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The muon $g-2$ and the bounds on the Higgs boson mass

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After a brief review of the muon $g-2$ status, we analyze the possibility that the present discrepancy between experiment and the standard model (SM) prediction may be due to hypothetical errors in the determination of the hadronic leading-order contribution to the latter. In particular, we show how an increase of the hadroproduction cross section in low-energy e^+e^- collisions could bridge the muon $g-2$ discrepancy, leading however to a decrease on the electroweak upper bound on M_H, the SM Higgs boson mass. That bound is currently $M_H \lesssim 150$ GeV (95% C.L.) based on the preliminary top quark mass $M_t = 172.6(1.4)$ GeV and the recent determination $\Delta\alpha_{\rm had}^{(5)}(M_Z) = 0.027\,68(22)$, while the direct-search lower bound is $M_H > 114.4$ GeV (95% C.L.). By means of a detailed analysis we conclude that this solution of the muon $g-2$ discrepancy is unlikely in view of current experimental error estimates. However, if this turns out to be the solution, the 95% C.L. upper bound on M_H is reduced to about 130 GeV which, in conjunction with the experimental lower bound, leaves a narrow window for the mass of this fundamental particle.

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I. INTRODUCTION

The measurement of the anomalous magnetic moment of the muon a_μ by the E821 experiment at Brookhaven, with a remarkable relative precision of 0.5 ppm [1], is challenging the standard model (SM) of particle physics. Indeed, as each sector of the SM contributes in a significant way to the theoretical prediction of $a_\mu = (g-2)/2$ (g is the muon's gyromagnetic factor), this measurement allows us to test the entire SM and provides a powerful tool to scrutinize viable "new physics" appendages to this theory [2].

The SM prediction of the muon $g-2$ is conveniently split into QED, electroweak (EW), and hadronic [leading- (HLO) and higher-order (HHO)] contributions: $a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm HLO} + a_\mu^{\rm HHO}$. The hadronic contributions dominate the present $a_\mu^{\rm SM}$ uncertainty. The QED prediction, computed up to four (and estimated at five) loops, currently stands at $a_\mu^{\rm QED} = 116\,584\,718.10(16) \times 10^{-11}$ [3,4], while the EW effects, suppressed by a factor $(m_\mu/M_W)^2$, provide $a_\mu^{\rm EW} = 154(2) \times 10^{-11}$ [5]. The most recent calculations of the hadronic leading-order contribution via the hadronic e^+e^- annihilation data, to be discussed later, are in very good agreement: $a_\mu^{\rm HLO} = 6909(44) \times 10^{-11}$ [6], $6894(46) \times 10^{-11}$ [7], $6921(56) \times 10^{-11}$ [8], and $6944(49) \times 10^{-11}$ [9]. The higher-order hadronic term is further divided into two parts: $a_\mu^{\rm HHO} = a_\mu^{\rm HHO}({\rm vp}) + a_\mu^{\rm HHO}({\rm lbl})$. The first one, $-98(1) \times 10^{-11}$ [7], is the $O(\alpha^3)$ contribution of diagrams containing hadronic vacuum polarization insertions [10]. The second term, also of $O(\alpha^3)$, is the hadronic light-by-light contribution; as it cannot be determined from data, its evaluation relies on specific models. Recent determinations of this term vary between $80(40) \times 10^{-11}$ [11] and $136(25) \times 10^{-11}$ [12]. The most recent one, $110(40) \times 10^{-11}$ [13], lies between them. If we add this result to the leading-order hadronic contribution, for example, the value of Ref. [7] (which also provides a recent calculation of the hadronic contribution to the effective fine-structure constant, later required for our analysis), and the rest of the SM contributions, we obtain $a_\mu^{\rm SM} = 116\,591\,778(61) \times 10^{-11}$. The difference with the experimental value $a_\mu^{\rm EXP} = 116\,592\,080(63) \times 10^{-11}$ [1] is $\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} = +302(88) \times 10^{-11}$, i.e., 3.4 standard deviations (all errors were added in quadrature). Similar discrepancies are obtained employing the values of the leading-order hadronic contribution reported in Refs. [6,8,9].

The term $a_\mu^{\rm HLO}$ can alternatively be computed incorporating hadronic τ-decay data, related to those of hadroproduction in e^+e^- collisions via isospin symmetry [14,15]. Unfortunately, there is a large difference between the e^+e^-- and τ-based determinations of $a_\mu^{\rm HLO}$, even if isospin violation corrections are taken into account [16]. The τ-based value is significantly higher, leading to a small

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($\sim 1\sigma$) Δa_μ difference. As the e^+e^- data are more directly related to the $a_\mu^{\rm HLO}$ calculation than the τ ones, the latest analyses do not include the latter. Also, we note that recently studied additional isospin-breaking corrections somewhat reduce the difference between these two sets of data (lowering the τ-based determination) [17,18], and a new analysis of the pion form factor claims that the τ and e^+e^- data are consistent after isospin violation effects and vector meson mixings are considered [19]. Recent reviews of the muon $g-2$ can be found in Refs. [4,20,21].

The 3.4σ discrepancy between the theoretical prediction and the experimental value of the muon $g-2$ can be explained in several ways. It could be due, at least in part, to an error in the determination of the hadronic light-by-light contribution. However, if this were the only cause of the discrepancy, $a_\mu^{\rm HHO}({\rm lbl})$ would have to move up by many standard deviations to fix it—roughly eight, if we use the $a_\mu^{\rm HHO}({\rm lbl})$ result of Ref. [13] (which includes all known uncertainties), and more than ten if the less conservative estimate of Ref. [12] is employed instead. Although the errors assigned to $a_\mu^{\rm HHO}({\rm lbl})$ are only educated guesses, this solution seems unlikely, at least as the dominant one.

Another possibility is to explain the discrepancy Δa_μ via the QED, EW, and hadronic higher-order vacuum polarization contributions; this looks very improbable, as one can immediately conclude inspecting their values and uncertainties reported above. If we assume that the $g-2$ experiment E821 is correct, we are left with two options: possible contributions of physics beyond the SM, or an erroneous determination of the leading-order hadronic contribution $a_\mu^{\rm HLO}$ (or combinations of the two). The first of these two options has been widely discussed in the literature; we will focus on the second one, and analyze its implications for the EW bounds on the mass of the Higgs boson.

II. SHIFTS OF $a_\mu^{\rm HLO}$ AND $\Delta\alpha_{\rm had}^{(5)}(M_Z)$

The evaluation of the hadronic leading-order contribution $a_\mu^{\rm HLO}$, due to the hadronic vacuum polarization correction to the one-loop QED diagram, involves long-distance QCD for which perturbation theory cannot be employed. However, using analyticity and unitarity, it was shown long ago that this term can be computed from hadronic e^+e^- annihilation data via the dispersion integral [22]

$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds K(s)\sigma(s), \tag{1}$$

where $\sigma(s)$ is the total cross section for e^+e^- annihilation into any hadronic state, with extraneous QED corrections subtracted off, and s is the squared momentum transfer. The kernel $K(s)$ is the well-known function

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2} \tag{2}$$

(see Ref. [23] for some of its explicit representations and their suitability for numerical evaluations). It decreases monotonically for increasing s and, for large s, it behaves as $m_\mu^2/(3s)$ to a good approximation. One finds that the low-energy region of the dispersive integral is enhanced by $\sim 1/s^2$. About 90% of the total contribution to $a_\mu^{\rm HLO}$ is accumulated at center-of-mass energies $\sqrt{s}$ below 1.8 GeV and roughly three-fourths of $a_\mu^{\rm HLO}$ is covered by the two-pion final state which is dominated by the $\rho(770)$ resonance [15]. Note that $a_\mu^{\rm HLO}$ is a positive definite quantity. Exclusive low-energy e^+e^- cross sections have been measured by experiments running at e^+e^- colliders in Frascati, Novosibirsk, Orsay, and Stanford, while at higher energies the total cross section has been measured inclusively. Perturbative QCD becomes applicable at higher loop momenta, so that at some energy scale one can switch from data to QCD [24].

Let us now assume that the discrepancy $\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} = +302(88) \times 10^{-11}$ is due to—and only to—hypothetical mistakes in $\sigma(s)$, and let us increase this cross section in order to raise $a_\mu^{\rm HLO}$, thus reducing Δa_μ. This simple assumption leads to interesting consequences. An upward shift of the hadronic cross section also induces an increase of the value of the hadronic contribution to the effective fine-structure constant at M_Z [25],

$$\Delta\alpha_{\rm had}^{(5)}(M_Z) = \frac{M_Z^2}{4\alpha\pi^2} P\int_{4m_\pi^2}^{\infty} ds \frac{\sigma(s)}{M_Z^2 - s} \tag{3}$$

(P stands for Cauchy's principal value). This integral is similar to the one we encountered in Eq. (1) for $a_\mu^{\rm HLO}$. There, however, the weight function in the integrand gives a stronger weight to low-energy data. The negligible contribution to $a_\mu^{\rm HLO}$ and $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ of the $\pi^0\gamma$ channel below the $\pi^+\pi^-$ threshold was ignored in Eqs. (1) and (3). Let us define

$$a = \int_{4m_\pi^2}^{s_u} ds f(s)\sigma(s), \tag{4}$$

$$b = \int_{4m_\pi^2}^{s_u} ds g(s)\sigma(s), \tag{5}$$

where the upper limit of integration is $s_u < M_Z^2$, and the kernels are $f(s) = K(s)/(4\pi^3)$ and $g(s) = [M_Z^2/(M_Z^2 - s)]/(4\alpha\pi^2)$. Equations (4) and (5) provide the contributions to $a_\mu^{\rm HLO}$ and $\Delta\alpha_{\rm had}^{(5)}(M_Z)$, respectively, in the region from the two-pion threshold up to s_u [see Eqs. (1) and (3)].

An increase of the cross section $\sigma(s)$ of the form

$$\Delta\sigma(s) = \epsilon\sigma(s) \tag{6}$$

in the energy range $\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$, where ϵ is a positive constant and $2m_\pi + \delta/2 < \sqrt{s_0} < \sqrt{s_u} - \delta/2$, increases a by $\Delta a(\sqrt{s_0}, \delta, \epsilon) = \epsilon \int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} 2t\sigma(t^2)f(t^2)dt$. If we assume that the muon $g-2$ discrepancy is entirely due to this increase in $\sigma(s)$ so that $\Delta a(\sqrt{s_0}, \delta, \epsilon) = \Delta a_\mu$, the parameter ϵ becomes

$$\epsilon = \frac{\Delta a_\mu}{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} 2tf(t^2)\sigma(t^2)dt}, \tag{7}$$

and the corresponding increase in $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ is

$$\Delta b(\sqrt{s_0}, \delta) = \Delta a_\mu \frac{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} g(t^2)\sigma(t^2)tdt}{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} f(t^2)\sigma(t^2)tdt}. \tag{8}$$

In the limiting case of a pointlike shift $\Delta\sigma(s) = \epsilon'\delta(s - s_0)$, with $2m_\pi < \sqrt{s_0} < \sqrt{s_u}$, the condition $\Delta a(\sqrt{s_0}, \epsilon') = \Delta a_\mu$, with $\Delta a(\sqrt{s_0}, \epsilon') = \epsilon' f(s_0)$, leads to

$$\Delta b(\sqrt{s_0}) = \Delta a_\mu[g(s_0)/f(s_0)]. \tag{9}$$

Following Ref. [15], to overcome the lack of precise data for $\sigma(s)$ at threshold energies, in the region $2m_\pi < \sqrt{s} < 500$ MeV one can adopt the polynomial parametrization for the pion form factor $F_\pi(s)$ inspired by chiral perturbation theory; the parameters are determined from a fit to the data for both timelike and spacelike momentum transfers [15,23,26]. The cross section below 500 MeV is therefore given by

$$\sigma(s) = \frac{\pi\alpha^2}{3s}\beta_\pi^3|F_\pi(s)|^2, \tag{10}$$

where $\beta_\pi = (1 - 4m_\pi^2/s)^{1/2}$, $F_\pi(s) = 1 + s\langle r^2\rangle_\pi/6 + s^2c_1 + s^3c_2$, $\langle r^2\rangle_\pi = (0.439 \pm 0.008)\ \mathrm{fm}^2$, $c_1 = (6.8 \pm 1.9)\ \mathrm{GeV}^{-4}$, and $c_2 = (-0.7 \pm 6.8)\ \mathrm{GeV}^{-6}$ (see Ref. [15] for the correlation matrix of these coefficients). Between 500 MeV and 1.4 GeV we use the cross section directly obtained combining the experimental results of the $\pi^+\pi^-$ [27], $\pi^+\pi^-\pi^0$ [28,29], K^+K^- [29,30], $K_L^0K_S^0$ [31], $2\pi^+2\pi^-$ [32], $\pi^0\pi^0\pi^+\pi^-$ [33], $\pi^0\gamma$ [34,35], and $\eta\gamma$ [35] channels. Between 1.4 and 2 GeV we employ the inclusive measurements of Ref. [36].

Figure 1 shows the shifts $\Delta b(\sqrt{s_0}, \delta = 210\ \mathrm{MeV})$ (histogram) and $\Delta b(\sqrt{s_0})$ (smooth curve) obtained from the increases $\Delta\sigma(s) = \epsilon\sigma(s)$ and $\Delta\sigma(s) = \epsilon'\delta(s - s_0)$, respectively. These shifts, shown as functions of $\sqrt{s_0}$, are added to the value $\Delta\alpha_{\rm had}^{(5)}(M_Z) = 0.027\,68(22)$ [7]. The uncertainty of the sum $\Delta\alpha_{\rm had}^{(5)}(M_Z) + \Delta b(\sqrt{s_0})$ is indicated by the light band. To compute it, we first note that the errors 46×10^{-11} in $a_\mu^{\rm HLO}$ and 22×10^{-5} in $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ [7] are strongly correlated since they arise mainly from the same source, namely, the uncertainty in the hadronic e^+e^- annihilation cross section (which includes the uncertainties associated with the radiative corrections applied to the experimental data). Taking this into account, and observing

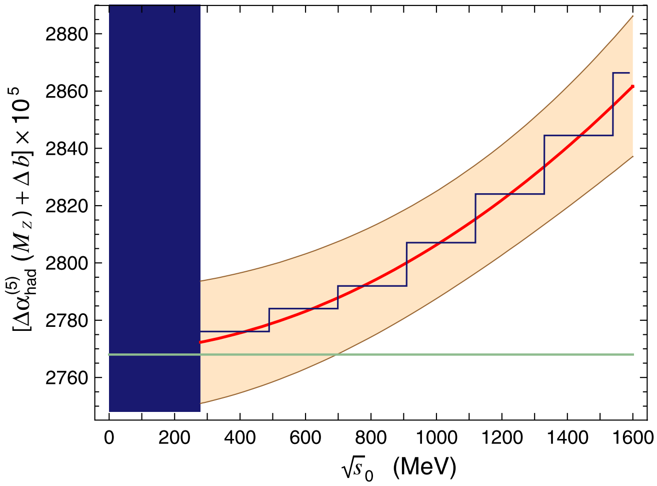

FIG. 1 (color online). Shifts of $\Delta\alpha_{\rm had}^{(5)}(M_Z)$. The histogram indicates the increase $\Delta b(\sqrt{s_0}, \delta)$ obtained varying the cross section by $\Delta\sigma(s) = \epsilon\sigma(s)$ in $\delta = 210$ MeV energy regions, while $\Delta b(\sqrt{s_0})$, obtained for pointlike increases, is plotted as a smooth curve. The shifts are added to $\Delta\alpha_{\rm had}^{(5)}(M_Z) = 0.027\,68(22)$ [7] (horizontal line). The uncertainty of the sum $\Delta\alpha_{\rm had}^{(5)}(M_Z) + \Delta b(\sqrt{s_0})$ is shown by the light band.

also that the error in $\Delta b(\sqrt{s_0})$ due to the $a_\mu^{\rm HLO}$ uncertainty is $-46 \times 10^{-11}[g(s_0)/f(s_0)]$, we add it linearly to 22×10^{-5}, and then combine in quadrature this result with the error in $\Delta b(\sqrt{s_0})$ induced by the remaining Δa_μ uncertainty. [We note that combining all errors in quadrature, ignoring their correlation, would enlarge the uncertainty of the sum $\Delta\alpha_{\rm had}^{(5)}(M_Z) + \Delta b(\sqrt{s_0})$, but would only induce minimal changes in our analysis.] The uncertainty of the sum $\Delta\alpha_{\rm had}^{(5)}(M_Z) + \Delta b(\sqrt{s_0}, \delta)$, for finite energy intervals, is computed analogously, neglecting the relative error of the ratio of integrals on the right-hand side of Eq. (8) with respect to the large relative error of Δa_μ. The dark area below $2m_\pi$, where m_π is the mass of the charged pion, denotes the kinematically forbidden region below the $\pi^+\pi^-$ threshold (the $\pi^0\gamma$ channel is neglected below this threshold).

III. CONNECTION WITH THE HIGGS BOSON MASS

The dependence of SM predictions, via quantum effects, on the mass of the Higgs boson M_H provides a powerful tool to set indirect bounds on the mass of this fundamental missing piece of the SM. Indeed, comparing calculated quantities with their precise experimental values, the present global fit of the LEP Electroweak Working Group (LEP-EWWG) leads to the value $M_H = 87^{+36}_{-27}$ GeV and to the 95% confidence level (C.L.) upper bound $M_H^{95} \simeq 160$ GeV [37]. This result is based on the very recent preliminary top quark mass $M_t = 172.6(1.4)$ GeV from a combined CDF-D0 fit [38] and the value

$\Delta\alpha_{\rm had}^{(5)}(M_Z) = 0.027\,58(35)$ [39]. The LEP direct-search lower bound is $M_H^{\rm LB} = 114.4$ GeV [40], also at the 95% C.L.

Although the global fit to the EW data employs a large set of observables, the M_H upper bound is strongly driven by the comparison of the theoretical predictions of the mass of the W boson and the effective EW mixing angle $\sin^2\theta_{\rm eff}^{\rm lept}$ with their precisely measured values [41]. Convenient formulas providing the SM prediction of M_W and $\sin^2\theta_{\rm eff}^{\rm lept}$ in terms of M_H, the top quark mass M_t, $\Delta\alpha_{\rm had}^{(5)}(M_Z)$, and $\alpha_s(M_Z)$, the value of the strong coupling constant at the scale M_Z, are given in Ref. [42]. Combining these M_W and $\sin^2\theta_{\rm eff}^{\rm lept}$ predictions by means of a numerical χ^2 analysis, and using the present world-average values $M_W = 80.398(25)$ GeV [43–45], $\sin^2\theta_{\rm eff}^{\rm lept} = 0.231\,53(16)$ [46], $M_t = 172.6(1.4)$ GeV [38], $\alpha_s(M_Z) = 0.118(2)$ [47], and the determination $\Delta\alpha_{\rm had}^{(5)}(M_Z) = 0.027\,58(35)$ [39] adopted by the LEP-EWWG, we obtain $M_H = 92^{+38}_{-28}$ GeV and $M_H^{95} = 161$ GeV. We see that indeed the M_H values obtained from the M_W and $\sin^2\theta_{\rm eff}^{\rm lept}$ predictions are quite close to the results of the global analysis.

The M_H dependence of the SM prediction of the muon $g-2$, via its EW contribution, is too weak to provide M_H bounds from the comparison with the measured value. Indeed, the shift of $a_\mu^{\rm SM}$ for M_H varying between 114.4 and 300 GeV is only of $O(10^{-11})$, which is negligible when compared with the hadronic and experimental uncertainties. On the other hand, $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ is one of the key inputs of the EW fits. For example, employing the recent (slightly higher) value $\Delta\alpha_{\rm had}^{(5)}(M_Z) = 0.027\,68(22)$ [7] instead of $\Delta\alpha_{\rm had}^{(5)}(M_Z) = 0.027\,58(35)$ [39], the M_H prediction shifts down to $M_H = 90^{+33}_{-25}$ GeV and $M_H^{95} = 150$ GeV. We note that M_H^{95} depends both on the central value and on the uncertainty of $\Delta\alpha_{\rm had}^{(5)}(M_Z)$. Henceforth, we employ the recent evaluation $\Delta\alpha_{\rm had}^{(5)}(M_Z) = 0.027\,68(22)$ [7]. (For the dependence of M_H and its bounds on $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ see Ref. [42].) Next we consider the new values of $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ obtained shifting 0.027 68(22) by $\Delta b(\sqrt{s_0})$ and $\Delta b(\sqrt{s_0}, \delta)$ (including their uncertainties as discussed in the previous section), and compute the corresponding new values of M_H^{95} by means of the combined χ^2-analysis based on the M_W and $\sin^2\theta_{\rm eff}^{\rm lept}$ inputs. The results are shown in Fig. 2. The lower region $M_H < 114.4$ GeV is excluded by the direct LEP searches at 95% C.L., while the upper one is excluded by the indirect EW 95% C.L. bound $M_H < 150$ GeV obtained with $\Delta\alpha_{\rm had}^{(5)}(M_Z) = 0.027\,68(22)$. [As in the case of $\Delta\alpha_{\rm had}^{(5)}(M_Z)$, the value adopted here for $a_\mu^{\rm HLO}$ is from the recent article in Ref. [7].] If we increase the hadronic cross section $\sigma(s)$ by $\epsilon'\delta(s-s_0)$ in order to bridge the muon $g-2$ discrepancy Δa_μ, M_H^{95} decreases, as shown by the continuous red line in Fig. 2, further restricting the already narrow allowed region for M_H. In particular, this curve falls below $M_H^{\rm LB}$ for $\sqrt{s_0} \gtrsim 1.1$ GeV. The two histograms show the M_H^{95} values when the analysis is repeated with $\Delta\sigma = \epsilon\sigma(s)$ shifts in $\delta = 210$ MeV and $\delta = 400$ MeV energy regions. We conclude that the hypothetical shifts $\Delta\sigma = \epsilon\sigma(s)$ (in $\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$) of the hadronic cross section that bridge the muon $g-2$ discrepancy, conflict with the LEP lower limit when $\sqrt{s_0} > (\sqrt{s_0})_{\rm thr} \sim 1.2$ GeV, for values of δ up to several hundreds of MeV. The threshold $(\sqrt{s_0})_{\rm thr}$ increases above $\sim$1.3 GeV for hypothetical shifts $\epsilon\sigma(s)$ in even wider energy regions $\delta \gtrsim 1$ GeV, but uniform shifts of the cross section in such wide energy ranges appear to be unrealistic.

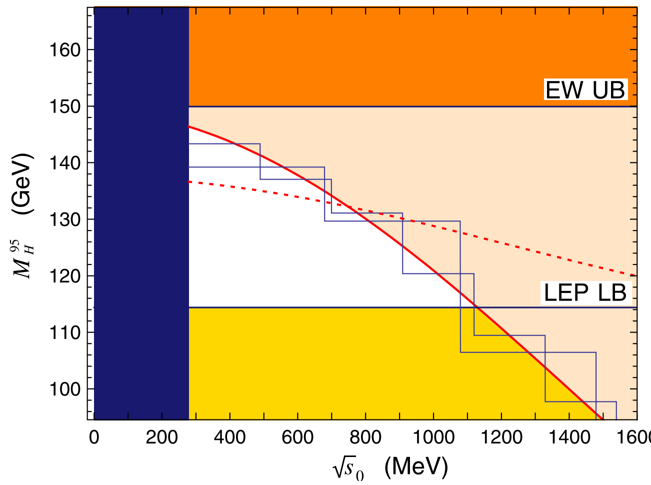

FIG. 2 (color online). The M_H^{95} values obtained via the M_W and $\sin^2\theta_{\rm eff}^{\rm lept}$ fits using as input for $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ the value 0.027 68(22) increased by $\Delta b(\sqrt{s_0})$ (smooth curve) and by $\Delta b(\sqrt{s_0}, \delta = 210$ MeV, 400 MeV) (histograms). The area below 114.4 GeV, partly yellow and partly pink, is excluded at 95% C.L. by the LEP direct lower bound, while the orange $M_H > 150$ GeV one is forbidden by the EW indirect upper bound. As in Fig. 1, the region $\sqrt{s_0} < 2m_\pi$ is excluded. The dotted line replaces the smooth one when τ data are incorporated in the determination of $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ and $a_\mu^{\rm SM}$.

If τ data are incorporated in the calculation of the dispersive integrals in Eqs. (1) and (3), the leading-order hadronic contribution to the muon $g-2$ significantly increases to $a_\mu^{\rm HLO} = 7110(58) \times 10^{-11}$ [15], the higher-order vacuum polarization term slightly decreases to $a_\mu^{\rm HHO}({\rm vp}) = -101(1) \times 10^{-11}$ [7,20], and the discrepancy with the experimental value drops to $\Delta a_\mu = +89(95) \times 10^{-11}$, i.e. roughly 1σ. While using τ data almost solves the muon $g-2$ discrepancy, it increases the value of $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ to 0.027 82(16) [15,48]. In Ref. [48] it was shown that this increase leads to a low M_H prediction which is suggestive of a near conflict with $M_H^{\rm LB}$, leaving a very narrow window for M_H. Indeed, with this value of $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ and the same above-discussed values of the other inputs of the χ^2 analysis, we find $M_H = 84^{+30}_{-23}$ GeV and an M_H^{95} value of only 138 GeV. The dotted line in Fig. 2 shows the M_H^{95} values obtained using τ data to compute $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ and Δa_μ, with the hadronic cross section $\sigma(s)$ increased by $\epsilon'\delta(s-s_0)$ in order to bridge the Δa_μ difference.

As we briefly mentioned in the Introduction, recently computed isospin-breaking violations, further improvements of the long-distance radiative corrections to the decay $\tau^- \to \pi^-\pi^0\nu_\tau$ [17], and differentiation of the neutral and charged ρ properties [18], reduce to some extent the difference between τ and e^+e^- data, lowering the τ-based determination of $a_\mu^{\rm HLO}$. Moreover, a recent analysis of the pion form factor below 1 GeV claims that τ data are consistent with the e^+e^- ones after isospin violation effects and vector meson mixings are considered [19]. In this case, one could therefore use the e^+e^- data below $\sim$1 GeV, confirmed by the τ ones, and assume that Δa_μ is accommodated by hypothetical errors occurring above $\sim$1 GeV, where disagreement persists between these two sets of data. Our previous analysis shows that this assumption would lead to values of M_H^{95} inconsistent with the LEP lower bound.

It is interesting to note that there are more complex scenarios where it is possible to bridge the Δa_μ discrepancy without significantly affecting M_H^{95}. For instance, we may envisage an increase of $\sigma(s)$ at low s combined with a decrease at high s in such a manner that their overall contribution to $\Delta\alpha_{\rm had}^{(5)}(M_Z)$, and therefore to M_H^{95}, approximately cancels. Since the contributions to $a_\mu^{\rm HLO}$ are more heavily weighted at low s, it is then possible to further adjust the positive and negative $\sigma(s)$ shifts to bridge the muon $g-2$ discrepancy. However, such a scheme requires two fine-tuning steps and a larger increase of $\sigma(s)$ at low s, and is therefore considerably more unlikely than the simplest scenarios, involving a single adjustable contribution, that are discussed in detail in this paper.

IV. HOW REALISTIC ARE THESE SHIFTS $\Delta\sigma(s)$?

In the above study, the hadronic cross section $\sigma(s)$ was shifted up by amounts required to adjust the muon $g-2$ discrepancy Δa_μ. Apart from the implications for the Higgs boson mass (and the restrictions deriving from them), these shifts may actually be inadmissibly large when compared with the quoted experimental uncertainties. For example, one of the histograms in Fig. 2 shows that a shift $\Delta\sigma$ in a 210 MeV bin centered just above the ρ peak could fix the muon $g-2$ discrepancy (lowering M_H^{95} to 131 GeV); but is such a shift of the precisely measured cross section at the ρ peak realistic?

To investigate this problem, we turn our attention to the parameter $\epsilon = \Delta\sigma(s)/\sigma(s)$, i.e. the ratio of the shift $\Delta\sigma(s)$ required to bridge the muon $g-2$ discrepancy and the cross section $\sigma(s)$, provided by Eq. (7). Clearly, the value of ϵ depends on the choice of the energy range $[\sqrt{s_0}-\delta/2, \sqrt{s_0}+\delta/2]$ where $\sigma(s)$ is increased and, for fixed $\sqrt{s_0}$, it increases when δ decreases. The minimum value of ϵ is roughly $+4\%$; it occurs if the hadronic cross section $\sigma(s)$ is multiplied by $(1+\epsilon)$ in the whole integration region of Eq. (1), from the $\pi^+\pi^-$ threshold to infinity [this minimum value of ϵ changes only negligibly whether the shift up of $\sigma(s)$ includes or not the high-energy region where perturbative QCD is employed]. Such a shift would lead to an M_H^{95} of roughly 75 GeV, well below the LEP lower bound.

Figure 3 shows the values of ϵ (in percent) for several bin widths δ and central values $\sqrt{s_0}$ (same length segments are of the same color). Also, next to each segment we quote the value of M_H^{95} (in GeV) obtained performing the shift $\Delta\sigma = \epsilon\sigma(s)$ in that energy range. A shift up of $\sigma(s)$ in the energy range from $2m_\pi$ to 850 MeV, to fix Δa_μ, leads to $\epsilon \sim 6\%$ and lowers M_H^{95} to 134 GeV. Higher values of ϵ are obtained for narrower energy bins, particularly if they do not include the ρ-ω resonance region. For example, a huge $\epsilon \sim 52\%$ increase is needed to accommodate Δa_μ with a shift of the cross section in the region from $2m_\pi$ up to 500 MeV (reducing M_H^{95} to 143 GeV), while an increase in a bin of the same size but centered at the ρ peak requires $\epsilon \sim 8\%$ (lowering M_H^{95} to 132 GeV). As the quoted experimental uncertainty of $\sigma(s)$ below 1 GeV is of the order of a few percent (or less, in some specific energy regions), the possibility to explain the muon $g-2$ discrepancy with these shifts $\Delta\sigma(s)$ appears to be unlikely. Figure 3 shows that for fixed δ (i.e., segments of the same color), lower values of ϵ are obtained if the shifts occur in energy ranges

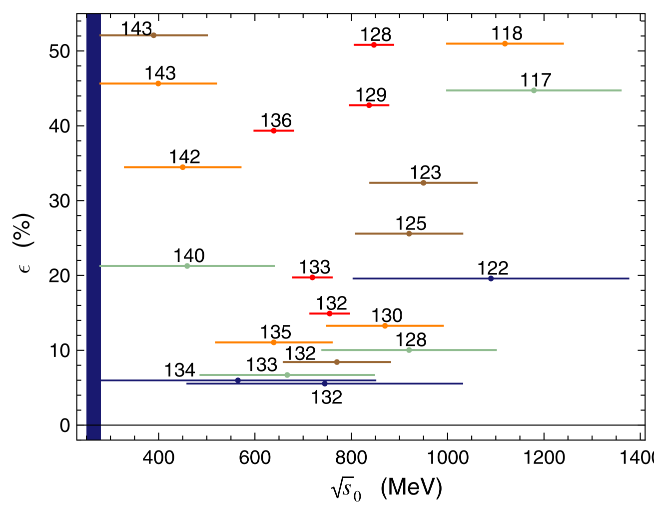

FIG. 3 (color online). Values of ϵ obtained increasing $\sigma(s)$ by $\epsilon\sigma(s)$, to bridge the Δa_μ discrepancy, in energy ranges $[\sqrt{s_0}-\delta/2, \sqrt{s_0}+\delta/2]$ for various values of $\sqrt{s_0}$ and δ. The number next to each segment indicates the M_H^{95} value (in GeV) induced by the $\epsilon\sigma(s)$ shift in that energy region. The same length segments are of the same color. The midpoint of each segment is displayed by a dot.

centered around the ρ-ω resonances; but also this possibility looks unlikely, since it requires variations of $\sigma(s)$ of at least $\sim 6\%$. If, however, we allow variations of the cross section up to $\sim 6\%$ (7%), M_H^{95} is reduced to less than ~ 134 GeV (135 GeV). For example, the $\sim 6\%$ shifts in the intervals [0.5,1.0] GeV or [0.6,1.2] GeV, required to fix Δa_μ (not represented in Fig. 3), lower M_H^{95} to 133 or 130 GeV, respectively.

We remind the reader that the present experimental results for $\sin^2\theta_{\rm eff}^{\rm lept}$ exhibit an intriguing dichotomy. Those based on the leptonic observables lead to $(\sin^2\theta_{\rm eff}^{\rm lept})_l = 0.231\,13(21)$, while the average of those derived from the hadronic sector is $(\sin^2\theta_{\rm eff}^{\rm lept})_h = 0.232\,22(27)$ [46]. The results within each group agree well with each other, but the averages of the two sectors differ by about 3.2σ. Our analysis, like the LEP-EWWG one, depends on the value of $\sin^2\theta_{\rm eff}^{\rm lept}$. For instance, if we were to use $(\sin^2\theta_{\rm eff}^{\rm lept})_h$, we would obtain a significantly higher SM prediction: $M_H = 129^{+53}_{-40}$ GeV, $M_H^{95} = 225$ GeV, and a continuous (red) line in Fig. 2 similarly shifted up. However, we note that in this scenario the M_H predictions from M_W and $(\sin^2\theta_{\rm eff}^{\rm lept})_h$ are inconsistent with one another unless one introduces additional "new physics" beyond the SM. For example, the difference could be associated with a value $S \sim 0.4$ to 0.5 of the S-parameter, an effect generally attributed to technicolorlike theories with additional heavy fermion chiral doublets [49]. Instead, if we were to employ $(\sin^2\theta_{\rm eff}^{\rm lept})_l$, the SM prediction would drop sharply to $M_H = 50^{+25}_{-18}$ GeV, $M_H^{95} = 97$ GeV, which is already in conflict with the direct-search lower bound. Thus, in that case, no shift $\Delta\sigma(s)$ could reconcile the $g-2$ discrepancy without violating the lower bound. In this paper we employ as input the world average of $\sin^2\theta_{\rm eff}^{\rm lept}$ since this is the value determined in the global analysis of the SM.

The M_H upper bounds presented in this article depend sensitively on the central value $M_t = 172.6$ GeV and its uncertainty $\delta M_t \delta = 1.4$ GeV. In the future, the former may still change and the latter will further decrease. We therefore provide the following simple formulas to translate easily the $M_H^{95} = 150$ GeV result of our numerical χ^2 analysis based on the M_W and $\sin^2\theta_{\rm eff}^{\rm lept}$ predictions, as well as the $M_H^{95}[0.6, 1.2] = 130$ GeV upper bound corresponding to the $\sim 6\%$ increase of $\sigma(s)$ in the interval [0.6,1.2] GeV (an illustrative case that accounts for Δa_μ), into the new values derived with different M_t and δM_t inputs:

$$M_H^{95} = (150.5 + 11.2x + 9.4y)\ \text{GeV} \tag{11}$$

$$M_H^{95}[0.6, 1.2] = (130.7 + 9.9x + 8.2y)\ \text{GeV} \tag{12}$$

with $x = M_t - 172.6$ GeV and $y = \delta M_t - 1.4$ GeV. Note that, in the case of a future rise of the M_t central value, the increase induced on the M_H upper bounds would be partially compensated by a reduction of the error δM_t. Equations (11) and (12) reproduce the results of the detailed numerical χ^2 analysis with maximum absolute deviations of roughly 1 GeV when $M_t \in [171, 174]$ GeV and $\delta M_t \in [1.0, 1.8]$ GeV.

V. CONCLUSIONS

The present discrepancy between the SM prediction of the anomalous magnetic moment of the muon and its experimental determination could be due to the contribution of new, yet undiscovered, physics beyond the SM, or to errors in the determination of the hadronic contributions. In this paper we considered the second hypothesis and, in particular, the possibility to accommodate the discrepancy $\Delta a_\mu = +302(88) \times 10^{-11}$ (3.4σ) by changes in the hadronic cross section $\sigma(s)$ used to determine the leading hadronic contribution $a_\mu^{\rm HLO}$. This option has important consequences on M_H^{95}, the 95% C.L. EW upper bound on the mass of the SM Higgs boson.

We first analyzed the effects induced by these hypothetical changes $\Delta\sigma(s)$ on the value of $\Delta\alpha_{\rm had}^{(5)}(M_Z)$, one of the key inputs of the EW fits with a strong influence on the SM M_H predictions. The comparison of the theoretical predictions of M_W and the effective EW mixing angle $\sin^2\theta_{\rm eff}^{\rm lept}$ with their precisely measured values allowed us to determine, via a combined χ^2 analysis, the variations of M_H^{95} induced by the shifts $\Delta\sigma(s)$. We concluded that if the hadronic cross section is shifted up in energy regions centered above ~ 1.2 GeV to bridge the muon $g-2$ discrepancy, the Higgs mass upper bound becomes inconsistent with the LEP lower limit.

If τ-decay data are incorporated in the calculation of $a_\mu^{\rm SM}$, the discrepancy Δa_μ drops to $+89(95) \times 10^{-11}$. While this almost solves the muon $g-2$ discrepancy, it raises the value of $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ leading to $M_H^{95} = 138$ GeV, increasing the tension with the LEP lower bound. One could also consider a scenario, suggested by recent studies, where the τ data confirm the e^+e^- ones below ~ 1 GeV, while a discrepancy between them persists at higher energies. If, in this case, Δa_μ is reconciled by hypothetical errors above ~ 1 GeV, where the data sets disagree, one also finds values of M_H^{95} inconsistent with the 114.4 GeV lower bound. For example, if $\sigma(s)$ is shifted in the interval [1.0,1.8] GeV, we obtain $M_H^{95} = 108$ GeV.

We then questioned the plausibility of the variations $\Delta\sigma(s) = \epsilon\sigma(s)$ required to fix Δa_μ. Their amounts clearly depend on the energy regions chosen for the change, but we showed that they are generally very large when compared with the actual experimental uncertainties. Given the small experimental uncertainty of $\sigma(s)$ below 1 GeV, the possibility to bridge the muon $g-2$ discrepancy with shifts of the hadronic cross section appears to be unlikely. Smaller values of ϵ (for fixed bin-widths δ) are needed when the shifts occur in energy regions centered around the ρ-ω resonances; but also this possibility looks unlikely since it requires variations of $\sigma(s)$ of at least $\sim$6%, a large modification given current experimental error estimates. However, if this turns out to be the solution of the Δa_μ discrepancy, we conclude that M_H^{95} is reduced to roughly 130 GeV which, in conjunction with the 114.4 GeV lower bound, leaves a narrow window for the mass of this fundamental particle. Simple formulas were also provided to translate M_H upper bounds derived in this paper into new values corresponding to M_t and δM_t inputs different from those employed here.

If the Δa_μ discrepancy is real, it points to "new physics," like low-energy supersymmetry. In fact, an intriguing explanation of Δa_μ is provided by some supersymmetric models, where it is reconciled by the additional contributions of supersymmetric partners [2] and one expects $M_H \lesssim 135$ GeV for the mass of the lightest scalar [50]. If, instead, the deviation is caused by an incorrect leading-order hadronic contribution, it leads to a larger $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ and, correspondingly, to low values of M_H^{95}, thus leaving a very narrow range for the SM Higgs boson mass.

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