Is there a $\pi\Lambda N$ bound state?

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We have searched for bound states in the $\pi\Lambda N$ system by solving the nonrelativistic Faddeev equations, as well as a relativistic version, with input separable πN, $\pi\Lambda$, and ΛN interactions. A bound-state solution, driven by the $\Delta(1232)$ and the $\Sigma(1385)$ p-wave meson-baryon resonances, was found in the channel $(I, J^P) = (\frac{3}{2}, 2^+)$, provided the Λ laboratory momentum at which the $\Lambda N\ ^3S_1$ phase shift becomes negative is larger than $p_{\rm lab} \sim 750$–800 MeV/c. Other strange and charmed $\pi BB'$ systems that might have bound states of a similar nature are listed.

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I. INTRODUCTION

Experimental searches for dibaryons have been inconclusive. In the nonstrange sector, pion-initiated reactions and pion-production reactions were used to search for low-lying narrow πNN resonances below the ΔN threshold, aiming particularly at channels with quantum numbers inaccessible to NN configurations [1]. Several broad NN resonances are known near the ΔN and $\Delta\Delta$ thresholds and may be attributed to quasibound states in these channels, as summarized recently [2]. In the strange sector, extensive searches have been conducted [3–5] for the H dibaryon, with strangeness $S = -2$ and quantum numbers $(I, J^P) = (0, 0^+)$, which originally was predicted to lie below the $\Lambda\Lambda$ threshold [6]. Only few dedicated searches for $S = -1$ dibaryons have been reported, for low-lying $L = 1$ ΛN resonances in singlet and triplet configurations that were predicted in a quark-model study by Mulders *et al.* [7] near the ΣN threshold, but negative results particularly for the singlet resonance were reported in K^--initiated experiments [8,9].

Here we look for low-lying $S = -1$ dibaryons associated with a "molecular" $\pi\Lambda N$ structure, by solving three-body Faddeev equations with pairwise phenomenological separable interactions. The ΛN system is known to be unbound, with s-wave forces in both singlet and triplet states that are overall attractive and which yield scattering lengths of order -2 fm [10]. The question is whether or not the pion is able to bind an s-wave ΛN pair within a $\pi\Lambda N$ bound state, or a resonance. Since the s-wave πN and $\pi\Lambda$ forces are very weak [11], we consider the p-wave resonances $\Delta(1232)$ $(\frac{3}{2}, \frac{3}{2}^+)$ and $\Sigma(1385)$ $(1, \frac{3}{2}^+)$, respectively, thus studying the $\pi\Lambda N$ three-body system with s-wave baryons and a p-wave pion in a $(\frac{3}{2}, 2^+)$ state, where the ΛN subsystem is necessarily in the $^3S_1$ configuration. For first orientation we neglect the $^3S_1 - {}^3D_1$ channel coupling which becomes important near and above the ΣN threshold.

For all three partitions of this $(\frac{3}{2}, 2^+)$ state of the $\pi\Lambda N$ system into an interacting pair and a spectator, the orbital angular momenta, spins, and isospins couple to their maximal values and, therefore, the spin and isospin recoupling coefficients are equal to one. This three-body state is likely to represent a state with maximum possible attraction. Furthermore, the fact that the spin and isospin recoupling coefficients are equal to one allows for a formal reduction of the present three-body problem to that of three spinless (and isospinless) particles. We comment that a similar choice of $(I, J^P) = (2, 2^+)$ for πNN, with each πN pair interacting in the $\Delta(1232)$-resonance $(\frac{3}{2}, \frac{3}{2}^+)$ channel, is impossible since a two-nucleon $I = 1$, $^3S_1$ state is forbidden by the Pauli principle.

Since we are interested in the bound-state region of the $\pi\Lambda N$ system, it is justified in first approximation to neglect the coupling to the higher-mass systems $\bar{K}NN$, $\pi\Sigma N$, and $K\Xi N$. The effect of the coupling to these higher-mass channels will be partly taken into account by adjusting the interactions within the $\pi\Lambda N$ system to the available experimental information on the two-body subsystems. Less justified is the neglect of the coupling to the lower-mass ΣN system, with a threshold about 60 MeV below that of $\pi\Lambda N$. This coupling renders $\pi\Lambda N$ bound states into quasibound states through shifting and broadening the zero-width bound states obtained when the coupling is disregarded, unless the binding energy exceeds approximately 60 MeV and the $\pi\Lambda N$ state is genuinely bound. In the present, exploratory calculation we ignore the coupling to ΣN. Potential models generally yield fairly weak ΣN interaction in the relevant $^1D_2$ and $^3D_2$ configurations [10]. The quark model of Ref. [7] does not have any $(\frac{3}{2}, 2^+)$ $S = -1$ dibaryon candidate in the vicinity of the $\pi\Lambda N$ threshold and below it.

The plan of this paper is as follows. In Sec. II we discuss the choice of two-body interactions and the three-body Faddeev equations solved in the nonrelativistic case, and

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report on the binding energies calculated for the $(\frac{3}{2}, 2^+)$ $\pi\Lambda N$ system. The corresponding analysis of, and the binding energies calculated in a relativistic version of the three-body model are discussed in Sec. III. The paper ends with a brief summary and discussion in Sec. IV, where additional strange and charmed $\pi BB'$ systems that might admit bound states of a similar nature are listed.

II. A NONRELATIVISTIC MODEL

A. The two-body subsystems

Since both $\pi\Lambda$ and πN subsystems are dominated by p-wave resonances, we assumed a rank-one separable meson-baryon interaction

$$V_i(p_i, p_i') = -g_i(p_i)g_i(p_i'). \quad (1)$$

The corresponding two-body t matrix is given by

$$t_i(p_i, p_i'; E) = -g_i(p_i)\tau_i(E)g_i(p_i'), \quad (2)$$

where E is the energy in the two-body center-of-mass (c.m.) system and

$$\tau_i^{-1}(E) = 1 + \int_0^\infty p_i^2 dp_i \frac{g_i^2(p_i)}{E - p_i^2/2\eta_i + i\epsilon}, \quad (3)$$

with $\eta_i = m_j m_k/(m_j + m_k)$, where $\epsilon_{ijk} \neq 0$. The form factors $g_i(p_i)$ are chosen of the form,

$$g_i(p_i) = \sqrt{\gamma_i} p_i (1 + p_i^2) e^{-p_i^2/\alpha_i^2}, \quad (4)$$

where the two parameters γ_i and α_i were adjusted to the position and width of the corresponding resonances, as given by the Particle Data Group [12]. These parameters are listed in Table I for the πN and $\pi\Lambda$ subsystems. We also constructed a second model of the πN interaction of the form

$$g_i(p_i) = \sqrt{\gamma_i} p_i [1 + (p_i/4.5)^2 + (p_i/1.35)^4] e^{-p_i^2/\alpha_i^2}, \quad (5)$$

which reproduces, in addition, the πN P_{33} scattering volume. The parameters of this model are also given in Table I. Note that p_i in Eqs. (4) and (5) assumes values in fm^{-1} units.

For the $^3S_1$ ΛN subsystem we assume a rank-two separable potential consisting of both attractive and repulsive terms:

$$V_i(p_i, p_i') = -g_i^a(p_i)g_i^a(p_i') + g_i^r(p_i)g_i^r(p_i'). \quad (6)$$

The corresponding two-body t matrix is given by

TABLE I. Parameters of the pion-baryon separable potentials Eqs. (4) and (5), α_i (in fm^{-1}) and γ_i (in fm^4), for the nonrelativistic model.

	$\alpha_{\pi N}$	$\gamma_{\pi N}$	$\alpha_{\pi\Lambda}$	$\gamma_{\pi\Lambda}$
Eq. (4)	2.021 352	0.021 16	2.523 999	0.005 64
Eq. (5)	1.560 768	0.062 44	—	—

$$t_i(p_i, p_i'; E) = -\sum_{\alpha=a,r}\sum_{\beta=a,r} g_i^\alpha(p_i)\tau_i^{\alpha\beta}(E)g_i^\beta(p_i'), \quad (7)$$

where

$$\tau_i^{ar}(E) = \tau_i^{ra}(E) = \frac{G_i^{ar}(E)}{[1 + G_i^{aa}(E)][1 - G_i^{rr}(E)] + [G_i^{ar}(E)]^2}, \quad (8)$$

$$\tau_i^{aa}(E) = \frac{1 - G_i^{rr}(E)}{[1 + G_i^{aa}(E)][1 - G_i^{rr}(E)] + [G_i^{ar}(E)]^2}, \quad (9)$$

$$\tau_i^{rr}(E) = -\frac{1 + G_i^{aa}(E)}{[1 + G_i^{aa}(E)][1 - G_i^{rr}(E)] + [G_i^{ar}(E)]^2}, \quad (10)$$

$$G_i^{\alpha\beta}(E) = \int_0^\infty p_i^2 dp_i \frac{g_i^\alpha(p_i)g_i^\beta(p_i)}{E - p_i^2/2\eta_i + i\epsilon}. \quad (11)$$

The form factors $g_i^\beta(p_i)$ are chosen to be of the Yamaguchi form

$$g_i^\beta(p_i) = \frac{\sqrt{\gamma_\beta}}{p_i^2 + \alpha_\beta^2} \qquad (\beta = a, r), \quad (12)$$

where the parameters α_a, γ_a, α_r, and γ_r are adjusted to reproduce given values of the ΛN $^3S_1$ scattering length and effective range for different values of the Λ laboratory momentum $p_{\rm lab}^{(0)}$ at which the $^3S_1$ ΛN phase shift becomes negative, changing sign from attraction at low momentum to repulsion at high momentum (as discussed in Sec. II C). The values of the scattering length and effective range adopted here are $a = -1.86$ fm and $r_0 = 3.13$ fm, respectively, corresponding to model ESC04d of Ref. [10]. These values are very close to those in models NSC97e,f [13] which have been widely used in Λ-hypernuclear calculations.

B. The three-body system

Since all the angular momenta, spins, and isospins are coupled to their maximal values, the recoupling coefficients of spin and isospin are equal to one, and the Faddeev equations depend only on the orbital angular momenta $\vec{\ell}$, $\vec{\lambda}$, $\vec{L}$, where $\vec{L} = \vec{\ell} + \vec{\lambda}$, with $L = 1$. The values of $\vec{\ell}$ and $\vec{\lambda}$ are $\ell = 1$, $\lambda = 0$ for configurations in which the pion interacts with one of the baryons while the other baryon is a spectator, and $\ell = 0$, $\lambda = 1$ for the configuration in which the two baryons interact while the pion is a spectator.

Below we denote the Λ hyperon as particle 1, the nucleon as particle 2, and the pion as particle 3. Thus, the Faddeev equations for the bound-state problem, using the separable potentials (1) and (6), are

$$T_i(q_i) = -\tau_i(E - q_i^2/2\nu_i)\sum_{j=1}^{2}\int_0^\infty dq'_j H_{ij}(q_i, q'_j)T_j(q'_j) \quad (i = 1, 2), \tag{13}$$

with $\nu_i = m_i(m_j + m_k)/(m_i + m_j + m_k)$, where $\epsilon_{ijk} \neq 0$, and

$$H_{ij}(q_i, q'_j) = (1 - \delta_{ij})K_{ij}(q_i, q'_j) - \sum_{\alpha=a,r}\sum_{\beta=a,r}\int_0^\infty dq_3 K^\alpha_{i3}(q_i, q_3) \times \tau_3^{\alpha\beta}(E - q_3^2/2\nu_3)K^\beta_{3j}(q_3, q'_j). \tag{14}$$

The kernels in Eq. (14) are given by

$$K_{12}(q_1, q_2) = \frac{1}{2}q_1q_2\int_{-1}^{1} d\cos\theta \frac{g_1(p_1)(\hat{p}_1\cdot\hat{p}_2)g_2(p_2)}{E - p_2^2/2\eta_2 - q_2^2/2\nu_2}, \tag{15}$$

$$K^\alpha_{31}(q_3, q_1) = \frac{1}{2}q_1q_3\int_{-1}^{1} d\cos\theta \frac{g_3^\alpha(p_3)(\hat{q}_3\cdot\hat{p}_1)g_1(p_1)}{E - p_1^2/2\eta_1 - q_1^2/2\nu_1}, \tag{16}$$

$$K^\alpha_{23}(q_2, q_3) = \frac{1}{2}q_2q_3\int_{-1}^{1} d\cos\theta \frac{g_2(p_2)(\hat{p}_2\cdot\hat{q}_3)g_3^\alpha(p_3)}{E - p_3^2/2\eta_3 - q_3^2/2\nu_3}. \tag{17}$$

From the three previous expressions one obtains the other three that correspond to $K_{ji}(q_j, q_i) = K_{ij}(q_i, q_j)$. One can calculate p_i, p_j, $(\hat{p}_1\cdot\hat{p}_2)$, $(\hat{q}_3\cdot\hat{p}_1)$, and $(\hat{p}_2\cdot\hat{q}_3)$ by using

$$\vec{p}_i = -\vec{q}_j - a_{ij}\vec{q}_i, \qquad \vec{p}_j = \vec{q}_i + a_{ji}\vec{q}_j, \tag{18}$$

where (i, j) is a cyclic pair, $\cos\theta = \hat{q}_i\cdot\hat{q}_j$, and

$$a_{ij} = \frac{\eta_i}{m_k}, \qquad a_{ji} = \frac{\eta_j}{m_k}. \tag{19}$$

In order to find the bound-state solutions of Eq. (13), integrals were replaced by sums applying numerical integration quadrature. In this way Eq. (13) becomes a set of homogeneous linear equations. This set has solutions only if the determinant of the matrix of its coefficients (the Fredholm determinant) vanishes at certain energies. Thus, the procedure to find the bound-state energies of the three-body system simply consists of searching for the zeros of the Fredholm determinant on the real energy axis. Some limiting situations are discussed in the appendix.

C. Results

In the last column of Table II, we list the calculated binding energies $B_{\pi\Lambda N}$ of the $\pi\Lambda N$ system in the $(I, J^P) = (\frac{3}{2}, 2^+)$ channel, for the $\pi\Lambda$ and πN interactions recorded in Table I and the various models of the ΛN interaction also listed in Table II. Most of the results are given for the choice Eq. (4) of the πN form factor, except for the $\alpha_a = 1.8\ \mathrm{fm}^{-1}$ runs for which listed in parentheses are also the binding energies obtained using the other choice Eq. (5). The dependence on the type of πN form factor is seen to be rather weak. We also checked the sensitivity to the strength parameter $\gamma_{\pi\Lambda}$; for example, the $\pi\Lambda N$ bound state for the case $B_{\pi\Lambda N} = 51$ MeV listed in the table disappears as soon as the standard value $\gamma_{\pi\Lambda} = 0.005\,64\ \mathrm{fm}^4$ from Table I is decreased to $0.005\,24\ \mathrm{fm}^4$. The dependence on the ΛN interaction is shown in detail in Table II. Essentially, the various ΛN models differ from each other by the amount of repulsion they contain. For a given value of range parameter α_a^{-1} for the attractive ΛN component, the calculated binding energy decreases as the repulsive component gets pushed inside and requires a larger strength. For a given value of range parameter α_r^{-1} for the repulsive component, the calculated binding energy decreases as the attractive component gets pushed inside, or equivalently as one lowers the momentum where the ΛN $^3S_1$ phase shift changes sign from positive (attraction) to negative (repulsion) values. It is seen that the bound state persists as long as this Λ laboratory momentum $p^{(0)}_{\mathrm{lab}}$ is larger than about 750–800 MeV/c. Incidentally, this is precisely the range of momenta at which the ΛN $^3S_1$ phase shift goes through zero in Nijmegen YN potential models that relegate the $^3S_1 - {}^3D_1$ attraction near and above the ΣN threshold to the $^3D_1$ channel [13].

TABLE II. Parameters of the ΛN $^3S_1$ potentials (12) α_β (in fm^{-1}), γ_β (in fm^{-2}) in the nonrelativistic model for $a = -1.86$ fm, $r_0 = 3.13$ fm, and the binding energies $B_{\pi\Lambda N}$ (in MeV) of the three-body $\pi\Lambda N$ system calculated using the πN and $\pi\Lambda$ potential parameters listed in Table I, Eq. (4) [the $B_{\pi\Lambda N}$ values in parentheses correspond to the πN parameters listed in Table I, Eq. (5)]. The momentum $p^{(0)}_{\mathrm{lab}}$ (in MeV/c) is the laboratory Λ momentum at which the ΛN $^3S_1$ phase shift becomes negative.

α_a	γ_a	α_r	γ_r	$p^{(0)}_{\mathrm{lab}}$	$B_{\pi\Lambda N}$
1.437	0.4179	—	—	—	140
1.6	0.8118	4.0	5.54	1184	111
1.6	0.8053	6.0	26.0	1069	96
1.6	0.8064	8.0	86.0	1045	86
1.7	1.195	4.0	10.0	975	92
1.7	1.186	6.0	51.0	910	66
1.7	1.190	8.0	190.0	899	52
1.8	1.735	4.0	15.5	877	72 (67)
1.8	1.718	6.0	86.0	834	38 (37)
1.8	1.745	8.0	405.0	826	21 (23)
1.9	2.513	4.0	22.7	814	51
1.9	2.501	6.0	145.0	784	9
1.9	2.573	8.0	1150.0	779	unbound
2.0	3.588	4.0	31.4	777	31
2.0	3.602	6.0	244.0	753	unbound
2.1	5.125	4.0	42.9	748	10
2.2	7.311	4.0	58.0	728	unbound

III. A RELATIVISTIC MODEL

Since the binding energies calculated nonrelativistically, for some of the cases listed in Table II are a sizable fraction of the pion mass, it appears necessary to take into account relativistic effects. Therefore, we will reformulate our model in terms of a relativistic on-mass-shell-spectator formalism [14–16]. In this formalism one starts with the Bethe-Salpeter equation for three particles which is set in a Faddeev form. The four-vector equations are then reduced to three-vector equations similar to the nonrelativistic Faddeev equations by putting all the spectator particles on the mass shell [15].

In order to reach a relativistic generalization of Eq. (13) we make two approximations. First, the negative-energy components of the fermion propagators are neglected; and second, the spin degrees of freedom are treated nonrelativistically by means of Racah coefficients (which are equal to one, as pointed out above). These two approximations are reasonable since the two fermions Λ and N are very heavy compared with the pion. Thus, as pointed out in the introduction, our model formally reduces to that of three spinless (and isospinless) particles interacting by pairwise separable interactions.

A. The two-body subsystems

In order to fit the p-wave resonance energy and width in the $\pi\Lambda$ and πN subsystems we considered the two-body Bethe-Salpeter equation for the pair jk with particle j (here the pion) on the mass shell interacting through a rank-one separable interaction defined by Eqs. (1) and (4). Recall that p_i, the magnitude of the relative three-momentum of the pair in the c.m. system, is Lorentz invariant since it is expressible in terms of the invariant mass of the relative momentum four-vector. The corresponding two-body t matrix in the c.m. system is given by

$$t_i(p_i, p_i'; \omega_0) = -g_i(p_i)\tau_i(\omega_0)g_i(p_i'), \qquad (20)$$

where ω_0 is the invariant mass of the two-body subsystem and

$$\tau_i^{-1}(\omega_0) = 1 + \int_0^\infty \frac{p_i^2 dp_i}{2\omega_j} \frac{g_i^2(p_i)}{(\omega_0 - \omega_j)^2 - \omega_k^2 + i\epsilon}, \qquad (21)$$

with $\omega_j = \sqrt{m_j^2 + p_i^2}$ and $\omega_k = \sqrt{m_k^2 + p_i^2}$. The parameters of these separable potentials are given in Table III. We did not pursue the option of keeping the respective baryon on-mass-shell, with an off-shell pion, because of the appearance of a persistent unphysical two-body bound state for this choice.

TABLE III. Parameters of the pion-baryon separable potential Eq. (4), α_i (in fm^{-1}) and γ_i (in fm^2), for the relativistic model with on-mass-shell π meson.

	$\alpha_{\pi N}$	$\gamma_{\pi N}$	$\alpha_{\pi\Lambda}$	$\gamma_{\pi\Lambda}$
Eq. (4)	2.231 357	0.219 260	2.720 821	0.083 916

For the ΛN subsystem we again used a rank-two separable potential defined by Eqs. (6) and (12) so that the two-body t matrix is given by Eqs. (7)–(10) with E replaced by ω_0 and $G_i^{\alpha\beta}(E)$ of Eq. (11) replaced by

$$G_i^{\alpha\beta}(\omega_0) = \int_0^\infty \frac{p_i^2 dp_i}{2\omega_j} \frac{g_i^\alpha(p_i)g_i^\beta(p_i)}{(\omega_0 - \omega_j)^2 - \omega_k^2 + i\epsilon}. \qquad (22)$$

The parameters of these separable potentials are listed below in Sec. III C.

B. The three-body system

The integral equations for the three-body problem are given by

$$T_i(q_i) = -\tau_i(W_0; q_i) \sum_{j=1}^{2} \int_0^{q_{\max}^{(j)}} dq_j' H_{ij}(q_i, q_j') T_j(q_j') \qquad (i = 1, 2), \qquad (23)$$

where W_0 is the invariant mass of the three-body system. The upper limit of integration

$$q_{\max}^{(j)} = \frac{W_0^2 - m_j^2}{2W_0}, \qquad (24)$$

is the momentum at which the invariant mass of the two-body subsystem j is equal to zero so that it then recoils with the speed of light [16]. The entity $\tau_i(W_0; q_i)$ corresponds to the t matrix (20) and (21) in an arbitrary frame where the spectator particle i (which is on-mass-shell) has momentum $\vec{q}_i$, particle j (which has also been put on-mass-shell) has momentum $\vec{q}_j$ and particle k (which is off the mass shell) has momentum $-\vec{q}_i - \vec{q}_j$. It is given by

$$\tau_i^{-1}(W_0; q_i) = 1 + \frac{1}{2} \int_{-1}^{1} d\cos\theta \int_0^\infty \frac{q_j^2 dq_j}{2\omega_j} \times \frac{g_i^2(p_i)}{(W_0 - \omega_i - \omega_j)^2 - \omega_k^2 + i\epsilon}, \qquad (25)$$

with

$$\omega_i = \sqrt{m_i^2 + q_i^2}, \qquad \omega_j = \sqrt{m_j^2 + q_j^2}, \qquad (26)$$

$$\omega_k = \sqrt{m_k^2 + q_i^2 + q_j^2 + 2q_i q_j \cos\theta}. \qquad (27)$$

The magnitude of the relative three-momentum $\vec{p}_i$ is a Lorentz invariant given by

$$p_i^2 = \frac{(P_{jk}^2 + m_j^2 - k_k^2)^2}{4P_{jk}^2} - m_j^2, \qquad (28)$$

where $P_{jk} = k_j + k_k$ is the total four momentum of the pair jk and k_k is the four momentum of particle k, *i.e.*,

$$P_{jk}^2 = (W_0 - \omega_i)^2 - q_i^2, \tag{29}$$

$$k_k^2 = (W_0 - \omega_i - \omega_j)^2 - q_i^2 - q_j^2 - 2q_iq_j\cos\theta. \tag{30}$$

Equation (25) reduces to Eq. (21) when $q_i = 0$. Similar expressions apply to the relativistic version of the ΛN t matrix in an arbitrary frame $\tau_3^{\alpha\beta}(W_0; q_3)$.

The kernel of Eq. (23) is given by Eqs. (14)–(17), where the upper limit ∞ in the integral of Eq. (14) is replaced by $q_{\rm max}^{(3)}$, and the following substitutions are made:

$$\frac{1}{E - p_j^2/2\eta_j - q_j^2/2\nu_j} \to \frac{1}{2\omega_j}\frac{1}{(W_0 - \omega_i - \omega_j)^2 - \omega_k^2}, \tag{31}$$

$$a_{ij} \to \frac{W_i^2 - q_i^2 + m_j^2 - k_k^2 + 2\omega_j\sqrt{W_i^2 - q_i^2}}{2\sqrt{W_i^2 - q_i^2}(W_i + \sqrt{W_i^2 - q_i^2})}, \tag{32}$$

$$a_{ji} \to \frac{W_j^2 - q_j^2 + m_i^2 - k_k^2 + 2\omega_i\sqrt{W_j^2 - q_j^2}}{2\sqrt{W_j^2 - q_j^2}(W_j + \sqrt{W_j^2 - q_j^2})}, \tag{33}$$

$$W_i = W_0 - \omega_i, \qquad W_j = W_0 - \omega_j. \tag{34}$$

Equation (31) is the propagator when the spectator particles i and j are on-mass-shell and the exchanged particle k is off-mass-shell. Equations (32)–(34) correspond to the relativitistic kinematics with particle k off the mass shell.

TABLE IV. Parameters of the ΛN $^3S_1$ potentials (12) α_β (in fm^{-1}), γ_β (in fm^{-4}) in the relativistic model with on-mass-shell nucleon, for $a = -1.86$ fm, $r_0 = 3.13$ fm, and the binding energies $B_{\pi\Lambda N}$ (in MeV) of the three-body $\pi\Lambda N$ system calculated using the πN and $\pi\Lambda$ potential parameters listed in Table III. The momentum $p_{\rm lab}^{(0)}$ (in MeV/c) is the laboratory Λ momentum at which the ΛN $^3S_1$ phase shift becomes negative.

α_a	γ_a	α_r	γ_r	$p_{\rm lab}^{(0)}$	$B_{\pi\Lambda N}$
2.0	318.2	4.0	2270	866	152
2.0	309.2	6.0	12 100	823	93
2.0	313.0	8.0	54 500	813	69
2.1	446.9	4.0	3080	823	121
2.1	434.3	6.0	18 000	788	59
2.1	440.8	8.0	105 000	783	35
2.2	626.6	4.0	4100	791	94
2.2	599.1	6.0	25 800	768	31
2.2	632.5	8.0	350 000	756	unbound
2.3	878.5	4.0	5400	766	69
2.3	845.8	6.0	40 700	746	unbound
2.4	1217	4.0	6930	750	48
2.4	1189	6.0	68 000	733	unbound
2.5	1728	4.0	9200	730	21
2.6	2354	4.0	11 400	728	6

C. Results

In the last column of Table IV, we list the calculated binding energies $B_{\pi\Lambda N}$ of the $\pi\Lambda N$ system in the $(I, J^P) = (\frac{3}{2}, 2^+)$ channel, for the $\pi\Lambda$ and πN interactions recorded in Table III and the various models of the ΛN interaction listed also in Table IV. The dependence of the calculated binding energies on the ranges of the repulsive and attractive components of the ΛN interaction is similar to that found in the nonrelativistic calculations. A bound state in the relativistic model persists as long as the Λ laboratory momentum at which the ΛN phase shift becomes negative, $p_{\rm lab}^{(0)}$, is larger than about 750 MeV/c. A comparison between Tables II and IV reveals that the relativistic model provides more attraction than the nonrelativistic one, in agreement with the slower increase of kinetic energy with momentum when relativistic kinematics is applied.

IV. SUMMARY AND DISCUSSION

We have used a nonrelativistic separable potential model and a relativistic version of it, solving three-body Faddeev equations, to search for $\pi\Lambda N$ bound states. In both models we found that a $(I, J^P) = (\frac{3}{2}, 2^+)$ bound state is likely to exist, provided the Λ laboratory momentum $p_{\rm lab}^{(0)}$ at which the $^3S_1$ ΛN phase shift becomes negative is larger than about 750–800 MeV/c. This agrees with the range of momenta at which Nijmegen YN potential models, where applicable [13], predict that the $^3S_1$ ΛN phase shift goes through zero. The Jülich '04 model [17] and the recent chiral EFT approach [18] predict that $p_{\rm lab}^{(0)} > 900$ MeV/c, so that the existence of a $\pi\Lambda N$ bound state in these models appears robust. The Nijmegen and Jülich YN potential models differ considerably from each other within the ΛN $J^P = 1^+$ coupled channels also in the behavior of the $^3D_1$ phase shift. The $^3S_1 - {}^3D_1$ coupling was neglected in the present exploratory three-body calculation, a neglect that might be justified in applications of the Jülich models where both the coupling and the size of the $^3D_1$ phase shift that builds up above the ΣN threshold at $p_{\rm lab} \approx 630$ MeV/c are weaker than in the Nijmegen models. However, all these YN models have been constructed to fit primarily low-energy scattering data which do not unambiguously constrain the short-range behavior of the $^3S_1$ ΛN system. The extent to which the two-body short-range repulsion varies between "soft" to "hard" is crucial for the three-body system's ability to bind, with the p-wave pion maximizing its attraction to each one of the baryons simultaneously.

More realistic three-body calculations will have to include Σ hyperons, extending the ΛN channel into $^3S_1 - {}^3D_1$ $\Lambda N - \Sigma N$ coupled channels, and the $\pi\Lambda$ channel into $\pi\Lambda - \pi\Sigma$ coupled channels. Although the $I = 1$ $\bar{K}N$ channel also couples to these πY coupled channels, in first approximation the three-body $\bar{K}NN$ channel is decoupled from the πYN coupled channels for $(I, J^P) = (\frac{3}{2}, 2^+)$ owing to the restrictions imposed by the Pauli principle on the two nucleons.

To search experimentally for a possible $I = \frac{3}{2}$, $J^P = 2^+$ $\pi\Lambda N$ dibaryon bound state or resonance, which we denote by $\mathcal{D}$, one could try in-flight (K^-, π^+) or (π^-, K^+) reactions on a deuteron target

$$K^- + d \to \mathcal{D}^- + \pi^+, \tag{35}$$

$$\pi^- + d \to \mathcal{D}^- + K^+. \tag{36}$$

These reactions lead automatically to the required value of isospin $I = \frac{3}{2}$ for the $\mathcal{D}$ dibaryon. The values required for spin-parity, $J^P = 2^+$, are also allowed. In terms of a coupled $\Sigma^- n$ system, the orbital angular momentum and Pauli spin are approximately conserved, resulting in two possibilities: ${}^3D_2$ and ${}^1D_2$. These could be explored by choosing an incident momentum and a meson scattering angle where the $K^- + p \to \Sigma^- + \pi^+$ or $\pi^- + p \to \Sigma^- + K^+$ underlying reactions are largely non-spin-flip $(\to {}^3D_2)$ or have a nonnegligible spin-flip component $(\to {}^1D_2)$. These experiments would be feasible at J-PARC.

The three-body calculations reported here for the $S = -1$ $\pi\Lambda N$ system may be extended to other three-body systems of the type $\pi B_1 B_2$, with $J^P = 2^+$ and a maximum value of isospin, consisting of a p-wave pion and $\frac{1}{2}^+$ baryons in a relative s-wave state. This precludes identical baryons: $B_1 \neq B_2$. Candidates may be classified as follows:

(i) $S = -2, -3$ strange systems obtained by substituting the SU(3)-octet Ξ hyperon for the Λ hyperon or for the nucleon in the $\pi\Lambda N$ three-body system, leading to $\pi\Xi N$ and $\pi\Lambda\Xi$, respectively. The new $\pi\Xi$ p-wave resonance here is the $\frac{3}{2}^+$ $\Xi(1530)$ belonging to the same SU(3) decuplet which contains the $\Delta(1232)$ and the $\Sigma(1385)$ considered in the present work.

(ii) $C = +1$ charmed systems made out of a pion, SU(3)-octet baryon (excluding the Σ hyperon) and $\frac{1}{2}^+$ charmed baryon (of the lowest mass for a given strangeness)

$$\pi N\Lambda_c(2286), \qquad \pi N\Xi_c(2470), \qquad \pi N\Omega_c(2700), \tag{37}$$

$$\pi \Lambda\Lambda_c(2286), \qquad \pi \Lambda\Xi_c(2470), \qquad \pi \Lambda\Omega_c(2700), \tag{38}$$

$$\pi \Xi\Lambda_c(2286), \qquad \pi \Xi\Xi_c(2470), \qquad \pi \Xi\Omega_c(2700). \tag{39}$$

(iii) $C = +2$ charmed systems made out of a pion and two $\frac{1}{2}^+$ singly charmed baryons, each of the lowest mass for a given strangeness

$$\pi\Lambda_c(2286)\Xi_c(2470), \qquad \pi\Lambda_c(2286)\Omega_c(2700), \qquad \pi\Xi_c(2470)\Omega_c(2700). \tag{40}$$

Note the appearance of the $\frac{1}{2}^+$ Ω_c baryon, of quark structure ssc. In the case of charmed baryons, the p-wave noncharmed SU(3)-decuplet $\frac{3}{2}^+$ resonances are replaced by charmed SU(3)-sextet members of the same extended SU(4) **20**-plet

$$\Sigma(1385) \to \Sigma_c(2520), \qquad \Xi(1530) \to \Xi_c(2645), \qquad \Omega(1670) \to \Omega_c(2770). \tag{41}$$

Here we limited listing to singly charmed baryons. The only observation we wish to make on a future charmed bound-state study is that the $\pi N\Lambda_c(2286)$ threshold lies *below* $N\Sigma_c(2455)$, where $\Sigma_c(2455)$ is the lowest lying known Σ_c, with assumed $J^P = \frac{1}{2}^+$. Therefore, if $\pi N\Lambda_c(2286)$ is bound, it will decay only by weak interactions. Hopefully, the study of these, and other charmed dibaryons will become feasible in due course.

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APPENDIX: LIMITING FADDEEV SOLUTIONS FOR $\pi\Lambda N$, πNN AND $\pi\Lambda\Lambda$

It is interesting to solve the coupled Faddeev Eqs. (13) in the limit of vanishing baryon-baryon interaction, $\tau_3^{\alpha\beta} = 0$. Equation (14) reduces then to $H_{ij} = (1 - \delta_{ij})K_{ij}$, for i, $j = 1, 2$, so that Eqs. (13) become

$$T_i = -\tau_i K_{ij} * T_j, \qquad (i \neq j), \tag{A1}$$

where the asterisk stands for convolution. Bound states are obtained by searching for zeros of the Fredholm determinant corresponding to the operator $(1 - \tau_1 K_{12} \tau_2 K_{21})$. Using πN and $\pi\Lambda$ interaction parameters from Table I, Eq. (4), a robust bound state is found at $B_{\pi\Lambda N} = 110$ MeV. From Table II we learn that a fully attractive ΛN interaction leads to a higher value of $B_{\pi\Lambda N}$, and that the introduction of a repulsive component quickly lowers the calculated $B_{\pi\Lambda N}$ values below that for a noninteracting ΛN pair.

Next, let's make the two baryons identical as far as their mass, spin-parity $\frac{1}{2}^+$, and interaction with the pion are concerned. Then, $\tau_1 = \tau_2 \equiv \tau$ and $K_{12} = K_{21} \equiv K$. Since one is looking for a *symmetric* spatial configuration for these two s-wave baryons, it is the symmetric combination of the T_i's that is required:

$$(T_1 + T_2) = -\tau K * (T_1 + T_2), \qquad \text{(A2)}$$

and the requirement of vanishing Fredholm determinant at bound-state energies becomes equivalent to searching for zeros of the operator $(1 + \tau K)$. The operator τ is positive definite for the attractive meson-baryon interactions considered in the present work, and the operator K is negative definite at energies below threshold. Thus, if the meson-baryon interaction is sufficiently strong, the operator $(1 + \tau K)$ will have a zero at a subthreshold energy. Indeed for such a fictitious $(I, J^P) = (2, 2^+)$ πNN system excluded by the Pauli principle, and using πN interaction parameters from Table I, Eq. (4), we get a bound state with binding energy $B_{\pi NN} = 29$ MeV.

For *physical* πNN and $\pi\Lambda\Lambda$ systems, with symmetric spin-isospin configurations chosen, the Pauli exclusion principle requires that the spatial configuration be *antisymmetric*, leading to the requirement of finding zeros of the operator $(1 - \tau K)$. Since τK, for the meson-baryon interactions considered here, is negative definite below threshold, this means that the operator $(1 - \tau K)$ assumes values higher than one below threshold, which is commonly interpreted in terms of three-body *repulsion*. It is unlikely that adding secondary interaction channels into this schematic calculation will change the conclusion that no bound states are expected for πBB systems with two identical $\frac{1}{2}^+$ baryons.

[1] W. Brodowski *et al.*, Phys. Lett. B **550**, 147 (2002).
[2] H. Clement *et al.*, Prog. Part. Nucl. Phys. **61**, 276 (2008).
[3] B. Bassalleck, Nucl. Phys. **A639**, 401c (1998).
[4] B. A. Cole, Nucl. Phys. **A639**, 407c (1998).
[5] H. J. Crawford, Nucl. Phys. **A639**, 417c (1998).
[6] R. L. Jaffe, Phys. Rev. Lett. **38**, 195 (1977).
[7] P. J. Mulders, A. T. Aerts, and J. J. de Swart, Phys. Rev. D **21**, 2653 (1980).
[8] K. Johnston *et al.*, Phys. Rev. C **46**, R1573 (1992).
[9] R. E. Chrien, C. B. Dover, and A. Gal, Czech. J. Phys. **42**, 1089 (1992).
[10] Th. A. Rijken and Y. Yamamoto, Phys. Rev. C **73**, 044008 (2006).
[11] S. Weinberg, Phys. Rev. Lett. **17**, 616 (1966).
[12] W.-M. Yao *et al.*, J. Phys. G **33**, 1 (2006).
[13] Th. A. Rijken, V. G. J. Stoks, and Y. Yamamoto, Phys. Rev. C **59**, 21 (1999).
[14] F. Gross, Phys. Rev. C **26**, 2226 (1982).
[15] H. Garcilazo, Phys. Rev. C **35**, 1804 (1987).
[16] A. Stadler, F. Gross, and M. Frank, Phys. Rev. C **56**, 2396 (1997).
[17] J. Haidenbauer and U.-G. Meißner, Phys. Rev. C **72**, 044005 (2005).
[18] H. Polinder, J. Haidenbauer, and U.-G. Meißner, Nucl. Phys. **A779**, 244 (2006).