Response of quark condensate to the chemical potential

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In this paper we propose a new method for calculating the response of the quark condensate to the chemical potential. Based on the method of calculating the dressed-quark propagator at finite chemical potential in the framework of the rainbow-ladder approximation of the Dyson-Schwinger approach proposed in [H. S. Zong, L. Chang, F. Y. Hou, W. M. Sun, and Y. X. Liu, Phys. Rev. C **71**, 015205 (2005).] and adopting the meromorphic form of the quark propagator given in [R. Alkofer, W. Detmold, C. S. Fischer, and P. Maris, Phys. Rev. D **70**, 014014 (2004).][M. S. Bhagwat, M. A. Pichowsky, and P. C. Tandy, Phys. Rev. D **67**, 054019 (2003).], the quark condensate at finite chemical potential $\langle\bar{q}q\rangle[\mu]$ is calculated analytically. The obtained expression for $\langle\bar{q}q\rangle[\mu]$ is real, which is different from the results in the previous literature. In addition, it is found that when the chemical potential μ is less than a critical one $\langle\bar{q}q\rangle[\mu]$ is kept unchanged from its vacuum value. A comparison is made between this behavior of the quark condensate and those reported in the previous literatures.

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It is well known that the quark condensate in medium plays a key role in understanding the behavior of hadron masses in medium and chiral symmetry restoration [1,2]. The previous calculations of the response of the quark condensate to chemical potential [3–10] have shown that the theoretical treatment of this quantity is subtle and different treatments can give quite different results. Therefore, the calculation of the quark condensate in medium requires more elaborate analyses. In this paper we shall study this problem in the framework of the rainbow-ladder approximation of the Dyson-Schwinger (DS) approach.

The quark condensate can be calculated from the dressed-quark propagator. A direct approach for obtaining the dressed-quark propagator is by solving the Dyson-Schwinger equation (DSE) of the quark propagator. Over the past few years, considerable progress has been made in the framework of the rainbow-ladder approximation of the DSEs [11–15], which provides a successful description of various nonperturbative aspects of strong interaction physics at zero temperature (T) and zero chemical potential (μ). Recently, the authors in Refs. [8,16,17] proposed a new method for calculating the dressed-quark propagator at finite μ in the framework of the rainbow-ladder approximation of the DSEs. It has been shown there that under the approximation of neglecting the μ-dependence of the dressed gluon propagator (We have assumed that the effect of chemical potential on the gluon propagator arising through quark loop insertions is small in comparison with that on the quark propagator. This is a commonly used approximation in calculating the dressed-quark propagator at finite chemical potential [12]) and the assumption that the dressed-quark propagator at finite μ is analytic in the neighborhood of $\mu = 0$, the dressed-quark propagator at finite μ is obtained from the one at $\mu = 0$ by a simple shift

$$\mathcal{S}[\mu](p) = \mathcal{S}(\tilde{p}), \tag{1}$$

where $\mathcal{S}[\mu](p)$ and $\mathcal{S}(p)$ denote the dressed-quark propagator at finite μ and zero μ, respectively, and $\tilde{p} = (\vec{p}, p_4 + i\mu)$.

In the chiral limit, the gauge-invariant expression for the renormalization-point-dependent vacuum quark condensate is defined as [18]

$$-\langle\bar{q}q\rangle_\zeta = Z_4(\zeta^2, \Lambda^2)\int\frac{d^4q}{(2\pi)^4}\,\mathrm{Tr}\mathcal{S}(q, \zeta). \tag{2}$$

Here ζ is the renormalization point and Λ is the regularization mass scale. $Z_4(\zeta^2, \Lambda^2) = Z_m(\zeta^2, \Lambda^2)Z_2(\zeta^2, \Lambda^2)$ with $Z_m(\zeta^2, \Lambda^2)$ being the mass renormalization constant and $Z_2(\zeta^2, \Lambda^2)$ the quark wave function renormalization constant.

In order to study chiral symmetry restoration at finite μ, one needs to generalize the above definition to finite μ. In the previous literature (see, for instance, Ref. [12]), the following definition is adopted:

$$-\langle\bar{q}q\rangle_\zeta[\mu] = Z_4(\zeta^2, \Lambda^2)\,\mathrm{Re}\int\frac{d^4q}{(2\pi)^4}\,\mathrm{Tr}\mathcal{S}[\mu](q, \zeta). \tag{3}$$

In this definition of quark condensate one has taken the real part. This is because previous numerical calculations show that at finite μ the integral appearing in the right-hand side of Eq. (3) is complex. Physically it is hard to understand that the quark condensate is complex. This is one reason that stimulates us to study the quark condensate at finite μ. As will be shown below, if one adopts Eq. (1) and properly considers the distribution of poles of $\mathrm{Tr}\mathcal{S}(q, \zeta)$ in the upper complex q_4 plane, the integral appearing in the right-hand side of Eq. (3) is real. So in this paper we directly generalize the definition (2) to finite μ as follows without taking the real part:

$$-\langle\bar{q}q\rangle_\zeta[\mu] = Z_4(\zeta^2, \Lambda^2)\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}\mathcal{S}[\mu](q,\zeta) = Z_4(\zeta^2, \Lambda^2)\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}\mathcal{S}(\tilde{q},\zeta), \tag{4}$$

where we have made use of Eq. (1). Making use of the identity

$$\int_{C_0} dq_4 f(q_4 + i\mu) = \int_{C_1} dq_4 f(q_4),$$

where C_0 and C_1 are the two integration paths as defined in Fig. 1, one can rewrite Eq. (4) as

$$-\langle\bar{q}q\rangle_\zeta[\mu] = Z_4(\zeta^2, \Lambda^2)\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}\mathcal{S}(\tilde{q},\zeta) = Z_4(\zeta^2, \Lambda^2)\int\frac{d^3\vec{q}}{(2\pi)^3}\int_{C_1}\frac{dq_4}{2\pi}\mathrm{Tr}\mathcal{S}(q,\zeta). \tag{5}$$

Let us use $z_n = \chi_n + i\omega_n(\omega_n > 0)$, $n = 1, 2, \ldots$ to denote the poles of the function $F(q_4) \equiv \mathrm{Tr}\mathcal{S}(q,\zeta)$ in the upper complex q_4 plane. Then from Eq. (5) and Cauchy's theorem we obtain the following:

$$\begin{aligned}-\langle\bar{q}q\rangle_\zeta[\mu] &= Z_4(\zeta^2, \Lambda^2)\int\frac{d^3\vec{q}}{(2\pi)^3}\int_{C_0}\frac{dq_4}{2\pi}\mathrm{Tr}\mathcal{S}(q,\zeta) - iZ_4(\zeta^2, \Lambda^2)\sum_n\int\frac{d^3\vec{q}}{(2\pi)^3}\theta(\mu-\omega_n) \times \mathrm{Res}(F, z_n)\\ &= -\langle\bar{q}q\rangle_\zeta - iZ_4(\zeta^2, \Lambda^2)\sum_n\int\frac{d^3\vec{q}}{(2\pi)^3} \times \theta(\mu-\omega_n)\mathrm{Res}(F, z_n).\end{aligned} \tag{6}$$

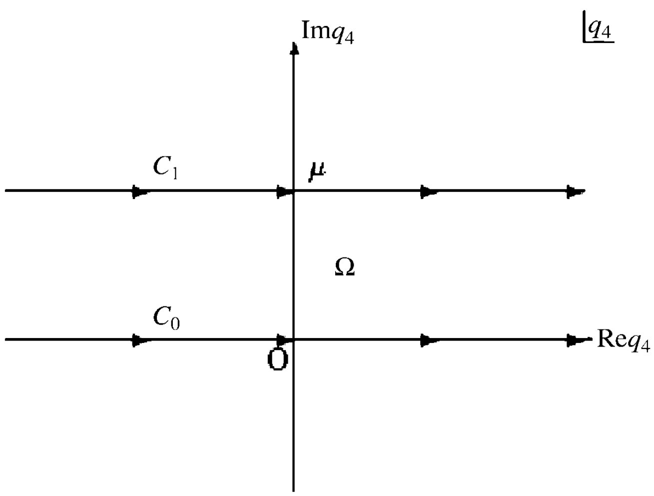

FIG. 1. The integration paths in the complex q_4 plane.

In order to calculate $\langle\bar{q}q\rangle_\zeta[\mu]$ using Eq. (6), one needs to specify the form of the dressed-quark propagator at zero chemical potential $\mathcal{S}(q,\zeta)$. In Ref. [19], guided by the solution of the coupled set of DSEs for the ghost, gluon, and quark propagator in the Landau gauge, the following meromorphic form of the renormalized quark propagator is proposed:

$$\mathcal{S}(q,\zeta) = Z_2^{-1}(\zeta^2, \Lambda^2)\sum_{j=1}^{n_P}\left(\frac{r_j}{i\not{q} + a_j + ib_j} + \frac{r_j}{i\not{q} + a_j - ib_j}\right), \tag{7}$$

where the renormalization scale is set to be $\zeta^2 = 16\ \mathrm{GeV}^2$. The propagator of this form has n_P pairs of complex conjugate poles located at $a_j \pm ib_j$. When some b_j is set to zero, the pair of complex conjugate poles degenerates to a real pole. The residues r_j are real (note that a similar meromorphic form of the quark propagator was previously proposed in Ref. [20], in which the residues in the two additive terms are complex conjugate of each other). In the chiral limit, the requirement that the dressed-quark propagator reduces to the free one in the large momentum limit entails that

$$\sum_{j=1}^{n_P} r_j = \frac{1}{2} \quad \text{and} \quad \sum_{j=1}^{n_P} r_j a_j = 0. \tag{8}$$

Then we have

$$\begin{aligned}F(q_4) &= \mathrm{Tr}\mathcal{S}(q,\zeta)\\ &= Z_2^{-1}(\zeta^2, \Lambda^2)\sum_{j=1}^{n_P} Nr_j\left[\frac{a_j + ib_j}{q^2 + (a_j + ib_j)^2} + \frac{a_j - ib_j}{q^2 + (a_j - ib_j)^2}\right]\\ &= Z_2^{-1}(\zeta^2, \Lambda^2)\sum_{j=1}^{n_P} Nr_j\left[\frac{a_j + ib_j}{q_4^2 + \vec{q}^2 + (a_j + ib_j)^2} + \frac{a_j - ib_j}{q_4^2 + \vec{q}^2 + (a_j - ib_j)^2}\right],\end{aligned} \tag{9}$$

where $N \equiv 4N_cN_f$ with N_c and N_f denoting the number of colors and flavors, respectively. Each additive term in $F(q_4)$ has two poles in the upper complex q_4 plane:

$$z_{j\pm} = \pm\chi_j + i\omega_j, \tag{10}$$

$$\omega_j = \sqrt{\frac{\vec{q}^2 + a_j^2 - b_j^2 + \sqrt{(\vec{q}^2 + a_j^2 - b_j^2)^2 + 4a_j^2b_j^2}}{2}}, \tag{11}$$

$$\chi_j = -\frac{a_jb_j}{\omega_j}. \tag{12}$$

Now let us analyze the θ-function in Eq. (6). From Eq. (11) we find that when $\mu < |a_j|$, ω_j will always be larger than μ irrespective of $\vec{q}$ and thus the contribution from the pole is zero due to the $\theta(\mu - \omega_j)$ function. When $\mu > |a_j|$, $\omega_j < \mu$ for $\vec{q}^2 < b_j^2 - a_j^2 + \mu^2 - (a_j^2 b_j^2/\mu^2)$ and $\omega_j > \mu$ for $\vec{q}^2 > b_j^2 - a_j^2 + \mu^2 - (a_j^2 b_j^2/\mu^2)$. Therefore we obtain the following:

$$-\langle\bar{q}q\rangle_\zeta[\mu] = -\langle\bar{q}q\rangle_\zeta - iZ_4(\zeta^2,\Lambda^2)\sum_{j=1}^{n_P}\theta(\mu - |a_j|) \times \int_0^{q_M}\frac{d|\vec{q}|}{(2\pi)^3}\cdot 4\pi\vec{q}^2(\mathrm{Res}(F, z_{j+}) + \mathrm{Res}(F, z_{j-})), \tag{13}$$

where $q_M \equiv \sqrt{b_j^2 - a_j^2 + \mu^2 - (a_j^2 b_j^2/\mu^2)}$. The residues are easily calculated to be

$$\begin{aligned}&\mathrm{Res}(F, z_{j+}) + \mathrm{Res}(F, z_{j-})\\ &= Z_2^{-1}(\zeta^2,\Lambda^2)Nr_j\left[\frac{a_j + ib_j}{2(\chi_j + i\omega_j)} + \frac{a_j - ib_j}{2(-\chi_j + i\omega_j)}\right]\\ &= Z_2^{-1}(\zeta^2,\Lambda^2)\frac{Nr_j(a_j\omega_j - b_j\chi_j)}{i(\chi_j^2+\omega_j^2)}\\ &= Z_2^{-1}(\zeta^2,\Lambda^2)\frac{Nr_j(\omega_j^2 + b_j^2)a_j\omega_j}{i(a_j^2 b_j^2 + \omega_j^4)}.\end{aligned} \tag{14}$$

Then Eq. (13) can be written as:

$$-\langle\bar{q}q\rangle_\zeta[\mu] = -\langle\bar{q}q\rangle_\zeta - Z_m(\zeta^2,\Lambda^2)\sum_{j=1}^{n_P}\theta(\mu - |a_j|)\frac{Nr_j a_j}{2\pi^2} \times \int_0^{q_M} d|\vec{q}|\frac{\vec{q}^2(\omega_j^2 + b_j^2)\omega_j}{a_j^2 b_j^2 + \omega_j^4}. \tag{15}$$

Now let us calculate the integral in Eq. (15). First let us introduce a new integration variable ω_j by Eq. (11):

$$|\vec{q}| = \frac{1}{\omega_j}\sqrt{(\omega_j^2 + b_j^2)(\omega_j^2 - a_j^2)}, \tag{16}$$

$$d|\vec{q}| = \frac{\omega_j^4 + a_j^2 b_j^2}{\omega_j^2\sqrt{(\omega_j^2 + b_j^2)(\omega_j^2 - a_j^2)}}d\omega_j. \tag{17}$$

Then we have the following:

$$\int_0^{q_M} d|\vec{q}|\frac{\vec{q}^2(\omega_j^2 + b_j^2)\omega_j}{a_j^2 b_j^2 + \omega_j^4} = \int_{|a_j|}^{\mu}\frac{(\omega_j^2 + b_j^2)\sqrt{(\omega_j^2 + b_j^2)(\omega_j^2 - a_j^2)}}{\omega_j^4}\omega_j d\omega_j. \tag{18}$$

We make a further change of variable $t = \sqrt{\omega_j^2 - a_j^2}$ and obtain

$$\begin{aligned}&\int_{|a_j|}^{\mu}\frac{(\omega_j^2 + b_j^2)\sqrt{(\omega_j^2 + b_j^2)(\omega_j^2 - a_j^2)}}{\omega_j^4}\omega_j d\omega_j\\ &= \int_0^{\sqrt{\mu^2 - a_j^2}} dt\frac{t^2(t^2 + a_j^2 + b_j^2)\sqrt{t^2 + a_j^2 + b_j^2}}{(t^2 + a_j^2)^2}\\ &= \int_0^{\sqrt{\mu^2 - a_j^2}}\left[1 + \frac{b_j^2 - a_j^2}{t^2 + a_j^2} - \frac{a_j^2 b_j^2}{(t^2 + a_j^2)^2}\right]\sqrt{t^2 + a_j^2 + b_j^2}dt\\ &= (I_1(t) + I_2(t) + I_3(t))|_0^{\sqrt{\mu^2 - a_j^2}},\end{aligned}$$

where

$$I_1(t) = \int\sqrt{t^2 + a_j^2 + b_j^2}dt = \frac{1}{2}t\sqrt{t^2 + a_j^2 + b_j^2} + \frac{1}{4}(a_j^2 + b_j^2) \times \ln\frac{\sqrt{t^2 + a_j^2 + b_j^2} + t}{\sqrt{t^2 + a_j^2 + b_j^2} - t}, \tag{19}$$

$$I_2(t) = \int\frac{b_j^2 - a_j^2}{t^2 + a_j^2}\sqrt{t^2 + a_j^2 + b_j^2}dt = \left|\frac{b_j}{a_j}\right|(b_j^2 - a_j^2)\arctan\sqrt{\frac{b_j^2 t^2}{a_j^2(t^2 + a_j^2 + b_j^2)}} + \frac{b_j^2 - a_j^2}{2}\ln\frac{\sqrt{t^2 + a_j^2 + b_j^2} + t}{\sqrt{t^2 + a_j^2 + b_j^2} - t}, \tag{20}$$

$$I_3(t) = -\int\frac{a_j^2 b_j^2}{(t^2 + a_j^2)^2}\sqrt{t^2 + a_j^2 + b_j^2}dt = -(a_j^2 + b_j^2)\left|\frac{b_j}{2a_j}\right|\left[\arctan\sqrt{\frac{b_j^2 t^2}{a_j^2(t^2 + a_j^2 + b_j^2)}} + \frac{\sqrt{a_j^2 b_j^2 t^2(t^2 + a_j^2 + b_j^2)}}{(a_j^2 + b_j^2)(t^2 + a_j^2)}\right]. \tag{21}$$

Thus we obtain

$$f_j(\mu) \equiv \int_0^{q_M} d|\vec{q}| \frac{\vec{q}^2(\omega_j^2 + b_j^2)\omega_j}{a_j^2 b_j^2 + \omega_j^4}$$
$$= \frac{1}{2}\left(1 - \frac{b_j^2}{\mu^2}\right)\sqrt{(\mu^2 - a_j^2)(\mu^2 + b_j^2)} + \frac{3b_j^2 - a_j^2}{4}$$
$$\times \ln \frac{\sqrt{\mu^2 + b_j^2} + \sqrt{\mu^2 - a_j^2}}{\sqrt{\mu^2 + b_j^2} - \sqrt{\mu^2 - a_j^2}} + \frac{b_j^2 - 3a_j^2}{2} \left| \frac{b_j}{a_j} \right.$$
$$\times \left| \arctan \sqrt{\frac{b_j^2(\mu^2 - a_j^2)}{a_j^2(\mu^2 + b_j^2)}}, \right. \tag{22}$$

and using this the quark condensate at finite μ can be expressed as

$$-\langle \bar{q}q \rangle_\zeta[\mu] = -\langle \bar{q}q \rangle_\zeta - Z_m(\zeta^2, \Lambda^2) \sum_{j=1}^{n_P} \theta(\mu - |a_j|) \times \frac{N r_j a_j}{2\pi^2} f_j(\mu). \tag{23}$$

From the above expression it can be seen that $\langle \bar{q}q \rangle_\zeta[\mu]$ is real, which is expected physically. Now it is interesting to compare our result with the results obtained in the previous literature. First, let us see the μ-dependence of the quark condensate. From Eq. (23) it can be seen that in our approach, when μ is less than the minimal one of $|a_j|$ ($1 \leq j \leq n_P$), the quark condensate is kept unchanged from its vacuum value. As was mentioned above, in the previous literatures the calculated value of the quark condensate at finite μ is complex and in this case one regards its real part as the chiral order parameter. So here we compare our result with the real part of the value obtained there. In Ref. [7], which employs the Nambu-Jona-Lasinio (NJL) model, it is reported that at finite μ the absolute value of the quark condensate decreases with increasing μ, while in the nonlocal, covariant extension of the NJL model (see, e.g., Ref. [21]), it is also reported that at $T = 0$ the quark condensate is kept unchanged from its vacuum value when μ is below some critical value. In the review article [12] on finite density and finite temperature Dyson-Schwinger equations, it is pointed out that in all models that preserve the momentum dependence of the dressed-quark self-energies, the quark condensate increases with increasing μ [3–6].

In the above calculations we have employed the meromorphic quark propagator (7) with real residues. In principle, we can do the same calculation for a more general form of meromorphic quark propagator with complex residues (see Ref. [20])

$$\mathcal{S}(q, \zeta) = Z_2^{-1}(\zeta^2, \Lambda^2) \sum_{j=1}^{n_P} \left(\frac{r_j}{iq + a_j + ib_j} + \frac{r_j^*}{iq + a_j - ib_j} \right). \tag{24}$$

In this case Eq. (9) changes into

$$F(q_4) = \mathrm{Tr}\mathcal{S}(q, \zeta)$$
$$= Z_2^{-1}(\zeta^2, \Lambda^2) \sum_{j=1}^{n_P} N \left[\frac{r_j(a_j + ib_j)}{q_4^2 + \vec{q}^2 + (a_j + ib_j)^2} + \frac{r_j^*(a_j - ib_j)}{q_4^2 + \vec{q}^2 + (a_j - ib_j)^2} \right]. \tag{25}$$

Obviously the location of poles does not change. The residues are calculated to be

$$\mathrm{Res}\,(F, z_{j+}) + \mathrm{Res}(F, z_{j-}) = Z_2^{-1}(\zeta^2, \Lambda^2) N \left[\frac{r_j(a_j + ib_j)}{2(\chi_j + i\omega_j)} + \frac{r_j^*(a_j - ib_j)}{2(-\chi_j + i\omega_j)} \right]$$
$$= Z_2^{-1}(\zeta^2, \Lambda^2) \left[\mathrm{Re}(r_j) \times \frac{N(\omega_j^2 + b_j^2) a_j \omega_j}{i(a_j^2 b_j^2 + \omega_j^4)} - \mathrm{Im}(r_j) \frac{N(\omega_j^2 - a_j^2) b_j \omega_j}{i(a_j^2 b_j^2 + \omega_j^4)} \right] \tag{26}$$

and the expression for the quark condensate is

$$-\langle \bar{q}q \rangle_\zeta[\mu] = -\langle \bar{q}q \rangle_\zeta - Z_m(\zeta^2, \Lambda^2) \sum_{j=1}^{n_P} \theta(\mu - |a_j|) \times \left[\frac{N a_j}{2\pi^2} \mathrm{Re}(r_j) \int_0^{q_M} d|\vec{q}| \frac{\vec{q}^2(\omega_j^2 + b_j^2)\omega_j}{a_j^2 b_j^2 + \omega_j^4} - \frac{N b_j}{2\pi^2} \mathrm{Im}(r_j) \int_0^{q_M} d|\vec{q}| \frac{\vec{q}^2(\omega_j^2 - a_j^2)\omega_j}{a_j^2 b_j^2 + \omega_j^4} \right]. \tag{27}$$

The integral involved can be readily calculated to be

$$g_j(\mu) \equiv \int_0^{q_M} d|\vec{q}| \frac{\vec{q}^2(\omega_j^2 - a_j^2)\omega_j}{a_j^2 b_j^2 + \omega_j^4}$$
$$= \frac{1}{2}\left(1 + \frac{a_j^2}{\mu^2}\right)\sqrt{(\mu^2 - a_j^2)(\mu^2 + b_j^2)}$$
$$+ \frac{b_j^2 - 3a_j^2}{4} \ln\frac{\sqrt{\mu^2 + b_j^2} + \sqrt{\mu^2 - a_j^2}}{\sqrt{\mu^2 + b_j^2} - \sqrt{\mu^2 - a_j^2}}$$
$$+ \frac{a_j^2 - 3b_j^2}{2} \left|\frac{a_j}{b_j}\right| \arctan\sqrt{\frac{b_j^2(\mu^2 - a_j^2)}{a_j^2(\mu^2 + b_j^2)}} \tag{28}$$

and the final result is

$$-\langle\bar{q}q\rangle_\zeta[\mu] = -\langle\bar{q}q\rangle_\zeta - Z_m(\zeta^2, \Lambda^2)\sum_{j=1}^{n_P}\theta(\mu - |a_j|)\frac{N}{2\pi^2}$$
$$\times [\mathrm{Re}(r_j)a_j f_j(\mu) - \mathrm{Im}(r_j)b_j g_j(\mu)]. \tag{29}$$

From the above expression it can be seen that for the model quark propagator (24) the calculated quark condensate still has the two features found previously: (1) $\langle\bar{q}q\rangle_\zeta[\mu]$ is real; (2) when μ is less than a critical value μ_c (i.e., the minimal one of $|a_j|$ $(1 \leq j \leq n_P)$), $\langle\bar{q}q\rangle_\zeta[\mu]$ is kept unchanged from its vacuum value.

Having obtained the analytic expression for the quark condensate at finite chemical potential, now let us analyze the range of applicability of this expression. From our calculation, it can be seen that the behavior of the quark condensate at finite μ depends strongly on the form of the dressed-quark propagator at zero μ. It is well known that the nonperturbative quark propagator is a result of the self-consistent solution of QCD DSEs, which reflects the dynamical chiral symmetry breaking of QCD vacuum with the quark condensate being the corresponding order parameter. The excitation of dynamical quarks leads to a melting of the quark condensate and therefore to a change of the QCD vacuum structure [22]. However, the approach in the present paper does not take into account such a change in the QCD vacuum structure. Therefore, one expects that the propagator $\mathcal{S}[\mu](q, \zeta)$ adopted in this paper is applicable at best for $\mu < \mu_c$. As a consequence, the obtained expression for the quark condensate is also only applicable for $\mu < \mu_c$.

In order to give a prediction of the critical chemical potential μ_c at zero temperature for a given parametrization of the model quark propagator (24), we need to specify its parameters. For definiteness we use three sets of parameters given in Ref. [19], which represent three forms of the propagator: three real poles (3R), two pairs of complex conjugate poles (2CC), and one real pole and one pair of complex conjugate poles (1R1CC). These parameters are listed in Table I.

From Eq. (2), in order to evaluate the vacuum quark condensate $\langle\bar{q}q\rangle_\zeta[\mu = 0]$, we have to know the mass renormalization constant $Z_m(\zeta^2, \Lambda^2)$. For the mass renormalization constant Z_m we take the one-loop perturbative result [12]

$$Z_m(\zeta^2, \Lambda^2) = \left[\frac{\alpha(\Lambda^2)}{\alpha(\zeta^2)}\right]^{\gamma_m}$$

with

$$\alpha(\zeta^2) = \frac{\pi}{-\frac{1}{2}\beta_1 \ln[\zeta^2/\Lambda_{\mathrm{QCD}}^2]}$$

being the running strong coupling constant (in this paper, following Ref. [19], we choose $\Lambda_{\mathrm{QCD}} = 0.5$ GeV) and $\gamma_m = 12/(33 - 2N_f)$ the mass anomalous dimension. In our numerical calculation the renormalization point is set to be $\zeta^2 = 16\ \mathrm{GeV}^2$ and the regularization mass scale Λ^2 is also set to be this value, which is large enough. So we have $Z_m(\zeta^2, \Lambda^2) = 1$. The value of the vacuum condensate $-\langle\bar{q}q\rangle^{1/3}[\mu = 0]$ and the critical chemical potential μ_c calculated using the three sets of parameters are given below 10 cm Table II. The quark condensate at $\mu = 0$ and critical chemical potential μ_c.

The values of the vacuum quark condensate given in Table II can be directly compared with the value of the quark condensate employed in the contemporary phenomenological studies [23]: $(0.236 \pm 0.008$ GeV). The critical chemical potential given in Table II is somewhat larger than the value obtained in Ref. [21] using a nonlocal, covariant extension of the NJL model (about 278 MeV).

To summarize, based on the method of calculating the dressed-quark propagator at finite chemical potential in the framework of the rainbow-ladder approximation of the Dyson-Schwinger approach proposed in Ref. [8] and adopting the meromorphic form of the quark propagator proposed in Refs. [19,20], an analytic expression for the quark condensate at finite chemical potential $\langle\bar{q}q\rangle[\mu]$ is obtained. In this model $\langle\bar{q}q\rangle[\mu]$ is totally determined by the distribution of poles of $\mathrm{Tr}\mathcal{S}(q)$ in the upper complex q_4 plane. It is found that when the chemical potential μ is less than a critical one μ_c, the quark condensate is kept unchanged from its vacuum value. This behavior of the quark

TABLE I. The parameters used to calculate the quark condensate. These parameters are taken directly from Table II of Ref. [19].

Parametrization	r_1	a_1 (GeV)	b_1 (GeV)	r_2	a_2 (GeV)	b_2 (GeV)	r_3	a_3 (GeV)
2CC	0.360	0.351	0.08	0.140	−0.899	0.463	—	—
1R1CC	0.354	0.377	—	0.146	−0.91	0.45	—	—
3R	0.365	0.341	—	1.2	−1.31	—	−1.06	−1.40

TABLE II. The quark condensate at $\mu = 0$ and critical chemical potential μ_c.

Parametrization	$-(\bar{q}q)^{1/3}[\mu = 0]$ (GeV)	μ_c (GeV)
2CC	0.30	0.351
1R1CC	0.30	0.377
3R	0.32	0.341

condensate at finite μ is compared with those reported in previous literatures [3–10]. In our analytic calculation, the distribution of poles of $\mathrm{Tr}\mathcal{S}(q)$ is properly considered and the resultant expression of $\langle\bar{q}q\rangle[\mu]$ is real without the need of taking the real part. The range of applicability of the obtained analytic expression for $\langle\bar{q}q\rangle[\mu]$ is discussed, and we argue that it is only applicable for $\mu < \mu_c$. For three different parametrizations of the model quark propagator given in Ref. [19] (three real poles (3R), two pairs of complex conjugate poles (2CC), and one real pole and one pair of complex conjugate poles (1R1CC)) the critical chemical potential is found to be 351 MeV, 377 MeV, and 341 MeV, respectively, which is somewhat larger than the value obtained in Ref. [21] using a nonlocal, covariant extension of the NJL model (about 275 MeV).

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