

Okubo-Zweig-Iizuka-rule violation and $B \rightarrow \eta^{(\prime)}K$ branching ratios

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We show that the few-percent Okubo-Zweig-Iizuka-rule violating effects in the quark-flavor basis for the $\eta - \eta'$ mixing can enhance the chiral scale associated with the η_q meson a few times. This enhancement is sufficient for accommodating the dramatically different data of the $B \rightarrow \eta'K$ and $B \rightarrow \eta K$ branching ratios. We comment on other proposals for resolving this problem, including flavor-singlet contributions, axial $U(1)$ anomaly, and nonperturbative charming penguins. Discrimination of the above proposals by means of the $B \rightarrow \eta^{(\prime)}\ell\nu$ and $B_s \rightarrow \eta^{(\prime)}\ell\ell$ data is suggested.

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I. INTRODUCTION

The large $B \rightarrow \eta'K$ and small $B \rightarrow \eta K$ branching ratios measured by the B factories are still not completely understood [1]:

$$\begin{aligned} B(B^\pm \rightarrow \eta'K^\pm) &= (70.2 \pm 2.5) \times 10^{-6}, \\ B(B^0 \rightarrow \eta'K^0) &= (64.9 \pm 3.1) \times 10^{-6}, \\ B(B^\pm \rightarrow \eta K^\pm) &= (2.7 \pm 0.3) \times 10^{-6}, \\ B(B^0 \rightarrow \eta K^0) &< 1.9 \times 10^{-6}. \end{aligned} \quad (1)$$

The predictions for $B(B \rightarrow \eta'K)$ from both the perturbative QCD (PQCD) [2] and QCD-improved factorization (QCDF) [3] approaches in the Feldmann-Kroll-Stech (FKS) scheme [4] for the $\eta - \eta'$ mixing are smaller than the data. Several resolutions to this puzzle have been proposed: a significant flavor-singlet contribution [3], a large $B \rightarrow \eta'$ transition form factor [5], a high chiral scale m_0^q [6] associated with the η_q meson, which is composed of the $u\bar{u}$ and $d\bar{d}$ content in the quark-flavor basis [4], an enhanced hadronic matrix element $\langle 0|\bar{s}\gamma_5 s|\eta' \rangle$ [7] of the strange-quark pseudoscalar density due to axial $U(1)$ anomaly [8], the long-distance charming penguin and gluonic charming penguin [9] in the soft-collinear effective theory (SCET) [10,11], and inelastic final-state interaction [12]. The motivation of [12] is to fix final-state interaction effects using the data in Eq. (1), and then to predict CP asymmetries in the $B \rightarrow \eta^{(\prime)}K$ decays.

A sizable gluonic content in the η' meson was indicated from a phenomenological analysis of the relevant data [13], and also by the recent KLOE measurement [14] (but see an opposite observation in [15]). The flavor-singlet contributions to the $B \rightarrow \eta^{(\prime)}K$ branching ratios, containing those from the $b \rightarrow sgg$ transition [16], from the spectator scattering [17,18], and from the weak annihilation,

have been taken into account in QCDF [3]. However, the gluonic contribution to the $B \rightarrow \eta'$ transition form factor was parameterized and increased arbitrarily up to 40% [3] in order to explain Eq. (1). This piece was also included in the parametrization for two-body nonleptonic B meson decay amplitudes based on SCET, but found to be destructive to the quark contribution from data fitting [9]. To settle this issue, we have examined the gluonic contribution in the PQCD approach [19–22] with the associated parameters being experimentally constrained, and observed that it is constructive and negligible (of a few percent at most) in the $B \rightarrow \eta^{(\prime)}$ transitions [23]. Our conclusion has been confirmed by the sum-rule analysis in [24]. If so, one has to clarify what mechanism is responsible for the increase of the $B \rightarrow \eta'$ form factor postulated in [5].

The chiral scale for the η_q meson is defined by $m_0^q \equiv m_{qq}^2/(2m_q)$ with the light-quark mass $m_q = m_u = m_d$ under the exact isospin symmetry. The mass m_{qq} was increased from its generally accepted value 0.11 GeV, close to the pion mass, to 0.22 GeV in [6]. This enhancement then gives a larger $B \rightarrow \eta_q K$ decay amplitude, a more destructive (constructive) interference with the $B \rightarrow \eta_s K$ amplitude [25], where the η_s meson is composed of the $s\bar{s}$ content in the quark-flavor basis, and thus a smaller $B \rightarrow \eta K$ (larger $B \rightarrow \eta'K$) branching ratio. It has been found that the PQCD results for the $B \rightarrow \eta^{(\prime)}K$ branching ratios corresponding to $m_{qq} = 0.22$ GeV agree with the data [6]. Note that the PQCD results for the $B \rightarrow \eta^{(\prime)}K^*$ branching ratios are also consistent with the data, which show a tendency opposite to Eq. (1): $B(B^\pm \rightarrow \eta'K^*)$ are smaller than $B(B^\pm \rightarrow \eta K^*)$ by about a factor 4 [1]. Whether there is any mechanism to achieve the enhancement of m_{qq} is not clear. We shall argue that a tiny effect violating the Okubo-Zweig-Iizuka (OZI) rule [26], which was neglected in the FKS scheme, could be the responsible mechanism.

The OZI-rule violation has been studied in, for example, exclusive $\eta^{(\prime)}$ productions from πN and NN scattering

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in a wide range of energy scales [27]. Most of the observed ratios of the cross sections $\sigma(\pi N, NN \rightarrow \eta X)/\sigma(\pi N, NN \rightarrow \eta' X)$ are in agreement with or slightly larger than the expectation around 1.5 from the FKS scheme, considering experimental uncertainties. The exceptions with significant OZI-rule violation appear in the $\eta^{(\prime)}$ productions at thresholds: the ratio $\sigma(pp \rightarrow pp\eta)/\sigma(pp \rightarrow pp\eta')$ was measured to be 37.0 ± 11.3 and 26.2 ± 5.4 with the proton energy being 2.9 MeV and 4.1 MeV, respectively [27,28]. The above tendency hints the possibility of a small OZI-rule violation in B and D meson decays into light final states, whose energy release is of order few GeV. The proposal in [7] relies on the large matrix element $\langle 0|\bar{s}\gamma_5 s|\eta'\rangle$, which strengthens its difference from $\langle 0|\bar{s}\gamma_5 s|\eta\rangle$, and the difference between the $B \rightarrow \eta'K$ and $B \rightarrow \eta K$ branching ratios through penguin contributions. It will be pointed out that this proposal demands larger OZI-rule violation, which is not obviously signaled in the $D_s \rightarrow \eta^{(\prime)}\ell\nu$ data.

In Sec. II, we show that few-percent OZI violating effects enhance the mass m_{qq} sufficiently, which accommodates the data of the $B \rightarrow \eta^{(\prime)}K$ branching ratios in the PQCD approach. In Sec. III, we make a critical review on other proposals for this subject from both theoretical and experimental points of view. Section IV contains a summary, in which experimental discrimination for all the proposed mechanisms is suggested.

II. OZI-RULE VIOLATION

We consider the following OZI-rule violating matrix elements in the quark-flavor basis

$$\begin{aligned}\langle 0|\bar{q}\gamma^\mu\gamma_5 q|\eta_s(P)\rangle &= \frac{i}{\sqrt{2}}f_{qs}P^\mu, \\ \langle 0|\bar{s}\gamma^\mu\gamma_5 s|\eta_q(P)\rangle &= if_{sq}P^\mu\end{aligned}\quad (2)$$

$$M^2 = \begin{pmatrix} m_\eta^2 & 0 \\ 0 & m_{\eta'}^2 \end{pmatrix}, \quad M_{qs}^2 = \begin{pmatrix} m_{qq}^2 + (\sqrt{2}/f_{qq})\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle & (1/f_{ss})\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle \\ (\sqrt{2}/f_{qq})\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle & m_{ss}^2 + (1/f_{ss})\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle \end{pmatrix}, \quad (8)$$

with the abbreviations

$$\begin{aligned}m_{qq}^2 &= \frac{\sqrt{2}}{f_{qq}}\langle 0|m_u\bar{u}i\gamma_5 u + m_d\bar{d}i\gamma_5 d|\eta_q\rangle, \\ m_{ss}^2 &= \frac{2}{f_{ss}}\langle 0|m_s\bar{s}i\gamma_5 s|\eta_s\rangle.\end{aligned}\quad (9)$$

Note that the matrix M_{qs}^2 becomes non-Hermitian, after including the OZI violating effects, or employing the two-angle mixing formalism [see Eq. (15) below]. In fact, this matrix is Hermitian only in the FKS scheme.

for the light quark $q = u$ or d , where the decay constants f_{qs} and f_{sq} are expected to be small and have been neglected in the FKS scheme. We also define the decay constants for the $\eta_{q,s}$ mesons and for the $\eta^{(\prime)}$ mesons

$$\begin{aligned}\langle 0|\bar{q}\gamma^\mu\gamma_5 q|\eta_q(P)\rangle &= \frac{i}{\sqrt{2}}f_{qq}P^\mu, \\ \langle 0|\bar{s}\gamma^\mu\gamma_5 s|\eta_s(P)\rangle &= if_{ss}P^\mu, \\ \langle 0|\bar{q}\gamma^\mu\gamma_5 q|\eta^{(\prime)}(P)\rangle &= \frac{i}{\sqrt{2}}f_{\eta^{(\prime)}}^q P^\mu, \\ \langle 0|\bar{s}\gamma^\mu\gamma_5 s|\eta^{(\prime)}(P)\rangle &= if_{\eta^{(\prime)}}^s P^\mu.\end{aligned}\quad (3)$$

The physical states η and η' are related to the flavor states η_q and η_s through

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = U(\phi)\begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \end{pmatrix}, \quad (4)$$

with the unitary matrix

$$U(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}. \quad (5)$$

The above decay constants are transformed into each other via

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = U(\phi)\begin{pmatrix} f_{qq} & f_{sq} \\ f_{qs} & f_{ss} \end{pmatrix}. \quad (6)$$

We repeat the derivation of Eq. (7) in [23], obtaining

$$M_{qs}^2 = U^\dagger(\phi)M^2U(\phi)\begin{pmatrix} 1 & Y_{sq} \\ Y_{qs} & 1 \end{pmatrix}, \quad (7)$$

where the OZI violating parameters are defined by $Y_{qs} \equiv f_{qs}/f_{qq}$ and $Y_{sq} \equiv f_{sq}/f_{ss}$, and the mass matrices are written as

Equation (7) determines the four elements in M_{qs}^2

$$\begin{aligned}m_{qq}^2 &= m_{qq}^{(0)2} + \left[Y_{qs}(m_{\eta'}^2 - m_\eta^2)\cos\phi\sin\phi \right. \\ &\quad \left. - \frac{\sqrt{2}f_{ss}}{f_{qq}}Y_{sq}(m_\eta^2\cos^2\phi + m_{\eta'}^2\sin^2\phi) \right], \\ m_{ss}^2 &= m_{ss}^{(0)2} + \left[Y_{sq}(m_{\eta'}^2 - m_\eta^2)\cos\phi\sin\phi \right. \\ &\quad \left. - \frac{f_{qq}}{\sqrt{2}f_{ss}}Y_{qs}(m_\eta^2\cos^2\phi + m_{\eta'}^2\sin^2\phi) \right],\end{aligned}\quad (10)$$

with the original solutions [4]

$$m_{qq}^{(0)2} = m_{\eta'}^2 \cos^2 \phi + m_{\eta}^2 \sin^2 \phi - \frac{\sqrt{2}f_{ss}}{f_{qq}}(m_{\eta'}^2 - m_{\eta}^2) \times \cos \phi \sin \phi, \quad (11)$$

$$m_{ss}^{(0)2} = m_{\eta'}^2 \cos^2 \phi + m_{\eta}^2 \sin^2 \phi - \frac{f_{qq}}{\sqrt{2}f_{ss}}(m_{\eta'}^2 - m_{\eta}^2) \times \cos \phi \sin \phi. \quad (12)$$

Substituting the parameters extracted in [4]

$$f_q = (1.07 \pm 0.02)f_{\pi}, \quad f_s = (1.34 \pm 0.06)f_{\pi}, \quad (13)$$

$$\phi = 39.3^\circ \pm 1.0^\circ,$$

for f_{qq} , f_{ss} , and ϕ in Eq. (11), respectively, and adopting the masses $m_{\eta} = 0.548$ GeV and $m_{\eta'} = 0.958$ GeV, we derive

$$m_{qq}^{(0)} \approx 0.11 \text{ GeV}, \quad m_{ss}^{(0)} \approx 0.71 \text{ GeV}. \quad (14)$$

The smallness of $m_{qq}^{(0)}$ is attributed to the strong cancellation between the two terms on the right-hand side of Eq. (11), where the second term is associated with the axial $U(1)$ anomaly. It is then expected that $m_{qq}^{(0)}$ is easily affected by the OZI violating contribution, while the larger $m_{ss}^{(0)}$ is stable. If stretching the uncertainties of f_q , f_s , and ϕ in Eq. (13), $m_{qq}^{(0)}$ could reach 0.22 GeV without the OZI violating effect. Nevertheless, the ranges of these parameters depend on data included for fit (different sets of data lead to different ranges), and on theoretical modelling of considered processes [29]. Here, we suggest a plausible mechanism, which easily modifies m_{qq} without stretching uncertainties.

The order of magnitude of $f_{qs,sq}$ can be estimated via the two-angle mixing formalism [30,31]

$$\begin{pmatrix} f_{\eta}^q & f_{\eta}^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = U_{qs} \begin{pmatrix} f_q & 0 \\ 0 & f_s \end{pmatrix}, \quad (15)$$

with the matrix

$$U_{qs} \equiv \begin{pmatrix} \cos \phi_q & -\sin \phi_s \\ \sin \phi_q & \cos \phi_s \end{pmatrix}. \quad (16)$$

If $\phi_q = \phi_s$, the above formalism reduces to the FKS scheme; that is, the OZI violating matrix elements give rise to the difference between ϕ_q and ϕ_s , or to the energy dependence of the mixing angle introduced in [32]. We insert a typical set of parameters [32]

$$f_q = 1.10f_{\pi}, \quad f_s = 1.46f_{\pi}, \quad (17)$$

$$\phi_q = 38.9^\circ, \quad \phi_s = 41.0^\circ$$

into Eq. (15), compute the left-hand side of Eq. (15), and then invert Eq. (6) to obtain the ϕ dependence of the OZI violating parameters $Y_{qs,sq}$. The ϕ dependences of $Y_{qs,sq}$ and of $m_{qq,ss}$ from Eq. (10) in a reasonable range of ϕ , roughly from 33° to 42° [33–36], are displayed in Fig. 1. It

indicates that the tiny $Y_{qs} = 0.036$ and $Y_{sq} = -0.073$ (for $\phi = 36.84^\circ$) reproduce the inputs in [6]

$$m_{qq} = 0.22 \text{ GeV}, \quad m_{ss} = 0.71 \text{ GeV}, \quad (18)$$

namely, give a factor-2 enhancement of m_{qq} and almost no impact on m_{ss} .

An updated fitting leads to similar results but with a higher $f_s \approx 1.66f_{\pi}$ compared with that in Eq. (17), which is mainly attributed to the change of the $\phi \rightarrow \eta'\gamma$ data [37]. In this case, m_{qq} reaches about 0.2 GeV for a lower value of $\phi \approx 34^\circ$. In general, we should introduce the additional OZI violating matrix elements into Eq. (8),

$$m_{qs}^2 = \frac{\sqrt{2}}{f_{qq}} \langle 0 | m_u \bar{u} i \gamma_5 u + m_d \bar{d} i \gamma_5 d | \eta_s \rangle, \quad (19)$$

$$m_{sq}^2 = \frac{2}{f_{ss}} \langle 0 | m_s \bar{s} i \gamma_5 s | \eta_q \rangle,$$

whose inclusion, however, modifies m_{qq} and m_{ss} only slightly as shown later. Besides, the isospin breaking effect from mixing with pions is also negligible. This effect, found to be of few percents [38], appears quadratically in the expressions of m_{qq}^2 , namely, at the 10^{-4} level.

Corresponding to Eq. (18), the $B \rightarrow \eta^{(\prime)}K$ branching ratios were found to be

$$B(B^\pm \rightarrow \eta'K^\pm) = 65.04(34.60) \times 10^{-6},$$

$$B(B^0 \rightarrow \eta'K^0) = 62.69(31.44) \times 10^{-6}, \quad (20)$$

$$B(B^\pm \rightarrow \eta K^\pm) = 1.52(5.66) \times 10^{-6},$$

$$B(B^0 \rightarrow \eta K^0) = 1.43(3.01) \times 10^{-6},$$

in the PQCD approach [6], where the results for $m_{qq} = 0.14$ GeV are quoted in the parentheses for comparison. Obviously, the agreement with the data in Eq. (1) has been greatly improved. Note that our point is not to claim the existence of the OZI violating effects in the $B \rightarrow \eta^{(\prime)}K$ decays, but that just few percents of such effects, which are very likely viewing the data of other $\eta^{(\prime)}$ involved processes [27], are sufficient for resolving the puzzle.

The consistency of the PQCD results [6] with the data of the $B \rightarrow \eta^{(\prime)}K^*$ branching ratios is also improved by increasing m_{qq} . The data [1]

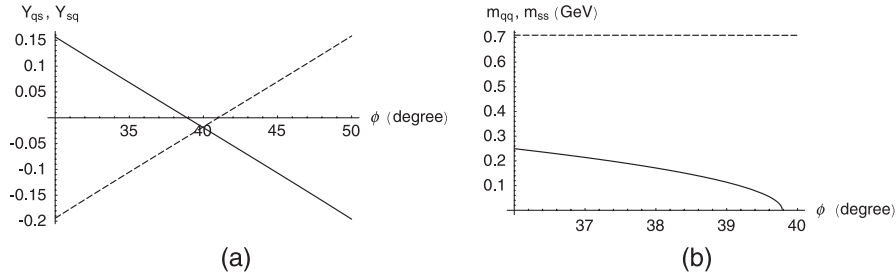
$$B(B^\pm \rightarrow \eta'K^{*\pm}) = (4.9_{-1.9}^{+2.1}) \times 10^{-6},$$

$$B(B^0 \rightarrow \eta'K^{*0}) = (3.8 \pm 1.2) \times 10^{-6}, \quad (21)$$

$$B(B^\pm \rightarrow \eta K^{*\pm}) = (19.3 \pm 1.6) \times 10^{-6},$$

$$B(B^0 \rightarrow \eta K^{*0}) = (15.9 \pm 1.0) \times 10^{-6}$$

exhibit a tendency opposite to that of the $B \rightarrow \eta^{(\prime)}K$ branching ratios in Eq. (1), which is attributed to the sign flip of the $(V-A)(V+A)$ penguin contribution in the $B \rightarrow \eta_s K^*$ decays (involving the $B \rightarrow K^*$ transition form factor) [3,39], i.e., to an opposite interference pattern between the $B \rightarrow \eta_q K^*$ and $B \rightarrow \eta_s K^*$ amplitudes.


 FIG. 1. Dependences of (a) Y_{qs} (solid line) and Y_{sq} (dashed line) and of (b) m_{qq} (solid line) and m_{ss} (dashed line) on ϕ .

Similarly, it is difficult to accommodate the factor-4 difference between the measured $B \rightarrow \eta' K^*$ and $B \rightarrow \eta K^*$ branching ratios in Eq. (21) in the FKS scheme. An additional mechanism, such as a significant flavor-singlet contribution [3] or a $B \rightarrow \eta_q K^*$ decay amplitude enhanced by a large m_{qq} [6], is required. We mention that the absolute $B \rightarrow \eta^{(\prime)} K^*$ branching ratios predicted in [3] in the default

scenario are smaller than the data in Eq. (21), which is a general trend of the QCDF approach to $B \rightarrow PV$ decays [40], with P (V) denoting a pseudoscalar (vector) meson.

The introduction of the OZI violating decay constants $f_{qs,sq}$ implies the additional twist-2 $\eta_{q,s}$ meson distribution amplitudes

$$\begin{aligned} \langle \eta_s(P) | \bar{q}_\gamma^a(z) q_\beta^b(0) | 0 \rangle &= -\frac{i}{2\sqrt{N_c}} \delta^{ab} \int_0^1 dx e^{ixP \cdot z} [\gamma_5 \not{P}]_{\beta\gamma} \phi_{qs}^A(x), \\ \langle \eta_q(P) | \bar{s}_\gamma^a(z) s_\beta^b(0) | 0 \rangle &= -\frac{i}{\sqrt{2N_c}} \delta^{ab} \int_0^1 dx e^{ixP \cdot z} [\gamma_5 \not{P}]_{\beta\gamma} \phi_{sq}^A(x). \end{aligned} \quad (22)$$

We show that these distribution amplitudes need not to be considered by taking the semileptonic decays $B \rightarrow \eta^{(\prime)} \ell \nu$ as an example. ϕ_{sq}^A is irrelevant at the current level of accuracy, since it contributes at next-to-leading order in α_s : it is involved in the diagram, where the light-quark pair from the B meson transition converts into a pair of valence strange quarks in the η_q meson through two-gluon exchanges. Therefore, we only examine the contribution from ϕ_{qs}^A to the $B \rightarrow \eta^{(\prime)}$ transition form factors $F_{+,0,T}$ defined via the matrix elements

$$\begin{aligned} \langle \eta^{(\prime)}(P_2) | \bar{b} \gamma_\mu u | B(P_1) \rangle &= F_{+,0,T}^{B\eta^{(\prime)}}(q^2) \left[(P_1 + P_2)_\mu - \frac{m_B^2 - m_{\eta^{(\prime)}}^2}{q^2} q_\mu \right] + F_0^{B\eta^{(\prime)}}(q^2) \frac{m_B^2 - m_{\eta^{(\prime)}}^2}{q^2} q_\mu, \\ \langle \eta^{(\prime)}(P_2) | \bar{b} i \sigma^{\mu\nu} q_\nu u | B(P_1) \rangle &= \frac{F_T^{B\eta^{(\prime)}}(q^2)}{m_B + m_{\eta^{(\prime)}}} [(m_B^2 - m_{\eta^{(\prime)}}^2) q^\mu - q^2 (P_1^\mu + P_2^\mu)], \end{aligned} \quad (23)$$

with the B meson momentum P_1 , the $\eta^{(\prime)}$ meson momentum P_2 , and the lepton-pair momentum $q = P_1 - P_2$.

The corresponding PQCD factorization formulas are referred to [23], and the Gegenbauer moments for the models of the η_q meson distribution amplitudes are the same as in [6,41]. We also compute the gluonic contribution for comparison [42]. Assuming the asymptotic form

$\phi_{qs}^A(x) = 3f_{qs}x(1-x)/\sqrt{6}$, the numerical results of the form factors $F_{+,0,T}^{B\eta}$ and $F_{+,0,T}^{B\eta'}$ with the parameters $f_{qs} = 5.14$ MeV, $\phi = 36.84^\circ$, and $m_{qq} = 0.22$ GeV selected from Fig. 1 are listed in Table I. The form factor values at zero recoil are larger than those in [23] due to the enhancement of m_{qq} . Consequently, the percentages of the gluonic contribution are lower here. It is found that the contribution from Eq. (22) is, like the gluonic one, unimportant. Hence, we can simply concentrate on the effect of the modified m_{qq} when studying the $B \rightarrow \eta^{(\prime)} K^{(*)}$ decays.

 TABLE I. Contributions to the $B \rightarrow \eta^{(\prime)}$ form factors at maximal recoil from the distribution amplitudes in Eq. (22) and from the gluonic content for $f_{qs} = 5.14$ MeV, $\phi = 36.84^\circ$, and $m_{qq} = 0.22$ GeV.

Form factors	$F_{+,0}^{B\eta}$	$F_T^{B\eta}$	$F_{+,0}^{B\eta'}$	$F_T^{B\eta'}$
$F(0)$	0.308	0.298	0.235	0.227
ϕ_{qs}^A contribution (%)	-0.386	-0.330	0.673	0.577
gluonic contribution (%)	0.196	0.169	1.24	1.07

III. CRITICAL REVIEW

As mentioned above, a large m_{qq} increases the $B \rightarrow \eta^{(\prime)}$ form factors and the $B \rightarrow \eta^{(\prime)} \ell \nu$ branching ratios. Based on the form factor values at maximal recoil in Table I and

the parametrization for the dependence on the lepton-pair invariant mass in [43], the branching ratios can be obtained. It has been verified that the predictions in PQCD [6]

$$\begin{aligned} B(B^+ \rightarrow \eta \ell^+ \nu) &= 1.27 \times 10^{-4}, \\ B(B^+ \rightarrow \eta' \ell^+ \nu) &= 0.62 \times 10^{-4}, \end{aligned} \quad (24)$$

obey the experimental bounds [44]

$$\begin{aligned} B(B^+ \rightarrow \eta \ell^+ \nu) &= (0.84 \pm 0.27 \pm 0.21) \\ &\times 10^{-4} < 1.4 \times 10^{-4} (90\% \text{ C.L.}), \\ B(B^+ \rightarrow \eta' \ell^+ \nu) &= (0.33 \pm 0.60 \pm 0.30) \\ &\times 10^{-4} < 1.3 \times 10^{-4} (90\% \text{ C.L.}). \end{aligned} \quad (25)$$

This check should apply to other proposals resorting to the enhancement of the $B \rightarrow \eta^{(\prime)}$ form factors, such as the inclusion of the flavor-singlet contribution.

Without the flavor-singlet contribution, one should have the ratio of the $B \rightarrow \eta^{(\prime)} \ell \nu$ branching ratios

$$R_{\ell\nu} \equiv \frac{B(B \rightarrow \eta' \ell \nu)}{B(B \rightarrow \eta \ell \nu)} \approx \tan^2 \phi, \quad (26)$$

which is less than unity in the FKS scheme. The PQCD results in Eq. (24) agree with this expectation. However, the recent CLEO measurement with $R_{\ell\nu} > 2.5$ [45] may indicate a significant flavor-singlet contribution in the $B \rightarrow \eta^{(\prime)}$ transitions. A simple estimate shows that the gluonic contribution must reach at least half of the quark one in order to satisfy CLEO's bound, in conflict with the implication from other data [4,46–48]. Furthermore, the ratio of the observed $D_s \rightarrow \eta^{(\prime)} \ell \nu$ branching ratios [49,50],

$$\frac{B(D_s \rightarrow \eta' \ell \nu)}{B(D_s \rightarrow \eta \ell \nu)} = 0.35 \pm 0.09 \pm 0.07, \quad (27)$$

does not reveal the same signal: under a monopole parametrization, it corresponds to the ratio of the $D_s \rightarrow \eta^{(\prime)}$ form factors at the maximal recoil [51]

$$\frac{F_+^{D_s \eta'}(0)}{F_+^{D_s \eta}(0)} = 1.14 \pm 0.17 \pm 0.13 \quad (28)$$

in agreement with the expectation from the FKS scheme.

The gluonic contribution to the $B \rightarrow \eta^{(\prime)}$ transitions also plays an essential role in the proposal of [9]. It is destructive to the quark contribution from the data fitting based on SCET, so that the $B \rightarrow \eta^{(\prime)}$ form factors have small values of $O(10^{-2})$. The $B \rightarrow \eta^{(\prime)} K$ branching ratios then receive contributions mainly from the nonperturbative charming penguin and gluonic charming penguin amplitudes. Especially, the gluonic charming penguin is responsible for the dominance of the $B \rightarrow \eta' K$ branching ratios over the $B \rightarrow \pi K$ ones. With the potentially sizable gluonic contribution, the ratio $R_{\ell\nu}$ in Eq. (26) could deviate from

$\tan^2 \phi$. However, due to the huge uncertainty of this contribution, no definite prediction for $R_{\ell\nu}$ can be made. Nevertheless, it is still possible to test the mechanism in [9] by measuring the semileptonic decays: the smallness of the $B \rightarrow \eta^{(\prime)}$ form factors leads to the small $B \rightarrow \eta^{(\prime)} \ell \nu$ branching ratios of $O(10^{-5})$, compared with $O(10^{-4})$ from the PQCD [6] and QCDF [3] approaches. Taking into account the uncertainty of solutions I and II in [9] to 1σ , we estimate, using the parametrization for form factors in [43], the rough upper bounds

$$\begin{aligned} B(B^+ \rightarrow \eta \ell^+ \nu) &< 5 \times 10^{-5}, \\ B(B^+ \rightarrow \eta' \ell^+ \nu) &< 3 \times 10^{-5}. \end{aligned} \quad (29)$$

The proposal in [7] resorts to the ratio of the matrix elements

$$\left| \frac{\langle 0 | \bar{s} \gamma_5 s | \eta' \rangle}{\langle 0 | \bar{s} \gamma_5 s | \eta \rangle} \right| \approx 2.1 \quad (30)$$

greater than $\cot \phi \approx 1.2$ in the FKS scheme. The matrix elements of the pseudoscalar density define the chiral mass scales, to which the two-parton twist-3 contributions are proportional. Therefore, the above ratio would affect Eq. (28) through these contributions in the theoretical frameworks based on the heavy-quark expansion and factorization theorems such as PQCD and SCET. However, the $D_s \rightarrow \eta^{(\prime)} \ell \nu$ data do not indicate a deviation from the FKS scheme. A more convincing discrimination can be achieved by measuring the $B_s \rightarrow \eta^{(\prime)} \ell^+ \ell^-$ decays, for which the heavy-quark expansion works better. If the mechanism in [7] is valid, a significant deviation from

$$R_{\ell\ell} \equiv \frac{B(B_s \rightarrow \eta' \ell \ell)}{B(B_s \rightarrow \eta \ell \ell)} \approx \cot^2 \phi \quad (31)$$

will be observed. According to [52], the twist-2 and twist-3 contributions are roughly equal in the B_s meson transition form factors. It is then likely that Eq. (30) doubles the ratio $R_{\ell\ell}$, leading to $R_{\ell\ell} \approx 3$.

On the other hand, the results in [7] can be examined from the viewpoint of the OZI-rule violation. The four matrix elements on the right-hand side of the following transformation have been derived in [7,53]

$$\begin{aligned} &\begin{pmatrix} \langle 0 | \bar{q} \gamma_5 q | \eta_q \rangle & \langle 0 | \bar{s} \gamma_5 s | \eta_q \rangle \\ \langle 0 | \bar{q} \gamma_5 q | \eta_s \rangle & \langle 0 | \bar{s} \gamma_5 s | \eta_s \rangle \end{pmatrix} \\ &= U^\dagger(\phi) \begin{pmatrix} \langle 0 | \bar{q} \gamma_5 q | \eta \rangle & \langle 0 | \bar{s} \gamma_5 s | \eta \rangle \\ \langle 0 | \bar{q} \gamma_5 q | \eta' \rangle & \langle 0 | \bar{s} \gamma_5 s | \eta' \rangle \end{pmatrix}. \end{aligned} \quad (32)$$

The matrix elements on the left-hand side of Eq. (32) define the OZI violating quantities

$$\begin{aligned} Z_{qs} &\equiv \frac{\langle 0 | \bar{q} \gamma_5 q | \eta_s \rangle}{\langle 0 | \bar{q} \gamma_5 q | \eta_q \rangle} = \frac{m_{qs}^2}{m_{qq}^2}, \\ Z_{sq} &\equiv \frac{\langle 0 | \bar{s} \gamma_5 s | \eta_q \rangle}{\langle 0 | \bar{s} \gamma_5 s | \eta_s \rangle} = \frac{m_{sq}^2}{m_{ss}^2}, \end{aligned} \quad (33)$$

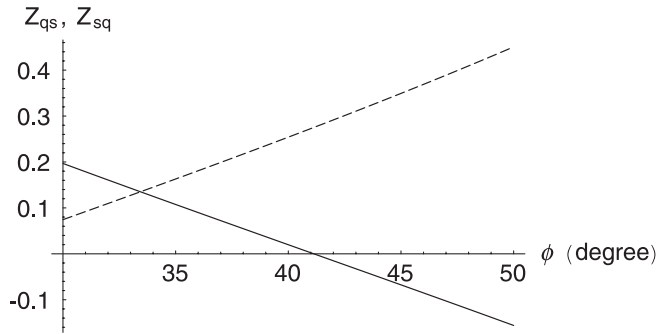


FIG. 2. Dependence of Z_{qs} (solid line) and Z_{sq} (dashed line) on ϕ in units of degrees.

which are related to the mass ratios via Eqs. (9) and (19). Figure 2 shows that either Z_{qs} or Z_{sq} remains sizable no matter how ϕ is varied in the range $30^\circ < \phi < 50^\circ$: for $\phi \approx 39.3^\circ$ [4] (32.7° adopted in [7]), we have $Z_{qs} \approx 3\%$ (15%) and $Z_{sq} \approx 24\%$ (12%). That is, the proposal in [7] demands more significant OZI-rule violation, compared with the few-percent violation in the decay constants considered in Sec. II. It is now clear that the mass m_{qs} makes a smaller impact on m_{qq} than $f_{qs,sq}$ do: few-percent Z_{qs} changes m_{qq} by only few percents following the formalism in Eqs. (7)–(19), while few-percent $Y_{qs,sq}$ increase m_{qq} by a factor 2. The neglect of $m_{qs,sq}$ in Eq. (10) is then justified.

IV. SUMMARY

In this work, we have surveyed various proposals for accommodating the dramatically different data of the $B \rightarrow \eta' K$ and $B \rightarrow \eta K$ branching ratios in Eq. (1). The flavor-

singlet contribution [3] seems to be insufficient for stretching the difference under the experimental constraints from other $\eta^{(\prime)}$ meson involved processes. If this contribution was the responsible mechanism, both the ratios $R_{\ell\nu}$ and $R_{\ell\ell}$ defined by Eqs. (26) and (31), respectively, would deviate from the FKS expectations by about a factor 2. Hence, it is crucial to settle down the discrepancy between the current *BABAR* [44] and *CLEO* [45] measurements of the $B \rightarrow \eta^{(\prime)} \ell \nu$ decays. The dominance of the charming penguin and gluonic charming penguin [9] implies the small $B \rightarrow \eta^{(\prime)}$ form factors and the $B \rightarrow \eta^{(\prime)} \ell \nu$ branching ratios of $O(10^{-5})$, which can be confronted with future data. The very different matrix elements $\langle 0 | \bar{s} \gamma_5 s | \eta \rangle$ and $\langle 0 | \bar{s} \gamma_5 s | \eta' \rangle$ caused by the axial $U(1)$ anomaly [7] demand larger OZI-rule violation, and render $R_{\ell\ell}$ to become twice that of $\cot^2 \phi$. The enhancement of the chiral scale associated with the η_q meson [6] requires only few-percent OZI-rule violation, and both Eqs. (26) and (31) hold. In summary, precise data of the $B \rightarrow \eta^{(\prime)} \ell \nu$ and $B_s \rightarrow \eta^{(\prime)} \ell^+ \ell^-$ decays will help discriminating the above proposals.

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