$B(B_s) \to D_{(s)}P$, $D_{(s)}V$, $D^*_{(s)}P$, and $D^*_{(s)}V$ decays in the perturbative QCD approach

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Two-body charmed decays $B_{(s)} \to D_{(s)}P$, $D^*_{(s)}P$, $D_{(s)}V$, and $D^*_{(s)}V$, where P and V denote the light pseudoscalar meson and vector meson, respectively, are analyzed in the perturbative QCD (pQCD) approach. Using the experimental data of six $B \to DP$ channels, we test the D meson wave function by χ^2 fit. We give the branching ratios of all the charmed B decay channels, most of which agree with experiments amazingly well. The predicted B_s decays can be confronted with the future experimental data. By straightforward calculations, our pQCD approach gives the right relative strong phase of a_2/a_1 that agrees with experiments. We also predict the percentage of transverse polarizations in $B_{(s)} \to D^*V$ decay channels.

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I. INTRODUCTION

B physics experiments not only provide a good test of the standard model but also put some constraints on new physics parameters. Recently, a lot of efforts have been made to the study of B meson decays both experimentally and theoretically. On the experimental side, enough data on B physics is expected from the future B factories, Tevatron and LHCb, whereas theoretically, a great improvement has been made to the study of exclusive decays of B mesons. Though the naive factorization [1] proves itself to be a successful method to explain branching ratios of various B decays [2], it failed to explain color-suppressed processes such as $\bar{B}^0 \to D^0\pi^0$ [3]. Currently, the perturbative QCD (pQCD) factorization approach [4] is one of the popular methods to deal with the two-body nonleptonic decays of B mesons. It explains the experimental data successfully, especially for the direct CP asymmetries [5] when the final states are two light mesons. This motivates the people to look for its validity when one of the final state mesons is heavy.

Comparing the decays of B (B_s) mesons to the light vector mesons, the charmed decays of B (B_s) are more complicated because of the hierarchy of the scale involved. For example, $B \to D$ transitions involve three scales: M_B, M_D, $\bar{\Lambda}$, which are strikingly different from each other. The factorization was proved in soft-collinear effective theory [6] but it is less predictive compared to the pQCD approach, since it needs more input than the pQCD approach. In $B \to$ light transition, the light spectator quark in the B meson is soft, while it is collinear in the final state meson, so that a hard gluon is needed to connect it to the four quark operator shown in Fig. 1. In $B \to D$ transitions the momentum square of the hard gluon connecting the spectator quark is only a factor of $(1 - m_D^2/m_B^2)$ to that of the $B \to$ light transitions, which ensures that pQCD should also work well in $B \to D$ transitions. In the pQCD approach, the hierarchy of the mass scale, $M_B \gg M_D \gg \bar{\Lambda}$, has already been incorporated [7]. Some separate calculations on $B \to D$ decays in the pQCD approach are carried out in Refs. [8,9] in the leading order of M_D/M_B and Λ/M_B expansions. It is found that the pQCD work well since the D meson recoils fast.

In this paper, we calculate all the processes of $B_{(s)}$ meson decays to a $D^{(*)}_{(s)}$ meson and a light pseudoscalar meson or vector meson. Only tree diagrams contribute to these processes which involve only one kind of Cabibbo-Kobayashi-Maskawa (CKM) matrix element which shows that there is no direct CP asymmetry in these decays. The light-cone distribution amplitudes of mesons are necessary inputs in the pQCD framework. Extensive studies have already been made on the calculation of the light meson's distribution amplitudes (DAs) using QCD sum rules [10]. Contrary to this, there are few studies made on the DAs of heavy mesons and especially for the D meson. In this paper we collect several distribution amplitudes of the D meson and then by fitting different parameters using the experimental results we will make a comparison among them.

The paper is organized as follows: Section II contains the conventions and notations that we adopt, together with all the wave functions used in this paper. The pQCD analytic formulas for the amplitudes are given in Sec. III. Section IV contains the numerical results and discussions. Finally, Sec. V summarizes the main outcome of this work.

II. ANALYTIC FORMULAS

For the charmed B decays under consideration, only the tree operators of the standard effective weak Hamiltonian contribute, which can be written as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V^*_{uq'} [C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)]. \quad (1)$$

with

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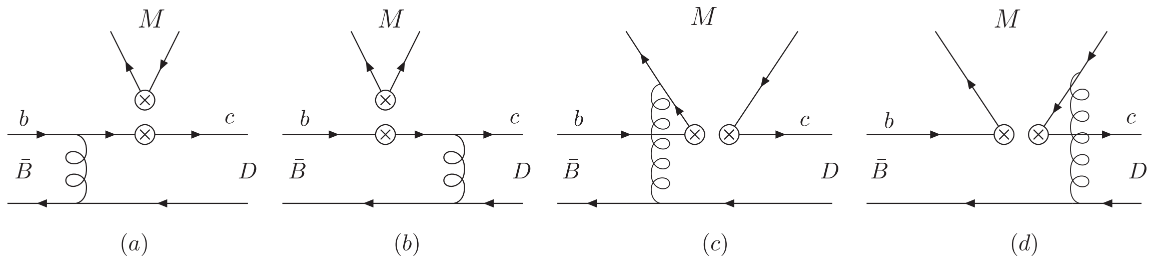

FIG. 1. Color allowed diagrams in the pQCD approach for $B \to DP$ decays.

$$O_1 = (\bar{c}_\alpha b_\beta)_{V-A}(\bar{q}'_\beta u_\alpha)_{V-A},$$
$$O_2 = (\bar{c}_\alpha b_\alpha)_{V-A}(\bar{q}'_\beta u_\beta)_{V-A}, \tag{2}$$

and $q' = d$, s. Here, α β are the color indices, $(\bar{q}_1 q_2)_{V-A} \equiv \bar{q}_1 \gamma^\mu (1-\gamma^5) q_2$ and the $V_{cb}V^*_{uq'}$ are the corresponding CKM matrix elements.

If one sandwiches the above Hamiltonian between initial and final state mesons and factorizes the matrix elements, then the combination of the Wilson coefficient usually appears. Conventionally, these are defined as

$$a_1 = C_2 + C_1/3, \qquad a_2 = C_1 + C_2/3, \tag{3}$$

where a_1 and a_2 correspond to the color favored and the color-suppressed contribution, respectively. Let us define two lightlike vectors n and v, with $n^2 = 0$, $v^2 = 0$, and $n \cdot v = 1$. Now, n can be written as $(1, 0, \mathbf{0}_T)$, and v is $(0, 1, \mathbf{0}_T)$. The momentum of B, D and the light mesons are, respectively, P_1, P_2, and P_3 and these are defined as

$$P_1 = \frac{M_B}{\sqrt{2}}(1, 1, \mathbf{0}_T), \qquad P_2 = \frac{M_B}{\sqrt{2}}(1, r^2, \mathbf{0}_T),$$
$$P_3 = \frac{M_B}{\sqrt{2}}(0, 1-r^2, \mathbf{0}_T). \tag{4}$$

The momenta of the light antiquark in B and D mesons are denoted by k_1 and k_2, respectively, whereas k_3 defines the momentum of the quark in the light meson. Their explicit expressions are

$$k_1 = \left(x_1 \frac{M_B}{\sqrt{2}}, 0, \mathbf{k}_{1\perp}\right) \text{ for color suppressed contributions,}$$
$$k_1 = \left(0, x_1 \frac{M_B}{\sqrt{2}}, \mathbf{k}_{1\perp}\right) \text{ for the others,}$$
$$k_2 = \left(x_2 \frac{M_B}{\sqrt{2}}, 0, \mathbf{k}_{2\perp}\right),$$
$$k_3 = \left(0, x_3 \frac{(1-r^2)M_B}{\sqrt{2}}, \mathbf{k}_{3\perp}\right). \tag{5}$$

Here, x_1, x_2, and x_3 are the momentum fractions, and $\mathbf{k}_{1\perp}$, $\mathbf{k}_{2\perp}$, and $\mathbf{k}_{3\perp}$ are the transverse momentum of the quark.

A. Wave functions of $B_{(s)}$ mesons

In the pQCD calculation, the light-cone wave functions of the mesons are needed as inputs. The B meson and the B_s meson have the similar structure of wave function, except different values of parameters characterizing a small SU(3) breaking effect. In general, the $B_{(s)}$ meson wave function is decomposed into the following Lorentz structures:

$$\int \frac{d^4 z}{(2\pi)^4} e^{ik_1 \cdot z} \langle 0 | b_\beta(0) \bar{q}_\alpha(z) | \bar{B}_{(s)}(P_1) \rangle$$
$$= \frac{i}{\sqrt{2N_c}} \left\{ (\not{P}_1 + M_{B_{(s)}}) \gamma_5 \left[\phi_{B_{(s)}}(k_1) - \frac{\not{n} - \not{v}}{\sqrt{2}} \bar{\phi}_{B_{(s)}}(k_1) \right] \right\}_{\beta\alpha}, \tag{6}$$

where, $\phi_{B_{(s)}}(k_1)$ and $\bar{\phi}_{B_{(s)}}(k_1)$ are the corresponding leading twist distribution amplitudes, and $q = u, d, s$. Here, $\bar{\phi}_{B_{(s)}}(k_1)$ gives a smaller contribution [11] and therefore we will neglect it in our calculation. Thus the final expression becomes

$$\Phi_{B_{(s)}} = \frac{i}{\sqrt{2N_c}} (\not{P}_1 + M_{B_{(s)}}) \gamma_5 \phi_{B_{(s)}}(k_1). \tag{7}$$

The distribution amplitude in the b space is

$$\phi_{B_{(s)}}(x, b) = N_{B_{(s)}} x^2 (1-x)^2 \exp\left[-\frac{1}{2}\left(\frac{x M_{B_{(s)}}}{\omega_b}\right)^2 - \frac{\omega_b^2 b^2}{2} \right], \tag{8}$$

with b as the conjugate space coordinate of $\mathbf{k}_{1\perp}$. $N_{B_{(s)}}$ is the normalization constant, which is determined by the normalization condition:

$$\int_0^1 dx \phi_{B_{(s)}}(x, b=0) = \frac{f_{B_{(s)}}}{2\sqrt{2N_c}}. \tag{9}$$

For parameter ω_b, the value 0.40 ± 0.05 GeV is usually taken for $\bar{B}^0_d$ and $B^\pm$ mesons, and 0.50 ± 0.05 GeV for the $\bar{B}^0_s$ meson, characterizing the small SU(3) breaking effect as argued in [12]. In this paper, we will use the ω_b as a parameter for fitting, with a range from 0.35 to 0.45 GeV.

B. Wave functions of light pseudoscalar mesons

The decay constant of the pseudoscalar meson is defined as

$$\langle 0|\bar{q}_1 \gamma_\mu \gamma_5 q_2 |P(P_3)\rangle = i f_P P_{3\mu}. \tag{10}$$

For our case these pseudoscalar mesons can be π and K and the respective decay constants are $f_\pi = 131$ MeV, $f_K = 160$ MeV.

The light-cone distribution amplitudes (for the outgoing state) for light pseudoscalar mesons is

$$\begin{aligned}\langle P(P_3)|q_{1\alpha}(0)\bar{q}_{2\beta}(z)|0\rangle = \frac{i}{\sqrt{2N_C}}\int_0^1 dx e^{ixP_3\cdot z}[\gamma_5 \not{P}_3 \phi^A(x) \\ + \gamma_5 m_0 \phi^P(x) \\ + m_0 \gamma_5 (\not{v}\not{n} - 1)\phi^T(x)]_{\alpha\beta},\end{aligned} \tag{11}$$

where v is the light-cone direction along which the light pseudoscalar meson's momentum is defined, and n is just opposite to it. Now the chiral scale parameter m_0 is defined as $m_0 = \frac{M_P^2}{m_{q_1}+m_{q_2}}$.

Usually the distribution amplitudes are expanded by the Gegenbauer polynomials and their expressions are

$$\begin{aligned}\phi_P^A(x) = \frac{3f_P}{\sqrt{2N_c}}x(1-x)[1 + a_1^A C_1^{3/2}(t) + a_2^A C_2^{3/2}(t) \\ + a_4^A C_4^{3/2}(t)],\end{aligned} \tag{12}$$

$$\phi_P^p(x) = \frac{f_P}{2\sqrt{2N_c}}[1 + a_2^p C_2^{1/2}(t) + a_4^p C_4^{1/2}(t)], \tag{13}$$

$$\phi_P^T(x) = -\frac{f_P}{2\sqrt{2N_c}}[C_1^{1/2}(t) + a_3^T C_3^{1/2}(t)], \tag{14}$$

with $t = 2x - 1$. The coefficients of the Gegenbauer polynomials are [13]

$$\begin{aligned} &a_{2\pi}^A = 0.44, \quad a_{4\pi}^A = 0.25, \quad a_{1K}^A = 0.17, \\ &a_{2K}^A = 0.2, \quad a_{2\pi}^p = 0.43, \quad a_{4\pi}^p = 0.09, \\ &a_{2K}^p = 0.24, \quad a_{4K}^p = -0.11, \quad a_{3\pi}^T = 0.55, \\ &a_{3K}^T = 0.35. \end{aligned} \tag{15}$$

For η and η', the mixing mechanism must be taken into consideration. Following the method presented in Ref. [14], where η_n and η_s are chosen as the basis of mixing,

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = U(\phi)\begin{pmatrix} |\eta_n\rangle \\ |\eta_s\rangle \end{pmatrix}, \tag{16}$$

with

$$|\eta_n\rangle = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d), \qquad |\eta_s\rangle = \bar{s}s, \tag{17}$$

$$U(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}, \tag{18}$$

and the mixing angle $\phi = 39.3° \pm 1.0°$.

The distribution amplitudes of η_n and η_s are assumed to be the same as that of the π meson, except different decay constants and chiral parameters. The decay constants of η_n and η_s are taken to be [14]

$$\begin{aligned} f_n &= (1.07 \pm 0.02) f_\pi = (139.1 \pm 2.6)\ \text{MeV}, \\ f_s &= (1.34 \pm 0.06) f_\pi = (174.2 \pm 7.8)\ \text{MeV} \end{aligned} \tag{19}$$

and the corresponding chiral parameters are given by

$$\begin{aligned} m_0^{\bar{n}n} = \frac{1}{2m_n}\Big[m_\eta^2 \cos^2\phi + m_{\eta'}^2 \sin^2\phi \\ - \frac{\sqrt{2}f_s}{f_n}(m_{\eta'}^2 - m_\eta^2)\cos\phi \sin\phi \Big], \end{aligned} \tag{20}$$

$$\begin{aligned} m_0^{\bar{s}s} = \frac{1}{2m_s}\Big[m_{\eta'}^2 \cos^2\phi + m_\eta^2 \sin^2\phi \\ - \frac{f_n}{\sqrt{2}f_s}(m_{\eta'}^2 - m_\eta^2)\cos\phi \sin\phi \Big]. \end{aligned} \tag{21}$$

C. Wave functions of light vector mesons

Following the same lines as we did for the pseudoscalar mesons, the decay constants for the vector mesons are defined by

$$\begin{aligned} \langle 0|\bar{q}_1 \gamma_\mu q_2|V(P_3, \epsilon)\rangle &= f_V m_V \epsilon_\mu, \\ \langle 0|\bar{q}_1 \sigma_{\mu\nu} q_2|V(P_3, \epsilon)\rangle &= i f_V^T (\epsilon_\mu P_{3\nu} - \epsilon_\nu P_{3\mu}). \end{aligned} \tag{22}$$

For the vector meson, the longitudinal decay constant can be extracted from the experiments [15], whereas the transverse one can be calculated by using the QCD sum rules [16]. The decay constants that we use in this work are defined in Table I.

Up to twist-3 the distribution amplitudes are

TABLE I. The decay constants of vector mesons (in MeV).

f_ρ	f_{K^*}	f_ω	f_ϕ	f_ρ^T	$f_{K^*}^T$	f_ω^T	f_ϕ^T
209 ± 2	217 ± 5	195 ± 3	231 ± 4	165 ± 9	185 ± 10	151 ± 9	186 ± 9

$$\langle V(P_3,\epsilon_L^*)|q_{1\alpha}(0)\bar{q}_{2\beta}(z)|0\rangle = -\frac{1}{\sqrt{2N_C}}\int_0^1 dxe^{ixP_3\cdot z}[M_V\not\epsilon_L^*\phi_V(x)+\not\epsilon_L^*\not P_3\phi_V^t(x)+M_V\phi_V^s(x)]_{\alpha\beta},$$
$$\langle V(P_3,\epsilon_T^*)|q_{1\alpha}(0)\bar{q}_{2\beta}(z)|0\rangle = -\frac{1}{\sqrt{2N_C}}\int_0^1 dxe^{ixP_3\cdot z}[M_V\not\epsilon_T^*\phi_V^v(x)+\not\epsilon_T^*\not P_3\phi_V^T(x)+M_V i\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu\epsilon_T^{*\nu}n^\rho v^\sigma\phi_V^a(x)]_{\alpha\beta},\tag{23}$$

where x is the momentum fraction of the q_2 quark. Contrary to the pseudoscalar case, here n defines the light-cone direction along which the momentum of light meson is taken and v is just the opposite light-cone direction. The twist-2 distribution amplitudes of vector mesons are defined as

$$\phi_V(x) = \frac{3f_V}{\sqrt{2N_C}}x(1-x)[1+a_1^{\parallel}C_1^{3/2}(t)+a_2^{\parallel}C_2^{3/2}(t)],$$
$$\phi_V^T(x) = \frac{3f_V}{\sqrt{2N_C}}x(1-x)[1+a_1^{\perp}C_1^{3/2}(t)+a_2^{\perp}C_2^{3/2}(t)],\tag{24}$$

and the corresponding values of the Gegenbauer moments are [17]

$$a_{2\rho}^{\parallel}=a_{2\omega}^{\parallel}=0.15\pm0.07,\qquad a_{1K^*}^{\parallel}=0.03\pm0.02,$$
$$a_{2K^*}^{\parallel}=0.11\pm0.09,\qquad a_{2\phi}^{\parallel}=0.18\pm0.08,$$
$$a_{2\rho}^{\perp}=a_{2\omega}^{\perp}=0.14\pm0.06,\qquad a_{1K^*}^{\perp}=0.04\pm0.03,$$
$$a_{2K^*}^{\perp}=0.10\pm0.08,\qquad a_{2\phi}^{\perp}=0.14\pm0.07.\tag{25}$$

For the other distribution amplitudes, we use the asymptotic form:

$$\phi_V^t(x)=\frac{3f_V^T}{2\sqrt{6}}t^2,\qquad \phi_V^s(x)=\frac{3f_V^T}{2\sqrt{6}}(-t),$$
$$\phi_V^v(x)=\frac{3f_V}{8\sqrt{6}}(1+t^2),\qquad \phi_V^a(x)=\frac{3f_V}{4\sqrt{6}}(-t).\tag{26}$$

D. Wave function of the $D^{(*)}$ meson

Up to twist-3 accuracy the two-particle light-cone distribution amplitudes of $D^{(*)}$ meson are defined as [7]

$$\langle D(P_2)|q_\alpha(z)\bar{c}_\beta(0)|0\rangle = \frac{i}{\sqrt{2N_C}}\int_0^1 dxe^{ixP_2\cdot z}\times[\gamma_5(\not P_2+M)\phi_D(x,b)]_{\alpha\beta}$$
$$\langle D^*(P_2)|q_\alpha(z)\bar{c}_\beta(0)|0\rangle = -\frac{1}{\sqrt{2N_C}}\int_0^1 dxe^{ixP_2\cdot z}\times[\not\epsilon(\not P_2+M_{D^*})\phi_{D^*}^L(x,b)+\not\epsilon_T(\not P_2+M_{D^*})\phi_{D^*}^T(x,b)]_{\alpha\beta}\tag{27}$$

with

$$\int_0^1 dx\phi_D(x,0)=\frac{f_D}{2\sqrt{2N_c}},$$
$$\int_0^1 dx\phi_{D^*}^L(x,0)=\frac{f_{D^*}}{2\sqrt{2N_c}},\tag{28}$$
$$\int_0^1 dx\phi_{D^*}^T(x,0)=\frac{f_{D^*}^T}{2\sqrt{2N_c}},$$

as the normalization conditions. In the heavy quark limit we have

$$f_{D^*}^T - f_{D^*}\frac{m_c+m_d}{M_{D^*}} \sim f_{D^*} - f_{D^*}^T\frac{m_c+m_d}{M_{D^*}} \sim O(\bar{\Lambda}/M_{D^*}).\tag{29}$$

Thus we will use $f_{D^*}^T = f_{D^*}$ in our calculation. Now, there are several models calculating the distribution amplitude for the D meson and we can collect them here as

$$\phi_D^{(\mathrm{Gen})}(x)=\frac{1}{2\sqrt{2N_c}}f_D 6x(1-x)[1+C_D(1-2x)],$$
$$\phi_D^{(\mathrm{MGen})}(x,b)=\frac{1}{2\sqrt{2N_c}}f_D 6x(1-x)[1+C_D(1-2x)]\exp\left[\frac{-\omega^2b^2}{2}\right],$$
$$\phi_D^{(\mathrm{KLS})}(x,b)=\frac{1}{2\sqrt{2N_c}}f_D N_D\sqrt{x(1-x)}\exp\left[-\frac{1}{2}\left(\frac{xM_D}{\omega}\right)^2-\frac{\omega^2b^2}{2}\right],$$
$$\phi_D^{(\mathrm{GN})}(x,b)=\frac{1}{2\sqrt{2N_c}}f_D N_D x\exp\left[-\frac{xM_D}{\omega}\right]\frac{1}{1+b^2\omega^2},\tag{30}$$
$$\phi_D^{(\mathrm{KKQT})}(x,b)=\frac{1}{2\sqrt{2N_c}}f_D N_D x\theta(x)\theta\left(\frac{2\Lambda_D}{M_D}-x\right)J_0\left(b\sqrt{x\left(\frac{2\Lambda_D}{M_D}-x\right)}\right),$$
$$\phi_D^{(\mathrm{Huang})}(x)=\frac{1}{2\sqrt{2N_c}}f_D N_D x(1-x)\exp\left[-\Lambda_D\frac{(1-x)m_d^2+xm_c^2}{x(1-x)}\right].$$

In Eq. (30) one can easily see that some DAs of the D meson we have collected here do not have b dependence. However, we use a b-dependent form in (27) without losing generality. In all the amplitudes defined above, the x is the momentum fraction of the light quark in the D meson. The first DA model $\phi_D^{(\mathrm{Gen})}$ was proposed in [7], which is the Gegenbauer polynomial-like form. In order to make it $k_\perp$ dependent, an exponential term is added to get $\phi_D^{(\mathrm{MGen})}$. The third candidate DA model $\phi_D^{(\mathrm{KLS})}$ was proposed in [18], which is a Gaussian-type model. The fourth one [19], which is an exponential model, and the fifth model [20], which is obtained by solving the equations of motion without three-parton contributions, were first proposed for the B meson. Here we use heavy quark symmetry and modify the parameters to make them D meson DAs. The sixth DA was proposed in [21], which is derived from the Brodsky-Huang-Lepage prescription [22], with $m_d = 0.35$ GeV, $m_c = 1.3$ GeV. In the next section we will try to fit out the best D meson wave function parameters with the experimental results. As for the D^* meson, we just assume that $\phi^L_{D^*} = \phi^T_{D^*} = \phi_{D^*}$ according to heavy quark symmetry.

III. CALCULATION OF DECAY AMPLITUDES IN pQCD APPROACH

A. Amplitudes for $B_{(s)} \to D_{(s)}P$ decays

There are three types of diagrams that may contribute to the $B \to D^{(*)}M$ decays: color allowed diagrams (we mark this kind of contribution with the subscript ext) shown in Fig. 1, color-suppressed diagrams (marked with int) shown in Fig. 2, and the annihilation-type diagrams (marked with exc) shown in Fig. 3. Now each type of diagram contains two categories: the one in which one meson can be factorized out (denoted as ξ) and the other in which no meson can be factorized out (denoted as M).

The first two diagrams in Figs. 1–3 involve only two meson wave functions, and their results are as follows:

$$\begin{aligned}\xi_{\mathrm{ext}} = {}& 8\pi C_F f_P \int_0^1 dx_1 dx_2 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \phi_B(x_1, b_1)\\ &\times \phi_D(x_2, b_2)[E_e(t_e^{(1)})h(x_1, x_2, b_1, b_2)\\ &\times S_t(x_2)(1 + x_2 + r)\\ &+ rE_e(t_e^{(2)})h(x_2, x_1, b_2, b_1)S_t(x_1)],\end{aligned} \tag{31}$$

$$\begin{aligned}\xi_{\mathrm{int}} = {}& 8\pi C_F f_D \int_0^1 dx_1 dx_3 \int_0^{1/\Lambda} b_1 db_1 b_3 db_3 \phi_B(x_1, b_1)\\ &\times \{[(2 - x_3)\phi_P(x_3) - r_0(1 - 2x_3)(\phi_P^p(x_3)\\ &- \phi_P^T(x_3))]E_i(t_i^{(1)})h(x_1, (1 - x_3)(1 - r^2), b_1, b_3)\\ &\times S_t(x_3) + 2r_0\phi_P^p(x_3)E_i(t_i^{(2)})\\ &\times h(1 - x_3, x_1(1 - r^2), b_3, b_1)S_t(x_1)\},\end{aligned} \tag{32}$$

$$\begin{aligned}\xi_{\mathrm{exc}} = {}& 8\pi C_F f_B \int_0^1 dx_2 dx_3 \int_0^{1/\Lambda} b_2 db_2 b_3 db_3 \phi_D(x_2, b_2)\\ &\times [-x_3\phi_P(x_3)E_a(t_a^{(1)})h_a(x_2, x_3(1 - r^2), b_2, b_3)\\ &\times S_t(x_3) + x_2\phi_P(x_3)E_a(t_a^{(2)})\\ &\times h_a(x_3, x_2(1 - r^2), b_3, b_2)S_t(x_2)],\end{aligned} \tag{33}$$

with the mass ratio $r_0 \equiv m_0/m_B$. f_P, f_B, and f_D are the decay constants of the light pseudoscalar meson, B meson, and D meson, respectively. The factors evolving with the scale t are given by

$$\begin{aligned}E_e(t) &= \alpha_s(t)a_1(t)\exp[-S_B(t) - S_D(t)],\\ E_i(t) &= \alpha_s(t)a_2(t)\exp[-S_B(t) - S_P(t)],\\ E_a(t) &= \alpha_s(t)a_2(t)\exp[-S_D(t) - S_P(t)].\end{aligned} \tag{34}$$

We adopt the expression of the Sudakov factor for D meson as suggested in Ref. [7], which is listed in Appendix A together with the expressions for $S_B(t)$, $S_P(t)$.

The functions h in the hard part of factorization formulas, derived from the factorizable diagrams, are given by

$$\begin{aligned}h(x_1, x_2, b_1, b_2) = {}& K_0(\sqrt{x_1 x_2}m_B b_1)[\theta(b_1 - b_2)\\ &\times K_0(\sqrt{x_2}m_B b_1)I_0(\sqrt{x_2}m_B b_2)\\ &+ \theta(b_2 - b_1)K_0(\sqrt{x_2}m_B b_2)\\ &\times I_0(\sqrt{x_2}m_B b_1)],\end{aligned} \tag{35}$$

$$\begin{aligned}h_a(x_2, x_3, b_2, b_3) = {}& \left(i\frac{\pi}{2}\right)^2 H_0^{(1)}(\sqrt{x_2 x_3}m_B b_2)[\theta(b_2 - b_3)\\ &\times H_0^{(1)}(\sqrt{x_3}m_B b_2)J_0(\sqrt{x_3}m_B b_3)\\ &+ \theta(b_3 - b_2)H_0^{(1)}(\sqrt{x_3}m_B b_3)\\ &\times J_0(\sqrt{x_3}m_B b_2)],\end{aligned} \tag{36}$$

where $H^{(1)}(z) = J_0(z) + iY_0(z)$. The hard scales t are de-

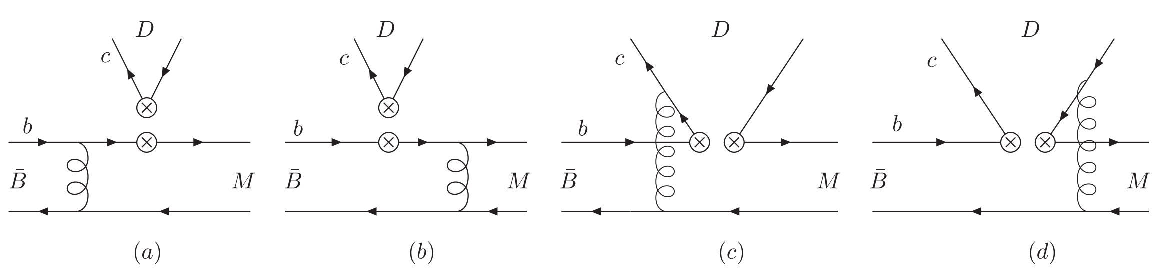

FIG. 2. Color-suppressed diagrams in the pQCD approach for $B \to DP$ decays.

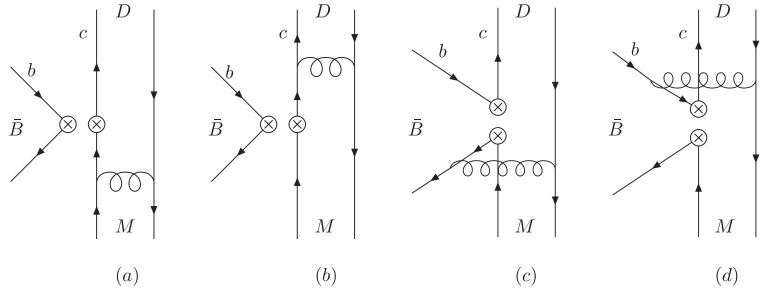

FIG. 3. Annihilation-type diagrams in the pQCD approach for $B \to DP$ decays.

termined by

$$\begin{aligned}
t_e^{(1)} &= \max(\sqrt{x_2} m_B, 1/b_1, 1/b_2),\\
t_e^{(2)} &= \max(\sqrt{x_1} m_B, 1/b_1, 1/b_2),\\
t_i^{(1)} &= \max(\sqrt{(1-x_3)(1-r^2)} m_B, 1/b_1, 1/b_3),\\
t_i^{(2)} &= \max(\sqrt{x_1(1-r^2)} m_B, 1/b_1, 1/b_3),\\
t_a^{(1)} &= \max(\sqrt{x_3(1-r^2)} m_B, 1/b_2, 1/b_3),\\
t_a^{(2)} &= \max(\sqrt{x_2(1-r^2)} m_B, 1/b_2, 1/b_3).
\end{aligned} \tag{37}$$

The formulas for the last two diagrams in Figs. 1–3 contain the kinematics variables of the three mesons. Their explicit expressions are given by

$$\begin{aligned}
\mathcal{M}_{\rm ext} = {}& 16\pi\sqrt{2N_c} C_F \int_0^1 [dx] \int_0^{1/\Lambda} b_1 db_1 b_3 db_3\\
&\times \phi_B(x_1, b_1)\phi_D(x_2, b_1)\phi_P(x_3)\\
&\times [x_3 E_b(t_b^{(1)}) h_b^{(1)}(x_i, b_i)\\
&- (1 - x_3 + x_2) E_b(t_b^{(2)}) h_b^{(2)}(x_i, b_i)],
\end{aligned} \tag{38}$$

$$\begin{aligned}
\mathcal{M}_{\rm int} = {}& 16\pi\sqrt{2N_c} C_F \int_0^1 [dx] \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \phi_B(x_1, b_1)\\
&\times \phi_D(x_2, b_2)[((x_3 - 1 - x_2)\phi_P(x_3)\\
&+ r_0(1-x_3)(\phi_P^p(x_3) - \phi_P^T(x_3))) E_d(t_d^{(1)}) h_d^{(1)}(x_i, b_i)\\
&+ [(1-x_2)\phi_P(x_3) + r_0(x_3 - 1)(\phi_P^p(x_3)\\
&+ \phi_P^T(x_3))] E_d(t_d^{(2)}) h_d^{(2)}(x_i, b_i)],
\end{aligned} \tag{39}$$

$$\begin{aligned}
\mathcal{M}_{\rm exc} = {}& 16\pi\sqrt{2N_c} C_F \int_0^1 [dx]\\
&\times \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \phi_B(x_1, b_1)\phi_D(x_2, b_2)\\
&\times [x_3 \phi_P(x_3) E_f(t_f^{(1)}) h_f^{(1)}(x_i, b_i)\\
&- x_2 \phi_P(x_3) E_f(t_f^{(2)}) h_f^{(2)}(x_i, b_i)],
\end{aligned} \tag{40}$$

with $[dx] \equiv dx_1 dx_2 dx_3$. The expressions for the evolution factors are

$$\begin{aligned}
E_b(t) &= \alpha_s(t)\frac{C_1(t)}{N_C}\exp[-S(t)|_{b_2=b_1}],\\
E_d(t) &= \alpha_s(t)\frac{C_2(t)}{N_C}\exp[-S(t)|_{b_3=b_1}],\\
E_f(t) &= \alpha_s(t)\frac{C_2(t)}{N_C}\exp[-S(t)|_{b_3=b_2}],
\end{aligned} \tag{41}$$

with the Sudakov exponent $S = S_B + S_D + S_P$.

The functions $h^{(j)}$, $j = 1$ and 2, in these amplitudes are

$$\begin{aligned}
h_b^{(j)} = {}& [\theta(b_1 - b_3) K_0(B m_B b_1) I_0(B m_B b_3)\\
&+ \theta(b_3 - b_1) K_0(B m_B b_3) I_0(B m_B b_1)]\\
&\times \begin{Bmatrix} K_0(\sqrt{|B_j^2|} m_B b_3) & \text{for } B_j^2 \geq 0\\ \frac{i\pi}{2} H_0^{(1)}(\sqrt{|B_j^2|} m_B b_3) & \text{for } B_j^2 \leq 0 \end{Bmatrix},
\end{aligned} \tag{42}$$

$$\begin{aligned}
h_d^{(j)} = {}& [\theta(b_1 - b_2) K_0(D m_B b_1) I_0(D m_B b_2)\\
&+ \theta(b_2 - b_1) K_0(D m_B b_2) I_0(D m_B b_1)]\\
&\times \begin{Bmatrix} K_0(\sqrt{|D_j^2|} m_B b_2) & \text{for } D_j^2 \geq 0\\ \frac{i\pi}{2} H_0^{(1)}(\sqrt{|D_j^2|} m_B b_2) & \text{for } D_j^2 \leq 0 \end{Bmatrix},
\end{aligned} \tag{43}$$

$$\begin{aligned}
h_f^{(j)} = {}& i\frac{\pi}{2}[\theta(b_1 - b_2) H_0^{(1)}(F m_B b_1) J_0(F m_B b_2)\\
&+ \theta(b_2 - b_1) H_0^{(1)}(F m_B b_2) J_0(F m_B b_1)]\\
&\times \begin{Bmatrix} K_0(\sqrt{|F_j^2|} m_B b_1) & \text{for } F_j^2 \geq 0\\ \frac{i\pi}{2} H_0^{(1)}(\sqrt{|F_j^2|} m_B b_1) & \text{for } F_j^2 \leq 0 \end{Bmatrix},
\end{aligned} \tag{44}$$

with the variables

$$
\begin{aligned}
&B^2 = x_1x_2, \qquad B_1^2 = x_1x_2 - x_2x_3(1-r^2),\\
&B_2^2 = x_1x_2 - x_2(1-x_3)(1-r^2),\\
&D^2 = x_1(1-x_3)(1-r^2),\\
&D_1^2 = (x_1-x_2)(1-x_3)(1-r^2),\\
&D_2^2 = (x_1+x_2)r^2 - (1-x_1-x_2)(1-x_3)(1-r^2),\\
&F^2 = x_2x_3(1-r^2), \qquad F_1^2 = x_2(x_1 - x_3(1-r^2)),\\
&F_2^2 = 1-(1-x_2)(1-x_1-x_3(1-r^2)).
\end{aligned} \tag{45}
$$

The scales $t^{(j)}$ are given by

$$
\begin{aligned}
t_b^{(j)} &= \max(Bm_B, \sqrt{|B_j^2|}m_B, 1/b_1, 1/b_3),\\
t_d^{(j)} &= \max(Dm_B, \sqrt{|D_j^2|}m_B, 1/b_1, 1/b_2),\\
t_f^{(j)} &= \max(Fm_B, \sqrt{|F_j^2|}m_B, 1/b_1, 1/b_2).
\end{aligned} \tag{46}
$$

The decay amplitudes of each $B_{(s)} \to D_{(s)}P$ channels are then

$$
A(B^- \to D^0\pi^-) = \frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*(\xi_{\rm ext} + \mathcal{M}_{\rm ext} + \xi_{\rm int} + \mathcal{M}_{\rm int}), \tag{47}
$$

$$
A(B^- \to D^0K^-) = \frac{G_F}{\sqrt{2}}V_{cb}V_{us}^*(\xi_{\rm ext} + \mathcal{M}_{\rm ext} + \xi_{\rm int} + \mathcal{M}_{\rm int}), \tag{48}
$$

$$
A(\bar{B}^0 \to D^+\pi^-) = \frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*(\xi_{\rm ext} + \mathcal{M}_{\rm ext} + \xi_{\rm exc} + \mathcal{M}_{\rm exc}), \tag{49}
$$

$$
A(\bar{B}^0 \to D^+K^-) = \frac{G_F}{\sqrt{2}}V_{cb}V_{us}^*(\xi_{\rm ext} + \mathcal{M}_{\rm ext}), \tag{50}
$$

$$
A(\bar{B}^0 \to D_s^+K^-) = \frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*(\xi_{\rm exc} + \mathcal{M}_{\rm exc}), \tag{51}
$$

$$
A(\bar{B}^0 \to D^0\pi^0) = \frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*\frac{1}{\sqrt{2}}(-(\xi_{\rm int} + \mathcal{M}_{\rm int}) + (\xi_{\rm exc} + \mathcal{M}_{\rm exc})), \tag{52}
$$

$$
A(\bar{B}^0 \to D^0\bar{K}^0) = \frac{G_F}{\sqrt{2}}V_{cb}V_{us}^*(\xi_{\rm int} + \mathcal{M}_{\rm int}), \tag{53}
$$

$$
A(\bar{B}^0 \to D^0\eta_n) = \frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*\frac{1}{\sqrt{2}}(\xi_{\rm int} + \mathcal{M}_{\rm int} + \xi_{\rm exc} + \mathcal{M}_{\rm exc}), \tag{54}
$$

$$
A(\bar{B}_s^0 \to D^+\pi^-) = \frac{G_F}{\sqrt{2}}V_{cb}V_{us}^*(\xi_{\rm exc} + \mathcal{M}_{\rm exc}), \tag{55}
$$

$$
A(\bar{B}_s^0 \to D_s^+\pi^-) = \frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*(\xi_{\rm ext} + \mathcal{M}_{\rm ext}), \tag{56}
$$

$$
A(\bar{B}_s^0 \to D_s^+K^-) = \frac{G_F}{\sqrt{2}}V_{cb}V_{us}^*(\xi_{\rm ext} + \mathcal{M}_{\rm ext} + \xi_{\rm exc} + \mathcal{M}_{\rm exc}), \tag{57}
$$

$$
A(\bar{B}_s^0 \to D^0\pi^0) = \frac{G_F}{\sqrt{2}}V_{cb}V_{us}^*\frac{1}{\sqrt{2}}(\xi_{\rm exc} + \mathcal{M}_{\rm exc}), \tag{58}
$$

$$
A(\bar{B}_s^0 \to D^0\bar{K}^0) = \frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*(\xi_{\rm int} + \mathcal{M}_{\rm int}), \tag{59}
$$

$$
A(\bar{B}_s^0 \to D^0\eta_n) = \frac{G_F}{\sqrt{2}}V_{cb}V_{us}^*\frac{1}{\sqrt{2}}(\xi_{\rm exc} + \mathcal{M}_{\rm exc}), \tag{60}
$$

$$
A(\bar{B}_s^0 \to D^0\eta_s) = \frac{G_F}{\sqrt{2}}V_{cb}V_{us}^*(\xi_{\rm int} + \mathcal{M}_{\rm int}). \tag{61}
$$

It should be noticed that, in (54), (60), and (61), the decay amplitudes are for the mixing basis of η and η'. For the physical state η and η', the decay amplitudes are

$$
A(\bar{B}^0 \to D^0\eta) = A(\bar{B}^0 \to D^0\eta_n)\cos\phi, \tag{62}
$$

$$
A(\bar{B}^0 \to D^0\eta') = A(\bar{B}^0 \to D^0\eta_n)\sin\phi, \tag{63}
$$

$$
A(\bar{B}_s^0 \to D^0\eta) = A(\bar{B}_s^0 \to D^0\eta_n)\cos\phi - A(\bar{B}_s^0 \to D^0\eta_s)\sin\phi, \tag{64}
$$

$$
A(\bar{B}_s^0 \to D^0\eta') = A(\bar{B}_s^0 \to D^0\eta_n)\sin\phi + A(\bar{B}_s^0 \to D^0\eta_s)\cos\phi. \tag{65}
$$

B. Amplitudes for $B_{(s)} \to D_{(s)}V$ and $B_{(s)} \to D^*_{(s)}P$ decays

For the processes $B_{(s)} \to D_{(s)}V$ and $D^*_{(s)}P$, the transverse polarization of the vector mesons will not contribute. In the leading power contribution, the formulas of $B \to DV$ and $B \to D^*P$ are the same as that of $B_{(s)} \to D_{(s)}P$ decays, except some substitutions.

For $B_{(s)} \to D_{(s)}V$, the following substitutions should be done for the formula ξ_i and $\mathcal{M}_i$:

$$
\begin{gathered}
\phi_P \to \phi_V, \qquad \phi_P^p \to -\phi_V^s, \qquad \phi_P^t \to -\phi_V^t,\\
m_0 \to m_V, \qquad f_P \to f_V.
\end{gathered} \tag{66}
$$

ϕ_V, ϕ_V^s and ϕ_V^t are the light-cone distribution amplitudes of vector mesons, which we defined before. m_V and f_V are the mass and the decay constant of the vector meson.

Similarly, for $B_{(s)} \to D^*_{(s)}P$, the substitutions in the formulas ξ_i and $\mathcal{M}_i$ should be done as

$$m_D \to m_{D^*}, \quad f_D \to f_{D^*}, \quad \phi_D(x_2) \to \phi_{D^*}(x_2). \tag{67}$$

Making the following substitutions in Eqs. (47)–(65), we can get the final decay amplitude for each $B \to D^* P$ decays:

$$D^+ \to D^{*+}, \qquad D^0 \to D^{*0}, \qquad D_s^+ \to D_s^{*+}. \tag{68}$$

The formulas for $B \to DV$ can be obtained through the substitutions

$$\pi \to \rho, \qquad K \to K^*, \qquad \eta_n \to \omega, \qquad \eta_s \to \phi \tag{69}$$

in Eqs. (47)–(61).

C. Amplitudes for $B_{(s)} \to D_{(s)}^* V$ decays

In $B \to D_{(s)}^* V$ decays, both longitudinal and transverse polarization can contribute. For the longitudinal polarization, the amplitudes can be obtained by carrying out the substitutions referred in Eqs. (66) and (67), when only the leading power contribution is taken into consideration. The transverse polarized contribution is suppressed by r or r_V, $r_V \equiv m_V/m_B$. Although the transverse polarization will not give the leading power contribution, to make the point more clear we list the analytic formulas for transverse polarizations $\xi_{\rm ext}^T$, $\xi_{\rm int}^T$, and $\xi_{\rm exc}^T$:

$$\begin{aligned}\xi_{\rm ext}^T = {}& 8\pi C_F m_B^4 f_V \int_0^1 dx_1 dx_2 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \\ &\times \phi_D^T(x_2, b_2) r_V [E_e(t_e^{(1)}) h(x_1, x_2, b_1, b_2) S_t(x_2) \\ &\times (-\epsilon^{n\bar{n}\epsilon_D^{*T}\epsilon_V^{*T}} - i\epsilon_D^{*T}\cdot\epsilon_V^{*T}(1+2r)) \\ &+ E_e(t_e^{(2)}) h(x_2, x_1, b_2, b_1) S_t(x_1) r(r+1) \\ &\times (-\epsilon^{n\bar{n}\epsilon_D^{*T}\epsilon_V^{*T}} - i\epsilon_D^{*T}\cdot\epsilon_V^{*T})],\end{aligned} \tag{70}$$

$$\begin{aligned}\xi_{\rm int}^T = {}& 8\pi C_F m_B^4 f_{D^*} \int_0^1 dx_1 dx_3 \int_0^{1/\Lambda} b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) r\{[\epsilon^{n\bar{n}\epsilon_D^{*T}\epsilon_V^{*T}}(-\phi_V^T(x_3) - r_V((x_3-3)\phi_V^a(x_3) \\ &+ (x_3-1)\phi_V^v(x_3))) - i\epsilon_D^{*T}\cdot\epsilon_V^{*T}(\phi_V^T(x_3) - r_V((x_3-1)\phi_V^a(x_3) - (x_3-3)\phi_V^v(x_3))))] \\ &\times E_i(t_i^{(1)}) h(x_1, x_3(1-r^2), b_1, b_3) S_t(x_3) + r_V[\epsilon^{n\bar{n}\epsilon_D^{*T}\epsilon_V^{*T}}(\phi_V^a(x_3) - \phi_V^v(x_3)) \\ &+ i\epsilon_D^{*T}\cdot\epsilon_V^{*T}(\phi_V^a(x_3) - \phi_V^v(x_3))] E_i(t_i^{(2)}) h(x_3, x_1(1-r^2), b_3, b_1) S_t(x_1)\},\end{aligned} \tag{71}$$

$$\begin{aligned}\xi_{\rm exc}^T = {}& 8\pi C_F m_B^4 f_B \int_0^1 dx_2 dx_3 \int_0^{1/\Lambda} b_2 db_2 b_3 db_3 \phi_D^T(x_2, b_2)[E_a(t_a^{(1)}) h_a(x_2, x_3(1-r^2), b_2, b_3) S_t(x_3)(\epsilon^{n\bar{n}\epsilon_D^{*T}\epsilon_V^{*T}}[r^2\phi_V^T(x_3) \\ &- r r_V(x_3+1)\phi_V^a(x_3) + r r_V(1-x_3)\phi_V^v(x_3)] + i\epsilon_D^{*T}\cdot\epsilon_V^{*T}[-r^2\phi_V^T(x_3) + r r_V(x_3-1)\phi_V^a(x_3) + r r_V(x_3+1)\phi_V^v(x_3)]) \\ &+ E_a(t_a^{(2)}) h_a(x_3, x_2(1-r^2), b_3, b_2) S_t(x_2) r r_V(\epsilon^{n\bar{n}\epsilon_D^{*T}\epsilon_V^{*T}}((1+x_2)\phi_V^a(x_3) + (1-x_2)\phi_V^v(x_3)) \\ &- i\epsilon_D^{*T}\cdot\epsilon_V^{*T}((1-x_2)\phi_V^a(x_3) + (1+x_2)\phi_V^v(x_3)))].\end{aligned} \tag{72}$$

The evolution factors in these amplitudes are the same as those in Eq. (34) after substituting $S_V(t)$ for $S_P(t)$. For the nonfactorizable amplitudes, the factorization formulas involve the kinematic variables of all three mesons. Their expressions are

$$\begin{aligned}\mathcal{M}_{\rm ext}^T = {}& 16\pi\sqrt{2N_c} C_F m_B^4 \int_0^1 [dx] \int_0^{1/\Lambda} b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) \phi_D^T(x_2, b_1) r_V [E_b(t_b^{(1)}) h_b^{(1)}(x_i, b_i)(\epsilon^{n\bar{n}\epsilon_D^{*T}\epsilon_V^{*T}} x_3(\phi_V^a(x_3) \\ &- \phi_V^v(x_3)) + i\epsilon_D^{*T}\cdot\epsilon_V^{*T} x_3(\phi_V^a(x_3) - \phi_V^v(x_3))) + E_b(t_b^{(2)}) h_b^{(2)}(x_i, b_i)\{\epsilon^{n\bar{n}\epsilon_D^{*T}\epsilon_V^{*T}}((1-x_3)(1-2r)\phi_V^a(x_3) \\ &+ (x_3-1)\phi_V^v(x_3)) - i\epsilon_D^{*T}\cdot\epsilon_V^{*T}((x_3-1)\phi_V^a(x_3) + (1-2r)(1-x_3)\phi_V^v(x_3))\}],\end{aligned} \tag{73}$$

$$\begin{aligned}\mathcal{M}_{\rm int}^T = {}& 16\pi\sqrt{2N_c} C_F m_B^4 \int_0^1 [dx] \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_D^T(x_2, b_2) r [E_d(t_d^{(2)}) h_d^{(2)}(x_i, b_i)(\epsilon^{n\bar{n}\epsilon_D^{*T}\epsilon_V^{*T}}((x_2-1)\phi_V^T(x_3) \\ &+ r_V(\phi_V^a(x_3) + \phi_V^v(x_3))) - i\epsilon_D^{*T}\cdot\epsilon_V^{*T}((1-x_2)\phi_V^T(x_3) + r_V(\phi_V^a(x_3) + \phi_V^v(x_3)))) + E_d(t_d^{(1)}) h_d^{(1)}(x_i, b_i) \\ &\times (\epsilon^{n\bar{n}\epsilon_D^{*T}\epsilon_V^{*T}}(2r_V(x_3-1)\phi_V^a(x_3) - x_2\phi_V^T(x_3)) - i\epsilon_D^{*T}\cdot\epsilon_V^{*T}(x_2\phi_V^T(x_3) + 2r_V(x_3-1)\phi_V^v(x_3)))],\end{aligned} \tag{74}$$

$$\begin{aligned}\mathcal{M}_{\rm exc}^T = {}& 16\pi\sqrt{2N_c} C_F m_B^4 \int_0^1 [dx] \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_D^T(x_2, b_2)\{E_f(t_f^{(1)}) h_f^{(1)}(x_i, b_i) \phi_V^T(x_3) \\ &\times (\epsilon^{n\bar{n}\epsilon_D^{*T}\epsilon_V^{*T}}(x_2 r^2 - r_V^2 x_3) - i\epsilon_D^{*T}\cdot\epsilon_V^{*T}(x_2 r^2 + r_V^2 x_3)) + E_f(t_f^{(2)}) h_f^{(2)}(x_i, b_i)[\epsilon^{n\bar{n}\epsilon_D^{*T}\epsilon_V^{*T}}(-2r r_V \phi_V^a(x_3) \\ &- r^2(x_2-1)\phi_V^T(x_3) + r_V^2(x_3-1)\phi_V^T(x_3)) + i\epsilon_D^{*T}\cdot\epsilon_V^{*T}(((x_2-1)r^2 + r_V^2(x_2-1))\phi_V^T(x_3) + 2r r_V \phi_V^v(x_3))]\}.\end{aligned} \tag{75}$$

The $h's$ and $h^{(j)}$ functions in the amplitudes here are the same as defined in Eqs. (35), (36), and (42)–(44).

Thus combining everything together, one can get the final decay amplitudes of each polarization for $B \to D^* V$ decays.

IV. NUMERICAL CALCULATIONS AND DISCUSSIONS

In this section, we do the numerical analysis of our calculation which we have done in the previous sections. The parameters of the $D^{(*)}_{(s)}$ meson we use are

$$\begin{aligned} m_D &= 1.869 \text{ GeV}, & m_{D_s^-} &= 1.968 \text{ GeV}, \\ m_{D^*} &= 2.010 \text{ GeV}, & m_{D_s^{*-}} &= 2.112 \text{ GeV}, \\ f_D &= 223 \text{ MeV}, & f_{D_s^-} &= 274 \text{ MeV}. \end{aligned} \tag{76}$$

A lot of study has been made on the decay constants of $D_{(s)}$ mesons. Here we use the values from Ref. [23]. Since there are no experimental results of the decay constants of $D^*_{(s)}$ mesons, we use the relations between f_D and f_{D^*} derived from heavy quark effective theory [24]:

$$f_{D^*} = \sqrt{\frac{m_D}{m_{D^*}}} f_D, \qquad f_{D_s^{*-}} = \sqrt{\frac{m_{D_s^-}}{m_{D_s^*-}}} f_{D_s^-}, \tag{77}$$

which is different from Ref. [25].

With the D meson wave functions at hand, the decay width is given by

$$\Gamma = \frac{1}{32\pi} m_B^7 (1 - r^2)|A|^2, \tag{78}$$

where A is the decay amplitude defined in Eqs. (47)–(61). Finally, the branch ratio is

$$\text{Br} = \Gamma\hbar/\tau_{B_{(s)}}, \tag{79}$$

with $\tau_{B_{(s)}}$ as the lifetime of the $B_{(s)}$ meson. We take $\tau_{B^-} = 1.674 \times 10^{-12} s$, $\tau_{\bar{B}^0} = 1.542 \times 10^{-12} s$, $\tau_{\bar{B}^0_s} = 1.466 \times 10^{-12} s$, and $G_F = 1.16639 \times 10^{-5}\ \text{GeV}^{-2}$.

A. Results of fitting

Since $B \to DP$ decay channels have been measured experimentally with high precision, we use these experimental results to fit out the parameters of the candidate D meson DAs. Here we do not use the experimental results containing η or η' in the final states because there are uncertainties from the mixing. The six decay channels $B^- \to D^0\pi^-$, $B^- \to D^0K^-$, $\bar{B}^0 \to D^+\pi^-$, $\bar{B}^0 \to D^+K^-$, $\bar{B}^0 \to D^0\pi^0$, and $\bar{B}^0 \to D^0\bar{K}^0$ are used to fit the D meson wave functions and the parameter ω_b in the B meson wave function. The experimental results of these channels are given in Ref. [26] and these are collected in Table III. There are three degrees in the fitting with $\Phi^{(\text{MGen})}$ (ω_b, C_D, and ω), and two degrees for the others. The formula we used for fitting is

$$\chi^2 = \sum_i \frac{(\text{Br}_i^{\text{ex}} - \text{Br}_i^{\text{th}})^2}{\sigma_i^2}. \tag{80}$$

Here, i means the summation over the six decay channels. Br_i^{ex} (Br_i^{th}) is the experimental (theoretical) value of branch ratio, and σ_i is the uncertainty of the experimental value. In Table II we list the smallest χ^2 we get for all the D meson DAs. Except $\Phi^{(\text{KLS})}$ and $\Phi^{(\text{GN})}$, all the other DAs have a small $\chi^2_{\min}$. The $\Phi^{(\text{MGen})}$ is the best one, with its parameters fixed as $\omega_b = 0.38$ GeV, $C_D = 0.5$, $\omega = 0.1$ GeV. The χ^2 for $\bar{B}^0 \to D^0\bar{K}^0$ is the largest, this may indicate the large SU(3) breaking effect. With this channel excluded, the results for the χ^2 are good enough. In the calculation we will use $\Phi^{(\text{MGen})}$ for our numerical analysis of all the decay channels. For the D_s meson, we use $C_D = 0.4$, $\omega = 0.2$ GeV, with a little SU(3) breaking effect. In this case, we can see from Fig. 4 that the $\bar{s}$ quark in the D_s meson has a slightly larger momentum fraction than the $\bar{d}/\bar{u}$ quark in the D meson, which characterizes the slightly larger mass of the s quark.

Keeping in mind that the mass difference between the vector meson $D^*_{(s)}$ and pseudoscalar meson $D_{(s)}$ is small, we adopt the same DA for them also.

TABLE II. The smallest χ^2 for each kind of the D meson DA and the corresponding χ_i^2 (i represents the six channels we used for fitting) for every channel.

	$\chi_i^2[\Phi^{(\text{Gen})}]$	$\chi_i^2[\Phi^{(\text{MGen})}]$	$\chi_i^2[\Phi^{(\text{KLS})}]$	$\chi_i^2[\Phi^{(\text{GN})}]$	$\chi_i^2[\Phi^{(\text{KKQT})}]$	$\chi_i^2[\Phi^{(\text{Huang})}]$
$B^- \to D^0\pi^-$	1.30	6.42	14.23	13.06	0.28	2.49
$B^- \to D^0K^-$	0.11	0.06	0.63	1.15	0.36	0.05
$\bar{B}^0 \to D^+\pi^-$	0.23	0.00	8.73	6.74	3.10	0.80
$\bar{B}^0 \to D^+K^-$	0.03	0.15	9.88	2.79	0.03	0.04
$\bar{B}^0 \to D^0\pi^0$	2.49	2.16	95.45	74.03	2.30	8.36
$\bar{B}^0 \to D^0\bar{K}^0$	21.77	17.02	34.15	37.03	22.80	17.72
$\chi^2_{\min}$ (total)	25.92	25.82	163.06	134.8	28.87	29.44

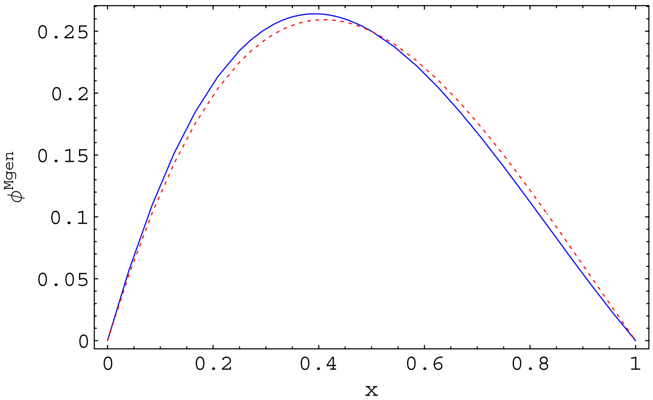

FIG. 4 (color online). The D meson distribution amplitude $\phi_D^{\rm MGen}(0.5, 0)$ (blue, solid line) and the D_s meson distribution amplitude $\phi_{D_s}^{\rm MGen}(0.4, 0)$ (red, dotted line).

B. Results for all the related channels and discussions

Our numerical results are listed in Tables III, IV, V, and VI. The first error in these entries is caused by the hadronic parameters in the $\bar{B}_{(s)}$ meson wave function (the decay constant and the shape parameter). We take $f_B = 0.19 \pm 0.025$ GeV and $f_{B_s} = 0.24 \pm 0.03$ GeV, $\omega_b^{B_s} = 0.50 \pm 0.05$ GeV. The second error arises from the higher order perturbative QCD corrections: the choice of the hard scales, defined in Eqs. (37) and (46), which vary from $0.75t$ to $1.25t$, and the uncertainty of $\Lambda_{\rm QCD}^{(4)} = 0.25 \pm 0.05$ GeV.

TABLE III. Branching ratios of $B_{(s)} \to DP$ decays calculated in the pQCD approach with experimental data (in units of 10^{-4}).

	Experimental results	Our results
$B^- \to D^0\pi^-$	47.5 ± 1.9	$51.9^{+14.6+3.2+1.5}_{-12.8-6.7-1.5}$
$B^- \to D^0K^-$	3.83 ± 0.45	$3.97^{+1.11+0.54+0.12}_{-0.98-0.91-0.12}$
$\bar{B}^0 \to D^+\pi^-$	26.5 ± 1.5	$26.9^{+7.5+5.2+0.8}_{-6.6-7.2-0.8}$
$\bar{B}^0 \to D^+K^-$	2.04 ± 0.57	$2.27^{+0.64+0.55+0.07}_{-0.56-0.65-0.07}$
$\bar{B}^0 \to D^0\pi^0$	2.61 ± 0.25	$2.14^{+0.60+0.61+0.06}_{-0.53-0.71-0.06}$
$\bar{B}^0 \to D^0\bar{K}^0$	0.523 ± 0.066	$0.23^{+0.06+0.07+0.01}_{-0.06-0.07-0.01}$
$\bar{B}^0 \to D^0\eta$	2.02 ± 0.21	$2.75^{+0.77+0.41+0.08}_{-0.68-0.38-0.08}$
$\bar{B}^0 \to D^0\eta'$	1.26 ± 0.21	$1.84^{+0.52+0.28+0.05}_{-0.45-0.25-0.05}$
$\bar{B}^0 \to D_s^+K^-$	0.269 ± 0.054	$0.73^{+0.20+0.19+0.02}_{-0.18-0.14-0.02}$
$\bar{B}_s^0 \to D^+\pi^-$		$(1.59^{+0.67+0.53+0.05}_{-0.30-0.23-0.05}) \times 10^{-2}$
$\bar{B}_s^0 \to D^0\pi^0$		$(0.88^{+0.19+0.18+0.03}_{-0.24-0.17-0.03}) \times 10^{-2}$
$\bar{B}_s^0 \to D^0\bar{K}^0$		$3.90^{+1.61+1.00+0.11}_{-1.16-0.97-0.11}$
$\bar{B}_s^0 \to D^0\eta$		$0.14^{+0.05+0.02+0.00}_{-0.05-0.04-0.00}$
$\bar{B}_s^0 \to D^0\eta'$		$0.33^{+0.11+0.04+0.01}_{-0.10-0.05-0.01}$
$\bar{B}_s^0 \to D_s^+\pi^-$	$38 \pm 3 \pm 13$	$19.6^{+10.6+6.3+0.6}_{-7.5-6.2-0.6}$
$\bar{B}_s^0 \to D_s^+K^-$		$1.70^{+0.87+0.53+0.05}_{-0.66-0.56-0.05}$

TABLE IV. Branching ratios of $B_{(s)} \to DV$ decays calculated in the pQCD approach with experimental data (in units of 10^{-4}).

	Experimental results	Our results
$B^- \to D^0\rho^-$	134 ± 18	$111^{+31.1+17.1+3.2}_{-27.3-23.2-3.2}$
$B^- \to D^0K^{*-}$	5.29 ± 0.45	$6.37^{+1.79+0.99+0.20}_{-1.57-1.48-0.20}$
$\bar{B}^0 \to D^+\rho^-$	75 ± 12	$67.0^{+18.8+11.2+2.0}_{-16.5-17.1-2.0}$
$\bar{B}^0 \to D^+K^{*-}$	4.60 ± 0.78	$3.83^{+1.07+0.82+0.12}_{-0.94-1.03-0.12}$
$\bar{B}^0 \to D^0\rho^0$	2.91 ± 0.40	$1.99^{+0.56+0.45+0.06}_{-0.49-0.61-0.06}$
$\bar{B}^0 \to D^0\omega$	2.60 ± 0.29	$4.08^{+1.14+0.63+0.12}_{-1.00-0.78-0.12}$
$\bar{B}^0 \to D^0\bar{K}^{*0}$	0.423 ± 0.064	$0.26^{+0.07+0.09+0.01}_{-0.06-0.07-0.01}$
$\bar{B}^0 \to D_s^+K^{*-}$	<8	$1.82^{+0.51+0.36+0.05}_{-0.45-0.46-0.05}$
$\bar{B}_s^0 \to D^+\rho^-$		$(7.88^{+2.83+1.99+0.24}_{-1.75-1.65-0.24}) \times 10^{-2}$
$\bar{B}_s^0 \to D^0\rho^0$		$(4.20^{+0.99+0.82+0.13}_{-1.17-0.89-0.13}) \times 10^{-2}$
$\bar{B}_s^0 \to D^0\bar{K}^{*0}$		$4.36^{+1.84+1.21+0.13}_{-1.31-1.14-0.13}$
$\bar{B}_s^0 \to D^0\omega$		$(3.61^{+0.97+0.50+0.11}_{-1.04-0.79-0.11}) \times 10^{-2}$
$\bar{B}_s^0 \to D^0\phi$		$0.30^{+0.11+0.07+0.01}_{-0.09-0.08-0.01}$
$\bar{B}_s^0 \to D_s^+\rho^-$		$47.0^{+24.9+15.3+1.36}_{-17.7-14.8-1.37}$
$\bar{B}_s^0 \to D_s^+K^{*-}$		$2.81^{+1.47+0.79+0.09}_{-1.09-0.85-0.09}$

The third error is from the uncertainties of the CKM matrix elements. In our calculation, we use

$$V_{cb} = (41.61^{+0.62}_{-0.63}) \times 10^{-3}, \qquad V_{ud} = 0.973\,85^{+0.00024}_{-0.00023}, \qquad V_{us} = 0.227\,15^{+0.00101}_{-0.00100}. \tag{81}$$

Among them, the hadronic inputs always give rise to the largest uncertainty, and the CKM matrix elements contribute little.

The first six channels in Table III are input values of the χ^2 fit program. Although we get a reasonable χ^2 in the fit, the branching ratio of $\bar{B}^0 \to D^0\bar{K}^0$ is about half of the experimental value. Comparing with the color-suppressed diagrams, the annihilation diagrams contribute little for $\bar{B}^0 \to D^0\bar{K}^0$ and $\bar{B}^0 \to D^0\pi^0$. So we can use them as a comparison. They have different CKM elements ($V_{cb}V_{us}^*$ for the former and $V_{cb}V_{ud}^*$ for the latter). Taking the factor $\frac{1}{\sqrt{2}}$ in the flavor wave function of the π meson into account, the ${\rm Br}(\bar{B}^0 \to D^0\bar{K}^0)$ is roughly one-tenth of ${\rm Br}(\bar{B}^0 \to D^0\pi^0)$. So this value of ${\rm Br}(\bar{B}^0 \to D^0\bar{K}^0)$ is theoretically reasonable. A similar argument is valid for ${\rm Br}(\bar{B}^0 \to D^{*0}\bar{K}^0)$.

Although we used only six $B \to DP$ channels to fix the D meson wave function, the results of other channels, especially those of $B \to DV$ and D^*P channels, agree very well with the current experimental measurements. It is easy to see that ${\rm Br}(\bar{B}^0 \to D^0\omega)$ is twice larger than ${\rm Br}(\bar{B}^0 \to D^0\rho)$, while their experimental values are close to each other. Both of the channels receive contributions from the color-suppressed diagrams and annihilation dia-

TABLE V. Branching ratios of $B_{(s)} \to D^*P$ decays calculated in the pQCD approach with experimental data (in units of 10^{-4}).

	Experimental results	Our results
$B^- \to D^{*0}\pi^-$	52.8 ± 2.8	$51.1^{+14.3+3.5+1.5}_{-12.6-6.5-1.5}$
$B^- \to D^{*0}K^-$	3.6 ± 1.0	$3.94^{+1.11+0.54+0.12}_{-0.97-0.90-0.12}$
$\bar{B}^0 \to D^{*+}\pi^-$	26.2 ± 1.3	$26.1^{+7.3+5.01+0.76}_{-6.4-7.01-0.76}$
$\bar{B}^0 \to D^{*+}K^-$	2.04 ± 0.47	$2.21^{+0.62+0.54+0.07}_{-0.54-0.63-0.07}$
$\bar{B}^0 \to D^{*0}\pi^0$	1.71 ± 0.28	$2.30^{+0.65+0.58+0.07}_{-0.57-0.76-0.07}$
$\bar{B}^0 \to D^{*0}\eta$	1.80 ± 0.31	$2.89^{+0.81+0.40+0.08}_{-0.71-0.40-0.08}$
$\bar{B}^0 \to D^{*0}\eta'$	1.21 ± 0.40	$1.94^{+0.54+0.27+0.06}_{-0.48-0.27-0.06}$
$\bar{B}^0 \to D^{*0}\bar{K}^0$	0.36 ± 0.12	$0.25^{+0.07+0.07+0.01}_{-0.06-0.07-0.01}$
$\bar{B}^0 \to D_s^{*+}K^-$	0.200 ± 0.064	$0.73^{+0.20+0.18+0.02}_{-0.18-0.16-0.02}$
$\bar{B}_s^0 \to D^{*+}\pi^-$		$(1.57^{+0.66+0.54+0.00}_{-0.29-0.23-0.00}) \times 10^{-2}$
$\bar{B}_s^0 \to D^{*0}\pi^0$		$(0.85^{+0.18+0.18+0.03}_{-0.22-0.16-0.03}) \times 10^{-2}$
$\bar{B}_s^0 \to D^{*0}\bar{K}^0$		$4.14^{+1.71+1.05+0.12}_{-1.18-0.96-0.12}$
$\bar{B}_s^0 \to D^{*0}\eta$		$0.15^{+0.06+0.02+0.00}_{-0.06-0.04-0.00}$
$\bar{B}_s^0 \to D^{*0}\eta'$		$0.35^{+0.12+0.04+0.01}_{-0.11-0.05-0.01}$
$\bar{B}_s^0 \to D_s^{*+}\pi^-$		$18.9^{+10.3+6.2+0.5}_{-7.2-5.9-0.6}$
$\bar{B}_s^0 \to D_s^{*+}K^-$		$1.64^{+0.84+0.51+0.05}_{-0.64-0.54-0.05}$

grams and these are of the same order of magnitude for the above-mentioned two processes. For color-suppressed diagrams, the $d\bar{d}$ of the flavor part contributes, whereas the $u\bar{u}$ part contributes to the annihilation diagrams. Amplitudes of these two kinds of diagrams have the same sign in the $\bar{B}^0 \to D^0\omega$ decay but different sign in the $\bar{B}^0 \to D^0\rho$ decay due to isospin. A similar situation exists for $\mathrm{Br}(\bar{B}^0 \to D^{*0}\omega)$ and $\mathrm{Br}(\bar{B} \to D^{*0}\rho)$.

The $\bar{B}^0 \to D_s^+K^-$ decay is a kind of pure annihilation-type decay dominant by the W exchange diagram. Our result is larger than the experiments and also than the previous pQCD calculations [27] due to the change of the choice of D meson wave functions. The annihilation-type diagrams are power suppressed in the pQCD approach, which is more sensitive to the hadronic wave functions.

For the decays $B \to D^*_{(s)}V$ in Table VI, we also estimate the ratios of transverse polarized contribution $R_T = |A_T|^2/(|A_T|^2 + |A_L|^2)$. We should mention that these results are just indicative, because transverse polarizations are power suppressed by r_V or r compared to the longitudinal contribution. Although the transverse polarization is suppressed in $B \to D^*V$ decays, in some channels, such as $\bar{B}^0 \to D^{*0}\rho^0$ and $\bar{B}^0 \to D^{*0}\bar{K}^{*0}$, etc., it has 40% contributions. The reason is that the dominant contribution in these channels is from $\mathcal{M}_{\mathrm{int}}$ in Eq. (39), which is x_3 suppressed, while the transverse contribution in Eq. (74) is only r suppressed. They are comparable numerically to make a large contribution for transverse polarizations in these color-suppressed channels. This mechanism is different from those charmless B decays where the dominant transverse polarizations are from the spacelike penguin (penguin annihilation) contributions [28]. Here the annihilation-type contributions are mainly from W ex-

TABLE VI. Predicted branching ratios of $B_{(s)} \to D^*V$ decays with experimental data (in units of 10^{-4}) together with the percentage of transverse polarizations R_T.

	Experimental Branching ratios		Branching ratios in pQCD	R_T
$B^- \to D^{*0}\rho^-$			$115^{+32.3+18.0+3.3}_{-28.3-24.0-3.3}$	0.04
$B^- \to D^{*0}K^{*-}$	8.3 ± 1.5	0.14	$6.70^{+1.88+1.10+0.20}_{-1.65-1.59-0.20}$	0.05
$\bar{B}^0 \to D^{*+}\rho^-$			$75.4^{+21.1+15.8+2.2}_{-18.5-18.4-2.2}$	0.15
$\bar{B}^0 \to D^{*+}K^{*-}$	3.20 ± 0.67		$4.63^{+1.30+1.01+0.14}_{-1.14-1.29-0.14}$	0.19
$\bar{B}^0 \to D^{*0}\bar{K}^{*0}$	3.73 ± 0.99		$4.05^{+1.14+0.51+0.12}_{-1.00-0.72-0.12}$	0.45
$\bar{B}^0 \to D^{*0}\omega$	2.68 ± 0.50		$5.72^{+1.60+0.78+0.17}_{-1.41-1.16-0.17}$	0.25
$\bar{B}^0 \to D^{*0}\bar{K}^{*0}$	<0.69		$0.53^{+0.15+0.08+0.02}_{-0.13-0.09-0.02}$	0.45
$\bar{B}^0 \to D_s^{*+}K^{*-}$			$1.92^{+0.54+0.26+0.06}_{-0.47-0.46-0.06}$	0.02
$\bar{B}_s^0 \to D^{*+}\rho^-$			$(8.17^{+2.28+1.22+0.25}_{-2.46-1.57-0.25}) \times 10^{-2}$	0.01
$\bar{B}_s^0 \to D^{*0}\rho^0$			$(4.09^{+1.19+0.69+0.12}_{-1.10-0.89-0.12}) \times 10^{-2}$	0.01
$\bar{B}_s^0 \to D^{*0}\bar{K}^{*0}$			$8.22^{+3.21+1.44+0.24}_{-2.68-1.90-0.24}$	0.42
$\bar{B}_s^0 \to D^{*0}\omega$			$(3.45^{+1.06+0.66+0.11}_{-0.77-0.64-0.11}) \times 10^{-2}$	0.01
$\bar{B}_s^0 \to D^{*0}\phi$			$0.50^{+0.19+0.08+0.02}_{-0.16-0.11-0.02}$	0.35
$\bar{B}_s^0 \to D_s^{*+}\rho^-$			$52.3^{+28.3+17.7+1.5}_{-19.5-16.6-1.5}$	0.13
$\bar{B}_s^0 \to D_s^{*+}K^{*-}$			$3.22^{+1.83+0.98+0.10}_{-1.24-0.95-0.10}$	0.17

change diagrams contributing little to transverse polarizations.

Our results are slightly different compared to the others calculated in literature using the pQCD approach [7,9] and the reason is the change of parameters. Most of the $\bar{B}^0(B^\pm)$ decay channels measured by the B factories are consistent with our calculations. For the $\bar{B}_s^0$ decays, only one channel is measured. Our predictions will soon be tested by the LHCb experiments.

For comparison with other methods, we also give the form factors at the maximal recoil:

$$\xi_+^{B\to D} = 0.51^{+0.15+0.05}_{-0.20-0.07}, \qquad \xi_+^{B_s\to D_s} = 0.44^{+0.11+0.06}_{-0.09-0.07}. \tag{82}$$

These are comparable with other methods [29].

If applying the naive factorization approach, we can get

$$A(\bar{B}^0 \to D^+\pi^-) = i\frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*(M_B^2 - M_D^2) \times f_\pi F^{B\to D}(M_\pi^2)a_1(D\pi), \tag{83}$$

$$\sqrt{2}A(\bar{B}^0 \to D^0\pi^0) = -i\frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*(M_B^2 - M_\pi^2) \times f_D F^{B\to\pi}(M_D^2)a_2(D\pi). \tag{84}$$

Substituting our results for $A(\bar{B}^0 \to D^+\pi^-)$ and $A(\bar{B}^0 \to D^0\pi^0)$ in Eqs. (83) and (84), we can extract the Bauer-Stech-Wirbel parameters a_1 and a_2 from our pQCD approach:

$$|a_2/a_1| = 0.48, \qquad \mathrm{Arg}(a_2/a_1) = -37.6°. \tag{85}$$

If the annihilation diagrams' contribution is excluded, the results are

$$|a_2/a_1| = 0.56, \qquad \mathrm{Arg}(a_2/a_1) = -55.7°. \tag{86}$$

Indeed, the large $|a_2/a_1|$ implies that the color-suppressed decays are not very much suppressed as previously expected [3]. The relative strong phase between the two contributions is not small as naive expectations. In pQCD, the strong phase of a_1 is mainly from the $\mathcal{M}_{\rm ext}$ and $\mathcal{M}_{\rm exc}$, while for a_2, the $\mathcal{M}_{\rm int}$ contribution is largest, even larger than $\mathcal{M}_{\rm exc}$'s. These results are consistent with recent direct studies from experiments [30]. But the difference is that our results come from direct dynamical calculation and not from fit.

V. CONCLUSION

In this paper, we have calculated the branch ratios of $B_{(s)} \to D_{(s)}P$, $D_{(s)}^*P$, $D_{(s)}V$, and $D_{(s)}^*V$ channels, with the D meson wave function obtained through fitting. We have also calculated the ratios of transverse polarized contributions in $B \to D^*V$ decays. Most of the results agreed well with the experiments. It seems that there is a disagreement with the experimental data in the relative size of branching ratios for $\bar{B}^0 \to D^{(*)0}\rho$ and $\bar{B}^0 \to D^{(*)0}\omega$. Some channels of the $B \to D^*V$ decays may receive a large contribution from the transverse polarization. The results obtained for $\bar{B}_s^0 \to D_{(s)}P$, $D_{(s)}V$, $D_{(s)}^*P$, and $D_{(s)}^*V$ decays will be tested in the future experiments.

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APPENDIX: pQCD FUNCTIONS

Jet function appears in Eqs. (31)–(33) and in Eqs. (70)–(72) is

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c. \tag{A1}$$

The value of c in the above equation is 0.5 in this paper and the $S_j(x_i)$ ($j = B$, C, P, or V) functions in Sudakov form factors in (34) and (41) are

$$S_B(t) = s\left(x_1\frac{m_B}{\sqrt{2}}, b_1\right) + 2\int_{1/b_1}^t \frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{A2}$$

$$S_C(t) = s\left(x_2\frac{m_B}{\sqrt{2}}, b_2\right) + 2\int_{1/b_2}^t \frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{A3}$$

$$\begin{aligned} S_V(t) &= S_P(t) \\ &= s\left(x_3\frac{m_B}{\sqrt{2}}, b_3\right) + s\left((1-x_3)\frac{m_B}{\sqrt{2}}, b_3\right) \\ &\quad + 2\int_{1/b_3}^t \frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \end{aligned} \tag{A4}$$

with the quark anomalous dimension $\gamma_q = -\alpha_s/\pi$. The explicit form for the function $s(Q, b)$ is

$$\begin{aligned} s(Q,b) &= \frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right) - \frac{A^{(1)}}{2\beta_1}(\hat{q}-\hat{b}) + \frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}} - 1\right) \\ &\quad - \left[\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1}\ln\left(\frac{e^{2\gamma_E-1}}{2}\right)\right]\ln\left(\frac{\hat{q}}{\hat{b}}\right) \\ &\quad + \frac{A^{(1)}\beta_2}{4\beta_1^3}\hat{q}\left[\frac{\ln(2\hat{q})+1}{\hat{q}} - \frac{\ln(2\hat{b})+1}{\hat{b}}\right] \\ &\quad + \frac{A^{(1)}\beta_2}{8\beta_1^3}[\ln^2(2\hat{q}) - \ln^2(2\hat{b})], \end{aligned} \tag{A5}$$

where the variables are defined by

$$\hat{q} \equiv \ln[Q/(\sqrt{2}\Lambda)], \qquad \hat{b} \equiv \ln[1/(b\Lambda)], \tag{A6}$$

and the coefficients $A^{(i)}$ and β_i are

$$\beta_1 = \frac{33 - 2n_f}{12}, \qquad \beta_2 = \frac{153 - 19n_f}{24}, \qquad A^{(1)} = \frac{4}{3},$$
$$A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_1 \ln\left(\frac{1}{2}e^{\gamma_E}\right), \qquad \text{(A7)}$$

n_f is the number of the quark flavors and γ_E is the Euler constant. We will use the one-loop running coupling constant, i.e. we pick up the four terms in the first line of the expression for the function $s(Q, b)$.

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