

# $B \rightarrow D\tau\nu$ branching ratios: Opportunity for lattice QCD and hadron colliders

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In the standard model, scalar contributions to leptonic and semileptonic decays are helicity suppressed. The hypothesis of additional physical neutral/charged Higgses can enhance such scalar contributions and give detectable effects especially in  $B$  physics. For the charged Higgs, experimental information on both  $\text{Br}(B \rightarrow D\tau\nu)$  and  $\text{Br}(B \rightarrow \tau\nu)$  has already become available, and in particular, the  $B \rightarrow D\tau\nu$  branching ratio measurements will be further improved in the coming years. Hadronic uncertainties of scalar contributions in semileptonic decays are already in much better shape than the ones plaguing the helicity-suppressed leptonic decays  $B \rightarrow \tau\nu$ . Combining existing experimental information from the  $B$  factories, we explore which existing and future lattice estimates will be useful to directly address new physics effects from measurements of  $\text{Br}(B_{u,d,s} \rightarrow D_{u,d,s}\tau\nu)$ , which can be performed also at hadron colliders.

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As is often stressed, in the near future the CERN LHC will represent the main avenue to establish the presence of new physics (NP) by directly detecting new particles at the TeV scale. On the other hand, virtual effects of these particles can affect low-energy observables, probed mainly by the flavor factories and soon by the LHCb. As has been proven by the  $B$  factories, the energy reach of such indirect searches can often surpass direct detection strategies, making them worthy of pursuit even at the opening of the new energy frontier. Among the possible new particles, the Higgs boson is the only one expected in the standard model (SM) picture. At the same time, we have to observe that the established SM parametrization of the Higgs sector is only a conservative example of a possible electroweak symmetry breaking mechanism. The present information on the massive  $W$  and  $Z$  bosons from electroweak precision tests only constrains the Goldstone modes [1] of the Higgs field while leaving space for an extended physical Higgs sector. Namely, additional neutral/charged Higgses appear in many models trying to solve the inconsistencies of the SM.

Therefore theoretical and experimental study of scalar effects in observables, mediated at tree level by neutral/charged bosons,<sup>1</sup> is vitally important in future experimental programs. In particular, effective density operators from charged scalar boson interactions have to be considered in the effective weak Hamiltonian, which for  $b \rightarrow q(u, c)$  transitions, for example, reads

$$\mathcal{H}_{\text{eff}}^{b \rightarrow q} = \frac{G_F}{\sqrt{2}} V_{qb} \sum_{l=e,\mu,\tau} [(\bar{q}\gamma_\mu(1-\gamma_5)b)(\bar{l}\gamma^\mu(1-\gamma_5)\nu) + C_{\text{NP}}^l(\bar{q}(1+\gamma_5)b)(\bar{l}(1-\gamma_5)\nu)] + \text{H.c.} \quad (1)$$

<sup>1</sup>Loop induced flavor changing neutral current processes, for example [2]  $b \rightarrow s\gamma$ , can be sensitive to additional Higgses, but this information is diluted by contributions from other particles, and final constraints are model dependent [3].

In the minimal flavor violating (MFV) extensions of the SM [4] by an additional Higgs doublet, the additional NP coupling can be written as

$$C_{\text{NP}}^l = -\frac{m_b m_l}{m_{H^\pm}^2} \frac{\tan^2 \beta}{1 + \epsilon_0 \tan \beta}, \quad (2)$$

where  $\tan \beta$  is the ratio of the two Higgs vacuum expectation values while  $\epsilon_0$  parametrizes possible Peccei-Quinn symmetry breaking corrections and is typically of the order of 1% in the MFV minimal supersymmetric SM (MSSM). Because of the suppression of quark and lepton Yukawa couplings in Eq. (2),  $B$ -helicity suppressed processes receive the largest effects from the charged Higgs. In this respect, the  $B \rightarrow \tau\nu$  decay branching ratio [5], given by

$$\text{Br}(B \rightarrow \tau\nu) = \frac{G_F^2 |V_{ub}|^2}{8\pi} m_\tau^2 f_B^2 m_B \tau_B \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \times \left|1 + \frac{m_B^2}{m_b m_\tau} C_{\text{NP}}^\tau\right|^2, \quad (3)$$

has often been stressed as a good candidate, and the recent  $B$  factory results have given important constraints on  $C_{\text{NP}}^\tau$ . Unfortunately, the presently established experimental precision is only about 30% and unlikely to improve in the near future as the perspectives to measure  $B \rightarrow \tau\nu$  at the Tevatron or LHCb are highly compromised. Furthermore, the SM expectation estimate presently suffers from sizable parametrical uncertainties induced by  $|V_{ub}|$  and  $f_B$ . This opens the door for alternative modes to be studied with the present experiments.

While Higgs effects in  $K$  and  $D$  modes are small and difficult to disentangle at present theoretical precision [6,7], the situation is much better in the case of semileptonic  $B \rightarrow Dl\nu$  decays [8–10]. The partial rate can be written in terms of  $w = v_B \cdot v_D$  as

$$\frac{d\Gamma(B \rightarrow D l \bar{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2 m_B^5}{192 \pi^3} \rho_V(w) \left[ 1 - \frac{m_l^2}{m_B^2} \right. \\ \left. \times \left| 1 + \frac{t(w)}{(m_b - m_c) m_l} C_{\text{NP}}^l \right|^2 \rho_S(w) \right], \quad (4)$$

where  $t(w) = m_B^2 + m_D^2 - 2wm_D m_B$  and we have decomposed the rate into the vector and scalar Dalitz density contributions

$$\rho_V(w) = 4 \left( 1 + \frac{m_D}{m_B} \right)^2 \left( \frac{m_D}{m_B} \right)^3 (w^2 - 1)^{3/2} \left( 1 - \frac{m_l^2}{t(w)} \right)^2 \\ \times \left( 1 + \frac{m_l^2}{2t(w)} \right) G(w)^2, \quad (5)$$

$$\rho_S(w) = \frac{3}{2} \frac{m_B^2}{t(w)} \left( 1 + \frac{m_l^2}{2t(w)} \right)^{-1} \frac{1+w}{1-w} \Delta(w)^2, \quad (6)$$

where  $G(w)$  and  $\Delta(w)$  encode our ignorance of the QCD dynamics. Even before analyzing the theoretical uncertainties of these modes let us note that the present constraints on  $C_{\text{NP}}^\tau$  from  $K \rightarrow \mu \nu$  [6] and  $B \rightarrow \tau \nu$  [11] decays<sup>2</sup> still allow for sizable new physics effects in Eq. (4) for the case of  $B \rightarrow D \tau \nu$  as represented in Fig. 1, where the allowed region of the helicity-suppressed contribution of Eq. (4) for  $B \rightarrow D \tau \nu$ , namely,

$$\rho_S^{\text{NP}}(w) = \left| 1 + \frac{t(w)}{(m_b - m_c) m_\tau} C_{\text{NP}}^\tau \right|^2 \rho_S(w), \quad (7)$$

is shown.

The main parametric uncertainties in Eq. (4) are represented by the modulus of  $V_{cb}$  and the hadronic form factors  $G(w)$  and  $\Delta(w)$ . Presently, the most accurate value of  $|V_{cb}| = 4.15(7)\%$  comes from the fit to inclusive  $B \rightarrow X_c l \nu$  decays which are insensitive to scalar contributions [9]. Because of charm states, information from heavy-quark expansion for the form factors is *a priori* unsatisfactory, since corrections to the static limit  $m_c, m_b \rightarrow \infty$ , formally parametrized by  $\xi = 1/m_b(1 - m_b/m_c)$  can be large and undetermined. More reliable information is expected from the lattice, and indeed a number of quenched as well as unquenched studies have already computed the normalization of the vector form factor  $G(w)$  at  $w = 1$  to a precision of a few percent [12], while a recent study extended its determination to a region of  $w \in [1, 1.2]$  [13]. These values must then be extrapolated over the entire kinematically accessible decay phase space, which is larger in the case of  $B \rightarrow D e \nu$  ( $w \in [1, 1.59]$ ) than for the tau mode ( $w \in [1, 1.43]$ ). For such an extrapolation, the Heavy Flavor Averaging Group (HFAG) adopts the

<sup>2</sup>In detail, the  $\epsilon_0 \tan\beta$  terms in Eq. (2) are set to be equal between  $B \rightarrow \tau \nu$  and  $K \rightarrow \mu \nu$ , as it happens in MFV MSSM.

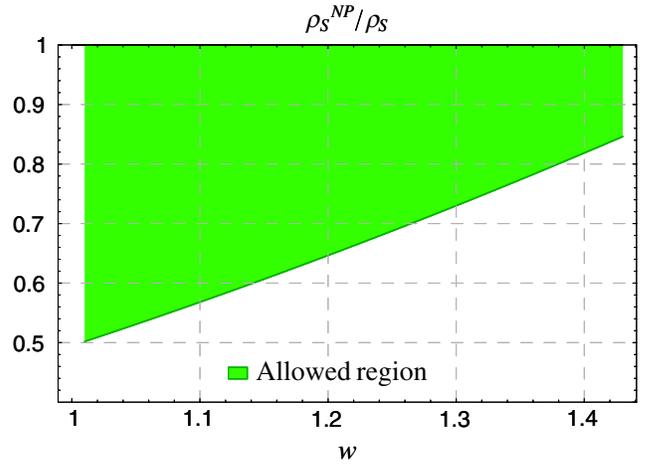


FIG. 1 (color online). In the shaded area (green) we plot as a function of  $w$  the allowed region for  $\rho_S^{\text{NP}}(w)/\rho_S(w)$  in Eq. (7), using constraints from both  $B \rightarrow \tau \nu$  and  $K \rightarrow \mu \nu$  decays [6,11]. A large deviation from the unity, the SM expectation, is still possible with respect to the SM. Note that  $\rho_S(w)$  contributes 50% to the  $\text{Br}(B \rightarrow D \tau \nu)$ .

parametrization of  $G(w)$  [14]<sup>3</sup>

$$G(w) = G(1) \times [1 - 8\rho^2 z(w) + (51\rho^2 - 10)z(w)^2 \\ - (252\rho^2 - 84)z(w)^3], \quad (8)$$

with  $z(w) = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$  in terms of two parameters: the normalization  $G(1)$  and the slope  $\rho^2$ . In addition, in the SM, as well as in its MFV extensions, only  $G(w)$  will actually contribute to  $B \rightarrow D e \nu$ , and one can use experimental information on the differential decay spectra in such an extrapolation.<sup>4</sup> At present, the HFAG [17] experimental information consists of relatively old publications by Belle [18] and Cleo [19]. We can nevertheless use this information to assess the relative precision obtainable from combining lattice information with experimental inputs efficiently. We compare in Fig. 2 the Belle [18] and Cleo [19] data on  $|V_{cb} G(w)|$  and the HFAG fit to the data from Eq. (8) [using  $|V_{cb}| G(1) = (42.3 \pm 4.5)10^{-3}$ ,  $\rho^2 = 1.17 \pm 0.18$  with correlation 0.93], together with the lattice data from Ref. [13] and the fit from Eq. (8) to the lattice

<sup>3</sup>Using analyticity and crossing symmetry, a general parametrization for semileptonic decays has been proposed in Ref. [15]. However, for modes such as  $B \rightarrow D$ , the smallness of  $z$ , and the judicious use of heavy-quark symmetry in Ref. [14], allows for an especially tailored parametrization in terms of Eq. (8).

<sup>4</sup>For completeness, the mechanism introduced in Ref. [16] to enhance electronic modes in  $K \rightarrow e \nu_\tau$  and  $B \rightarrow e \nu_\tau$  by orders of magnitude gives negligible effects less than 0.1% for the partial rate of  $B \rightarrow D l \nu$ , once the  $K \rightarrow e \nu_\tau$  bound [6] is taken into account.

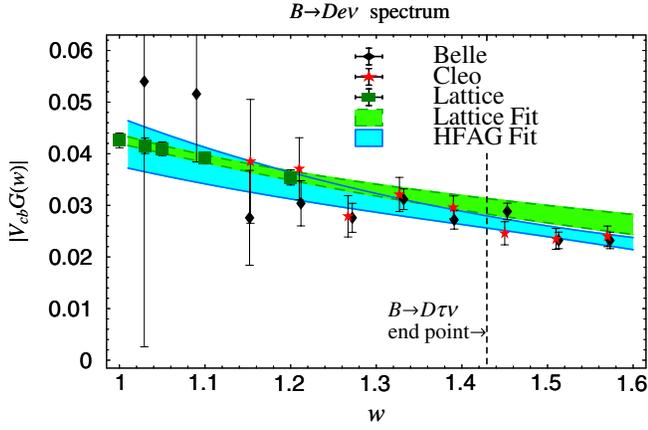


FIG. 2 (color online). Comparison of  $B \rightarrow De\nu$  form factor determination from Belle [18], Cleo [19], and lattice QCD. The latter data points have been multiplied by the HFAG world average value of  $|V_{cb}|$  from inclusive measurements. The HFAG average fit [17] to Eq. (8) is also shown.

results of  $G(w)$  [13]<sup>5</sup> both multiplied by the HFAG value of  $|V_{cb}|$  mentioned above. The two sets are in agreement at present precision (roughly 10% on the normalization and 15% on the slope), while improvement could come from several sources: *BABAR* has already announced its intention to improve the measurement of the differential decay rate to allow for extraction of form factor shape parameters to below 10% by reducing the statistics error of Belle by a factor of 4 [20].<sup>6</sup> However, to be able to apply this precision to the integrated rates, one would need to precisely determine either  $G(w)$  on the lattice while using inclusive determination of  $|V_{cb}|$  or consider ratios, where at least the overall normalization factors of  $|V_{cb}G(1)|$  cancel.

On the other hand, the uncertainties coming from  $\Delta(w)$ , which regulates the helicity-suppressed terms, are already much smaller, especially than those plaguing the dimensional variable  $f_B$  entering  $B \rightarrow l\nu$  decays. In other words, the current (quenched) lattice estimate of  $\Delta(w)$  for  $w$  in the range 1–1.2 is at a few percent precision, consistent with a constant value of  $\Delta(w) = 0.46(2)$ .<sup>7</sup> Mainly, such an

<sup>5</sup>Numerically the fit yields  $G(1) = 1.03(1)$ ,  $\rho^2 = 0.97(14)$ ; however, these values and especially their errors should not be taken at face value as correlations among different lattice points as well as quenching and chiral extrapolation errors are not taken into account. We only use them as estimates to make qualitative comparisons.

<sup>6</sup>At this level of precision, nonhelicity-suppressed NP contributions to the  $b \rightarrow ce\nu$  transition could be constrained for the first time (for example,  $R$  parity violating MSSM [21]).

<sup>7</sup>Unknown correlations among lattice values at different  $w$  are taken into account by considering a conservative estimate of the error to be the average of individual values' errors. Quenching and chiral extrapolation errors have not been taken into account in Ref [13]. However, similar studies on the  $B \rightarrow D^*l\nu$  form factors at zero recoil [22] have estimated them to be roughly 20% of the deviation from the heavy-quark limit value, which in the case of  $\Delta(w)$  is still much below the current precision.

achievement on the lattice was possible by introducing double ratios of lattice correlators [12] and  $\theta$  boundary conditions [23]. Moreover, this precision can further be improved by studies involving unquenched simulations and lighter sea quark masses. In particular, a measurement of  $B_s \rightarrow D_s l\nu$  will opt for lattice data on  $B_s \rightarrow D_s$  form factors including scalar contributions. These however no longer require chiral extrapolations for the valence quarks, eliminating important sources of systematics. Finally, since  $\Delta(w)$  only contributes significantly to the decays involving taus, the extrapolation from the region presently probed by lattice simulations to the complete kinematically accessible region is not large as is the case for the  $G(w)$  form factor in  $B \rightarrow De\nu$  transitions.

We finally combine these lessons and try to project the present sensitivity of  $B \rightarrow Dl\nu$  decays to scalar contributions into the near future. We start with the ratio  $\text{Br}(B \rightarrow D\tau\nu)/\text{Br}(B \rightarrow De\nu)$  [9,10] which, as stressed above, even in the presence of NP scalar contributions only depends on two hadronic quantities, the precision of which can furthermore be improved in the near future:  $G(w)$  shape ( $\rho^2$ ) and  $\Delta(w)$ . By integrating Eq. (4) with the use of Eq. (8), the fitted lattice results for the form factor,  $\Delta(w)$  and the HFAG value of  $\rho^2$  as determined from the  $B \rightarrow De\nu$  spectrum, we average over the  $B_{d,u} \rightarrow D_{d,u}$  modes to obtain

$$\frac{\text{Br}(B \rightarrow D\tau\nu)}{\text{Br}(B \rightarrow De\nu)} = (0.28 \pm 0.03) \times [1 + 1.38(6) \text{Re}(C_{\text{NP}}^\tau)] + 0.88(4)|C_{\text{NP}}^\tau|^2. \quad (9)$$

We see that the SM prediction uncertainty is already around 10% and is expected to be improved soon with the new *BABAR* data on the  $G(w)$  shape. Furthermore, future unquenched lattice studies of  $\Delta(w)$  could confirm and reduce the presently quoted errors entering NP contributions estimate. Interestingly, *BABAR* has already published a value [24] for the above ratio with uncertainties of 30%, making it possible to compare with the  $B \rightarrow \tau\nu$  measurement and its bound on  $C_{\text{NP}}^\tau$  in Fig. 3. Even more importantly, unlike  $B \rightarrow \tau\nu$ , this measurement can be improved at hadron colliders together with  $B_s \rightarrow D_s\tau\nu$ . Therefore we plot the present exclusion region in the  $\tan\beta$ - $m_{H^\pm}$  plane in Fig. 4 together with the percentage deviation from the SM prediction for  $\text{Br}(B \rightarrow D\tau\nu)/\text{Br}(B \rightarrow De\nu)$  in the presently allowed region. For reference: the  $B \rightarrow D\tau\nu$  bound presently allows for (real) NP contributions in the range  $(-2.25 < C_{\text{NP}}^\tau < -1.04) \times \mathbf{U}(-0.52 < C_{\text{NP}}^\tau < 0.69)$  at 95% C.L.

An even more prospective observable, perhaps, may be represented by the ratio of partial  $B \rightarrow D\tau(e)\nu$  decay widths integrated over the same kinematical  $w$  region. Since in the case of  $B \rightarrow D\tau\nu$  the kinematically available region is much smaller than for the  $B \rightarrow De\nu$ , one can just consider the full  $\text{Br}(B \rightarrow D\tau\nu)$  [25], while imposing a kinematical cut of  $w < 1.43$  in the light lepton case. In this way one avoids the large extrapolation away from the

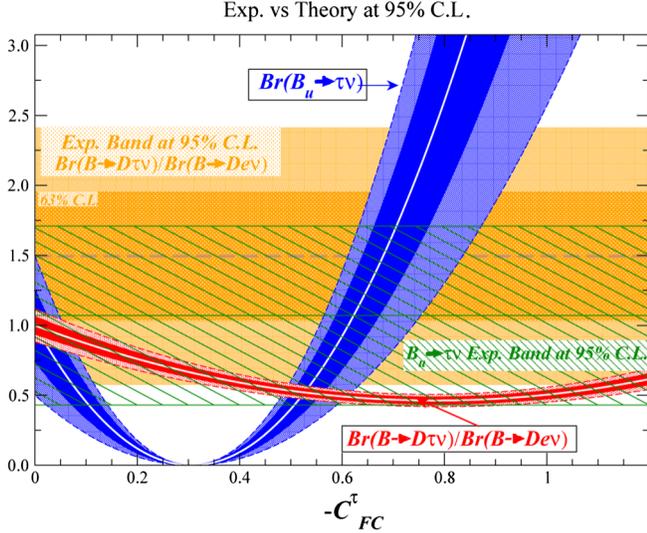


FIG. 3 (color online). The ratio  $\text{Br}(B \rightarrow D\tau\nu)/\text{Br}(B \rightarrow \text{Dev})$  is shown together with the  $\text{Br}(B \rightarrow \tau\nu)$  as a function of  $C_{\text{NP}}^\tau$ , Eq. (2). Both curves have been normalized to their SM central values. Error bands on the curves represent the theoretical uncertainties at 63% and at 95% C.L. The horizontal bands represent the corresponding experimental values [11,24].

lattice data points and further reduces the uncertainty due to the  $\rho^2$  parameter. Presently such a ratio can be estimated at  $\text{Br}(B \rightarrow D\tau\nu)/\text{Br}(B \rightarrow \text{Dev})|_{w < 1.43} = (0.56 \pm 0.03) \times [1 + 1.38(6) \text{Re}(C_{\text{NP}}^\tau) + 0.88(4)|C_{\text{NP}}^\tau|^2]$  with an error on the SM value of only 5%, while the relative new physics contributions are not affected by the cut at all, since they only appear in the tau mode. Once the experimental precision for this observable would approach the above theoretical errors, one could further restrict the kinematical

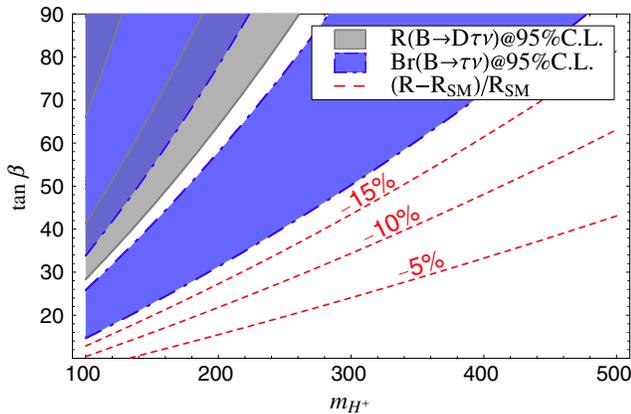


FIG. 4 (color online). Exclusion region in the  $m_{H^+} - \tan\beta$  plane due to the present determination of  $B \rightarrow \tau\nu$  (in dark blue with dash-dotted border) and  $\text{Br}(B \rightarrow D\tau\nu)/\text{Br}(B \rightarrow \text{Dev})$  (in light gray with solid border). Note that the small allowed band in the middle is excluded by the  $K \rightarrow \mu\nu$  determination [6] (not shown). Dashed lines (red) represent percentage deviation from the SM prediction of  $R = \text{Br}(B \rightarrow D\tau\nu)/\text{Br}(B \rightarrow \text{Dev})$  in the presently allowed region.

region considered closer to the one accessible to the lattice studies or finally consider binned or differential rates, where the parametrization dependence would be greatly reduced and direct comparisons with lattice possible.

In existing literature, the differential rates [9,10] are often stressed as also being highly sensitive to scalar contributions in  $B \rightarrow D$  transitions compared to the integrated rate. However, such measurements will only become available with the advent of the super flavor factories, where both the  $B \rightarrow \text{Dev}$  and  $B \rightarrow D\tau\nu$  spectra will be available at a few percent level in several  $w$  bins. Then, measuring the ratio of  $B \rightarrow D\tau\nu$  and  $B \rightarrow \text{Dev}$  differential distributions [9] integrated over given  $w$  bins gives direct access to  $\rho_S^{\text{NP}}(w)$  which can be compared with the lattice estimates of  $\rho_S(w)$  in the same bins to obtain bounds on  $C_{\text{NP}}^\tau$  by reducing ambiguities due to  $G(w)$  estimates and  $w$  parametrizations. We project the potentialities of measuring  $\rho_S^{\text{NP}}(w)$  in Eq. (7) with respect to  $\text{Br}(B \rightarrow \tau\nu)$  in Fig. 5.

In the meantime, the ratio of (partially) integrated rates  $\text{Br}(B \rightarrow D\tau\nu)/\text{Br}(B \rightarrow \text{Dev})$  seems to represent the best strategy for indirectly probing charged Higgs contributions to low-energy observables at the Tevatron and LHCb. Even if  $\text{Br}(B \rightarrow D\tau\nu)/\text{Br}(B \rightarrow \text{Dev})|_{w < 1.43}$  cannot be measured directly, precise data on  $V_{cb}$  and  $B \rightarrow \text{Dev}$  decay spectra from the  $B$  factories can be used to obtain comparable precision directly on the  $B \rightarrow D\tau\nu$  branching ratio. Moreover, since the bounds from  $B \rightarrow \tau\nu$  are affected by larger theoretical uncertainties, the  $B \rightarrow D\tau\nu$  modes allow for an important cross-check. Let us mention that at 95% with the present central value and with a smaller experimental error of 20%, the exclusion region from  $B \rightarrow D\tau\nu$  is already competitive to the one from  $B \rightarrow \tau\nu$ , while at

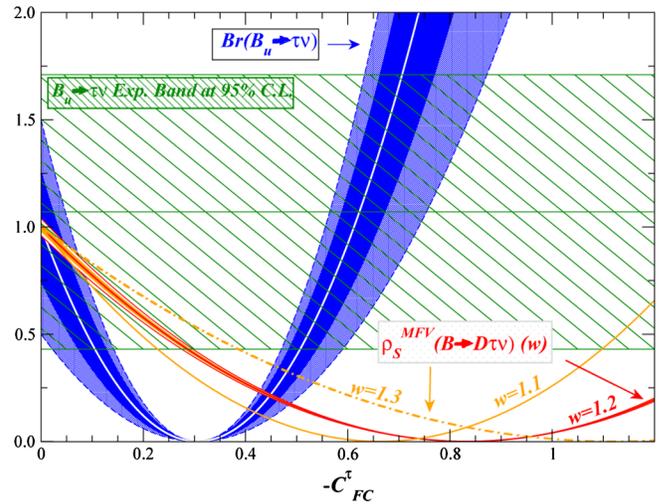


FIG. 5 (color online). The quantity  $\rho_S^{\text{NP}}(w)$  from Eq. (7) is shown for three values of  $w$  as a function of  $C_{\text{NP}}^\tau$ , Eq. (2). The values of  $w = 1.1, 1.2, 1.3$  are chosen to coincide with the presently available lattice data [13]. Experimentally,  $\rho_S^{\text{NP}}(w)$  can be accessed at a super flavor factory via the measurement of  $d\Gamma(B \rightarrow D\tau\nu)/d\Gamma(B \rightarrow D\nu)$ , Eq. (4) in those  $w$  bins.

5% error the SM and the MFV MSSM would actually be excluded. Thus such a precise measurement of  $\text{Br}(B \rightarrow D\tau\nu)$  together with further lattice studies of  $G(w)$  away from  $w = 1$  and  $\Delta(w)$  would be highly welcome since both the central values as well as an accurate estimation of their errors are essential to obtain valid bounds on new physics.

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