Isospin analysis of D^0 decay to three pions

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The final state of the decay $D^0 \to \pi^+\pi^-\pi^0$ is analyzed in terms of isospin eigenstates. It is shown that the final state is dominated by the isospin-0 component. This suggests that isospin considerations may provide insight into this and perhaps other D^0-meson decays. We also discuss the isospin nature of the nonresonant contribution in the decay, which can be further understood by studying the decay $D^0 \to \pi^0\pi^0\pi^0$.

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I. INTRODUCTION

An analysis of the resonant substructure in the decay $D^0 \to \pi^+\pi^-\pi^0$ was recently performed by the *BABAR* Collaboration [1]. The Dalitz-plot distribution of the $D^0 \to \pi^+\pi^-\pi^0$ events (Fig. 1) shows a clear six-fold symmetry, with the probability density function vanishing along three axes. As first described by Zemach [2] and noted in Ref. [1], this behavior is indicative of a final state with isospin $I = 0$.

In the *BABAR* analysis, the Dalitz-plot distribution is described by a probability density function formed from a wave function taken to be the sum of N_r contributions,

$$\psi(s_+, s_-) = \sum_r^{N_r} B_r g_r(s_+, s_-), \tag{1}$$

where $s_+ \equiv (p_{\pi^+} + p_{\pi^0})^2$ and $s_- \equiv (p_{\pi^-} + p_{\pi^0})^2$ are the squared invariant masses of the $\pi^+\pi^0$ and $\pi^-\pi^0$ pairs, respectively, B_r is a complex coefficient, and $g_r(s_+, s_-)$ is the distribution of contribution r, whose functional form is outlined in Ref. [1]. The definitions of $g_r(s_+, s_-)$ used here differ from that of Ref. [1], in that we define these functions to be normalized over the Dalitz plot,

$$\int ds_+ ds_- |g_r(s_+, s_-)|^2 = 1. \tag{2}$$

The values for the B_r coefficients consistent with Eqs. (1) and (2) are reproduced in Table I.

The goal of this paper is to quantify the extent to which the $I = 0$ component dominates the final state and learn about the contributions of the other isospin eigenstates. In Sec. II we perform an isospin analysis of the $\pi^+\pi^-\pi^0$ final state. The observed dominance of the $I = 0$ component suggests that isospin considerations are useful for developing an understanding of this decay. In Sec. III we discuss our results, the nature of the nonresonant contribution to the decay, a possible mechanism for the observed $I = 0$ dominance, and further measurements that will help clarify outstanding questions.

II. ISOSPIN DECOMPOSITION

Next, we analyze the decay $D^0 \to \pi^+\pi^-\pi^0$ in terms of isospin eigenstates. The 3-pion final state can be described in terms of the total isospin I, the isospin I_{12} of two of the three pions, and the z-projection I^z, which is always 0 for this final state. The seven eigenstates $|I(I_{12})\rangle$ of these

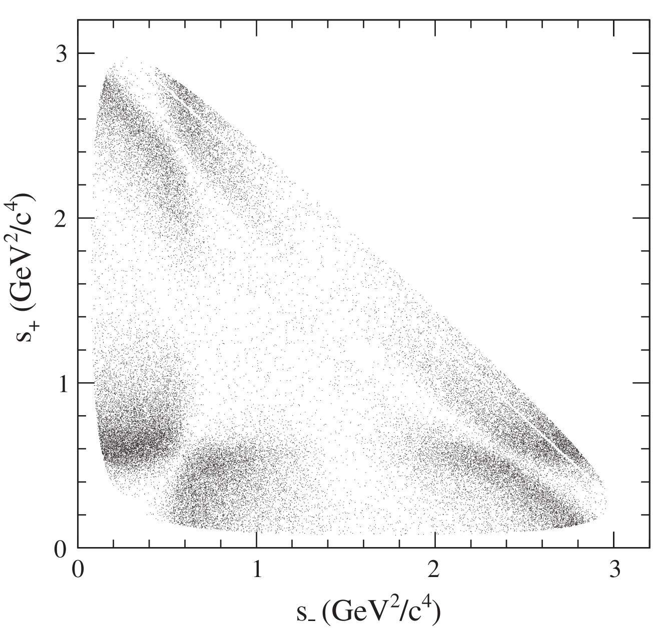

FIG. 1. The Dalitz-plot distribution of $D^0 \to \pi^+\pi^-\pi^0$ events, from Ref. [1]. The fine diagonal line at low $\pi^+\pi^-$ mass corresponds to the decays $D^0 \to K_S^0\pi^0$, which have been removed.

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TABLE I. Amplitude coefficients $B_r = |B_r|e^{i\phi_r}$ of the contributing final states of the decay $D^0 \to \pi^-\pi^+\pi^0$, adapted from Ref. [1]. The $f_0(400)$ was labeled $\sigma(400)$ in Ref. [1].

Final state r	Amplitude $\lvert B_r \rvert$	Phase ϕ_r (°)
Nonresonant	$0.106 \pm 0.013 \pm 0.014$	$-11 \pm 4 \pm 2$
$\rho(770)^+\pi^-$	1	0.0
$\rho(770)^0\pi^0$	$0.588 \pm 0.006 \pm 0.002$	$16.2 \pm 0.6 \pm 0.4$
$\rho(770)^-\pi^+$	$0.714 \pm 0.008 \pm 0.002$	$-2.0 \pm 0.6 \pm 0.6$
$\rho(1450)^+\pi^-$	$0.040 \pm 0.011 \pm 0.024$	$-146 \pm 18 \pm 24$
$\rho(1450)^0\pi^0$	$0.062 \pm 0.012 \pm 0.007$	$10 \pm 8 \pm 13$
$\rho(1450)^-\pi^+$	$0.154 \pm 0.010 \pm 0.007$	$16 \pm 3 \pm 3$
$\rho(1700)^+\pi^-$	$0.236 \pm 0.019 \pm 0.014$	$-17 \pm 2 \pm 3$
$\rho(1700)^0\pi^0$	$0.267 \pm 0.016 \pm 0.014$	$-17 \pm 2 \pm 2$
$\rho(1700)^-\pi^+$	$0.210 \pm 0.012 \pm 0.007$	$-50 \pm 3 \pm 3$
$f_0(980)\pi^0$	$0.056 \pm 0.005 \pm 0.006$	$-59 \pm 5 \pm 4$
$f_0(1370)\pi^0$	$0.072 \pm 0.010 \pm 0.010$	$156 \pm 9 \pm 6$
$f_0(1500)\pi^0$	$0.074 \pm 0.007 \pm 0.007$	$12 \pm 9 \pm 4$
$f_0(1710)\pi^0$	$0.072 \pm 0.010 \pm 0.011$	$51 \pm 8 \pm 7$
$f_2(1270)\pi^0$	$0.130 \pm 0.005 \pm 0.026$	$-171 \pm 3 \pm 4$
$f_0(400)\pi^0$	$0.104 \pm 0.008 \pm 0.017$	$8 \pm 4 \pm 8$

quantum numbers that also satisfy $I^z = 0$ can be written as a linear combination of the three-pion final states using the appropriate Clebsh-Gordan coefficients:

$$\begin{aligned}
|3(2)\rangle &= \frac{1}{\sqrt{10}}(|+0-\rangle + |0+-\rangle + |+-0\rangle + |-+0\rangle \\
&\quad + |0-+\rangle + |-0+\rangle + 2|000\rangle), \\
|2(2)\rangle &= \frac{1}{2}(|+0-\rangle + |0+-\rangle - |0-+\rangle - |-0+\rangle), \\
|1(2)\rangle &= \frac{1}{\sqrt{60}}[3(|+0-\rangle + |0+-\rangle + |0-+\rangle + |-0+\rangle) \\
&\quad - 2(|+-0\rangle + |-+0\rangle) - 4|000\rangle], \\
|2(1)\rangle &= \frac{1}{\sqrt{12}}[|+0-\rangle - |0+-\rangle + 2(|+-0\rangle - |-+0\rangle) \\
&\quad + |0-+\rangle - |-0+\rangle], \\
|1(1)\rangle &= \frac{1}{2}(|+0-\rangle - |0+-\rangle - |0-+\rangle + |-0+\rangle), \\
|0(1)\rangle &= \frac{1}{\sqrt{6}}(|+0-\rangle - |0+-\rangle - |+-0\rangle + |-+0\rangle \\
&\quad + |0-+\rangle - |-0+\rangle), \\
|1(0)\rangle &= \frac{1}{\sqrt{3}}(|+-0\rangle - |000\rangle + |-+0\rangle),
\end{aligned} \tag{3}$$

where we have used the notation

$$\begin{aligned}
|+0-\rangle &= |1,1\rangle|1,0\rangle|1,-1\rangle = |\pi^+\rangle|\pi^0\rangle|\pi^-\rangle, \\
|000\rangle &= |1,0\rangle|1,0\rangle|1,0\rangle = |\pi^0\rangle|\pi^0\rangle|\pi^0\rangle,
\end{aligned} \tag{4}$$

etc., and it is implied that the first two pions are in an isospin eigenstate whose eigenvalue is indicated by the bracketed number I_{12}.

The three states in Eq. (3) for which $I_{12} = 1$ are identified as those with a $\rho(770)$, $\rho(1450)$, or $\rho(1700)$. We denote these states as $\rho_n\pi$ according to their radial excitation quantum number $n \in \{1, 2, 3\}$, and use ρ^+, ρ^0, and ρ^- to indicate any linear combination of these states with specific electric charge. We define the ρ states to be

$$\begin{aligned}
|\rho^+\rangle &= |1,1\rangle = \frac{1}{\sqrt{2}}(|+0\rangle - |0+\rangle), \\
|\rho^0\rangle &= -|1,0\rangle = \frac{1}{\sqrt{2}}(|-+\rangle - |+-\rangle), \\
|\rho^-\rangle &= |1,-1\rangle = \frac{1}{\sqrt{2}}(|0-\rangle - |-0\rangle),
\end{aligned} \tag{5}$$

where the minus sign in the $|\rho^0\rangle$ definition implies that there is no sign change under cyclic permutations of the three pions, maintaining consistency with the definitions used in Ref. [1]. Given Eq. (5), the $I_{12} = 1$ states in Eq. (3) can be written as

$$\begin{aligned}
|2(1)\rangle &= \frac{1}{\sqrt{6}}(|\rho^+\pi^-\rangle - 2|\rho^0\pi^0\rangle + |\rho^-\pi^+\rangle), \\
|1(1)\rangle &= \frac{1}{\sqrt{2}}(|\rho^+\pi^-\rangle - |\rho^-\pi^+\rangle), \\
|0(1)\rangle &= \frac{1}{\sqrt{3}}(|\rho^+\pi^-\rangle + |\rho^0\pi^0\rangle + |\rho^-\pi^+\rangle),
\end{aligned} \tag{6}$$

where the sign of each $|\rho\pi\rangle$ state is such that it is symmetric under cyclic permutations of the three pions and antisymmetric under the exchange of any pair of pions.

The $\pi^+\pi^-\pi^0$ part of the state $|1(0)\rangle$ is identified as the sum of the contributions involving the two-body, $I = 0$ resonances f_i, with $i = 0, 2$. We therefore write

$$|1(0)\rangle = \frac{1}{\sqrt{3}}(\sqrt{2}|f\pi^0\rangle - |000\rangle). \tag{7}$$

Since there are no $I = 2$ resonances in Table I, the $I = 2$ states in Eq. (3) have no resonant contributions. However, the symmetry of the $\pi^+\pi^-\pi^0$ components of $|3(2)\rangle$ indicates that it may be identified with the nonresonant contribution of Table I. Alternatively, the nonresonant contribution may constitute the $\pi^+\pi^-\pi^0$ component of the symmetric $I = 1$ state

$$\begin{aligned}
|1(S)\rangle &\equiv \frac{2}{3}|1(2)\rangle + \frac{\sqrt{5}}{3}|1(0)\rangle \\
&= \frac{1}{\sqrt{15}}(|+0-\rangle + |0+-\rangle + |+-0\rangle + |-+0\rangle \\
&\quad + |0-+\rangle + |-0+\rangle - 3|000\rangle).
\end{aligned} \tag{8}$$

In principle, the observed nonresonant state may be a superposition of $|1(S)\rangle$ and $|3(2)\rangle$. However, the $|1(S)\rangle$ state is expected to dominate, due to the following argument. The four-quark final state produced by the weak decay $c \to d\bar{d}u$, shown in Fig. 2, cannot have $I = 3$. Since production

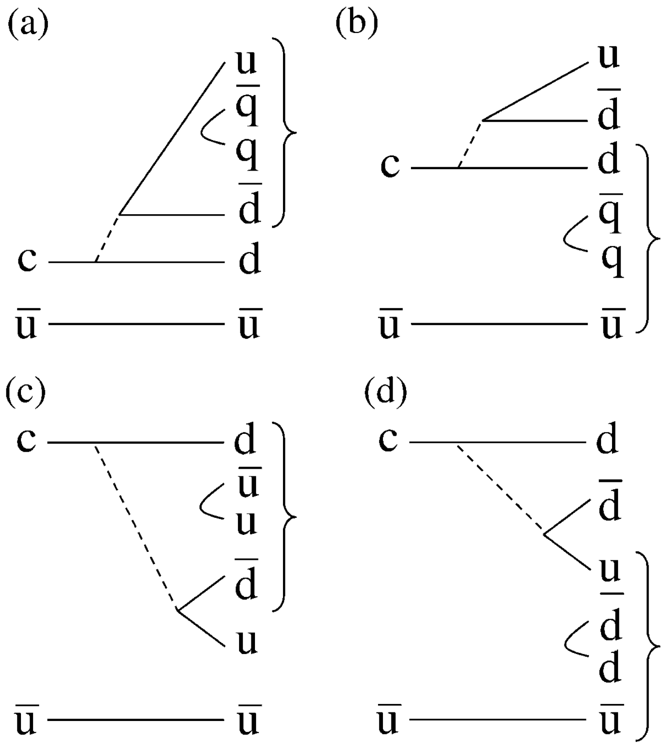

FIG. 2. Feynman diagrams for the decay $D^0 \to \pi^+\pi^-\pi^0$. With curly brackets indicating a resonance, the diagrams correspond to the decays (a) $D^0 \to \rho^+\pi^-$, (b) $D^0 \to \pi^+\rho^-$, and (c, d) $D^0 \to \rho^0\pi^0$ or $D^0 \to f\pi^0$.

of the third $q\bar{q}$ pair will be dominated by the strong-interaction, it will not change the total isospin. Therefore, $I = 3$ is disfavored. It is also possible that a very broad, $\pi^+\pi^-$ S-wave resonance is present in these decays, and that it was partly described by the constant nonresonant term in the fit in Ref. [1]. In that case, it would contribute only to the $|1(0)\rangle$ isospin eigenstate.

In what follows, we take the nonresonant contribution $|NR\rangle$ to be due only to $|1(S)\rangle$. Then Eqs. (7) and (8) yield the relation

$$|1(2)\rangle = \frac{3}{\sqrt{10}}|NR\rangle - \sqrt{\frac{5}{6}}|f\pi^0\rangle - \frac{2}{\sqrt{15}}|000\rangle. \quad (9)$$

We now reorder the terms of Eq. (1) according to their I_{12} eigenvalues:

$$\begin{aligned}\psi(s_+, s_-) = {}& B_{\rm NR} g_{\rm NR}(s_+, s_-) + B_{\rho^+\pi^-} g_{\rho^+\pi^-}(s_+, s_-) \\ &+ B_{\rho^0\pi^0} g_{\rho^0\pi^0}(s_+, s_-) \\ &+ B_{\rho^-\pi^+} g_{\rho^-\pi^+}(s_+, s_-) + B_{f\pi^0} g_{f\pi^0}(s_+, s_-),\end{aligned} \quad (10)$$

where the first term is the nonresonant term, the last is a sum over the six final states with $I_{12} = 0$ resonances listed at the bottom of Table I, and each of the second, third, and fourth terms is a sum over the three $I_{12} = 1$ $\rho\pi$ states. For example,

$$g_{\rho^+\pi^-}(s_+, s_-) \equiv \frac{S_{\rho^+\pi^-}}{N_{\rho^+\pi^-}} \exp[-i\delta_{\rho^+\pi^-}], \quad (11)$$

where

$$\begin{aligned} S_{\rho^+\pi^-} &\equiv \sum_{n=1}^{3} B_{\rho_n^+\pi^-} g_{\rho_n^+\pi^-}(s_+, s_-), \\ \delta_{\rho^+\pi^-} &\equiv \arg(S_{\rho^+\pi^-}), \\ N_{\rho^+\pi^-} &\equiv \sqrt{\int\int ds_+ ds_- |S_{\rho^+\pi^-}|^2}, \end{aligned} \quad (12)$$

and ρ_n ($n = 1, 2, 3$) indicates the three ρ resonances of Table I. With these definitions, the wave function $g_{\rho^+\pi^-}(s_+, s_-)$ is explicitly normalized and has vanishing average phase. Requiring that Eq. (10) be identical to (1) leads to the following values for the coefficients of Eq. (10):

$$\begin{aligned} B_{\rm NR} &= 0.1066 e^{-i11.4^\circ}, \\ B_{\rho^+\pi^-} &\equiv N_{\rho^+\pi^-} \exp[i\delta_{\rho^+\pi^-}] = 1.1976 e^{-i4.3^\circ}, \\ B_{\rho^0\pi^0} &\equiv N_{\rho^0\pi^0} \exp[i\delta_{\rho^0\pi^0}] = 0.8867 e^{i6.3^\circ}, \\ B_{\rho^-\pi^+} &\equiv N_{\rho^-\pi^+} \exp[i\delta_{\rho^-\pi^+}] = 1.0077 e^{-i8.2^\circ}, \\ B_{f\pi^0} &\equiv N_{f\pi^0} \exp[i\delta_{f\pi^0}] = 0.0700 e^{i40.0^\circ}, \end{aligned} \quad (13)$$

where the symbols N_s and δ_s for final state s are defined analogously to Eq. (12). The value of $B_{\rm NR}$ is taken from Table I and the rest are calculated numerically as in Eqs. (11) and (12). The phase convention is that of Table I, namely, $\delta_{\rho_1^+\pi^-} \equiv 0$.

Next, we write the wave function of Eq. (10) as a sum over the Dalitz-plot representations of the eigenstates of I and I_{12} of Eq. (3):

$$\begin{aligned}\psi(s_+, s_-) = {}& C_{1(2)} M_{1(2)}(s_+, s_-) + C_{2(1)} M_{2(1)}(s_+, s_-) \\ &+ C_{1(1)} M_{1(1)}(s_+, s_-) + C_{0(1)} M_{0(1)}(s_+, s_-) \\ &+ C_{1(0)} M_{1(0)}(s_+, s_-),\end{aligned} \quad (14)$$

where $M_{I(I_{12})}(s_+, s_-)$ is the normalized distribution function of the eigenstate $|I(I_{12})\rangle$, obtained by linearly combining the functions $g_x(s_+, s_-)$ of Eq. (10) with the coefficients of either Eq. (6) and (7), or (9). Terms for $|3(2)\rangle$ and $|2(2)\rangle$ were not included in Eq. (14), as reasoned earlier. Then from the definition of $M_{I(I_{12})}(s_+, s_-)$ follows the desired transformation between the resonance-based fit coefficients and the isospin coefficients:

TABLE II. Correlation matrix for the $C_{I(I12)}$ amplitude coefficients of Eq. (16).

	$\|C_{1(2)}\|$	$\arg(C_{1(2)})\|$	$\|C_{2(1)}\|$	$\arg(C_{2(1)})$	$\|C_{1(1)}\|$	$\arg(C_{1(1)})$	$\|C_{1(0)}\|$	$\arg(C_{1(0)})$
$\|C_{1(2)}\|$	1	−0.120	0.105	−0.018	0.631	0.110	0.279	0.657
$\arg(C_{1(2)})$	−0.120	1	0.062	0.106	−0.211	0.539	−0.760	0.136
$\|C_{2(1)}\|$	0.105	0.062	1	0.008	0.179	0.029	−0.017	0.078
$\arg(C_{2(1)})$	−0.018	0.106	0.008	1	0.148	0.333	0.110	0.151
$\|C_{1(1)}\|$	0.631	−0.211	0.179	0.148	1	0.050	0.259	0.288
$\arg(C_{1(1)})$	0.110	0.539	0.029	0.333	0.050	1	−0.296	0.097
$\|C_{1(0)}\|$	0.279	−0.760	−0.017	0.110	0.259	−0.296	1	0.077
$\arg(C_{1(0)})$	0.657	0.136	0.078	0.151	0.288	0.097	0.077	1

$$\begin{aligned} C_{1(2)} &= \frac{\sqrt{10}}{3} B_{\mathrm{NR}}, \\ C_{2(1)} &= \frac{1}{\sqrt{6}} (B_{\rho^+\pi^-} - 2B_{\rho^0\pi^0} + B_{\rho^-\pi^+}), \\ C_{1(1)} &= \frac{1}{\sqrt{2}} (B_{\rho^+\pi^-} - B_{\rho^-\pi^+}), \\ C_{0(1)} &= \frac{1}{\sqrt{3}} (B_{\rho^+\pi^-} + B_{\rho^0\pi^0} + B_{\rho^-\pi^+}), \\ C_{1(0)} &= \sqrt{\frac{3}{2}} B_{f\pi^0} + \sqrt{\frac{5}{6}} C_{1(2)}, \end{aligned} \tag{15}$$

where the expressions for $C_{1(0)}$ and $C_{1(2)}$ were chosen so as to satisfy the $\pi^+\pi^-\pi^0$ projection of Eqs. (7) and (9).

Taking the numerical values of the B_r coefficients from Eq. (13) and Table I, Eq. (15) gives

$$\begin{aligned} C_{1(2)} &= (0.0629 \pm 0.0028) \exp[i(-8.9 \pm 2.6)°], \\ C_{2(1)} &= (0.1395 \pm 0.0016) \exp[i(-42.5 \pm 0.7)°], \\ C_{1(1)} &= (0.0814 \pm 0.0023) \exp[i(18.0 \pm 2.0)°], \\ C_{0(1)} &\equiv 1, \\ C_{1(0)} &= (0.0954 \pm 0.0052) \exp[i(14.5 \pm 2.4)°], \end{aligned} \tag{16}$$

where we have normalized the coefficients so that $C_{0(1)} = 1$. The errors reflect the full error matrix of the results presented in Table I [3]. The correlation matrix for these coefficients are given in Table II.

Equation (16) quantifies the observation, made qualitatively in Ref. [1] on the basis of the symmetry exhibited by the Dalitz-plot distribution, that the final state of the decay $D^0 \to \pi^+\pi^-\pi^0$ is dominated by an $I = 0$ component.

III. DISCUSSION AND CONCLUSIONS

We have analyzed the relative contributions of different components to the decay $D^0 \to \pi^+\pi^-\pi^0$ using results published by *BABAR* [1]. It appears that isospin considerations may form a solid basis for understanding the observed decay pattern, as the amplitude of the $|0(1)\rangle$ final state dominates by factors of seven or more over the other isospin components. This dominance has no natural explanation in the decay mechanisms suggested by the factorization-motivated diagrams of this decay, shown in Fig. 2. While factorization is useful in predicting the behavior of B-meson decays, it is not as successful when applied to the lighter D mesons. The observed $|0(1)\rangle$ dominance in the decay $D^0 \to \pi^+\pi^-\pi^0$ may lead to a better general understanding of charmed meson decays. Alternatively, perhaps the $I = 0$ component is enhanced by the presence of a yet-unknown and possibly broad state with this quantum number, which couples strongly to three pions. An inclusive search for such a state may answer this question.

In conducting the isospin analysis, we took only the $\pi^+\pi^-\pi^0$ projections of the isospin-eigenstates $|1(2)\rangle$ and $|1(0)\rangle$. The CLEO Collaboration [4] has set an upper limit of 3.4×10^{-4} on the branching fraction $\mathcal{B}(D^0 \to \pi^0\pi^0\pi^0)$. Together with the *BABAR* [5] measurement of $\mathcal{B}(D^0 \to \pi^+\pi^-\pi^0) = (1.493 \pm 0.057)\%$, this implies an upper limit on the amplitude ratio $A(D^0 \to \pi^0\pi^0\pi^0)/A(D^0 \to \pi^+\pi^-\pi^0) < 0.15$, consistent with the suppression seen in the coefficients $C_{1(2)}$ and $C_{1(0)}$, and the expectation from Eqs. (7) and (9).

As discussed above, the $\pi^+\pi^-\pi^0$ nonresonant amplitude may be a combination of $|3(2)\rangle$, $|1(S)\rangle$, and a broad $\pi^+\pi^-$ resonance term in $|1(0)\rangle$. If it is due only to the $|3(2)\rangle$, Eq. (3) predicts the ratio between the nonresonant $\pi^0\pi^0\pi^0$ and $\pi^+\pi^-\pi^0$ amplitudes to be $R_{\mathrm{NR}} = \sqrt{2/3}$. By contrast, $|1(S)\rangle$-dominance leads to $R_{\mathrm{NR}} = \sqrt{3/2}$, from Eq. (8). In the $|1(0)\rangle$ case, the ratio between the nonresonant $\pi^0\pi^0\pi^0$ amplitude and the sum of the $f\pi^0$ and nonresonant $\pi^+\pi^-\pi^0$ amplitudes should be $1/\sqrt{2}$. We note that the ratio $R_{\mathrm{NR}} = \sqrt{1.556 \pm 0.012}$ is observed in K_L decays to three pions, where the nonresonant contribution accounts for over 95% of the branching fractions. The same situation exists in the decay $\eta \to \pi^+\pi^-\pi^0$. This strengthens the justification of our choice to identify the nonresonant contribution with the $|1(S)\rangle$ state. In any case, the arguments given here demonstrate that a measurement of the branching fraction $\mathcal{B}(D^0 \to \pi^0\pi^0\pi^0)$ and, possibly, an analysis of this mode's Dalitz-plot distribution should shed more light on the role of isospin symmetry in D^0 decays to three-pion final states.

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[1] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **99**, 251801 (2007).
[2] C. Zemach, Phys. Rev. **133**, B1201 (1964).
[3] K. Mishra, Ph.D. thesis, University of Cincinnati [SLAC-Report-893, 72-77, 2008].
[4] P. Rubin *et al.* (CLEO Collaboration), Phys. Rev. Lett. **96**, 081802 (2006).
[5] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **74**, 091102 (2006).