_L c \bar{c} \not{P}_L b_v | \bar{B}(p) \rangle), \\
 2M_B T_2(\mu) &= \frac{(4\pi)^2}{3} (4 \langle \bar{B}(p) | \bar{b}_v v P_L c \bar{c} \not{P}_L b_v | \bar{B}(p) \rangle \\
 &\quad - \langle \bar{B}(p) | \bar{b}_v \gamma_\mu P_L c \bar{c} \gamma^\mu P_L b_v | \bar{B}(p) \rangle). \tag{B1}
 \end{aligned}$$

Together with the Darwin term they are used to define a simplified operator basis

$$\begin{aligned}
 \mathcal{O}_{\rho_D} &= \bar{b}_v (iD_\mu) (i\nu D) (iD^\mu) b_v, \\
 \mathcal{O}_{T_1} &= (4\pi)^2 \mu^{2\epsilon} \frac{1}{3} (\bar{b}_v \gamma_\mu P_L c \bar{c} \gamma^\mu P_L b_v \\
 &\quad - \bar{b}_v \not{P}_L c \bar{c} \not{P}_L b_v), \\
 \mathcal{O}_{T_2} &= (4\pi)^2 \mu^{2\epsilon} \frac{1}{3} (4 \bar{b}_v \not{P}_L c \bar{c} \not{P}_L b_v \\
 &\quad - \bar{b}_v \gamma_\mu P_L c \bar{c} \gamma^\mu P_L b_v). \tag{B2}
 \end{aligned}$$

Notice that for convenience, we have extracted a factor $(4\pi)^2 \mu^{2\epsilon}$, in order to have a simple, dimensionless anomalous-dimension matrix [26].

Calculating the one-loop matrix elements of the operators $\mathcal{O}_{T_{1,2}}$ for the partonic transition $b \rightarrow b$ in the presence of a soft background field $A_\mu(k)$, see Fig. 1(b), and comparing with the tree-level matrix element of the Darwin-term operator, we obtain the following results in $D = 4 - 2\epsilon$ dimensions,

$$\begin{aligned}
 \langle b | \mathcal{O}_{T_1} | b \rangle^{(0)} &= + \frac{1}{3} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m_c^2} \right) \langle b | \mathcal{O}_{\rho_D} | b \rangle_{\text{tree}}, \\
 \langle b | \mathcal{O}_{T_2} | b \rangle^{(0)} &= - \frac{2}{3} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m_c^2} \right) \langle b | \mathcal{O}_{\rho_D} | b \rangle_{\text{tree}}, \tag{B3}
 \end{aligned}$$

where the one-gluon matrix element of the Darwin-term operator on parton level is given by

$$\begin{aligned}
 \langle b | \mathcal{O}_{\rho_D} | b \rangle_{\text{tree}} &= \frac{1}{2} \langle b | \bar{b}_v [iD_\mu, [(i\nu \cdot D), iD^\mu]] b_v | b \rangle_{\text{tree}} \\
 &\quad + \mathcal{O}(1/m_b) \\
 &= \frac{g}{2} ((\nu \cdot k)(k \cdot A) - k^2(\nu \cdot A)) \bar{u}_b u_b + \dots \tag{B4}
 \end{aligned}$$

From (B3) we read off the desired elements of the anomalous-dimension matrix

$$\gamma = \begin{pmatrix} 0 & 0 & 0 \\ -2/3 & 0 & 0 \\ 4/3 & 0 & 0 \end{pmatrix} + \mathcal{O}(\alpha_s), \quad (\text{B5})$$

where the neglected higher-order terms describe the mixing of “intrinsic-charm” operators into themselves and of the Darwin term into itself, which are not explicitly needed for the discussion in the body of the text.

2. Dimension-seven

A similarly simplified analysis can be performed for the mixing of the dimension-seven “intrinsic-charm” operators into the dimension-seven two-quark operator $m_c^4 \bar{b}_v \not{b}_v$. As before, defining

$$\begin{aligned} \mathcal{O}_2 &= m_c^4 \bar{b}_v \not{b}_v, \\ \mathcal{O}_{T_3} &= (4\pi)^2 \mu^{2\epsilon} \frac{1}{3} ((i\nu \cdot \partial \bar{b}_v \gamma_\mu P_L c) (\bar{c} \gamma^\mu P_L b_v) - (i\nu \cdot \partial \bar{b}_v \not{P}_L c) (\bar{c} \not{P}_L b_v)), \\ \mathcal{O}_{T_4} &= (4\pi)^2 \mu^{2\epsilon} \frac{1}{3} ((i\partial_\alpha \bar{b}_v \not{P}_L c) (\bar{c} \gamma^\alpha P_L b_v) - (i\nu \cdot \partial \bar{b}_v \not{P}_L c) (\bar{c} \not{P}_L b_v)), \\ \mathcal{O}_{T_5} &= (4\pi)^2 \mu^{2\epsilon} \frac{1}{3} ((i\partial_\alpha \bar{b}_v \gamma^\alpha P_L c) (\bar{c} \not{P}_L b_v) - (i\nu \cdot \partial \bar{b}_v \not{P}_L c) (\bar{c} \not{P}_L b_v)), \\ \mathcal{O}_{T_6} &= (4\pi)^2 \mu^{2\epsilon} \frac{1}{3} (6(i\nu \cdot \partial \bar{b}_v \not{P}_L c) (\bar{c} \not{P}_L b_v) - (i\nu \cdot \partial \bar{b}_v \gamma_\mu P_L c) (\bar{c} \gamma^\mu P_L b_v) \\ &\quad - (4\pi)^2 \mu^{2\epsilon} \frac{1}{3} ((i\partial_\alpha \bar{b}_v \not{P}_L c) (\bar{c} \gamma^\alpha P_L b_v) + (i\partial_\alpha \bar{b}_v \gamma^\alpha P_L c) (\bar{c} \not{P}_L b_v)), \\ \mathcal{O}_{T_7} &= (4\pi)^2 \mu^{2\epsilon} \frac{1}{6} \epsilon^{\mu\nu\alpha\beta} \nu_\beta (i\partial_\alpha \bar{b}_v \gamma_\nu P_L c) (\bar{c} \gamma_\mu P_L b_v), \end{aligned} \quad (\text{B6})$$

we calculate the contributions to the two-parton matrix elements from the tadpole diagram in Fig. 1(a) as

$$\begin{aligned} \langle b | \mathcal{O}_{T_3} | b \rangle^{(0)} &= +\frac{1}{8} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m_c^2} + \dots \right) \langle b | \mathcal{O}_2 | b \rangle_{\text{tree}}, \\ \langle b | \mathcal{O}_{T_4} | b \rangle^{(0)} &= \langle b | \mathcal{O}_{T_5} | b \rangle^{(0)} = -\frac{1}{8} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m_c^2} + \dots \right) \langle b | \mathcal{O}_2 | b \rangle_{\text{tree}}, \\ \langle b | \mathcal{O}_{T_6} | b \rangle^{(0)} &= 0, \\ \langle b | \mathcal{O}_{T_7} | b \rangle^{(0)} &= -\frac{1}{8} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m_c^2} + \dots \right) \langle b | \mathcal{O}_2 | b \rangle_{\text{tree}}, \end{aligned} \quad (\text{B7})$$

from which we read off the elements of the anomalous-dimension matrix entering (26).

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- [1] J. Chay, H. Georgi, and B. Grinstein, Phys. Lett. B **247**, 399 (1990).
 - [2] I. I. Y. Bigi, M. A. Shifman, N. G. Uraltsev, and A. I. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993); B. Blok, L. Koyrakh, M. A. Shifman, and A. I. Vainshtein, Phys. Rev. D **49**, 3356 (1994); **50**, 3572(E) (1994).
 - [3] A. V. Manohar and M. B. Wise, Phys. Rev. D **49**, 1310 (1994).
 - [4] T. Mannel, Nucl. Phys. **B413**, 396 (1994).
 - [5] D. Benson, I. I. Bigi, T. Mannel, and N. Uraltsev, Nucl. Phys. **B665**, 367 (2003).
 - [6] P. Gambino and N. Uraltsev, Eur. Phys. J. C **34**, 181 (2004).
 - [7] C. W. Bauer, Z. Ligeti, M. Luke, and A. V. Manohar, Phys. Rev. D **67**, 054012 (2003).
 - [8] A. H. Hoang, Z. Ligeti, and A. V. Manohar, Phys. Rev. D **59**, 074017 (1999).
 - [9] K. Melnikov, arXiv:0803.0951.
 - [10] A. Pak and A. Czarnecki, Phys. Rev. Lett. **100**, 241807 (2008).
 - [11] T. Becher, H. Boos, and E. Lunghi, J. High Energy Phys. **12** (2007) 062; H. Boos, Ph.D. thesis, Siegen, 2007.
 - [12] B. M. Dassinger, T. Mannel, and S. Turczyk, J. High Energy Phys. **03** (2007) 087.
 - [13] I. I. Bigi, N. Uraltsev, and R. Zwicky, Eur. Phys. J. C **50**, 539 (2007).
 - [14] S. J. Brodsky and S. Gardner, Phys. Rev. D **65**, 054016 (2002).
 - [15] O. Buchmüller and H. Flücher, Phys. Rev. D **73**, 073008 (2006).

- [16] C. W. Bauer, A. F. Falk, and M. E. Luke, *Phys. Rev. D* **54**, 2097 (1996).
- [17] T. Mannel and M. Neubert, *Phys. Rev. D* **50**, 2037 (1994).
- [18] More precisely, the “intrinsic-charm” operators correspond to local operators for semihard charm quarks, i.e., quarks with all momentum components of order m_c . This is to be distinguished from the hard-collinear (jet) modes for the charm-quark which appear in nonlocal operators describing the shape-function region [19,27] for $b \rightarrow c\ell\nu$.
- [19] H. Boos, T. Feldmann, T. Mannel, and B. D. Pecjak, *Phys. Rev. D* **73**, 036003 (2006); *J. High Energy Phys.* 05 (2006) 056.
- [20] Strictly speaking, these are $N^{-1}\text{LL}$. To resum the leading logarithms of the order $\alpha_s^n \times \ln^n \rho$, we would need the $\mathcal{O}(\alpha_s)$ mixing of the Darwin term into itself and the complete set of “intrinsic-charm” operators into themselves, which goes beyond the scope of this work (see, however, [16] for a complete leading-log analysis for the dimension-seven contributions to the total rate).
- [21] I. I. Y. Bigi and N. G. Uraltsev, *Nucl. Phys.* **B423**, 33 (1994).
- [22] P. Gambino, P. Giordano, G. Ossola, and N. Uraltsev, *J. High Energy Phys.* 10 (2007) 058; P. Gambino and P. Giordano, arXiv:0805.0271.
- [23] K. S. M. Lee and I. W. Stewart, *Nucl. Phys.* **B721**, 325 (2005).
- [24] S. W. Bosch, M. Neubert, and G. Paz, *J. High Energy Phys.* 11 (2004) 073; M. Neubert, *Eur. Phys. J. C* **44**, 205 (2005).
- [25] M. Beneke, F. Campanario, T. Mannel, and B. D. Pecjak, *J. High Energy Phys.* 06 (2005) 071.
- [26] With this convention, the anomalous-dimension matrix is of order $(\alpha_s)^0$. In order to have it in the standard form, one would have to extract a factor $g_s^2 = 4\pi\alpha_s\mu^{2\epsilon}$, instead.
- [27] T. Mannel and F. J. Tackmann, *Phys. Rev. D* **71**, 034017 (2005).