Generalization of Friedberg-Lee symmetry

Chao-Shang Huang,[3] Tianjun Li,[3,4] Wei Liao,[1,2,*] and Shou-Hua Zhu[5]

[1]*Institute of Modern Physics, East China University of Science and Technology, P.O. Box 532, 130 Meilong Road, Shanghai 200237, People's Republic of China*

[2]*Center for High Energy Physics, Peking University, Beijing 100871, People's Republic of China*

[3]*Institute of Theoretical Physics, Chinese Academy of Sciences, P. O. Box 2735, Beijing 100080, People's Republic of China*

[4]*George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, Texas 77843, USA*

[5]*Institute of Theoretical Physics, School of Physics, Peking University, Beijing 100871, People's Republic of China*

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We study the possible origin of Friedberg-Lee symmetry. First, we propose the generalized Friedberg-Lee symmetry in the potential by including the scalar fields in the field transformations, which can be broken down to the Friedberg-Lee symmetry spontaneously. We show that the generalized Friedberg-Lee symmetry allows a typical form of Yukawa couplings, and the realistic neutrino masses and mixings can be generated via the seesaw mechanism. If the right-handed neutrinos transform nontrivially under the generalized Friedberg-Lee symmetry, we can have the testable TeV scale seesaw mechanism. Second, we present two models with the $SO(3) \times U(1)$ global flavor symmetry in the lepton sector. After the flavor symmetry breaking, we can obtain the charged lepton masses, and explain the neutrino masses and mixings via the seesaw mechanism. Interestingly, the complete neutrino mass matrices are similar to those of the above models with generalized Friedberg-Lee symmetry. So the Friedberg-Lee symmetry is the residual symmetry in the neutrino mass matrix after the $SO(3) \times U(1)$ flavor symmetry breaking.

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I. INTRODUCTION

Recent developments in neutrino physics [1–6] have stimulated many interesting new ideas [7–10]. One beautiful approach towards understanding neutrino masses and mixings was presented by Friedberg and Lee [10–12]. They showed that there may be a hidden symmetry in the neutrino mass matrix with tri-bimaximal mixings, *i.e.*, the invariance under the translation in the space of Grassmann number

$$\nu_{e,\mu,\tau} \rightarrow \nu_{e,\mu,\tau} + \theta. \quad (1)$$

The symmetry was later used to explain the quark masses and mixings [11]. Instead of a universal translation for all fermions, they introduced different coefficients in translation of different flavors of quarks

$$q_i \rightarrow q_i + \xi_i \theta. \quad (2)$$

And this symmetry implies that one family of the standard model (SM) fermions is massless. Explicit symmetry breaking terms are introduced to reproduce the masses for the light SM fermions. Research along this approach has been performed by several groups [13,14].

On the other hand, it is generally acknowledged that the seesaw mechanism [15–19] is a powerful method to understand the tiny masses of the active neutrinos. The seesaw mechanism needs a symmetry to guarantee the masslessness of the neutrinos at leading order. The masses of light neutrinos are generated after symmetry breaking. In this respect it is natural to ask what kind symmetry can implement the seesaw mechanism in such a way that the Friedberg-Lee (FL) symmetry is the residual symmetry hidden in the neutrino mass matrix.

In this article, we first generalize the FL symmetry in a simple way by including the scalar fields in the left-handed neutrino field transformations. The generalized Friedberg-Lee (gFL) symmetry naturally incorporates the FL symmetry. And the FL symmetry of Eq. (1) or Eq. (2) is obtained after the larger gFL symmetry breaking. The masslessness of three light neutrinos is a direct consequence of the gFL symmetry. After the gFL symmetry is broken down to FL symmetry, the light neutrinos get masses via the seesaw mechanism, and their masses and mixings are intimately related to the residual FL symmetry. We show that the observed neutrino masses and mixings can be reproduced via the seesaw mechanism. Also, if the transformations of the right-handed neutrinos under the gFL symmetry is similar to those of the left-handed neutrinos, the testable TeV scale seesaw mechanism can be realized. Moreover, we briefly discuss how to embed the models with gFL symmetry into the extensions of the SM. Second, we propose two models with the $SO(3) \times U(1)$ global flavor symmetry in the lepton sector. After the flavor symmetry breaking, the charged lepton masses can be obtained, and the neutrino masses and mixings can be generated via the seesaw mechanism. Interestingly, the complete neutrino mass matrices for the left-handed and right-handed neutrinos are similar to those of the above models with gFL symetry. So the FL symmetry is the

*Corresponding author.
liaow@ecust.edu.cn

residual symmetry in the neutrino mass matrix after the $SO(3) \times U(1)$ flavor symmetry breaking.

The content of this article is organized as follows. In Sec. II we propose the gFL symmetry and study the models with gFL symmetry. In Sec. III, we consider the models with $SO(3) \times U(1)$ flavor symmetry in the lepton sector. Our conclusions and discussions are in Sec. IV.

II. GENERALIZED FRIEDBERG-LEE SYMMETRY

We consider two models with the generalization of FL symmetry. One model has the usual seesaw mechanism where only the left-handed neutrinos transform nontrivially under the gFL symmetry, and the other model has the testable TeV scale seesaw mechanism in which both the left-handed and right-handed neutrinos transform nontrivially under the gFL symmetry.

A. Usual seesaw mechanism

We consider three families of the left-handed neutrinos ν_{Li}, right-handed neutrinos ν^c_{Ri}, and three SM singlet scalar fields ϕ_i, where $i = 1, 2, 3$. We introduce the generalized Friedberg-Lee symmetry by including scalar fields in the field transformations of ν_{Li}. We introduce the following gFL symmetry transformation:

$$\nu_{Li} \to \nu_{Li} + \phi_i\theta, \qquad \nu^c_{Ri} \to \nu^c_{Ri}, \qquad \phi_i \to \phi_i, \tag{3}$$

where θ is a Grassmann number.[1] We require that the neutrino mass terms and Yukawa terms be invariant under this symmetry transformation.

The FL symmetry is obtained after the gFL symmetry breaks spontaneously. This can be achieved by assuming that the potential of ϕ_i triggers the spontaneous symmetry breaking. We assume ϕ_i to have a potential as follows:

$$-\Delta\mathcal{L} = \xi\left(\sum_{i=1}^{3}|\phi_i|^2 - v^2\right)^2, \tag{4}$$

where $\xi > 0$. Then, ϕ_i get the vacuum expectation values (VEVs) at the minimum of the potential

$$\langle\phi_i\rangle = v_i, \tag{5}$$

where $v^2 = \sum_{i=1}^{3}|v_i|^2$. The induced transformation is as follows:

$$\nu_{Li} \to \nu_{Li} + v_i\theta, \qquad \nu^c_{Ri} \to \nu^c_{Ri}. \tag{6}$$

Because the coefficients v_i in the above equation are space-time independent, we obtain the FL symmetry as a residual symmetry.

The mass term and Yukawa terms invariant under the gFL transformation are

$$-\Delta\mathcal{L} = \tfrac{1}{2}(m_0)_{ij}\nu^{cT}_{Ri}i\sigma_2\nu^c_{Rj} + \lambda_{ijk}\nu^{cT}_{Ri}i\sigma_2\nu_{Lj}\phi_k + \tfrac{1}{2}\eta_{ijk}\nu^{cT}_{Ri}i\sigma_2\nu^c_{Rj}\phi_k + \tfrac{1}{2}\eta'_{ijk}\nu^{cT}_{Ri}i\sigma_2\nu^c_{Rj}\phi^\dagger_k + \text{H.c.}, \tag{7}$$

where λ_{ijk}, η_{ijk}, and η'_{ijk} are Yukawa couplings, and ν^T means the transpose of ν. Also, we have to impose

$$\lambda_{ijk} = -\lambda_{ikj}, \tag{8}$$

$$(m_0)_{ij} = (m_0)_{ji}, \qquad \eta_{ijk} = \eta_{jik}, \qquad \eta'_{ijk} = \eta'_{jik}. \tag{9}$$

The first, the third, and the fourth terms in Eq. (7) are obviously invariant under the gFL transformation in Eq. (3). Equation (8) is required to make the second term invariant under the gFL transformation. Using Eq. (8), the second term transforms to

$$\lambda_{ijk}\nu^{cT}_{Ri}i\sigma_2\nu_{Lj}\phi_k + \lambda_{ijk}\nu^{cT}_{Ri}i\sigma_2\theta\phi_j\phi_k = \lambda_{ijk}\nu^{cT}_{Ri}i\sigma_2\nu_{Lj}\phi_k, \tag{10}$$

so it is invariant under the gFL symmetry. However, the other terms, *e.g.*, $\nu^T_{Li}i\sigma_2\nu_{Lj}$ and $\nu^T_{Li}i\sigma_2\nu_{Lj}\phi_k$, etc., are not invariant under the gFL transformation and are killed by the gFL symmetry.

We see that the mass term $\nu^T_{Li}i\sigma_2\nu_{Lj}$ is killed by the gFL symmetry defined in Eq. (3). If gFL symmetry is not broken to the FL symmetry neutrinos, the ν_{Li} will not be able to get masses. In this sense, the masslessness of the three ν_{Li} is a direct consequence of the gFL symmetry. Neutrinos ν_{Li} get seesaw type masses after ϕ_i get VEVs, and gFL symmetry in Eq. (3) is broken to the residual FL symmetry in Eq. (6). The generation of the seesaw masses for ν_{Li} is shown in the following.

After the gFL symmetry is broken down to the FL symmetry, we obtain the following neutrino mass terms:

$$-\Delta\mathcal{L} = \tfrac{1}{2}(m_R)_{ij}\nu^{cT}_{Ri}i\sigma_2\nu^c_{Rj} + \Lambda_{ij}\nu^{cT}_{Ri}i\sigma_2\nu_{Lj} + \text{H.c.}, \tag{11}$$

where

$$(m_R)_{ij} = (m_0)_{ij} + \sum_k\eta_{ijk}v_k + \sum_k\eta'_{ijk}v^*_k. \tag{12}$$

$$\Lambda_{ij} = \sum_k\lambda_{ijk}v_k. \tag{13}$$

It is obvious that Eq. (11) is invariant under the residual FL symmetry transformation in Eq. (6). And we can write the neutrino mass matrix in the basis $(\nu_L, \nu^c_R)^T$ as follows:

$$\mathcal{M} = \begin{pmatrix} 0_{3\times3}, & \Lambda^T \\ \Lambda, & m_R \end{pmatrix}, \tag{14}$$

where Λ and m_R are 3×3 matrices, and their matrix elements are Λ_{ij} and $(m_R)_{ij}$, respectively.

Assuming the mass scale of Λ is much lower than that of m_R we get the seesaw mass matrix for the light neutrinos

[1] This gFL symmetry is introduced for neutrinos after electroweak breaking. One may consider that θ carries an isospin number. Extension to doublet is discussed in Sec. II D

$$m_\nu = -\Lambda^T(m_R^{-1})\Lambda. \tag{15}$$

Thus, using the gFL symmetry we have implemented the seesaw mechanism. It is clear that the gFL symmetry protects the masslessness of neutrinos ν_L. Right-handed neutrinos ν_R^c are allowed to have masses and are heavy. Only one typical form of the neutrino Dirac Yukawa couplings is allowed by the gFL symmetry. This type of the Yukawa couplings introduces the mixings of ν_L and ν_R^c. After the gFL symmetry is spontaneously broken down to the FL symmetry we get a seesaw type mass matrix for $(\nu_L, \nu_R^c)^T$ and the seesaw mass matrix for the light neutrinos, which are shown in Eqs. (14) and (15), respectively.

B. Neutrino masses and mixings

In this subsection, we give examples which can reproduce the realistic neutrino masses and mixings. For simplicity, we assume v_i are real, and Λ and m_R are real matrices. For illustration we will try to obtain the following tri-bimaximal neutrino mixing matrix [20–22]:

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}, \tag{16}$$

which has $\theta_{13} = 0$, $\theta_{23} = \pi/4$, and $\tan^2\theta_{12} = 0.5$. More realistic textures can be done by following the discussions in this subsection.

A direct consequence of the residual FL symmetry in Eq. (6) is that one light neutrino is massless. This can be seen by noting that under the transformation $\nu_{Li} \to \nu_{Li} + v_i\theta$ the seesaw mass term of the light neutrinos is transformed to (after rearrangement)

$$\begin{aligned} \tfrac{1}{2}(m_\nu)_{ij}\nu_{Li}^T i\sigma_2 \nu_{Lj} + \text{H.c.} \to \tfrac{1}{2}(m_\nu)_{ij}[&\nu_{Li}^T i\sigma_2\nu_{Lj} \\ &+ 2v_j\nu_{Li}^T i\sigma_2\theta + v_i v_j \theta^T i\sigma_2\theta] \\ &+ \text{H.c.} \end{aligned} \tag{17}$$

The invariance under the FL symmetry transformation says that the second term in the bracket of the right-hand side of Eq. (17) gives zero. Hence we obtain

$$m_\nu \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = 0. \tag{18}$$

So neutrinos ν_{Li} have one eigenstate with zero mass. The eigenvector is $(v_1, v_2, v_3)^T$.[2] Equation (18) can also be obtained by using Eqs. (8), (13), and (15) directly.

[2]A difference between the gFL symmetry and the FL symmetry is that the coefficients ϕ_i in the transformation law of the gFL symmetry (hence the coefficients in the eigenvector) can take arbitrary values instead of fixed constants v_i in the FL transformation. So gFL symmetry makes three neutrinos massless and the FL symmetry only guarantees one neutrino massless.

We shall present two examples. The first example has inverted hierarchy. For simplicity, we assume that m_R is a unit matrix, *i.e.*, $m_R = m_s\mathbf{1}$. And we choose

$$(v_1, v_2, v_2)^T = \frac{v}{\sqrt{2}}(0, 1, 1)^T. \tag{19}$$

Using Eq. (19) we get

$$m_\nu = -\frac{v^2}{2m_s}\begin{pmatrix} F_2, & F_\lambda, & -F_\lambda \\ F_\lambda, & \lambda_2, & -\lambda_2 \\ -F_\lambda, & -\lambda_2, & \lambda_2 \end{pmatrix}, \tag{20}$$

where

$$F_2 = \sum_i(\lambda_{i12} + \lambda_{i13})^2, \qquad F_\lambda = \sum_i \lambda_{i23}(\lambda_{i12} + \lambda_{i13}),$$
$$\lambda_2 = \sum_i \lambda_{i23}^2. \tag{21}$$

We find that m_ν is diagonalized by U

$$U^T m_\nu U = -\frac{v^2}{2m_s}\text{diag}\{F_2 - F_\lambda, F_2 + 2F_\lambda, 0\}, \tag{22}$$

provided that the following condition is satisfied

$$F_2 + F_\lambda = 2\lambda_2. \tag{23}$$

And we get

$$\begin{aligned} \Delta m_{21}^2 &= 3F_\lambda(2F_2 + F_\lambda)\frac{v^4}{4m_s^2}, \\ \Delta m_{31}^2 &= -(F_2 - F_\lambda)^2\frac{v^4}{4m_s^2}. \end{aligned} \tag{24}$$

The realistic neutrino mass square differences can be obtained since we have enough independent parameters to fit two Δm^2.

The second example has normal hierarchy; we take Λ antisymmetric and m_R diagonal

$$m_R = \text{diag}\{m_{r1}, m_{r2}, m_{r3}\}. \tag{25}$$

We choose

$$(v_1, v_2, v_3)^T = \frac{v}{\sqrt{6}}(2, -1, 1)^T. \tag{26}$$

Using $\lambda_{ijk} = \lambda\epsilon_{ijk}$ we get

$$\Lambda = \frac{v}{\sqrt{6}}\begin{pmatrix} 0 & \lambda & \lambda \\ -\lambda & 0 & 2\lambda \\ -\lambda & -2\lambda & 0 \end{pmatrix}. \tag{27}$$

And we find

$$m_\nu = -\frac{\lambda^2 v^2}{6}\begin{pmatrix} \frac{1}{m_{r2}} + \frac{1}{m_{r3}}, & \frac{2}{m_{r3}}, & -\frac{2}{m_{r2}} \\ \frac{2}{m_{r3}}, & \frac{1}{m_{r1}} + \frac{4}{m_{r3}}, & \frac{1}{m_{r1}} \\ -\frac{2}{m_{r2}}, & \frac{1}{m_{r1}}, & \frac{1}{m_{r1}} + \frac{4}{m_{r2}} \end{pmatrix}. \tag{28}$$

If the condition

$$m_{r2} = m_{r3} \tag{29}$$

is satisfied, we find

$$U^T m_\nu U = -\frac{\lambda^2 v^2}{3}\mathrm{diag}\left\{0, \frac{3}{m_{r2}}, \frac{1}{m_{r1}} + \frac{2}{m_{r2}}\right\}. \tag{30}$$

Hence we get

$$\Delta m_{21}^2 = \frac{\lambda^4 v^4}{m_{r2}^2}, \qquad \Delta m_{31}^2 = \frac{\lambda^4 v^4}{9}\left(\frac{1}{m_{r1}} + \frac{2}{m_{r2}}\right)^2. \tag{31}$$

Using the hierarchy in neutrino mass $\Delta m_{31}^2 \approx 25\Delta m_{21}^2$, we find

$$m_{r2} \approx 13 m_{r1}. \tag{32}$$

C. Testable TeV scale seesaw mechanism

In recent years, there has been some interest in the TeV scale seesaw mechanism [23,24]. The mechanism suggests that the mixings of the left-handed and right-handed neutrinos are independent of the hierarchy in the Dirac-type and Majorana-type masses. This makes the seesaw mechanism testable at the future colliders or in rare decay processes. In this subsection, we show that we can also realize the testable TeV scale seesaw mechanism via the generalized Friedberg-Lee symmetry.

Instead of Eq. (3), we introduce the following gFL symmetry transformation under which the right-handed neutrinos transform nontrivially as well:

$$\nu_{Li} \to \nu_{Li} + \frac{1}{\sqrt{1+|\alpha_i|^2}}\phi_i\theta, \tag{33}$$

$$\nu_{Ri}^c \to \nu_{Ri}^c + \frac{\alpha_i}{\sqrt{1+|\alpha_i|^2}}\phi_i\theta, \tag{34}$$

$$\phi_i \to \phi_i, \tag{35}$$

where $\alpha_i (i = 1, 2, 3)$ are complex numbers.

We introduce neutrinos $\nu_\perp$ and $\nu_\top$ in an orthogonal basis

$$\nu_{\perp i} = \frac{1}{\sqrt{1+|\alpha_i|^2}}(\nu_{Li} + \alpha_i^* \nu_{Ri}^c), \tag{36}$$

$$\nu_{\top i} = \frac{1}{\sqrt{1+|\alpha_i|^2}}(-\alpha_i \nu_{Li} + \nu_{Ri}^c). \tag{37}$$

It is easy to see that under Eqs. (33) and (34) we have

$$\nu_{\perp i} \to \nu_{\perp i} + \phi_i\theta, \qquad \nu_{\top i} \to \nu_{\top i}. \tag{38}$$

Thus, in the new basis the Eq. (3) is reproduced. Then the discussions on the seesaw mechanism and the neutrino masses and mixings are similar to those in Secs. II A and II B. The only difference with the previous case is that the mixings between the left-handed and right-handed neutrinos are no longer suppressed by the mass hierarchy in the seesaw type mass matrix in Eq. (14). Denoting the neutrino mass eigenstates as $(\nu, \nu_H)^T$, we can find that

$$\begin{aligned}\begin{pmatrix}\nu_L\\ \nu_R^c\end{pmatrix} &\approx \begin{pmatrix}\mathcal{A}_0, & -\mathcal{A}_1^\dagger\\ \mathcal{A}_1, & \mathcal{A}_0\end{pmatrix}\begin{pmatrix}U, & 0\\ 0, & U_H\end{pmatrix}\begin{pmatrix}\nu\\ \nu_H\end{pmatrix}\\ &= \begin{pmatrix}\mathcal{A}_0 U, & -\mathcal{A}_1^\dagger U_H\\ \mathcal{A}_1 U, & \mathcal{A}_0 U_H\end{pmatrix}\begin{pmatrix}\nu\\ \nu_H\end{pmatrix},\end{aligned} \tag{39}$$

where U is the mixing matrix of the light neutrinos ν, U_H is the mixing matrix of heavy neutrinos ν_H, and

$$\mathcal{A}_0 = \mathrm{diag}\left\{\frac{1}{\sqrt{1+|\alpha_1|^2}}, \frac{1}{\sqrt{1+|\alpha_2|^2}}, \frac{1}{\sqrt{1+|\alpha_3|^2}}\right\}, \tag{40}$$

$$\mathcal{A}_1 = \mathrm{diag}\left\{\frac{\alpha_1}{\sqrt{1+|\alpha_1|^2}}, \frac{\alpha_2}{\sqrt{1+|\alpha_2|^2}}, \frac{\alpha_3}{\sqrt{1+|\alpha_3|^2}}\right\}. \tag{41}$$

We find that the mixings of the left-handed and right-handed neutrinos are determined by α_i which is independent of the mass hierarchy between the Dirac-type and Majorana-type masses. A_1 determines the strength of unitarity violation of the mixings of light neutrinos [25]. This kind of scenario may possibly be tested at the future colliders [26] and neutrino oscillation experiments [27].

D. Embedding into the extensions of the SM

We can embed the above models into the extensions of the SM. Let us denote the SM lepton doublets as L_i, and the SM Higgs field as H. Also, we introduce three SM singlet scalar fields ϕ_i. By the way, the following discussions can be easily generated to the supersymmetric standard models by changing

$$H \to H_u, \qquad \tilde{H} \to H_d, \tag{42}$$

where $\tilde{H} = i\sigma_2 H^*$, and H_u and H_d are one pair of the Higgs doublets in the supersymmetric standard models.

(A) For the usual seesaw mechanism, we introduce the following gFL symmetry:

$$\begin{aligned} &L_i \to L_i + \phi_i\chi, \qquad \nu_{Ri}^c \to \nu_{Ri}^c,\\ &\phi_i \to \phi_i, \qquad H \to H,\end{aligned} \tag{43}$$

where χ is an $SU(2)_L$ doublet and has two components of Grassmann constant. And the relevant neutrino Lagrangian is

$$\begin{aligned}-\Delta\mathcal{L} = &\frac{1}{2}(m_0)_{ij}\nu_{Ri}^{cT} i\sigma_2 \nu_{Rj}^c + \lambda_{ijk}\bar{\nu}_{Ri}L_j\frac{\phi_k}{M_*}H\\ &+ \frac{1}{2}\eta_{ijk}\nu_{Ri}^{cT} i\sigma_2\nu_{Rj}^c\phi_k\\ &+ \frac{1}{2}\eta'_{ijk}\nu_{Ri}^{cT} i\sigma_2\nu_{Rj}^c\phi_k^\dagger + \mathrm{H.c.},\end{aligned} \tag{44}$$

where $\lambda_{ijk} = -\lambda_{ikj}$, and $(m_0)_{ij}$, η_{ijk}, and η'_{ijk} are symmetric for i and j, and M_* is the cutoff scale of the gFL symmetry. Because the Lagrangian in Eq. (44) is similar to that in Eq. (7), we can embed the model with the usual seesaw mechanism into the extension of the SM.

As a remark, the most naive approach is that we introduce three Higgs doublets H_i, and define the following gFL symmetry:

$$L_i \to L_i + \tilde{H}_i\theta, \qquad \nu^c_{Ri} \to \nu^c_{Ri}, \qquad H_i \to H_i, \tag{45}$$

where $\tilde{H}_i = i\sigma_2 H_i^*$. However, the neutrino Dirac Yukawa couplings $\bar{\nu}_{Ri}L_jH_k$ are not invariant under the above gFL symmetry. And then we can not explain the neutrino masses and mixings via the seesaw mechanism. In short, this approach does not work.

(B) For the testable TeV scale seesaw mechanism, we have to embed the three right-handed neutrinos into three fermionic doublets L'_i. To cancel the anomaly, we introduce three fermionic doublets $\tilde{L}'_i$ which are the Hermitian conjugate of L'_i. And we introduce the gFL symmetry transformation as follows:

$$L_i \to L_i + \frac{1}{\sqrt{1+|\alpha_i|^2}}\phi_i\chi, \tag{46}$$

$$L'_i \to L'_i + \frac{\alpha_i}{\sqrt{1+|\alpha_i|^2}}\phi_i\chi, \tag{47}$$

$$\phi_i \to \phi_i, \qquad H \to H, \qquad \tilde{L}'_i \to \tilde{L}'_i. \tag{48}$$

And we define

$$L_{\perp i} = \frac{1}{\sqrt{1+|\alpha_i|^2}}(L_i + \alpha_i^* L'_i), \tag{49}$$

$$L_{\top i} = \frac{1}{\sqrt{1+|\alpha_i|^2}}(-\alpha_i L_i + L'_i). \tag{50}$$

It is easy to see that under the above gFL symmetry we have

$$L_{\perp i} \to L_{\perp i} + \phi_i\chi, \qquad L_{\top i} \to L_{\top i}. \tag{51}$$

And then we obtain the major relevant neutrino Lagrangian

$$\begin{aligned} -\Delta\mathcal{L} = {}& \frac{1}{M_I}\bigg(\lambda^\nu_{ijkl}L_{\perp i}L_{\perp j}\frac{\phi_k}{M_*}\frac{\phi_l}{M_*}H^2 \\ & + y^\nu_{ijk}L_{\top i}L_{\perp j}\frac{\phi_k}{M_*}H^2 + \lambda_{ij}\tilde{L}'_i\tilde{L}'_j\tilde{H}\tilde{H}\bigg) \\ & + M_{ij}L_{\top i}\tilde{L}'_j + y^L_{ijk}L_{\top i}\tilde{L}'_j\phi_l + \text{H.c.}, \end{aligned} \tag{52}$$

where M_I is an intermediate scale and the Yukawa couplings λ^ν_{ijkl} satisfy $\lambda^\nu_{ijkl} = -\lambda^\nu_{kjil} = -\lambda^\nu_{ilkj}$ or $\lambda^\nu_{ijkl} = -\lambda^\nu_{ljki} = -\lambda^\nu_{ikjl}$, and the Yukawa coupling y^ν_{ijk} is antisymmetric for j and k. Interestingly, the neutrino mass matrix proposed by Friedberg and Lee can be generated by the first term in Eq. (52). Even if this term is zero, *i.e.*, $\lambda^\nu_{ijkl} = 0$, the observed neutrino masses and mixings can be generated by the double seesaw mechanism [28,29]. Here, we emphasize that we neglect the other high-dimensional operators that are not important in the discussions of the neutrino masses and mixings.

In addition, the first three terms in Eq. (52) are nonrenormalizable and can be obtained by the seesaw mechanism. For example, if we introduce three SM singlet fermions N_i, the first three terms can be obtained due to the following Lagrangian via the seesaw mechanism:

$$\begin{aligned} -\Delta\mathcal{L} = {}& \frac{1}{2}(M_N)_{ij}\bar{N}^c_iN_j + \lambda_{li}\bar{N}_lL_{\top i}H + \eta_{ljk}\bar{N}_lL_{\perp j}\frac{\phi_k}{M_*}H \\ & + \lambda_{li}N_l\tilde{L}'_i\tilde{H} + \text{H.c.}, \end{aligned} \tag{53}$$

where $(M_N)_{ij}$ is symmetric, and $\eta_{ljk} = -\eta_{lkj}$. M_I is around the mass scales of N_i.

III. $SO(3) \times U(1)$ FLAVOR SYMMETRY IN THE LEPTON SECTOR

To explain the SM fermion masses and mixings, we usually use the Froggatt-Nielsen mechanism [30] by introducing the global flavor symmetry. Thus, the FL symmetry could also be a residual symmetry after the flavor symmetry breaking. In this section, we consider the $SO(3) \times U(1)$ flavor symmtry in the lepton sector.

Let us explain the convention in detail. We denote the SM Higgs doublet as H, the left-handed lepton doublets as L_i, and the right-handed charged leptons as E_i. To break the $SO(3) \times U(1)$ flavor symmetry, we also introduce three Higgs doublets H_i, and nine SM singlet scalar field Φ, Φ_i, and Φ_{ij}. We assume that the L_i, E_i, H_i, and Φ_i form the fundamental representation of $SO(3)$, and Φ_{ij} form the symmetric representation of $SO(3)$. We shall present two concrete models in the following subsections: In Model I, ν_{Ri} are singlets under $SO(3)$, while in Model II, ν_{Ri} form the fundamental representation of $SO(3)$ and we do not need the Φ_i fields.

A. FL symmetry with seesaw mechanism

Before we study the $SO(3) \times U(1)$ flavor symmetry, let us consider the FL model with the seesaw mechanism. We consider the FL symmetry as follows:

$$L_i \to L_i + \xi_i\chi, \qquad \nu_{Ri} \to \nu_{Ri}, \qquad H \to H, \tag{54}$$

where we obtain the original FL symmetry by choosing $\xi_1 = \xi_2 = \xi_3$. The neutrino Lagrangian, which is invariant under above FL symmetry, is

$$-\Delta\mathcal{L} = \tfrac{1}{2}(m_0')_{ij}\bar{\nu}^c_{Ri}\nu_{Rj} + y_{ijk}\bar{\nu}_{Ri}(\xi_k L_j - \xi_j L_k)H. \quad (55)$$

Following the usual procedure [15–19], we realize the seesaw mechanism with FL symmetry in the light neutrino mass matrix. Therefore, in order to generalize the FL symmetry, we need to construct the models that can reproduce the above Lagrangian in Eq. (55) after the generalized symmetry breaking. As an example to explain the main idea, we introduce three SM Higgs doublets and consider $\xi_i H$ as H_i. Then the above neutrino Lagrangian becomes

$$-\Delta\mathcal{L} = \tfrac{1}{2}(m_0')_{ij}\bar{\nu}^c_{Ri}\nu_{Rj} + \tfrac{1}{2}(m_0')_{ij}\bar{\nu}^c_{Ri}\nu_{Rj} + y_{ijk}\bar{\nu}_{Ri}(L_j H_k - H_j L_k). \quad (56)$$

Therefore, we can obtain the neutrino mass matrix with FL symmetry if the neutrino Dirac Yukawa couplings y_{ijk} are antisymmetric for the lepton doublet indices j and Higgs field indices k, *i.e.*, $y_{ijk} = -y_{ikj}$.

B. Model I

We assume that under the $U(1)$ symmetry, ν_{Ri} has charge 0, L_i has charge 1, E_i has charge $-1/2$, H has charge $1/2$, H_i has charge 2, Φ_i has charge -3, and Φ and Φ_{ij} have charges -1. The $SO(3)\times U(1)$ invariant Lagrangian is

$$-\Delta\mathcal{L} = \frac{1}{2}(m_0')_{ij}\bar{\nu}^c_{Ri}\nu_{Rj} + \frac{1}{M_{\rm Pl}}(y^\nu_{ijkl}\bar{\nu}_{Ri}L_j H_k\Phi_l + \lambda^E\bar{E}_i L_i\tilde{H}\Phi + y^E_{ij}\bar{E}_i L_j\tilde{H}\Phi_{ij}) + \text{H.c.}, \quad (57)$$

where the Yukawa couplings y^ν_{ijkl} are antisymmetric for their indices j, k, and l due to the $SO(3)$ invariance. For simplicity, we assume that the SM Higgs field H has VEVs close to 174 GeV, while the Higgs fields H_i have small VEVs, for example, a few GeVs. In addition, we assume that Φ, Φ_i, and Φ_{ij} have VEVs around the grand unification scale 2.4×10^{16} or higher, so that the dimension-5 operators can generate the masses for the charged leptons and neutrinos. It is not difficult to show that we do have enough degrees of freedom to explain the charged lepton masses, and the neutrino masses and mixings.

After the $SO(3)\times U(1)$ flavor symmetry breaking, we obtain that the neutrino mass matrix for the left-handed and right-handed neutrinos from the Lagrangian in Eq. (57) is the same as that from the Lagrangian in Eq. (7) by choosing the following relations:

$$(m_0)_{ij} + \eta_{ijk}\langle\phi_k\rangle + \eta'_{ijk}\langle\phi^*_k\rangle = (m_0')_{ij}, \qquad \lambda_{ijk}\langle\phi_i\rangle = \frac{1}{M_{\rm Pl}}y^\nu_{ijkl}\langle H_k\rangle\langle\Phi_l\rangle. \quad (58)$$

Similar to the discussions in Sec. II B, we can explain the realistic neutrino masses and mixings. Interestingly, the $SO(3)\times U(1)$ flavor symmetry is broken down to the FL symmetry. In other words, the FL symmetry is the residual symmetry in the neutrino mass matrix from the flavor symmetry breaking.

Moreover, the FL symmetry can be broken only by the dimension-7 or higher operators. And the dimension-7 operators that break the FL symmetry are

$$-\Delta\mathcal{L} = \frac{1}{M^3_{\rm Pl}}\bar{\nu}_{Ri}L_j H_k(\Phi^3\delta_{jk} + \Phi^2\Phi_{jk} + \Phi_{jl}\Phi_{lm}\Phi_{mk} + \Phi_{jk}\Phi_{lm}\Phi_{lm}) + \text{H.c.}, \quad (59)$$

where, for simplicity, we neglect the Yukawa couplings. Thus, the FL symmetry is a very good approximate symmetry in the neutrino mass matrix.

By the way, the VEVs of Φ, Φ_i, and Φ_{ij} break the $U(1)$ symmetry down to the Z_2 symmetry. Under this Z_2 symmetry, E_i and H are odd while the other fields are even. This Z_2 symmetry forbids the Dirac Yukawa couplings between H and neutrinos. Otherwise, the discussions will become very complicated because the VEVs of H are much larger than those of H_i while the VEVs of Φ, Φ_i, and Φ_{ij} are close to the Planck scale. Also, this $U(1)$ symmetry will not affect the quark Yukawa couplings if we assign the $U(1)$ charges $1/2$ and $-1/2$ to the right-handed up-type and down-type quarks, respectively.

C. Model II

We assume that under the $U(1)$ symmetry, ν_{Ri} has charge -1, L_i has charge 1, E_i has charge $-3/2$, H has charge $1/2$, H_i has charge -2, and Φ and Φ_{ij} have charges -2. The $SO(3)\times U(1)$ invariant Lagrangian is

$$-\Delta\mathcal{L} = \frac{1}{2}\lambda^N\bar{\nu}^c_{Ri}\nu_{Ri}\Phi^\dagger + \frac{1}{2}y^N_{ij}\bar{\nu}^c_{Ri}\nu_{Rj}\Phi^\dagger_{ij} + y^\nu_{ijk}\bar{\nu}_{Ri}L_j H_k + \frac{1}{M_{\rm Pl}}(\lambda^E\bar{E}_i L_i\tilde{H}\Phi + y^E_{ij}\bar{E}_i L_j\tilde{H}\Phi_{ij}) + \text{H.c.}, \quad (60)$$

where the Yukawa coupling y^ν_{ijk} is antisymmetric for their indices i, j, and k. Similar to the above subsection, we have enough degrees of freedom to explain the charged lepton masses.

After the $SO(3)\times U(1)$ flavor symmetry breaking, we obtain that the neutrino mass matrix for the left-handed and right-handed neutrinos from the Lagrangian in Eq. (60) is a special case of that from the Lagrangian in Eq. (7) by choosing the following relations:

$$(m_0)_{ij} + \eta_{ijk}\langle\phi_k\rangle + \eta'_{ijk}\langle\phi^*_k\rangle = \lambda^N\langle\Phi^\dagger\rangle\delta_{ij} + y^N_{ij}\langle\Phi^\dagger_{ij}\rangle, \qquad \lambda_{ijk}\langle\phi_i\rangle = y^\nu_{ijk}\langle H_k\rangle. \quad (61)$$

The point is that the Yukawa coupling y^ν_{ijk} is antisymmetric for i, j, and k while λ_{ijk} is only antisymmetric for j and k. Similar to the second example in Sec. II B, we can explain the observed neutrino masses and mixings. And the FL symmetry is the residual symmetry from the $SO(3)\times U(1)$

flavor symmetry breaking as well. Unlike Model I, it is very difficult to break the FL symmetry via the higher dimensional operators, so the FL symmetry may be a symmetry in the neutrino mass matrix.

IV. CONCLUSIONS AND DISCUSSIONS

In summary, we study the possible origin of the FL symmetry. First, we generalize the FL symmetry to the gFL symmetry by including the scalar fields in the field transformations. The FL symmetry is the residual symmetry after the larger gFL symmetry breaking. A direct consequence of the gFL symmetry is the masslessness of three light neutrinos, which obtain masses via the seesaw mechanism after the gFL symmetry breaking. We also show that the observed neutrino masses and mixings can be generated. Also, if the transformations of the right-handed neutrinos under the gFL symmetry are similar to those of the left-handed neutrinos, we can have the testable TeV scale seesaw mechanism. Moreover, the models with gFL symmetry can be embedded into the extensions of the SM. Second, we propose two models with the $SO(3) \times U(1)$ global flavor symmetry in the lepton sector. After the flavor symmetry breaking, we can obtain the charged lepton masses, and explain the neutrino masses and mixings via the seesaw mechanism. In particular, the complete neutrino mass matrices are similar to those of the above models with gFL symetry. So, the $SO(3) \times U(1)$ flavor symmetry is broken down to the FL symmetry which is the residual symmetry in the neutrino mass matrix.

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