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Role of meson loops in the $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ decays into $V\gamma$

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We investigate the relevance of the meson loops in the $f_0(1710)$ scalar meson decay into one photon and one vector meson, (ρ, ω, and ϕ). In particular we estimate the size of the loops coming from the decay of the $f_0(1710)$ into two pseudoscalar mesons, containing three pseudoscalar mesons in the loop or two pseudoscalar mesons and one vector meson. The results, despite having large uncertainties, manifest that the contribution of the meson loops to these radiative decays is quite relevant and should be taken into account by the theoretical calculations which use these observables as a test of the possible glueball nature of this resonance. At the same time we also evaluate the relevance of the same loops in the two lighter scalar mesons, the $f_0(1370)$ and $f_0(1500)$, providing information of relevance in the search for the nature of the second nonet of scalar mesons.

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I. INTRODUCTION

One of the striking features of QCD is the possibility of getting meson states formed from bound states of gluons and glueballs, and some lattice calculations hint at their existence [1] with masses around $1.3 \sim 1.7$ GeV. In Refs. [2–4] the $f_0(1710)$ scalar meson is suggested as a possible glueball. In [3] it is argued that chiral symmetry has as a consequence that the glueball decays predominantly to $s\bar{s}$ and the ratio of decay amplitudes $G \to \bar{s}s$ to $G \to \bar{u}d + \bar{d}d$ is of the order of $m_s/2\hat{m} \simeq m_K^2/m_\pi^2$, with m_K and m_π the kaon and pion masses; while in [4] $G \to \bar{q}q\bar{q}q$ is claimed to be the dominant mechanism to reproduce the BES observed $\pi\pi/\bar{K}K$ ratio $\sim$0.41 for $f_0(1710)$ [5]. In a recent work, using multiple meson-meson channels to build up the scalar resonances [6], it is found that the $f_0(1710)$ scalar meson couples very strongly to $\bar{s}s$ and then it is suggested that it could be indeed a glueball. Yet, one of the tests which is supposed to be very relevant to find out the nature of the meson resonances is their radiative decay [7,8]. Indeed, in [7] a thorough investigation of the radiative decay of the $f_0(1710)$ into $\rho\gamma$ and $\phi\gamma$ is done and large decay widths are obtained depending on the amount of mixing with other nonglueball components, as suggested in [9,10].

The importance of the radiative decay to investigate the nature of resonances has also been pointed out in [11–13] in connection with the scalar $f_0(980)$ and $a_0(980)$ resonances, which in chiral unitary theories appear as dynamically generated [14–19]. With the assumption that the $f_0(980)$ and $a_0(980)$ resonances are dynamically generated, hence basically molecules of the $\pi\pi$ and $K\bar{K}$ coupled channels, the radiative decay of these resonances is done by photon emission from the meson components, which technically appear as loop contributions. In [11] the important contribution of the $K\bar{K}$ loops to the radiative decay of those scalar resonances into $\rho^0\gamma$ and $\omega\gamma$ was evaluated. In [12] additional pion loops were considered and in [13] the contribution of intermediate vector meson channels was also taken into account, which in the case of the $a_0(980)$ radiative decay turned out to be important.

There is another more general motivation for the present study, which is to offer useful information that can help in the long-standing search for the nature of the scalar mesons. Quite a general consensus has been reached about the four quark nature of the low lying scalar mesons (below 1 GeV), with the quarks rearranged as pseudoscalar mesons as in the case of dynamically generated resonances [14–19], large meson-meson components coming from unitarization of a seed of $q\bar{q}$ with the meson-meson states [20,21], $K\bar{K}$ molecules for the $f_0(980)$ and $a_0(980)$ [22], or other sorts of four quark rearrangement [23–27]. The issue is far more debated in the case of the higher mass second nonet of scalars. Some models would advocate a $L = 1$, $q\bar{q}$ nature for the $a_0(1450)$, $K_0^*(1430)$, $f_0(1370)$, and $f_0(1710)$, which would form a SU(3) nonet [28]. But this poses problems with the low masses of the $L = 1$ $q\bar{q}$, 1^{++}, and 2^{++} states. One way to sort out this problem is to invoke a mixing of these higher energy states with the lower energy ones [29–31]. The possible mixing of the higher lying scalar states with a glueball state has also been the object of consideration in [30–33], and in a different framework, using chiral effective Lagrangians at tree level, in [34]. A different mixing of $qq\bar{q}\,\bar{q}$ and $q\bar{q}$ states is also proposed in [35] to explain the two nonets of scalar mesons, and the couplings of the different resonances to pairs of pseudoscalar mesons are evaluated there. With no doubt, radiative decays will be a source of information in the future to help unravel the puzzle of the nature of the second nonet of scalars. The present work shows results for radiative decays stemming from the meson cloud of these states, evaluated by means of loop diagrams tested successfully in other reactions.

Given the fact that the loops gave sizable contributions to the radiative decay widths of the $f_0(980)$ and $a_0(980)$ resonances, it looks important that a determination of their strength for higher mass scalar mesons is carried out. This is the purpose of the present paper where we determine for the case of the $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ the contribution of the loops which were evaluated in [13] for the case of the $f_0(980)$ and $a_0(980)$ resonances. Unlike the case of the $f_0(980)$ and $a_0(980)$ resonances, where the couplings of the resonances to the different channels is provided by the chiral unitary theories, in the present case we have to resort to phenomenology. The rest of the information needed for the evaluation follows closely the steps of [13]. The present work is further motivated because its experimental study is feasible at BESIII in the near future and plans to do it are under way with partial wave analysis tools ready [36].

II. FORMALISM

According to the Particle Data Group (PDG) [37], the $f_0(1710)$ scalar resonance decays mostly into two pseudoscalar mesons, and among the pseudoscalar mesons the $K\bar{K}$ channel is the dominant one. From the experimental value $\Gamma_{K\bar{K}}/\Gamma_{\rm total} = 0.38^{+0.09}_{-0.19}$ [37,38] and from the expression of the decay of the $f_0(1710)$ into $K\bar{K}$,

$$\Gamma_{K\bar{K}} = \frac{\beta}{16\pi M_f}|g_{K\bar{K}}|^2, \tag{1}$$

where M_f is the mass of the $f_0(1710)$ and $\beta = \sqrt{1-4m_K^2/s}$, the coupling of the $f_0(1710)$ to the $K\bar{K}$ isospin $I=0$ state results $g_{K\bar{K}} \simeq 2350 \pm 500$ MeV. It is worth noting that this value compares very well with that obtained with the coupled channel analysis of [6,39], $g_{K\bar{K}} \simeq 2862 \pm 440$ MeV.

The idea to follow in order to evaluate the radiative decay width is similar to the procedure used in Ref. [40], which we summarize here adapted to the present case. Once we have the value of the coupling of the $f_0(1710)$ to the $K\bar{K}$, the philosophy is to consider the transition from the $f_0(1710)$ mesons to the $K\bar{K}$ states at one loop and then attach the photon to the possible allowed lines, considering that a vector meson in the final state has to be produced. By using arguments of gauge invariance, it can be shown that we only need to evaluate the diagrams depicted in Fig. 1, despite the fact that other diagrams are in principle needed like, e.g., those with a photon emitted from the initial and final vertices.

The channels we will consider are $f_0(1710) \to \rho^0\gamma$, $f_0(1710) \to \omega\gamma$, and $f_0(1710) \to \phi\gamma$. In Fig. 1 P_1, P_2, represent the kaons in the loop and V_1, V_2, vector mesons.

In Tables I and II we show the different allowed $P_1P_2V_1V_2$ particles of the diagrams in Fig. 1, together with the corresponding coefficients for each channel to be explained latter on. In Fig. 1, P, q, k, and Q represent the momentum of the different lines that will be used in the evaluation of the loop function.

Next we show how gauge invariance is invoked in order to simplify the calculations. We follow a similar procedure as done in Refs. [41,42] in the evaluation of the radiative ϕ decay, in Refs. [43,44] for the radiative axial-vector meson decays, and in Ref. [13] for the radiative decays of the $f_0(980)$ and $a_0(980)$ scalar mesons.

We can write the general expression of the amplitude for the radiative decay of the $f_0(1710)$ meson into a vector meson and a photon ($f_0(1710) \to V\gamma$) as

$$T = \epsilon_{V\mu}\epsilon_\nu T^{\mu\nu} \tag{2}$$

with

$$T^{\mu\nu} = a g^{\mu\nu} + bQ^\mu Q^\nu + cQ^\mu k^\nu + dk^\mu Q^\nu + ek^\mu k^\nu, \tag{3}$$

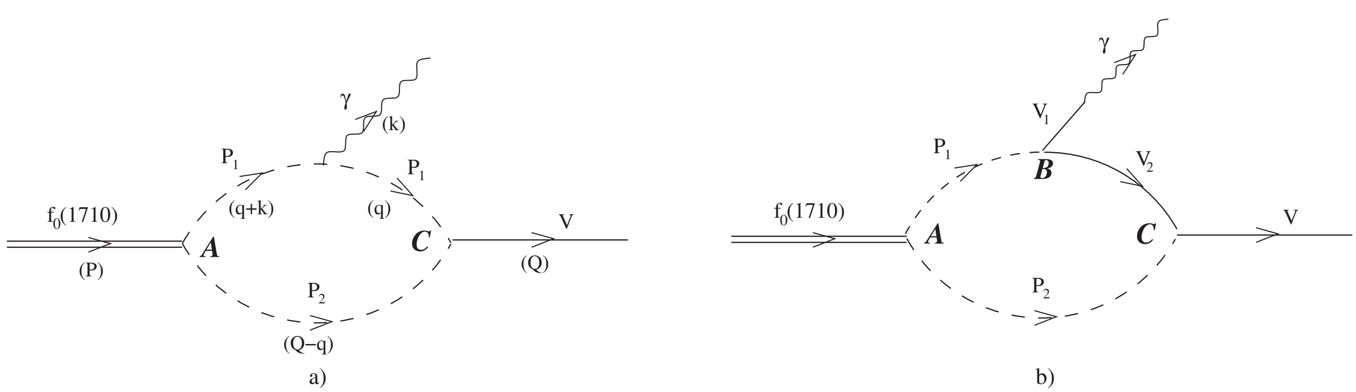

FIG. 1. Feynman diagrams contributing to the $f_0(1710)$ radiative decay. The variables within brackets represent the momenta; P_1, P_2 the different allowed kaons; and V, V_1, V_2 the different vector mesons. A, B, and C are coefficients explained in the text.

TABLE I. Coefficients A and C for type-a diagrams.

Decay	P_1P_2	A	C
$f_0(1710) \to \rho^0\gamma$	K^+K^-	$-1/\sqrt{2}$	$-1/\sqrt{2}$
	K^-K^+	$-1/\sqrt{2}$	$1/\sqrt{2}$
$f_0(1710) \to \omega\gamma$	K^+K^-	$-1/\sqrt{2}$	$-1/\sqrt{2}$
	K^-K^+	$-1/\sqrt{2}$	$1/\sqrt{2}$
$f_0(1710) \to \phi\gamma$	K^+K^-	$-1/\sqrt{2}$	1
	K^-K^+	$-1/\sqrt{2}$	-1

where Q is the final vector meson momentum and k the photon momentum. In Eq. (2), ϵ_V and ϵ are the vector meson and photon polarization vectors, respectively.

TABLE II. Coefficients A, B, and C for the different allowed type-b diagrams. Where no number is given, it should be understood that it is the same as in the row above.

Decay	$P_1P_2V_2V_1$	A	B	C
$f_0 \to \rho^0\gamma$	$K^+K^-K^{*+}\rho^0$	$-1/\sqrt{2}$	$1/\sqrt{2}$	$1/\sqrt{2}$
	ω		$1/\sqrt{2}$	
	ϕ		1	
	$K^-K^+K^{*-}\rho^0$	$-1/\sqrt{2}$	$1/\sqrt{2}$	$1/\sqrt{2}$
	ω		$1/\sqrt{2}$	
	ϕ		1	
	$K^0\bar{K}^0K^{*0}\rho^0$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	$-1/\sqrt{2}$
	ω		$1/\sqrt{2}$	
	ϕ		1	
	$\bar{K}^0K^0\bar{K}^{*0}\rho^0$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	$-1/\sqrt{2}$
	ω		$1/\sqrt{2}$	
	ϕ		1	
$f_0 \to \omega\gamma$	$K^+K^-K^{*+}\rho^0$	$-1/\sqrt{2}$	$1/\sqrt{2}$	$1/\sqrt{2}$
	ω		$1/\sqrt{2}$	
	ϕ		1	
	$K^-K^+K^{*-}\rho^0$	$-1/\sqrt{2}$	$1/\sqrt{2}$	$1/\sqrt{2}$
	ω		$1/\sqrt{2}$	
	ϕ		1	
	$K^0\bar{K}^0K^{*0}\rho^0$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	$1/\sqrt{2}$
	ω		$1/\sqrt{2}$	
	ϕ		1	
	$\bar{K}^0K^0\bar{K}^{*0}\rho^0$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	$1/\sqrt{2}$
	ω		$1/\sqrt{2}$	
	ϕ		1	
$f_0 \to \phi\gamma$	$K^+K^-K^{*+}\rho^0$	$-1/\sqrt{2}$	$1/\sqrt{2}$	1
	ω		$1/\sqrt{2}$	
	ϕ		1	
	$K^-K^+K^{*-}\rho^0$	$-1/\sqrt{2}$	$1/\sqrt{2}$	1
	ω		$1/\sqrt{2}$	
	ϕ		1	
	$K^0\bar{K}^0K^{*0}\rho^0$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	1
	ω		$1/\sqrt{2}$	
	ϕ		1	
	$\bar{K}^0K^0\bar{K}^{*0}\rho^0$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	1
	ω		$1/\sqrt{2}$	
	ϕ		1	

Because of the Lorenz condition, $\epsilon_{V\mu}Q^\mu = 0$, $\epsilon_\nu k^\nu = 0$, the a and d terms are the only ones in Eq. (3) which do not vanish. On the other hand, gauge invariance implies that $T^{\mu\nu}k_\nu = 0$, from where one gets

$$a = -dQ\cdot k. \tag{4}$$

Therefore, the amplitude gets the general form

$$T = -d(Q.kg^{\mu\nu} - k^\mu Q^\nu)\epsilon_{V\mu}\epsilon_\nu. \tag{5}$$

This implies that we only need to evaluate those diagrams contributing to the d-term, which are those having a final structure $k^\mu Q^\nu$. The advantage to evaluating only the d coefficient is that only the loop diagrams of Fig. 1 contribute since other diagrams, like those involving photon couplings to the vertices which are necessary to fulfill gauge invariance, do not give contribution to the d coefficient [13,41–43].

In Ref. [13] it was shown that the d coefficients are finite for the type-a diagrams of Fig. 1. By doing the same calculation as in Ref. [13] (see that reference for the explicit expressions of the Lagrangians needed) the d-coefficient for the type-a diagrams is given by the finite expression

$$d_a = ACQ_1 g_{K\bar{K}}\sqrt{2}e\frac{M_V G_V}{f^2}\frac{1}{8\pi^2}\int_0^1 dx\int_0^x dy\frac{(1-x)y}{s+i\varepsilon}, \tag{6}$$

where $s = Q^2x(1-x) + 2Q\cdot k(1-x)y + (m_2^2 - m_1^2)x - m_2^2$ and A are coefficients given in Table I needed to relate the $f_0(1710)P_1P_2$ coupling in charge basis with those in isospin basis, $g_{K\bar{K}}$. In Eq. (6), f is the pion decay constant for which we take the same value as in Ref. [13], Q_1 is the sign of the charge of the P_1 pseudoscalar meson, e is taken positive, G_V is the VPP coupling constant defined in [45] and for which we use the numerical value $G_V = 55 \pm 5$ MeV from Ref. [46], $m_1(m_2)$ is the mass of the $P_1(P_2)$ pseudoscalar meson, M_V the mass of the final vector meson, and C are coefficients coming from the vector-pseudoscalar-pseudoscalar (VPP) Lagrangian of Ref. [13] after performing the trace of the matrix and which depend on the particular P_1, P_2, and V particles [specifically, C is the coefficient of $\langle V^\mu[\partial_\mu P, P]\rangle$ of the VPP Lagrangian defined as $C(P_1\partial P_2 - P_2\partial P_1)$]. The different A, C coefficients are given in Table I.

In Ref. [13] it was shown that the type-b loops of Fig. 1 played a relevant role in the radiative decays into $V\gamma$ of the $f_0(980)$ and $a_0(980)$ scalar resonances. Thus we also evaluate the contribution of this kind of loop. The Lagrangians used for the vertices containing fields other than the $f_0(1710)$ are the same as in Ref. [13]. In that reference it was also shown that, despite the type-b loop being apparently quadratically divergent, it could be reduced to a logarithmic divergence which can be easily identified and regularized by expressing it in terms of the two pseudoscalar mesons loop function. Actually the expression of the d-coefficient for the type-b mechanisms can be split into a convergent part

$$d_b^{\rm con} = -ABC g_{K\bar{K}} \frac{N_B N_C G'^2 F_V}{2M_1} e\lambda_{V_1} \frac{1}{32\pi^2} \times \int_0^1 dx \int_0^x dy \frac{1}{s' + i\varepsilon}(Q^2(1-x)^2 - M_2^2) \quad (7)$$

and a divergent one

$$d_b^{\rm div} = -ABC g_{K\bar{K}} \frac{N_B N_C G'^2 F_V}{4M_1} e\lambda_{V_1} G(P^2, m_1, m_2), \quad (8)$$

written in terms of the two pseudoscalar mesons loop function, $G(P^2, m_1, m_2)$, which can be properly regularized either with a cutoff [14] or with dimensional regularization [18,19]. Loops with vector mesons are regularized in [47] with dimensional regularization and in [13] also with a cutoff. The results obtained with the cutoff method are very similar with those using the procedure outlined before, which is the one we follow here.

In Eqs. (7) and (8), $s' = Q^2 x(1-x) + 2Q\cdot k(1-x)y + (m_2^2 - M_2^2)x + (M_2^2 - m_1^2)y - m_2^2$; $M_1(M_2)$ is the mass of the $V_1(V_2)$ vector meson; λ_V is 1, 1/3, $-\sqrt{2}/3$ for $V = \rho$, ω, ϕ, respectively, and F_V is a coupling constant in the normalization of [45] for which we use the value $F_V = 156 \pm 5$ MeV [46]. The coefficient A has the same meaning as in the type-a loop case and B is the coefficient of the $P_1V_1V_2$ vertex obtained after performing the $SU(3)$ trace in $\langle VVP\rangle$ (see Ref. [13]) defined as $BP_1\bar{V}_1\bar{V}_2$. Analogously, C is the coefficient coming from the P_2V_2V vertex defined as $CP_2\bar{V}V_2$ from the resulting expression after taking the trace in $\langle VVP\rangle$. The N_B and N_C coefficients are normalization factors for the B and C VVP vertices (with coupling G') such that the $V\to P\gamma$ decays agree with experiment, as explained in Ref. [13].

The total amplitude for the radiative decay process is given by Eq. (5) where $d = d_a + d_b^{\rm con} + d_b^{\rm div}$ from Eqs. (6)–(8).

The radiative decay width of the $f_0(1710)$ resonance into a vector meson and a photon is given, in the narrow resonance limit, by

$$\Gamma(M_f, M_V) = \frac{|\vec{k}|}{8\pi M_f^2}\Sigma|T|^2\theta(M_f - M_V) = \frac{M_f^3}{32\pi}\left(1 - \frac{M_V^2}{M_f^2}\right)^3 |d|^2 \theta(M_f - M_V), \quad (9)$$

where θ is the step function.

The finite widths of the $f_0(1710)$ resonance and the final vector meson are taken into account by folding the previous expression with their corresponding mass distributions, in a similar way as explained in Refs. [13,40].

III. RESULTS

In Table III we show our results for the contributions of the type-a and type-b loops to the radiative decay widths under consideration. The theoretical errors quoted in our final results have been obtained by doing a Monte Carlo sampling of the parameters of the model within their uncertainties. It is worth stressing that our results must be viewed only as qualitative since we are not considering loops coming from decay channels of the $f_0(1710)$ other than $K\bar{K}$.

From the results in Table III we can see that the contribution of the loops considered in the present work to the different $f_0(1710)$ radiative decays under consideration is very relevant, especially for the $\rho^0\gamma$ decay channel. The reason why for the $\rho^0\gamma$ channel the type-b loops is much larger than for the $\omega\gamma$ and $\phi\gamma$ is that for the $\rho^0\gamma$ channel the interference of the loop containing charged kaons with those containing neutral kaons is constructive (see Table II) for the mechanisms with a ρ meson attached to the photon (the dominant one). On the contrary, for the $\omega\gamma$ and $\phi\gamma$ decay channels this interference is destructive.

In Table IV we compare our result with other theoretical determinations using quark models [7,8]. The L, M, and H labels in the columns for the results of Refs. [7,8] refer to three different versions of their model (light-weight, medium-weight, and heavy-weight glueball, respectively).

We can see the large dispersion in the values obtained between the different models and even within the same model in Refs. [7,8]. Our results are of the same order of magnitude than most of the results of the quark models. This means that the meson loops play a very important role in these decays and cannot be neglected in realistic calculations.

Since we see that at least in the case of decay to $\rho\gamma$ the role of the vector mesons in the loop is relevant, this raises the question of possible contributions of higher mass channels. This is an issue that will have to be faced in the future. The results of [6], where unfortunately no information is provided on the coupling of the $f_0(1710)$ to the different channels, should be relevant in this respect and the new channels considered there in the scattering problem might provide also a sizable contribution to the radiative decay.

Finally, we would like to show the meson loop contributions to the radiative decays of the $f_0(1370)$ and $f_0(1500)$ mesons. Since these lighter scalar mesons also have large decay rates to $K\bar{K}$ and also $\pi\pi$ channels, the

TABLE III. Contribution of the different meson loops to the radiative decay widths. All the units are in KeV.

	Loop a	Loop b	Total
$f_0(1710)\to\rho^0\gamma$	4.3	75.5	100 ± 40
$f_0(1710)\to\omega\gamma$	4.5	2.34	3.3 ± 1.2
$f_0(1710)\to\phi\gamma$	6.9	2.9	15 ± 5

TABLE IV. Comparison of the radiative decay widths into $\rho\gamma$ and $\phi\gamma$ with other theoretical predictions. All the units are in KeV.

	[7]			[8]			Present work
	L	M	H	L	M	H	
$f_0(1710) \to \rho^0\gamma$	42	94	705	24	55^{+16}_{-14}	410^{+200}_{-160}	100 ± 40
$f_0(1710) \to \phi\gamma$	800	718	78	450	400^{+20}_{-20}	36^{+17}_{-14}	15 ± 5

meson loops are expected to give relevant contributions to their radiative decay widths. We can apply the same formalism to these two scalar mesons as the $f_0(1710)$ case. We summarize the coupling constants of the $f_0(1370)$ and $f_0(1500)$ to these channels together with the partial decay widths used in the estimations in Table V. The large errors of the couplings of the $f_0(1370)$ come from the errors of decay rates and also from the large uncertainties of its mass $m_{f_0(1370)} = 1200$–1500 MeV and of its total width $\Gamma_{f_0(1370)} = 200$–500 MeV [37]. In Table VI we show the contributions of the $K\bar{K}$ loops to the $f_0(1370)$ and $f_0(1500)$ radiative decays. We can see again that those loop contributions are large for $\rho^0\gamma$ channels for the same reason as the $f_0(1710)$ decays.

We also estimate the contributions with two pions as P_1P_2 in loops depicted in Fig. 1 and show the considered channels and their coefficients in Tables VII and VIII. In Table IX we show the $\pi\pi$ loop contributions to the $f_0(1370)$ and $f_0(1500)$ decays. We can see significantly large contributions for $\rho^0\gamma$ and $\omega\gamma$ channels due to the large couplings of the $f_0(1370/1500)$ to the $\pi\pi$ channels while for the $\phi\gamma$ channel the $\pi\pi$ contribution is not allowed. We cannot calculate the interference between $K\bar{K}$ and $\pi\pi$ channels because of the lack of information on the relative phase, but we consider that these loop contributions should be taken into account in evaluations of the radiative decays.

Complete evaluations of the role of meson loops in the decays of the f_0 resonances are not possible at the present time, but the calculations carried out here, for the likely most important channels, show clearly that the role of loops in the decays of these resonances is something to deal with whenever one wishes to make accurate evaluations of the radiative decays of those resonances to draw conclusions on their nature.

TABLE V. Summary of the partial decay rates $f_0(1370)$ and $f_0(1500)$ to the $K\bar{K}$ and $\pi\pi$ channels in Ref. [37] and estimated absolute values of coupling constants $g_{f_0\to PP}$ in isospin base and in units of MeV.

	$\Gamma_{PP}/\Gamma_{\text{total}}$	$g_{f_0\to PP}$
$f_0(1370) \to K\bar{K}$	$(35 \pm 13)\%$	3500 ± 1400
$f_0(1370) \to \pi\pi$	$(26 \pm 9)\%$	2500 ± 900
$f_0(1500) \to K\bar{K}$	$(8.6 \pm 1)\%$	970 ± 170
$f_0(1500) \to \pi\pi$	$(35 \pm 2)\%$	1700 ± 200

TABLE VI. Contribution of the different meson loops with $K\bar{K}$ channel to the radiative decay widths. All the units are in KeV.

	Loop a	Loop b	Total
$f_0(1370) \to \rho^0\gamma$	8.6	51	79 ± 40
$f_0(1370) \to \omega\gamma$	9.3	1.6	7 ± 3
$f_0(1370) \to \phi\gamma$	8	1.7	11 ± 6
$f_0(1500) \to \rho^0\gamma$	0.7	4.9	8 ± 1
$f_0(1500) \to \omega\gamma$	0.8	0.2	0.6 ± 0.2
$f_0(1500) \to \phi\gamma$	0.9	0.1	1.2 ± 0.2

TABLE VII. Coefficients A and C for type-a diagrams for $\pi\pi$ channel.

Decay	P_1P_2	A	C
$f_0 \to \rho^0\gamma$	$\pi^+\pi^-$	$-\sqrt{2/3}$	$-\sqrt{2}$
	$\pi^-\pi^+$	$-\sqrt{2/3}$	$\sqrt{2}$

TABLE VIII. Coefficients A, B, and C for the different allowed type-b diagrams for $\pi\pi$ channels.

Decay	$P_1P_2V_2V_1$	A	B	C
$f_0 \to \rho_0\gamma$	$\pi^0\pi^0\omega\rho^0$	$-\sqrt{2/3}$	$\sqrt{2}$	$\sqrt{2}$
$f_0 \to \omega\gamma$	$\pi^+\pi^-\rho^+\omega$	$-\sqrt{2/3}$	$\sqrt{2}$	$\sqrt{2}$
	$\pi^-\pi^+\rho^-\omega$	$-\sqrt{2/3}$	$\sqrt{2}$	$\sqrt{2}$
	$\pi^0\pi^0\rho^0\omega$	$-\sqrt{2/3}$	$\sqrt{2}$	$\sqrt{2}$

TABLE IX. Contribution of the different meson loops with $\pi\pi$ channel to the radiative decay widths. All the units are in KeV.

	Loop a	Loop b	Total
$f_0(1370) \to \rho^0\gamma$	5.7	130	125 ± 80
$f_0(1370) \to \omega\gamma$	-	128	128 ± 80
$f_0(1370) \to \phi\gamma$	-	-	-
$f_0(1500) \to \rho^0\gamma$	3.4	80	77 ± 8
$f_0(1500) \to \omega\gamma$	-	79	79 ± 8
$f_0(1500) \to \phi\gamma$	-	-	-

IV. SUMMARY

In summary, we have studied the relevance of the kaon loops in the radiative decay of the $f_0(1710)$ scalar resonance. Given the dominance of the $K\bar{K}$ channel in the decay of this resonance, we have evaluated the loops stemming from this decay which contain either three pseudoscalar mesons (type-a in Fig. 1) or two pseudoscalars and one vector meson (type-*b*), in analogy to what was used in previous works regarding the radiative decay of the $f_0(980)$ and $a_0(980)$ resonances. We also have evaluated the effects of the $K\bar{K}$ and $\pi\pi$ meson loops in the $f_0(1370)$ and $f_0(1500)$ decays.

By using arguments of gauge invariance, it can be shown that only type-a and type-b loops need to be evaluated and that type-a is convergent while the logarithmic divergence of the type-b loop can be written in terms of the well-known two pseudoscalar mesons loop function.

The contribution of these meson loops to the radiative decay width is very relevant and should be taken into account in the theoretical calculations. With no intention to make a precise evaluation of the role of meson loops, lacking information from the coupling of the f_0 resonance to higher mass meson-meson channels and/or the relative phase between different meson channels, the present work has the value of showing the order of magnitude of what should be expected, which is sufficient to claim that their consideration is important for a proper understanding of the radiative decay widths. Experimental measurements of these radiative decays would be mostly welcome to deepen into the understanding of these controversial scalar resonances and could be accessible at experimental facilities like BESIII.

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