Viscosity bound violation in higher derivative gravity

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(Received 3 March 2008; published 11 June 2008)

Motivated by the vast string landscape, we consider the shear viscosity to entropy density ratio in conformal field theories dual to Einstein gravity with curvature square corrections. After field redefinitions these theories reduce to Gauss-Bonnet gravity, which has special properties that allow us to compute the shear viscosity nonperturbatively in the Gauss-Bonnet coupling. By tuning of the coupling, the value of the shear viscosity to entropy density ratio can be adjusted to any positive value from infinity down to zero, thus violating the conjectured viscosity bound. At linear order in the coupling, we also check consistency of four different methods to calculate the shear viscosity, and we find that all of them agree. We search for possible pathologies associated with this class of theories violating the viscosity bound.

DOI: 10.1103/PhysRevD.77.126006 PACS numbers: 11.25.Tq, 12.38.Mh, 25.75.−q

I. INTRODUCTION

The AdS/conformal field theory (CFT) correspondence [1–4] has yielded many important insights into the dynamics of strongly coupled gauge theories. Among numerous results obtained so far, one of the most striking is the universality of the ratio of the shear viscosity η to the entropy density s [5–8]

$$\frac{\eta}{s} = \frac{1}{4\pi} \tag{1.1}$$

for all gauge theories with an Einstein gravity dual in the limit $N \to \infty$ and $\lambda \to \infty$. Here, N is the number of colors and λ is the 't Hooft coupling. It was further conjectured in [8] that (1.1) is a universal lower bound [the Kovtun-Starinets-Son (KSS) bound] for all materials. So far, all known substances including water and liquid helium satisfy the bound. The systems coming closest to the bound include the quark-gluon plasma created at Relativistic Heavy Ion Collider (RHIC) [9–14] and certain cold atomic gases in the unitarity limit (see e.g. [15]). η/s for pure gluon QCD slightly above the deconfinement temperature has also been calculated on the lattice recently [16] and is about 30% larger than (1.1). See also [17]. See [18–22] for other discussions of the bound.

Now, as stated above, the ratio (1.1) was obtained for a class of gauge theories whose holographic duals are dictated by classical Einstein gravity (coupled to matter). More generally, string theory (or any quantum theory of gravity) contains higher derivative corrections from stringy or quantum effects, the inclusion of which will modify the ratio. In terms of gauge theories, such modifications correspond to $1/\lambda$ or $1/N$ corrections. As a concrete example, let us take $\mathcal{N} = 4$ super-Yang-Mills theory, whose dual corresponds to type IIB string theory on $AdS_5 \times S^5$. The leading order correction in $1/\lambda$ arises from stringy corrections to the low-energy effective action of type IIB supergravity, schematically of the form $\alpha'^3 R^4$. The correction to η/s due to such a term was calculated in [23,24]. It was found that the correction is positive, consistent with the conjectured bound.

In this paper, instead of limiting ourselves to specific known string theory corrections, we explore the modification of η/s due to generic higher derivative terms in the holographic gravity dual. The reason is partly pragmatic: other than in a few maximally supersymmetric circumstances, very little is known about forms of higher derivative corrections generated in string theory. Given the vastness of the string landscape [25], one expects that generic corrections do occur. Restricting to the gravity sector in AdS_5, the leading order higher derivative corrections can be written as[1]

$$I = \frac{1}{16\pi G_N} \int d^5x \sqrt{-g} (R - 2\Lambda + L^2(\alpha_1 R^2 + \alpha_2 R_{\mu\nu} R^{\mu\nu} + \alpha_3 R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma})), \tag{1.2}$$

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[1]Our conventions are those of [26]. In this section we suppress Gibbons-Hawking surface terms.

where $\Lambda = -\frac{6}{L^2}$ and for now we assume that $\alpha_i \sim \frac{\alpha'}{L^2} \ll 1$. Other terms with additional derivatives or factors of R are naturally suppressed by higher powers of $\frac{\alpha'}{L^2}$. String loop (quantum) corrections can also generate such terms, but they are suppressed by powers of g_s and we will consistently neglect them by taking the $g_s \to 0$ limit.[2] To lowest order in α_i the correction to η/s will be a linear combination of α_i's, and the viscosity bound is then violated for one side of the half-plane. Specifically, we will find

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 - 8\alpha_3) + O(\alpha_i^2) \tag{1.3}$$

and hence the bound is violated for $\alpha_3 > 0$. Note that the above expression is independent of α_1 and α_2. This can be inferred from a field redefinition argument (see Sec. II C).

How do we interpret these violations? Possible scenarios are:

(1) The bound can be violated. For example, this scenario would be realized if one explicitly finds a well-defined string theory on AdS_5 which generates a stringy correction with $\alpha_3 > 0$. (See [27] for a plausible counterexample to the KSS bound.)

(2) The bound is correct (for example, if one can prove it using a field theoretical method) and a bulk gravity theory with $\alpha_3 > 0$ cannot have a well-defined boundary CFT dual.

(a) The bulk theory is manifestly inconsistent as an effective theory. For example, it could violate bulk causality or unitarity.

(b) It is impossible to generate such a low-energy effective classical action from a consistent quantum theory of gravity. In modern language we say that the theory lies in the swampland of string theory.

Any of these alternatives, if realized, is interesting. Needless to say, possibility 1 would be interesting. Given that recent analyses from RHIC data [10–14] indicate the η/s is close to (and could be even smaller than) the bound, this further motivates to investigate the universality of the KSS bound in holographic models.

Possibility 2(a) should help clarify the physical origin of the bound by correlating bulk pathologies and the violation of the bound. Possibility 2(b) could provide powerful tools for constraining possible higher derivative corrections in the string landscape. Note that while there are some nice no-go theorems which rule out classes of nongravitational effective field theories [28] (also see [29]), the generalization of the arguments of [28] to gravitational theories is subtle and difficult. Thus, constraints from AdS/CFT based on the consistency of the boundary theory would be valuable.

[2]Note that to calculate g_s corrections, all the light fields must be taken into account. In addition, the calculation of η/s could be more subtle once we begin to include quantum effects.

In investigating the scenarios above, Gauss-Bonnet (GB) gravity will provide a useful model. Gauss-Bonnet gravity, defined by the classical action of the form [30]

$$I = \frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\Big[R - 2\Lambda + \frac{\lambda_{\rm GB}}{2}L^2(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})\Big], \tag{1.4}$$

has many nice properties that are absent for theories with more general ratios of the α_i's. For example, expanding around flat Minkowski space, the metric fluctuations have exactly the same quadratic kinetic terms as those in Einstein gravity. All higher derivative terms cancel [30]. Similarly, expanding around the AdS black brane geometry, which will be the main focus of the paper, there are also only second derivatives on the metric fluctuations. Thus small metric fluctuations can be quantized for finite values of the parameter $\lambda_{\rm GB}$.[3] Furthermore, crucial for our investigation is its remarkable feature of solvability: sets of exact solutions to the classical equation of motion have been obtained [31,32] and the exact form of the Gibbons-Hawking surface term is known [33].

Given these nice features of Gauss-Bonnet gravity, we will venture outside the regime of the perturbatively corrected Einstein gravity and study the theory with finite values of $\lambda_{\rm GB}$. To physically motivate this, one could envision that somewhere in the string landscape $\lambda_{\rm GB}$ is large but all the other higher derivative corrections are small. One of the main results of the paper is a value of η/s for the CFT dual of Gauss-Bonnet gravity, *nonperturbative* in $\lambda_{\rm GB}$:[4]

$$\frac{\eta}{s} = \frac{1}{4\pi}[1 - 4\lambda_{\rm GB}]. \tag{1.5}$$

We emphasize that this is not just a linearly corrected value. In particular, the viscosity bound is badly violated as $\lambda_{\rm GB} \to \frac{1}{4}$. As we will discuss shortly, $\lambda_{\rm GB}$ is bounded above by $\frac{1}{4}$ for the theory to have a boundary CFT, and η/s never decreases beyond 0.

Given the result (1.5) for Gauss-Bonnet, if the possibility 2(a) were correct, we would expect that pathologies would become easier to discern in the limit where $\eta/s \to 0$. We will investigate this line of thought in Sec. IV. On the other hand, thinking along the line of possibility 1, the Gauss-Bonnet theory with $\lambda_{\rm GB}$ arbitrarily close to $\frac{1}{4}$ may have a concrete realization in the string landscape. In this case, there exists no lower bound for η/s, and investigating the CFT dual of Gauss-Bonnet theory should clarify how to

[3]Generic theories in (1.2) contain four derivatives and a consistent quantization is not possible other than treating higher derivative terms as perturbations.

[4]We have also computed the value of η/s for Gauss-Bonnet gravity for any spacetime dimension D and the expression is given in (3.26).

evade the heuristic mean free path argument for the existence of the lower bound (presented in, e.g., [8]).

The plan of the paper is as follows. In Sec. II, we review various properties of two-point correlation functions and outline the real-time AdS/CFT calculation of the shear viscosity. We then explicitly calculate the shear viscosity for Gauss-Bonnet theory in Sec. III. In Sec. IV, we seek possible pathologies associated with theories violating the viscosity bound. There, we will find a curious new metastable state for large enough $\lambda_{\rm GB}$. Finally in Sec. V, we conclude with various remarks and speculations. To make the paper fairly self-contained, various appendices are added. In particular, quasinormal mode calculations of the shear viscosity are presented in Appendix B and one using the membrane paradigm in Appendix D.

II. SHEAR VISCOSITY IN R^2 THEORIES: PRELIMINARIES

A. Two-point correlation functions and viscosity

Let us begin by collecting various properties of two-point correlation functions, following [34–36] (see also [37]). Consider retarded two-point correlation functions of the stress energy tensor $T_{\mu\nu}$ of a CFT in $3+1$-dimensional Minkowski space at a finite temperature T

$$G_{\mu\nu,\alpha\beta}(\omega,\vec{q}) = -i\int dtd\vec{x}e^{i\omega t-i\vec{q}\cdot\vec{x}}\theta(t) \times\langle[T_{\mu\nu}(t,\vec{x}),T_{\alpha\beta}(0,0)]\rangle. \tag{2.1}$$

They describe linear responses of the system to small disturbances. It turns out that various components of (2.1) can be expressed in terms of three independent scalar functions. For example, if we take spatial momentum to be $\vec{q}=(0,0,q)$, then

$$G_{12,12}=\frac{1}{2}G_3(\omega,q),\qquad G_{13,13}=\frac{1}{2}\frac{\omega^2}{\omega^2-q^2}G_1(\omega,q),$$
$$G_{33,33}=\frac{2}{3}\frac{\omega^4}{(\omega^2-q^2)^2}G_2(\omega,q), \tag{2.2}$$

and so on. At $\vec{q}=0$ all three functions $G_{1,2,3}(\omega,0)$ are equal to one another as a consequence of rotational symmetry.

When $\omega, |\vec{q}| \ll T$ one expects the CFT plasma to be described by hydrodynamics. The scalar functions $G_{1,2,3}$ encode the hydrodynamic behavior of shear, sound, and transverse modes, respectively. More explicitly, they have the following properties:

(i) G_1 has a simple diffusion pole at $\omega=-iDq^2$, where

$$D=\frac{\eta}{\epsilon+P}=\frac{1}{T}\frac{\eta}{s} \tag{2.3}$$

with ϵ and s being the energy and entropy density, and P the pressure of the gauge theory plasma.

(ii) G_2 has a simple pole at $\omega=\pm c_s q-i\Gamma_s q^2$, where c_s is the speed of sound and Γ_s is the sound damping constant, given by (for conformal theories)

$$\Gamma_s=\frac{2}{3T}\frac{\eta}{s} \tag{2.4}$$

(iii) η can also be obtained from $G_{1,2,3}$ at zero spatial momentum by the Kubo formula, e.g.,

$$\eta=\lim_{\omega\to 0}\frac{1}{\omega}\,\mathrm{Im}\,G_{12,12}(\omega,0) \tag{2.5}$$

Equations (2.3), (2.4), and (2.5) provide three independent ways of extracting η/s. We provide calculations utilizing the first two in Appendix B. A calculation utilizing the Kubo formula (2.5) is easier, and we will explicitly implement it for Gauss-Bonnet theory in Sec. III. In the next subsection, we outline how to obtain retarded two-point functions within the framework of the real-time AdS/CFT correspondence.

B. AdS/CFT calculation of shear viscosity: Outline

The stress tensor correlators for a boundary CFT described by (1.2) or (1.4), can be computed from gravity as follows. One first finds a black brane solution (i.e. a black hole whose horizon is $\mathbf{R}^3$) to the equations of motion of (1.2) or (1.4). Such a solution describes the boundary theory on $\mathbf{R}^{3,1}$ at a temperature T, which can be identified with the Hawking temperature of the black brane. The entropy and energy density of the boundary theory are given by the corresponding quantities of the black brane. The fluctuations of the boundary theory stress tensor are described in the gravity language by small metric fluctuations $h_{\mu\nu}$ around the black brane solution. In particular, after taking into account of various symmetries and gauge degrees of freedom, the metric fluctuations can be combined into three independent scalar fields ϕ_a, $a=1,2,3$, which are dual to the three functions G_a of the boundary theory.

To find G_a, one could first work out the bulk two-point retarded function for ϕ_a and then take both points to the boundary of the black brane geometry. In practice it is often more convenient to use the prescription proposed in [38], which can be derived from the real-time AdS/CFT correspondence [39]. Let us briefly review it here:

(1) Solve the linearized equation of motion for $\phi_a(r;k)$ with the following boundary conditions:

(a) Impose the infalling boundary condition at the horizon. In other words, modes with timelike momenta should be falling into the horizon and modes with spacelike momenta should be regular.

(b) Take r to be the radial direction of the black brane geometry with the boundary at $r=\infty$. Require

$$\phi_a(r;k)|_{r=1/\epsilon} = J_a(k), \qquad k = (\omega, q), \tag{2.6}$$

where $\epsilon \to 0$ imposes an infrared cutoff near the infinity of the spacetime and $J_a(k)$ is an infinitesimal boundary source for the bulk field $\phi_a(r;k)$.

(2) Plug in the above solution into the action, expanded to quadratic order in $\phi_a(r;k)$. It will reduce to pure surface contributions. The prescription instructs us to pick up only the contribution from the boundary at $r = \frac{1}{\epsilon}$. The resulting action can be written as

$$S = -\frac{1}{2}\int \frac{d^4k}{(2\pi)^4} J_a(-k)\mathcal{F}_a(k,r)J_a(k)\Big|_{r=1/\epsilon}. \tag{2.7}$$

Finally the retarded function $G_a(k)$ in momentum space for the boundary field dual to ϕ_a is given by

$$G_a(k) = \lim_{\epsilon\to 0}\mathcal{F}_a(k,r)|_{r=1/\epsilon}. \tag{2.8}$$

Using the Kubo formula (2.5), we can get the shear viscosity by studying a mode ϕ_3 with $\vec{q} = 0$ in the low-frequency limit $\omega \to 0$. We will do so in the next section. Alternatively, using (2.3) or (2.4), we can read off the viscosity from pole structures of retarded two-point functions. Such a calculation is a bit more involved and will be performed in Appendix B.

The above prescription for computing retarded functions in AdS/CFT works well if the bulk scalar field has only two derivatives as in the Gauss-Bonnet case (1.4). If the bulk action contains more than two derivatives, complications could arise even if one treats the higher derivative parts as perturbations. For example, one needs to add Gibbons-Hawking surface terms to ensure a well-defined variational problem. A systematic prescription for doing so is, however, not available at the moment beyond the linear order. Thus there are potential ambiguities in implementing (2.8).[5] Clearly these are important questions which should be explored more systematically. At the R^2 level, as we describe below in Sec. II C, all of our calculations can be reduced to the Gauss-Bonnet case in which these potential complications do not arise.

C. Field redefinitions in R^2 theories

We now show that to linear order in α_i, η/s for (1.2) is independent of α_1 and α_2. It is well known that to linear order in α_i, one can make a field redefinition to remove the R^2 and $R_{\mu\nu}R^{\mu\nu}$ term in (1.2). More explicitly, in (1.2) set $\alpha_3 = 0$ and take

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} + \alpha_2 L^2 \tilde{R}_{\mu\nu} - \frac{L^2}{3}(\alpha_2 + 2\alpha_1)\tilde{g}_{\mu\nu}\tilde{R}, \tag{2.9}$$

where $\tilde{R}$ denotes the Ricci scalar for $\tilde{g}_{\mu\nu}$ and so on. Then (1.2) becomes

$$\begin{aligned} I &= \frac{1}{16\pi G_N}\int\sqrt{-\tilde{g}}((1+\mathcal{K})\tilde{R} - 2\Lambda) + O(\alpha^2) \\ &= \frac{1+\mathcal{K}}{16\pi G_N}\int\sqrt{-\tilde{g}}(\tilde{R} - 2\tilde{\Lambda}) + O(\alpha^2) \end{aligned} \tag{2.10}$$

with

$$\mathcal{K} = \frac{2\Lambda L^2}{3}(5\alpha_1 + \alpha_2), \qquad \tilde{\Lambda} = \frac{\Lambda}{1+\mathcal{K}}. \tag{2.11}$$

It follows from (2.9) that a background solution $g^{(0)}$ to (1.2) (with $\alpha_3 = 0$) is related to a solution $\tilde{g}^{(0)}$ to (2.10) by

$$ds_0^2 = A^2 d\tilde{s}_0^2, \qquad A = 1 - \frac{\mathcal{K}}{3}. \tag{2.12}$$

The scaling in (2.12) does not change the background Hawking temperature. The diffusion pole (2.3) calculated using (2.10) around $\tilde{g}^{(0)}$ then gives the standard result $D = \frac{1}{4\pi T}$ [34]. Thus we conclude that $\eta/s = \frac{1}{4\pi}$ for (1.2) with $\alpha_3 = 0$. Then to linear order in α_i, η/s can only depend on α_3. To find this dependence, it is convenient to work with the Gauss-Bonnet theory (1.4). Gauss-Bonnet gravity is not only much simpler than (1.2) with generic $\alpha_3 \neq 0$, but also contains only second derivative terms in the equations of motion for $h_{\mu\nu}$, making the extraction of boundary correlators unambiguous.

III. SHEAR VISCOSITY FOR GAUSS-BONNET GRAVITY

In this section, after briefly reviewing the thermodynamic properties of the black brane solution, we compute the shear viscosity for Gauss-Bonnet gravity (1.4) nonperturbatively in $\lambda_{\rm GB}$. Here, we follow the outline presented in the previous section, with the Kubo formula (2.5) in mind. In Appendix B, we extract η/s from the shear channel (2.3) and the sound channel (2.4) (perturbatively in $\lambda_{\rm GB}$). There we also find that the sound velocity remains at the conformal value $c_s^2 = \frac{1}{3}$ as it should. In Appendix D, we provide a membrane paradigm calculation, again nonperturbatively in $\lambda_{\rm GB}$. All four methods give the same result.

A. Black brane geometry and thermodynamics

Exact solutions and thermodynamic properties of black objects in Gauss-Bonnet gravity (1.4) were discussed in [32] (see also [40–43]). Here we summarize some features relevant for our discussion below. The black brane solution can be written as

$$ds^2 = -f(r)N_\sharp^2 dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{L^2}\Big(\sum_{i=1}^{3} dx_i^2\Big), \tag{3.1}$$

where

[5] In [24], such additional terms do not appear to affect the calculation at the order under discussion there.

$$f(r) = \frac{r^2}{L^2}\frac{1}{2\lambda_{\rm GB}}\left[1 - \sqrt{1 - 4\lambda_{\rm GB}\left(1 - \frac{r_+^4}{r^4}\right)}\right]. \quad (3.2)$$

In (3.1), $N_\sharp$ is an arbitrary constant which specifies the speed of light of the boundary theory. Note that as $r \to \infty$,

$$f(r) \to \frac{r^2}{a^2 L^2}, \quad \text{with } a^2 \equiv \frac{1}{2}(1 + \sqrt{1 - 4\lambda_{\rm GB}}). \quad (3.3)$$

It is straightforward to see that the AdS curvature scale of these geometries is aL.[6] If we choose $N_\sharp = a$, then the boundary speed of light is unity. However, we will leave it unspecified in the following. We assume that $\lambda_{\rm GB} \leq \frac{1}{4}$. Beyond this point, (1.4) does not admit a vacuum AdS solution, and cannot have a boundary CFT dual. In passing, we note that while the curvature singularity occurs at $r = 0$ for $\lambda_{\rm GB} \geq 0$, it shifts to $r = r_+(1 - \frac{1}{4\lambda_{\rm GB}})^{-1/4}$ for $\lambda_{\rm GB} < 0$.

The horizon is located at $r = r_+$ and the Hawking temperature, entropy density, and energy density of the black brane are[7]

$$T = N_\sharp \frac{r_+}{\pi L^2}, \quad (3.4)$$

$$s = \frac{1}{4G_N}\left(\frac{r_+}{L}\right)^3 = \frac{(\pi L)^3}{4G_N}\frac{(T)^3}{N_\sharp^3}, \qquad \epsilon = \frac{3}{4}Ts. \quad (3.5)$$

If we fix the boundary theory temperature T and the speed of light to be unity (taking $N_\sharp = a$), the entropy and energy density are monotonically increasing functions of $\lambda_{\rm GB}$, reaching a maximum at $\lambda_{\rm GB} = \frac{1}{4}$ and going to zero as $\lambda_{\rm GB} \to -\infty$.

To make our discussion self-contained, in Appendix A, we compute the free energy of the black brane and derive the entropy density. In particular, we show that the contribution from the Gibbons-Hawking surface term to the free energy vanishes.

B. Action and equation of motion for the scalar channel

To compute the shear viscosity, we now study small metric fluctuations $\phi = h^1{}_2$ around the black brane background of the form

$$ds^2 = -f(r)N_\sharp^2 dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{L^2}\left(\sum_{i=1}^{3} dx_i^2 + 2\phi(t, \vec{x}, r)dx_1 dx_2\right). \quad (3.6)$$

We will take ϕ to be independent of x_1 and x_2 and write

$$\phi(t, \vec{x}, r) = \int \frac{d\omega dq}{(2\pi)^2}\phi(r; k)e^{-i\omega t + iqx_3}, \qquad k = (\omega, 0, 0, q), \qquad \phi(r; -k) = \phi^*(r; k). \quad (3.7)$$

For notational convenience, let us introduce

$$z = \frac{r}{r_+}, \qquad \tilde{\omega} = \frac{L^2}{r_+}\omega, \qquad \tilde{q} = \frac{L^2}{r_+}q, \qquad \tilde{f} = \frac{L^2}{r_+^2}f = \frac{z^2}{2\lambda_{\rm GB}}\left(1 - \sqrt{1 - 4\lambda_{\rm GB} + \frac{4\lambda_{\rm GB}}{z^4}}\right). \quad (3.8)$$

Then, at quadratic order, the action for ϕ can be written as

$$S = \int \frac{dk_1 dk_2}{(2\pi)^2}S(k_1, k_2) \quad \text{with}$$
$$S(k_1 = 0, k_2 = 0) = -\frac{1}{2}C\int dz \frac{d\omega dq}{(2\pi)^2} \times (K(\partial_z\phi)^2 - K_2\phi^2 + \partial_z(K_3\phi^2)), \quad (3.9)$$

where

$$C = \frac{1}{16\pi G_N}\left(\frac{N_\sharp r_+^4}{L^5}\right), \qquad K = z^2\tilde{f}(z - \lambda_{\rm GB}\partial_z\tilde{f}),$$
$$K_2 = K\frac{\tilde{\omega}^2}{N_\sharp^2\tilde{f}^2} - \tilde{q}^2 z(1 - \lambda_{\rm GB}\partial_z^2\tilde{f}), \quad (3.10)$$

and ϕ^2 should be understood as a shorthand notation for $\phi(z; k)\phi(z, -k)$. Here, S is the sum of the bulk action (1.4) and the associated Gibbons-Hawking surface term [33]. The explicit expression for K_3 will not be important for our subsequent discussion.

The equation of motion following from (3.9) is[8]

$$K\phi'' + K'\phi' + K_2\phi = 0, \quad (3.11)$$

where primes indicate partial derivatives with respect to z. Using the equation of motion, the action (3.9) reduces to the surface contributions as advertised in Sec. II B,

$$S(k_1 = 0, k_2 = 0) = -\frac{1}{2}C\int \frac{d\omega dq}{(2\pi)^2}(K\phi'\phi + K_3\phi^2)|_{\rm surface}. \quad (3.12)$$

The prescription described in Sec. II B instructs us to pick up the contribution from the boundary at $z \to \infty$. Here, the term proportional to K_3 will give rise to a real divergent contact term, which is discarded.

[6]Here we note that the Gauss-Bonnet theory also admits another background with the curvature scale $\tilde{a}L$ where $\tilde{a}^2 = \frac{1}{2} \times (1 - \sqrt{1 - 4\lambda_{\rm GB}})$. Even though this remains an asymptotically AdS solution for $\lambda_{\rm GB} > 0$, we do not consider it here because this background is unstable and contains ghosts [31].

[7]Note that for *planar* black branes in Gauss-Bonnet theory, the area law for entropy still holds [44]. This is not the case for more general higher-derivative-corrected black objects.

[8]An easy way to get the quadratic action (3.9) is to first obtain the linearized equation of motion and then read off K and K_2 from it.

A curious thing about (3.9) is that for all values of z, both K and K_2 (but not K_3) are proportional to $\frac{1}{4} - \lambda_{\rm GB}$.[9] Thus other than the boundary term the whole action (3.9) vanishes identically at $\lambda_{\rm GB} = \frac{1}{4}$. Nevertheless, the equation of motion (3.11) remains nontrivial in the limit $\lambda_{\rm GB} \to \frac{1}{4}$ as the $\frac{1}{4} - \lambda_{\rm GB}$ factor cancels out. Note that the correlation function does not necessarily go to zero in this limit since it also depends on the behavior of the solution to (3.11) and the limiting procedure (3.12). As we will see momentarily, at least in the small frequency limit it does become zero with a vanishing shear viscosity.

C. Low-frequency expansion and the viscosity

General solutions to the equation of motion (3.11) can be written as

$$\phi(z;k) = a_{\rm in}(k)\phi_{\rm in}(z;k) + a_{\rm out}(k)\phi_{\rm out}(z;k), \tag{3.14}$$

where $\phi_{\rm in}$ and $\phi_{\rm out}$ satisfy infalling and outgoing boundary conditions at the horizon, respectively. They are complex conjugates of each other, and we normalize them by requiring them to approach 1 as $z \to \infty$. Then, the prescription of Sec. II B corresponds to setting

$$a_{\rm in}(k) = J(k), \qquad a_{\rm out}(k) = 0, \tag{3.15}$$

where $J(k)$ is an infinitesimal boundary source for the bulk field ϕ.

More explicitly, as $z \to 1$, various functions in (3.11) have the following behavior

$$\begin{aligned} \frac{K_2}{K} &\approx \frac{\tilde{\omega}^2}{16N_\sharp^2(z-1)^2} + O((z-1)^{-1}) + O(\tilde{q}^2), \\ \frac{K'}{K} &= \frac{1}{z-1} + O(1). \end{aligned} \tag{3.16}$$

It follows that near the horizon $z = 1$, Eq. (3.11) can be solved by (for $\vec{q} = 0$)

$$\phi(z) \sim (z-1)^{\pm i\tilde{\omega}/4N_\sharp} \sim (z-1)^{\pm i\omega/4\pi T} \tag{3.17}$$

with the infalling boundary condition corresponding to the negative sign. To solve (3.11) in the small frequency limit, it is convenient to write

$$\begin{aligned} \phi_{\rm in}(z;k) = {} & e^{-i(\tilde{\omega}/4N_\sharp)\ln(a^2\tilde{f}/z^2)} \\ & \times \left(1 - i\frac{\tilde{\omega}}{4N_\sharp} g_1(z) + O(\tilde{\omega}^2, \tilde{q}^2)\right), \end{aligned} \tag{3.18}$$

where we require $g_1(z)$ to be nonsingular at the horizon $z = 1$. We show in Appendix C that g_1 is a nonsingular function with the large z expansion

$$g_1(z) = \frac{4\lambda_{\rm GB}}{\sqrt{1-4\lambda_{\rm GB}}}\frac{a^2}{z^4} + O(z^{-8}). \tag{3.19}$$

Therefore, with our boundary conditions (3.15), we find

$$\begin{aligned} \phi(z;k) = J(k)\Bigg[& 1 + \frac{i\tilde{\omega}}{4N_\sharp} a^2\sqrt{1-4\lambda_{\rm GB}}\left(\frac{1}{z^4} + O(z^{-8})\right) \\ & + O(\tilde{\omega}^2, \tilde{q}^2)\Bigg]. \end{aligned} \tag{3.20}$$

This is the right asymptotic behavior for the bulk field ϕ describing metric fluctuations since the CFT stress tensor has conformal dimension 4.

Plugging (3.20) into (3.12) and using the expressions for C and K in (3.10), the prescription described in Sec. II B gives

$$\text{Im } G_{12,12}(\omega, 0) = \omega\frac{1}{16\pi G_N}\left(\frac{r_+^3}{L^3}\right)(1 - 4\lambda_{\rm GB}) + O(\omega^2). \tag{3.21}$$

Then, the Kubo formula (2.5) yields

$$\eta = \frac{1}{16\pi G_N}\left(\frac{r_+^3}{L^3}\right)(1 - 4\lambda_{\rm GB}). \tag{3.22}$$

Finally, taking the ratio of (3.5) and (3.22) we find that

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 - 4\lambda_{\rm GB}). \tag{3.23}$$

This is *nonperturbative* in $\lambda_{\rm GB}$. Especially, the linear correction is the only nonvanishing term.[10]

We now conclude this section with various remarks:

(1) Based on the field redefinition argument presented in Sec. II C, one finds from (3.23) that for (1.2),

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 - 8\alpha_3) + O(\alpha_i^2). \tag{3.24}$$

We have also performed an independent calculation of η/s (without using field redefinitions) for (1.2) using all three methods outlined in Sec. II A and confirmed (3.24).

(2) The ratio η/s dips below the viscosity bound for $\lambda_{\rm GB} > 0$ in Gauss-Bonnet gravity and for $\alpha_3 > 0$ in (1.2). In particular, the shear viscosity approaches zero as $\lambda_{\rm GB} \to \frac{1}{4}$ for Gauss-Bonnet. Note that the whole off-shell action becomes zero in this limit. It is likely the on-shell action also vanishes, implying that the correlation function could become identically zero in this limit.

(3) Fixing the temperature T and the boundary speed of light to be unity, as we take $\lambda_{\rm GB} \to -\infty$, $\eta \sim (-\lambda_{\rm GB})^{1/4} \to \infty$. In contrast the entropy density decreases as $s \sim (-\lambda_{\rm GB})^{-3/4} \to 0$.

[9] This can be seen by using the following equation in K and K_2

$$\tilde{f}'(z) = \frac{2z(2z^2 - \tilde{f})}{z^2 - 2\lambda_{\rm GB}\tilde{f}}. \tag{3.13}$$

[10] It would be interesting to find an explanation for vanishing of higher order corrections.

(4) The shear viscosity of the boundary conformal field theory is associated with absorption of transverse modes by the black brane in the bulk. This is a natural picture since the shear viscosity measures the dissipation rate of those fluctuations: the quicker the black brane absorbs them, the higher the dissipation rate will be. For example, as $\lambda_{\rm GB} \to -\infty$, η/s approaches infinity; this describes a situation where every bit of the black brane horizon devours the transverse fluctuations very quickly. In this limit the curvature singularity at $z = (1 - \frac{1}{4\lambda_{\rm GB}})^{-1/4}$ approaches the horizon and the tidal force near the horizon becomes strong. On the other hand, as $\lambda_{\rm GB} \to \frac{1}{4}$, $\eta/s \to 0$ and the black brane very slowly absorbs transverse modes.[11]

(5) The calculation leading to (3.23) can be generalized to general D spacetime dimensions and one finds for $D \geq 4 + 1$[12]

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - 2\frac{(D-1)}{(D-3)}\lambda_{\rm GB}\right]. \tag{3.26}$$

Here again $\lambda_{\rm GB}$ is bounded above by $\frac{1}{4}$. Thus for $D > 4 + 1$, η never approaches zero within Gauss-Bonnet theory. For $D = 3 + 1$ or $2 + 1$, in which case the Gauss-Bonnet term is topological, there is no correction to η/s.

(6) In Appendix D, we obtain the same result (3.23) using the membrane paradigm [6]. Thus when embedded into the AdS/CFT correspondence, the membrane paradigm correctly captures the infrared (hydrodynamic) sector of the boundary thermal field theory. Further, we see something interesting in its derivation. There, the diffusion constant is expressed as the product of a factor evaluated at the horizon (D22) and an integral from the horizon to infinity (D23). In the limit $\lambda_{\rm GB} \to \frac{1}{4}$, it is the former that approaches zero.

IV. CAUSALITY IN BULK AND ON BOUNDARY

In this section we investigate if there are causality problems in the bound-violating theories discussed above. First we will discuss the bulk causal structure. Then we discuss a curious high-momentum metastable state in the bulk graviton wave equation that may have consequences for boundary causality. The analysis in this section is refined in [45] where we indeed see a precise signal of causality violation for $\lambda_{\rm GB} > \frac{9}{100}$.

A. Graviton cone tipping

As a consequence of higher derivative terms in the gravity action, graviton wave packets in general do not propagate on the light cone of a given background geometry. For example, when $\lambda_{\rm GB} \neq 0$, Eq. (3.11) for the propagation of a transverse graviton differs from that of a minimally coupled massless scalar field propagating in the same background geometry (3.1). To make the discussion precise, let us write (we will consider only $x_{1,2}$-independent waves)

$$\phi(t, r, x_3) = e^{-i\omega t + ik_r r + iqx_3}\phi_{\rm en}(t, r, x_3). \tag{4.1}$$

Here, $\phi_{\rm en}$ is a slowly-varying envelope function, and we take the limit $k = (\omega, k_r, 0, 0, q) \to \infty$. In this limit, the equation of motion (3.11) reduces to

$$k^\mu k^\nu g^{\rm eff}_{\mu\nu} \approx 0, \tag{4.2}$$

where

$$\begin{aligned} ds^2_{\rm eff} &= g^{\rm eff}_{\mu\nu}dx^\mu dx^\nu \\ &= f(r)N_\sharp^2\left(-dt^2 + \frac{1}{c_g^2}dx_3^2\right) + \frac{1}{f(r)}dr^2. \end{aligned} \tag{4.3}$$

In (4.3)

$$c_g^2(z) = \frac{N_\sharp^2\tilde{f}(z)}{z^2}\frac{1 - \lambda_{\rm GB}\tilde{f}''}{1 - \frac{\lambda_{\rm GB}\tilde{f}'}{z}} \equiv c_b^2\frac{1 - \lambda_{\rm GB}\tilde{f}''}{1 - \frac{\lambda_{\rm GB}\tilde{f}'}{z}} \tag{4.4}$$

can be interpreted as the local "speed of graviton" on a constant r-hypersurface. $c_b^2 \equiv \frac{N_\sharp^2\tilde{f}(z)}{z^2}$ introduced in the second equality in (4.4) is the local speed of light as defined by the background metric (3.1). Thus the graviton cone in general does not coincide with the standard null cone or light cone defined by the background metric.[13] A few more

[11] We note that for $\lambda_{\rm GB} = \frac{1}{4}$ in $4 + 1$ spacetime dimension, the radial direction of the background geometry resembles a Bañados-Teitelboim-Zanelli (BTZ) black brane.

[12] For general dimensions we use the convention

$$\begin{aligned} S = \frac{1}{16\pi G_N}\int d^Dx\sqrt{-g} \\ \times [R - 2\Lambda + \alpha_{\rm GB}L^2(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})] \end{aligned} \tag{3.25}$$

with $\Lambda = -\frac{(D-1)(D-2)}{2L^2}$ and $\lambda_{\rm GB} = (D-3)(D-4)\alpha_{\rm GB}$.

[13] Note that

$$\frac{c_g^2}{c_b^2} = \frac{1 - \lambda_{\rm GB}\tilde{f}''}{1 - \frac{\lambda_{\rm GB}\tilde{f}'}{z}} = \frac{1 - 4\lambda_{\rm GB} + 12\frac{\lambda_{\rm GB}}{z^4}}{1 - 4\lambda_{\rm GB} + 4\frac{\lambda_{\rm GB}}{z^4}}, \tag{4.5}$$

and, in particular, the ratio is greater than 1 for $\lambda_{\rm GB} > 0$. Note that bulk causality and the existence of a well-posed Cauchy problem do not crucially depend on reference metric light cones and such tipping is not a definitive sign of causality problems. Also for any value of $\lambda_{\rm GB}$, the graviton cone coincides with the light cone in the radial direction. If not, we could have argued for the violation of the second law of thermodynamics following [46,47]. Further note that for $\lambda_{\rm GB} < -\frac{1}{8}$, there exists a region outside the horizon where $c_g^2 < 0$ which will lead to the appearance of tachyonic modes, following [48]. We have not explored the full significance of this instability here since it is not correlated with the viscosity bound.

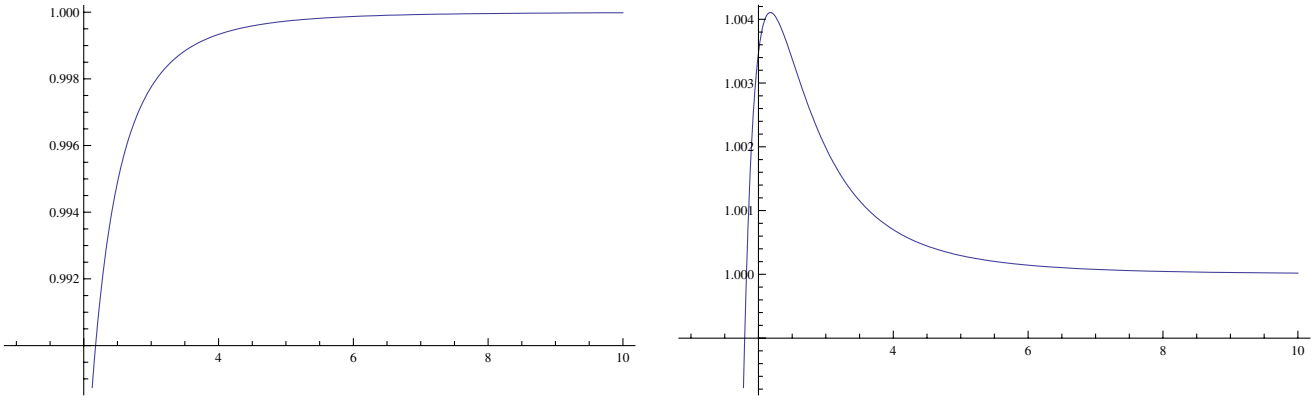

FIG. 1 (color online). $c_g^2(z)$ (vertical axis) as a function of z (horizontal axis) for $\lambda_{\rm GB} = 0.08$ (left panel) and $\lambda_{\rm GB} = 0.1$ (right panel). For $\lambda_{\rm GB} < \frac{9}{100}$, c_g^2 is a monotonically increasing function of z. When $\lambda_{\rm GB} > \frac{9}{100}$, as one decreases z from infinity, c_g^2 increases from 1 to a maximum value at some $z > 1$ and then decreases to 0 as $z \to 1$ (horizon).

comments about the graviton cone are found at the end of Appendix D.

In the nongravitational boundary theory there is an invariant notion of light cone and causality. At a heuristic level, a graviton wave packet moving at speed $c_g(z)$ in the bulk should translate into disturbances of the stress tensor propagating with the same velocity in the boundary theory. It is thus instructive to compare c_g and c_b with the boundary speed of light, which we now set to unity by taking $N_\sharp = a$ (a was defined in (3.3)). At the boundary ($z = \infty$) one finds that $c_g(z) = c_b(z) = 1$. In the bulk, the background local speed of light c_b is always smaller than 1, which is related to the redshift of the black hole geometry. The local speed of graviton $c_g(z)$, however, can be greater than 1 for a certain range of z if $\lambda_{\rm GB}$ is sufficiently large. To see this, we can examine the behavior of c_g^2 near $z = \infty$,

$$c_g^2(z) - 1 = \frac{b_1}{z^4} + O(z^{-8}), \qquad z \to \infty,$$
$$b_1(\lambda_{\rm GB}) = -\frac{1 + \sqrt{1 - 4\lambda_{\rm GB}} - 20\lambda_{\rm GB}}{2(1 - 4\lambda_{\rm GB})}. \tag{4.6}$$

$b_1(\lambda_{\rm GB})$ becomes positive and thus c_g^2 increases above 1 if $\lambda_{\rm GB} > \frac{9}{100}$. For such a $\lambda_{\rm GB}$, as we decrease z from infinity, c_g^2 will increase from 1 to a maximum at some value of z and then decrease to zero at the horizon. See Fig. 1 for the plot of $c_g^2(z)$ as a function of z for two values of $\lambda_{\rm GB}$. When $\lambda_{\rm GB} = \frac{9}{100}$ one finds that the next order term in (4.6) is negative and thus c_g^2 does not go above 1. Also note that $\lambda_{\rm GB} \to \frac{1}{4}$, $b_1(\lambda_{\rm GB})$ goes to plus infinity.[14] Thus heuristically, in the boundary theory there is a potential for superluminal propagation of disturbances of the stress tensor.

In [45] we explore whether such bulk graviton cone behavior can lead to boundary causality violation by studying the behavior of graviton null geodesics in the effective geometry. There, we indeed see causality violation for $\lambda_{\rm GB} > \frac{9}{100}$.

B. New metastable states at high momenta ($\lambda_{\rm GB} > \frac{9}{100}$)

We now study the behavior of the full graviton wave equation. Let us recast Eq. (3.11) in Schrödinger form. For this purpose, we introduce

$$\frac{dy}{dz} = \frac{1}{N_\sharp \tilde{f}(z)}, \qquad \psi = B\phi, \qquad B = \sqrt{\frac{K}{\tilde{f}}}. \tag{4.7}$$

Then (3.11) becomes

$$-\partial_y^2\psi + V(y)\psi = \tilde{\omega}^2\psi \tag{4.8}$$

with

$$V(y) = \tilde{q}^2 c_g^2(z) + V_1,$$
$$V_1(y) = \frac{\partial_y^2 B}{B} = \frac{N_\sharp^2 \tilde{f}^2}{B}\left(B'' + \frac{\tilde{f}'}{\tilde{f}}B'\right), \tag{4.9}$$

where $c_g^2(z)$ was defined in (4.4). The advantage of using (4.8) is that qualitative features of the full graviton propagation (including the radial direction) can be inferred from the potential $V(y)$, since we have intuition for solutions of the Schrödinger equation. Since y is a monotonic function of z, below we will use the two coordinates interchangeably in describing the qualitative behavior of $V(y)$.

One can check that $V_1(z)$ is a monotonically increasing function for any $\lambda_{\rm GB} > 0$ (note $V_1(z) \to +\infty$ as $z \to \infty$). For $\lambda_{\rm GB} \le \frac{9}{100}$, $c_g^2(z)$ is also a monotonically increasing function as we discussed in the last subsection and the whole $V(z)$ is monotonic. When $\lambda_{\rm GB} > \frac{9}{100}$, there exists a range of z where $c_g^2(z)$ decreases with increasing z for sufficiently large z. Thus $V(z)$ can now have a local minimum for sufficiently large $\tilde{q}$. For illustration, see Fig. 2 for the plot of $V(z)$ as a function z for two values of $\lambda_{\rm GB}$.

[14] In fact coefficients of all higher order terms in $1/z$ expansion become divergent in this limit.

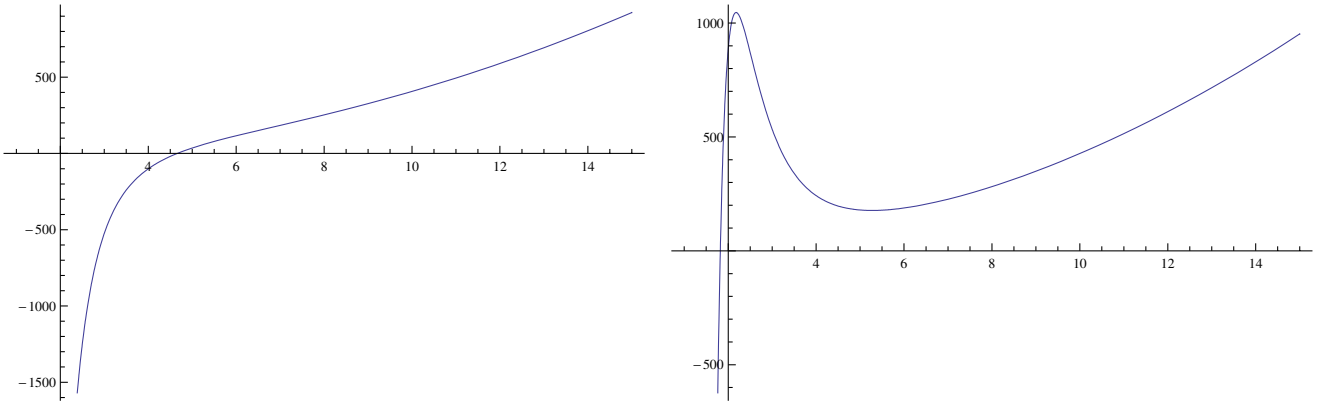

FIG. 2 (color online). $V(z) - q^2$ (vertical axis) as a function of z (horizontal axis) for $\lambda_{\rm GB} = 0.08$ and $\tilde{q} = 500$ (left panel) and for $\lambda_{\rm GB} = 0.1$ and $\tilde{q} = 500$ (right panel). $V(z)$ is a monotonically increasing function of z for $\lambda_{\rm GB} \leq \frac{9}{100}$, but develops a local minimum for $\lambda_{\rm GB} > \frac{9}{100}$ with large enough $\tilde{q}$.

Generically, a graviton wave packet will fall into the black brane very quickly, within the time scale of the inverse temperature $\frac{1}{T}$ (since this is the only scale in the boundary theory). Here, however, precisely when the local speed of graviton c_g can exceed 1 (i.e. for $\lambda_{\rm GB} > \frac{9}{100}$), $V(z)$ develops a local minimum for large enough $\tilde{q}$ and the Schrödinger equation (4.8) can have metastable states living around the minimum. Their lifetime is determined by the tunneling rate through the barrier which separates the minimum from the horizon. For very large $\tilde{q}$ this barrier becomes very high and an associated metastable state has lifetime parametrically larger than the time scale set by the temperature. In the boundary theory, these metastable states translate into poles of the retarded Green function for T_{xy} in the lower half-plane. The imaginary part of such a pole is given by the tunneling rate of the corresponding metastable state. Thus for $\lambda_{\rm GB} > \frac{9}{100}$, in boundary theory we find new quasiparticles at high momenta with a small imaginary part.[15]

In [45], we confirm that those long-lived quasiparticles give rise to causality violation for $\lambda_{\rm GB} > \frac{9}{100}$.

V. DISCUSSION

In this paper we have computed η/s for Gauss-Bonnet gravity using a variety of techniques. We have found that the viscosity bound is violated for $\lambda_{\rm GB} > 0$ and have looked for pathologies correlated to this violation. For small positive $\lambda_{\rm GB}$ we have not found any. The violation of the bound becomes extreme as $\lambda_{\rm GB} \to \frac{1}{4}$ where η vanishes. We have focused our attention on this region to find what unusual properties of the boundary theory could yield a violation not only of the bound but also of the qualitative intuitions suggesting a lower bound on η/s. Above we also have discussed a novel quasiparticle excitation. In [45], causality violation is firmly established for $\lambda_{\rm GB} > \frac{9}{100}$.

It is also instructive to examine the behavior of the zero temperature theory as $\lambda_{\rm GB} \to \frac{1}{4}$. Basic parameters describing the boundary CFT are the coefficients of the 4D Euler and Weyl densities called a and c, respectively. These have been computed first in [50], and for Gauss-Bonnet gravity in [51]. Their results indicate that

$$c \sim (1 - 4\lambda_{\rm GB})^{1/2}, \tag{5.1}$$

$$a \sim (3(1 - 4\lambda_{\rm GB})^{1/2} - 2). \tag{5.2}$$

The parameter c is related to the two-point function of a boundary stress tensor which is forced by unitarity to be positive. (5.1) shows that c vanishes at $\lambda_{\rm GB} = \frac{1}{4}$ demonstrating the sickness of this point.[16] For $\lambda_{\rm GB}$ a bit less than $\frac{1}{4}$ the stress tensor couples very weakly in a system with a large number of degrees of freedom. This is peculiar indeed. In the bulk it seems that gravity is becoming strongly coupled there.

The coefficient a vanishes at $\lambda_{\rm GB} = \frac{5}{36}$. The significance of this is unclear.

More generally, we believe it would be valuable to explore how generic higher derivative corrections modify various gauge theory observables. This is important not only for seeing how reliable it is to use the infinite 't Hooft coupling approximation for questions relevant to QCD, but also for achieving a more balanced conceptual picture of the strong coupling dynamics. Furthermore, this may generate new effective tools for separating the swampland from the landscape.

As a cautionary note we should mention that pathologies in the boundary theory in regions that violate the viscosity bound may not be visible in gravitational correlators, at least when $g_s = 0$. As an example consider the $\alpha'^3 R^4$

[15] A similar type of long-lived quasiparticles exist for $\mathcal{N} = 4$ SYM theory on S^3 [49], but not on $\mathbf{R}^3$.

[16] This can also be seen from the derivations in Sec. III.

terms discussed in [24]. For positive α', the physical case, the viscosity bound is preserved. But the bulk effective action can equally be studied for α' negative. Here gravitational correlators can be computed and will violate the viscosity bound. The only indication of trouble in the boundary theory at $g_s = 0$ will come from correlators of string scale massive states, whose mass and CFT conformal weight $\sim 1/(\alpha')^{1/2}$, an imaginary number.

ACKNOWLEDGMENTS

We thank A. Adams, N. Arkani-Hamed, R-G. Cai, A. Dymarsky, Q. Ejaz, T. Faulkner, H. Jockers, P. Kovtun, J. Liu, D. Mateos, H. Meyer, K. Rajagopal, D. T. Son, A. Starinets, L. Susskind, and B. Zwiebach for discussions. H. L. also wishes to thank J. Liu for collaboration at the initial stages of the work. We would also like to thank Yevgeny Katz and Pavel Petrov for sharing a draft of their work [27]. M. B. and H. L. are partly supported by the U.S. Department of Energy (DOE) under cooperative research agreement No. DE-FG02-05ER41360. H. L. is also supported in part by the A. P. Sloan Foundation and the U.S. Department of Energy (DOE) OJI program. H. L. is also supported in part by the Project of Knowledge Innovation Program (PKIP) of Chinese Academy of Sciences. H. L. would like to thank KITPC (Beijing) for hospitality during the last stage of this project. Research at the Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Research & Innovation. R. C. M. also acknowledges support from an NSERC Discovery grant and from the Canadian Institute for Advanced Research. S. S. is supported by NSF Grant No. 9870115 and the Stanford Institute for Theoretical Physics. S. Y. is supported by the Albion Walter Hewlett Stanford Graduate program and the Stanford Institute for Theoretical Physics.

APPENDIX A: THERMODYNAMIC PROPERTIES OF GB BLACK HOLES

1. Free energy

It is easy to confirm that the following metric is a stationary point of the Gauss-Bonnet action (1.4)

$$ds^2 = -f(r)N_\sharp^2 dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{L^2}\left(\sum_{i=1}^{3} dx_i^2\right) \quad \text{with}$$

$$f(r) = \frac{r^2}{L^2}\frac{1}{2\lambda_{\rm GB}}\left(1 - \sqrt{1 - 4\lambda_{\rm GB} + 4\lambda_{\rm GB}\left(\frac{r_+}{r}\right)^4}\right). \tag{A1}$$

First note that the Hawking temperature is

$$T(r_+) = \frac{1}{2\pi}\left[\frac{1}{\sqrt{g_{rr}}}\frac{d}{dr}\sqrt{g_{tt}}\right]\Bigg|_{r=r_+} = N_\sharp \frac{1}{\pi}\frac{r_+}{L^2}. \tag{A2}$$

To get the free energy $F[T]$ of the macroscopic configuration (A1), we note the following correspondence in the classical limit:

$$e^{-(1/T)F[T]} = Z[T] = e^{-I[T]}. \tag{A3}$$

Here, $I[T]$ is the Euclidean action of the configuration with temperature T. Evaluating the Euclideanized bulk action for Gauss-Bonnet gravity (1.4) with the background metric (A1), we find

$$\begin{aligned} I_{\rm bulk}[T(r_+)] &= -\frac{1}{16\pi G_N}\int_{r_+}^{r_{\rm max}} dr \int_0^{1/T} dt_E \int d^3x_i \sqrt{g_E} \\ &\quad \times [R - 2\Lambda + \frac{\lambda_{\rm GB}}{2}L^2(R^2 - 4R_{\mu\nu}R^{\mu\nu} \\ &\quad + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})] \\ &= \frac{1}{16\pi G_N} V_3 \frac{N_\sharp}{T}\frac{r_+^4}{L^5}\frac{1}{\lambda_{\rm GB}} \\ &\quad \times \left[\frac{r_{\rm max}^4}{r_+^4}(12\lambda_{\rm GB} - 5 + 5\sqrt{1 - 4\lambda_{\rm GB}}) \right. \\ &\quad \left. - 4\lambda_{\rm GB} + \frac{2\lambda_{\rm GB}}{\sqrt{1 - 4\lambda_{\rm GB}}}\right]. \end{aligned} \tag{A4}$$

We regulate this result by subtracting the Euclidean action of the $\lambda_{\rm GB}$-modified pure AdS space (obtained by setting $r_+ = 0$ in (A1))

$$\begin{aligned} I_{\rm bulk}^{\rm pure}[T'(T(r_+))] &= \frac{1}{16\pi G_N} V_3 \frac{N_\sharp}{T'}\frac{r_+^4}{L^5}\frac{1}{\lambda_{\rm GB}} \\ &\quad \times \left[\frac{r_{\rm max}^4}{r_+^4}(12\lambda_{\rm GB} - 5 + 5\sqrt{1 - 4\lambda_{\rm GB}})\right] \end{aligned} \tag{A5}$$

with $T'(T)$ chosen so that the geometries at $r = r_{\rm max}$ agree [4]. Quantitatively,

$$\begin{aligned} &\frac{1}{T'}\sqrt{\frac{r_{\rm max}^2}{L^2}\frac{1}{2\lambda_{\rm GB}}(1 - \sqrt{1 - 4\lambda_{\rm GB}})} \\ &\quad = \frac{1}{T}\sqrt{\frac{r_{\rm max}^2}{L^2}\frac{1}{2\lambda_{\rm GB}}\left(1 - \sqrt{1 - 4\lambda_{\rm GB} + 4\lambda_{\rm GB}\frac{r_+^4}{r_{\rm max}^4}}\right)}. \end{aligned} \tag{A6}$$

All in all, we get

$$\begin{aligned} F[T] &= T(I_{\rm bulk}[T] - I_{\rm bulk}^{\rm pure}[T'(T)]) \\ &= -\frac{1}{4G_N}V_3(\pi L T)^3\left(\frac{T}{4}\right)\frac{1}{N_\sharp^3}. \end{aligned} \tag{A7}$$

The entropy density is then given by

$$\begin{aligned} s[T] &= \frac{1}{V_3}\left(-\frac{d}{dT}F[T]\right) = \frac{1}{4G_N}(\pi L T)^3\frac{1}{N_\sharp^3} \\ &= \frac{1}{4G_N}\left(\frac{r_+}{L}\right)^3. \end{aligned} \tag{A8}$$

2. Vanishing of Gibbons-Hawking contribution

To be complete, we need to show that there is no contribution to the free energy from the Gibbons-Hawking surface term when we regulate with the background subtraction method presented above. This can be shown explicitly. For the black brane solution, the Gibbons-Hawking contribution is[17]:

$$I_{\rm GH}[T(r_+)] = -\frac{1}{16\pi G_N} V_3 \frac{N_\sharp}{T}\left(\frac{r}{L}\right)^3\left[6(\partial_r f)\left(\frac{f}{r}\right) - 6\lambda_{\rm GB}\left\{3L^2\left(\frac{\partial_r f}{r}\right) + 2L^2\left(\frac{f}{r^2}\right)\right\}\left(\frac{f}{r}\right)\right]\Bigg|_{r=r_{\rm max}}$$
$$= -\frac{1}{4\pi G_N} V_3 \frac{N_\sharp}{T}\frac{r_+^4}{L^5}(-2+3\sqrt{1-4\lambda_{\rm GB}})\left[\frac{r_{\rm max}^4}{r_+^4}\left(\frac{1-\sqrt{1-4\lambda_{\rm GB}}}{\lambda_{\rm GB}}\right) - \frac{1}{\sqrt{1-4\lambda_{\rm GB}}}\right]. \quad \text{(A9)}$$

A similar expression is obtained for pure AdS space. With the choice (A6), we obtain

$$I_{\rm GH}[T] - I_{\rm GH}^{\rm pure}[T'(T)] = 0. \quad \text{(A10)}$$

APPENDIX B: η/s FROM SHEAR AND SOUND CHANNEL POLES

Our calculation in this appendix follows the techniques developed in [36].

Consider a perturbation of the background metric of the form $h_{\mu\nu} = h_{\mu\nu}(r)e^{-i\omega t+iqx_3}$, with $\mu, \nu = t, r, x_1, x_2, x_3$. We can label various kinds of perturbations according to their transformations under the symmetry group of rotations in the 1-2 plane. There are three types of decoupled excitations corresponding to spin 2 (scalar channel), spin 1 (shear channel), and spin 0 (sound channel).

1. Shear channel

The shear channel excitations involve $h_{t\alpha}$, $h_{r\alpha}$, and $h_{3\alpha}$ with $\alpha = 1, 2$. Choosing the radial gauge $h_{\mu r} = 0$, the shear channel equations can be reduced to a single equation for $Z(r) = qg^{11}h_{t1} + \omega g^{11}h_{31}$. At first order in $\lambda_{\rm GB}$, $Z(r)$ satisfies the equation (below we use the notations introduced in the main text, see (3.8))

$$0 = Z''(z) + \frac{Z'(z)}{z}\left(\frac{5z^4-1}{z^4-1} + \frac{4\tilde{q}^2}{\tilde{q}^2(-z^4+1)+z^4\frac{\tilde{\omega}^2}{N_\sharp^2}}\right)$$
$$+ Z(z)\left(\frac{\tilde{q}^2(-z^4+1)+z^4\frac{\tilde{\omega}^2}{N_\sharp^2}}{(z^4-1)^2}\right)$$
$$+ \frac{\lambda_{\rm GB}}{2}\left[Z'(z)\left(-\frac{8(2\tilde{q}^4(z^4-1)^2+4\tilde{q}^2z^4\frac{\tilde{\omega}^2}{N_\sharp^2}-3z^8\frac{\tilde{\omega}^4}{N_\sharp^4})}{z^5(\tilde{q}^2(z^4-1)-z^4\frac{\tilde{\omega}^2}{N_\sharp^2})^2}\right)\right.$$
$$\left. + Z(z)\left(2\frac{\tilde{q}^2(z^4+3)-2z^4\frac{\tilde{\omega}^2}{N_\sharp^2}}{z^4(z^4-1)}\right)\right]. \quad \text{(B1)}$$

Following a similar analysis to that at the beginning of Sec. III C, we find that the solution to (B1) which satisfies an infalling boundary condition at the horizon $z = 1$ can be written as

$$Z(z) = \left(1-\frac{1}{z^4}\right)^{-i(\tilde{\omega}/4N_\sharp)} g(z), \quad \text{(B2)}$$

where g is regular at $z = 1$. In order to find the hydrodynamical poles, it is enough to find $g(z)$ for small values of $\tilde{\omega}$ and $\tilde{q}$, which we will assume are of the same order. For this purpose, we introduce a scaled quantity $W = \frac{\tilde{\omega}}{\tilde{q}N_\sharp}$ and expand $g(z)$ as a power series of $\tilde{q}$. The solution can be readily found to be

$$g(z) = 1 + \frac{i\tilde{q}}{4W}\left(1-\frac{1}{z^4}\right)\left[1+\lambda_{\rm GB}\left(3(W^2-1)-\frac{1}{z^4}\right)\right]$$
$$+ O(\tilde{q}^2, \lambda_{\rm GB}^2). \quad \text{(B3)}$$

We thus find near infinity $Z(z)$ can be expanded in $1/z$ as

$$Z(z) \approx \mathcal{A} + \mathcal{B}z^{-4} + O(z^{-8}), \qquad z \to \infty, \quad \text{(B4)}$$

where

$$\mathcal{A} = 1 + \frac{i\tilde{q}}{4W} + 3i\frac{\tilde{q}}{4W}\lambda_{\rm GB}(W^2-1) + O(\tilde{q}^2)$$
$$= 1 + \frac{iN_\sharp^2}{4\pi T}(1-3\lambda_{\rm GB})\frac{q^2}{\omega} + \frac{3i\lambda_{\rm GB}\omega}{4\pi T} + \cdots, \quad \text{(B5)}$$

$$\mathcal{B} = -\frac{i\tilde{q}}{4W} + i\frac{W\tilde{q}}{4} + i\frac{\lambda_{\rm GB}\tilde{q}}{W}\left(\frac{1}{2} - 3\frac{W^2}{4}\right) + O(\tilde{q}^2)$$
$$= \frac{i}{4\pi T}\frac{1-3\lambda_{\rm GB}}{\omega}\left(\omega^2 - \frac{N_\sharp^2}{1-\lambda_{\rm GB}}q^2\right) + \cdots. \quad \text{(B6)}$$

Carrying out the procedure (2.6), (2.7), and (2.8) one finds that

$$G_R(k) \propto \frac{\mathcal{B}}{\mathcal{A}}. \quad \text{(B7)}$$

In particular one can show that the poles of $G_R(k)$ solely arise from zeros of $\mathcal{A}$.

[17] A quick way to get the first equality is to consider the action of the most general static planar symmetric metrics, vary it, and focus on the terms involving second derivatives. Note that with this approach, we have also accounted here for the possible contribution of the higher derivative terms in the generalized Gibbons-Hawking term [33].

The Dirichlet boundary condition corresponding to $\mathcal{A} = 0$ determines the hydrodynamical pole as[18]

$$\omega = -iDq^2 + O(q^3), \qquad D = \frac{N_\sharp^2}{4\pi T}(1 - 3\lambda_{\rm GB}). \quad \text{(B8)}$$

Note that in the relation (2.3) between the diffusion constant D and η/s, the boundary speed of light c has been set to unity (otherwise the right-hand side should be multiplied by c^2). Choosing $N_\sharp^2 = a^2 \approx 1 - \lambda_{\rm GB}$ (see Eq. (3.3)) so that the boundary speed of light is unity, we find that

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 - 4\lambda_{\rm GB}) + O(\lambda_{\rm GB}^2). \quad \text{(B9)}$$

2. Sound channel

The sound channel excitations involve h_{tt}, h_{t3}, h_{33}, $h_{11} + h_{22}$, h_{rr}, h_{tr}, h_{r3}. Choosing the radial gauge $h_{\mu r} = 0$, the sound channel equations can be reduced to a single equation for the variable

$$Z_s(r) = \frac{4q}{\omega} g^{33} h_{t3} + 2g^{33} h_{33} - (g^{22} h_{22} + g^{11} h_{11}) \times \left(1 - \frac{q^2}{\omega^2}\frac{\partial_r g_{tt}}{\partial_r g_{11}}\right) + 2\frac{q^2}{\omega^2}\frac{h_{tt}}{g_{11}}. \quad \text{(B10)}$$

At first order in $\lambda_{\rm GB}$, the equation for $Z_s(z)$ can be written as (we use the same notation as in the main text)

$$\begin{aligned} 0 = {} & Z_s''(z) + Z_s'(z)\left(\frac{3\frac{\tilde\omega^2}{N_\sharp^2} z^4(1 - 5z^4) + \tilde q^2(9 - 16z^4 + 15z^8)}{z(-1 + z^4)(-3\frac{\tilde\omega^2}{N_\sharp^2} z^4 + \tilde q^2(-1 + 3z^4))}\right) \\ & + Z_s(z)\left(\frac{-3\frac{\tilde\omega^4}{N_\sharp^4} z^{10} + 2\tilde q^2 \frac{\tilde\omega^2}{N_\sharp^2} z^6(-2 + 3z^4) - \tilde q^2(-1 + z^4)(-16 + \tilde q^2 z^2(-1 + 3z^4))}{z^2(-1 + z^4)^2(-3\frac{\tilde\omega^2}{N_\sharp^2} z^4 + \tilde q^2(-1 + 3z^4))}\right) \\ & + \lambda_{\rm GB}\Bigg[Z_s'(z)\left(\frac{4(27\frac{\tilde\omega^4}{N_\sharp^4} z^8 + 6\tilde q^2 \tilde\omega^2 z^4(-11 + z^4) + \tilde q^4(-11 + 66z^4 - 27z^8))}{z^5(-3\frac{\tilde\omega^2}{N_\sharp^2} z^4 + \tilde q^2(-1 + 3z^4))^2}\right) \\ & + \frac{Z_s(z)}{z^6(-1 + z^4)(-3\frac{\tilde\omega^2}{N_\sharp^2} z^4 + \tilde q^2(-1 + 3z^4))^2}\Bigg(-18\frac{\tilde\omega^6}{N_\sharp^6} z^{14} + 3\tilde q^2 \frac{\tilde\omega^4}{N_\sharp^4} z^{10}(17 + 15z^4) + \tilde q^4(\tilde q^2(7 + z^4)(z - 3z^5)^2 \\ & + 32(4 - 23z^4 + 15z^8)) - 4\frac{\tilde q^2 \tilde\omega^2}{N_\sharp^2} z^4(-180 + 132z^4 + \tilde q^2 z^2(-10 + 9z^4(3 + z^4)))\Bigg)\Bigg]. \end{aligned} \quad \text{(B11)}$$

Again the solution satisfying the infalling boundary condition at the horizon $z = 1$ can be written as

$$Z(z) = \left(1 - \frac{1}{z^4}\right)^{-i(\tilde\omega/4N_\sharp)} s(z). \quad \text{(B12)}$$

Defining as above the quantity $W = \frac{\tilde\omega}{\tilde q N_\sharp}$, and expanding $s(z)$ in $\tilde q$, we find that

$$\begin{aligned} s(z) = {} & \frac{3W^2 z^4 - (1 + z^4)}{(3W^2 - 2)z^4} - \lambda_{\rm GB}\frac{-3 + 2z^4 + z^8}{z^8(3W^2 - 2)} \\ & + i\tilde q\Bigg[\frac{W(z^4 - 1)}{z^4(3W^2 - 2)} + \lambda_{\rm GB} W\left(1 - \frac{1}{z^4}\right) \\ & \times \frac{(-7 + 3(3W^2 - 5)z^4)}{4z^4(3W^2 - 2)}\Bigg] + O(\tilde q^2). \end{aligned} \quad \text{(B13)}$$

The leading asymptotic behavior close to the boundary at infinity is

$$Z_s(z) = \mathcal{A}_s + \mathcal{B}_s z^{-4} + O(z^{-8}),$$

with

$$\mathcal{A}_s \propto q^2(1 + \lambda_{\rm GB}) - \frac{i}{\pi T} q^2 \omega\left(1 - \frac{15}{4}\lambda_{\rm GB}\right) - \frac{3\omega^2}{N_\sharp^2} - \frac{i9\lambda_{\rm GB}}{4\pi T}\frac{\omega^3}{N_\sharp^2} + \cdots \quad \text{(B14)}$$

Again, the hydrodynamical pole is found by setting $\mathcal{A}_s = 0$, leading to

$$\omega_{\rm sound} = \pm c_s q - i\Gamma_s q^2, \quad \text{(B15)}$$

$$c_s = \frac{1}{\sqrt{3}} N_\sharp\left(1 + \frac{\lambda_{\rm GB}}{2}\right), \quad \text{(B16)}$$

$$\Gamma_s = \frac{2}{3}\frac{N_\sharp^2}{4\pi T}(1 - 3\lambda_{\rm GB}). \quad \text{(B17)}$$

By choosing the boundary speed of light to be unity, i.e. $N_\sharp = a \approx (1 - \frac{\lambda_{\rm GB}}{2})$, we thus find that $c_s = \frac{1}{\sqrt{3}}$ and from (2.4)

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 - 4\lambda_{\rm GB}) + O(\lambda_{\rm GB}^2) \quad \text{(B18)}$$

[18]We now need to assume $\omega \sim O(q^2)$.

in agreement with the results obtained from the shear channel and the main text.

APPENDIX C: DERIVATION OF (3.19)

In this appendix we give some details for obtaining $g_1(z)$ in Eq. (3.19). Plugging (3.18) into the equation of motion (3.12) one finds a fairly complicated ordinary differential equation (ODE) for $g_1(z)$. But, by changing variable a few times, it reduces to a simpler one. Namely, defining

$$u = \sqrt{1 - 4\lambda_{\rm GB} + 4\lambda_{\rm GB}\frac{1}{z^4}}, \qquad v = 1 - u, \tag{C1}$$

we get

$$(1 - v)(\partial_v(v\partial_v g_1 + 1)) + 2(v\partial_v g_1 + 1) = 0. \tag{C2}$$

Here, we note that $-\ln(v)$ is a (singular) solution, as one can also show from more abstract reasoning. In fact, this led to our choice of change of variable. Now, we will solve this equation. Defining

$$h_1(u) = (u - 1)\partial_u g_1 + 1, \tag{C3}$$

we have

$$u\partial_u h_1 = 2h_1, \tag{C4}$$

which leads to

$$h_1 = c_1 u^2, \tag{C5}$$

where c_1 is an integration constant. Thus we find that

$$\partial_u g_1 = \frac{c_1 u^2 - 1}{u - 1} = u + 1 \quad \text{choosing } c_1 = 1. \tag{C6}$$

Note in order for $g_1(u)$ to be nonsingular at the horizon $u = 1$, we need to choose $c_1 = 1$ as we have done above. Thus we have

$$g_1 = \tfrac{1}{2}u^2 + u + c_2. \tag{C7}$$

We will choose the integration constant c_2 so that $g_1 \to 0$ as $z \to \infty$. This then leads to (3.19).

APPENDIX D: STRETCHED HORIZON APPROACH

In this section, we calculate η/s for Gauss-Bonnet gravity by extending the stretched horizon approach of [6] (see also [37]). Along the way, we also explicitly show that η/s is independent of α_1 and α_2 at linear order, as expected from the field redefinition argument in Sec. II C. As a spin-off of this work, the framework constructed here allows us to consider tipping of the graviton cone in a more abstract way than that presented in Sec. IV A.

1. Kaluza-Klein reduction

The stretched horizon calculation of [37] begins with an effective Kaluza-Klein reduction of the AdS black hole metric and treating a certain class of off-diagonal metric perturbations as a vector in the reduced geometry. In order to develop the effective Maxwell action for $h_{\mu y}$, we reduce along the y-direction:

$$ds^2 = \tilde{g}_{\mu\nu}dx^\mu dx^\nu + e^{2\rho}(dy + A_\mu dx^\mu)^2. \tag{D1}$$

To construct the theory for a higher curvature action, we need to evaluate the various components of the Riemann tensor. This is most efficiently done using an orthonormal frame, i.e. $ds^2 = \eta_{AB}E^A E^B$, which we can conveniently choose as

$$\begin{aligned} E^a &= e^a{}_\mu dx^\mu \quad \text{with } a = \hat{t}, \hat{x}, \hat{z}, \hat{r} \\ E^{\hat{y}} &= e^\rho(dy + A_\mu dx^\mu), \end{aligned} \tag{D2}$$

where $e^a{}_\mu$ are some choice of tetrad components for the reduced metric $\tilde{g}_{\mu\nu}$, which need not be specified.

Straightforward calculations then yield the following results:

$$\begin{aligned} R_{abcd} &= \tilde{R}_{abcd} - \tfrac{1}{2}e^{2\rho}(F_{a[c}F_{b|d]} - F_{ab}F_{cd}) \\ &= [R^B]_{abcd} - \tfrac{1}{2}e^{2\rho}(F_{a[c}F_{b|d]} - F_{ab}F_{cd}), \\ R_{a\hat{y}b\hat{y}} &= -\tilde{\nabla}_a\tilde{\nabla}_b\rho - \tilde{\nabla}_a\rho\tilde{\nabla}_b\rho + \tfrac{1}{4}e^{2\rho}F_{ac}F_b{}^c \\ &= [R^B]_{a\hat{y}b\hat{y}} + \tfrac{1}{4}e^{2\rho}F_{ac}F_b{}^c, \\ R_{abc\hat{y}} &= -\tfrac{1}{2}e^\rho(\tilde{\nabla}_c F_{ab} + 2\tilde{\nabla}_c\rho F_{ab} + \tilde{\nabla}_b\rho F_{ac} - \tilde{\nabla}_a\rho F_{bc}). \end{aligned} \tag{D3}$$

Our notation here is such that $\tilde{R}_{abcd}$ and $\tilde{\nabla}_a$ denote the curvature components and covariant derivative of the four-dimensional geometry specified by $\tilde{g}_{\mu\nu}$. We have also presented the first two curvature components using the notation $[R^B]_{abcd}$ which denotes to the background curvature, i.e. the curvature of the full five-dimensional geometry with $A_\mu = 0$. Hence, for example, $[R^B]_{abc\hat{y}} = 0$.

For later convenience, we also present the components of the Ricci tensor and scalar here:

$$\begin{aligned} R_{ab} &= R^c{}_{acb} + R^{\hat{y}}{}_{a\hat{y}b} \\ &= \tilde{R}_{ab} - \tilde{\nabla}_a\tilde{\nabla}_b\rho - \tilde{\nabla}_a\rho\tilde{\nabla}_b\rho - \tfrac{1}{2}e^{2\rho}F_{ac}F_b^c \\ &= [R^B]_{ab} - \tfrac{1}{2}e^{2\rho}F_{ac}F_b{}^c, \\ R_{\hat{y}\hat{y}} &= R^a{}_{\hat{y}a\hat{y}} = -\tilde{\nabla}^2\rho - (\tilde{\nabla}\rho)^2 + \tfrac{1}{4}e^{2\rho}F^2 \\ &= [R^B]_{\hat{y}\hat{y}} + \tfrac{1}{4}e^{2\rho}F^2, \\ R_{a\hat{y}} &= R^b{}_{ab\hat{y}} = -\tfrac{1}{2}e^\rho(\tilde{\nabla}^a F_{ab} + 3\tilde{\nabla}^a\rho F_{ab}), \\ R &= R^a{}_a + R^{\hat{y}}{}_{\hat{y}} \\ &= \tilde{R} - 2\tilde{\nabla}^2\rho - 2(\tilde{\nabla}\rho)^2 - \tfrac{1}{4}e^{2\rho}F^2 \\ &= [R^B] - \tfrac{1}{4}e^{2\rho}F^2. \end{aligned} \tag{D4}$$

2. Curvature-squared theories

Given the above results, we can begin to apply the stretched horizon approach to the various curvature-squared theories considered above. First, we will confirm that for the R^2 and $R_{\mu\nu}R^{\mu\nu}$ theories η/s remains unchanged to leading order. Hence we begin with the action (1.2) with $\alpha_3 = 0$:

$$I = \frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\Big(\frac{12}{L^2} + R + L^2(\alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu})\Big). \tag{D5}$$

The background geometry is a planar AdS black hole with metric as in (3.1):

$$ds^2 = -f(r)N_\sharp^2 dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2}(dx^2 + dy^2 + dz^2), \tag{D6}$$

where the event horizon appears at $f(r = r_+) = 0$. (Note that for the present purposes, we do not have to specify $f(r)$ in further detail.) We introduce a metric perturbation $h^y{}_\mu = A_\mu$ and perform a Kaluza-Klein (KK) reduction on y as above. Then we wish to expand the action (D5) to second order in the perturbation. Keeping only the quadratic terms, the resulting action is

$$I_{\rm vec} \simeq \int d^4x\sqrt{-\tilde g}e^{3\rho}\Big(-\frac{1}{4}F^2 - \frac{L^2}{2}[\alpha_1[R^B]F^2 + \alpha_2(2[R^B]^{ab}F_{ac}F_b{}^c - [R^B]^{\hat y\hat y}F^2 - (e^{-3\rho}\tilde\nabla^a(e^{3\rho}F_{ab}))^2)]\Big). \tag{D7}$$

Now we begin by noting that we are working perturbatively to linear order in $\alpha_{1,2}$ and that the leading order equation of motion for the vector perturbation is: $\tilde\nabla^a(e^{3\rho}F_{ab}) = O(\alpha_i)$. As a result, we easily see that the contribution of the last term in the above action to the equations of motion will necessarily be $O(\alpha_i^2)$. Hence this term can be dropped in the present analysis. Further, since the background metric (D6) will satisfy Einstein's equations to leading order, $[R^B]_{\mu\nu} = -(4/L^2)\tilde g_{\mu\nu} + O(\alpha_i)$. We can make this replacement for the background curvatures appearing in the $O(\alpha_i)$ terms in the action, with the result:

$$I_{\rm vec} \simeq \int d^4x\sqrt{-\tilde g}e^{3\rho}\Big(-\frac{1}{4}F^2[1 + 40\alpha_1 - 8\alpha_2]\Big) + O(\alpha_i^2). \tag{D8}$$

Thus, to linear order, the only effect of these two curvature-squared terms is to change the normalization of the effective Maxwell action. The subsequent analysis will be identical to that presented in [6] with the standard result that $\eta/s = 1/4\pi$.

Next we need to construct the effective action for vector perturbation in Gauss-Bonnet theory (1.4). For this purpose, we can use the contributions calculated in the above action (D7) with $\alpha_1 = \lambda_{\rm GB}/2$ and $\alpha_2 = -2\lambda_{\rm GB}$. Next we must determine the contribution coming from the Riemann-squared term. Using the results in (D3), we have

$$I'_{\rm vec} \simeq \int d^4x\sqrt{-\tilde g}e^{3\rho}L^2\alpha_3\Big[-\frac{3}{2}[R^B]^{abcd}F_{ab}F_{cd} + 2[R^B]^{\hat y a\hat y b}F_{ac}F_b{}^c + (\tilde\nabla_c F_{ab} + 2\tilde\nabla_c\rho F_{ab} + \tilde\nabla_b\rho F_{ac} - \tilde\nabla_a\rho F_{bc})^2\Big], \tag{D9}$$

where in the end we will substitute $\alpha_3 = \lambda_{\rm GB}/2$. The first term has already been simplified using the cyclic identity, $R_{[abc]d} = 0$. Now the second line above can be simplified by judiciously integrating by parts, applying various identities and using the results in (D3) and (D4). For example, up to total derivatives, we have

$$e^{3\rho}(\tilde\nabla_c F_{ab})^2 = 2e^{-3\rho}(\tilde\nabla^a(e^{3\rho}F_{ab}))^2 + e^{3\rho}([R^B]^{abcd}F_{ab}F_{cd} - 2[R^B]^{ab}F_{ac}F_b{}^c) + e^{3\rho}(4\tilde\nabla^a\tilde\nabla^b\rho - 2\tilde\nabla^a\rho\tilde\nabla^b\rho)F_{ac}F_b{}^c. \tag{D10}$$

In any event, the final result is

$$I'_{\rm vec} \simeq \int d^4x\sqrt{-\tilde g}e^{3\rho}L^2\alpha_3\Big[-\frac{1}{2}[R^B]^{abcd}F_{ab}F_{cd} - 2([R^B]^{ab} + [R^B]^{\hat y a\hat y b})F_{ac}F_b{}^c + 3[R^B]^{\hat y\hat y}F^2 + 2(e^{-3\rho}\tilde\nabla^a(e^{3\rho}F_{ab}))^2\Big]. \tag{D11}$$

The quadratic action for the vector potential arising from the Gauss-Bonnet theory (1.4) is thus

$$I_{\rm vec}^{\rm GB} \simeq \int d^4x\sqrt{-\tilde g}e^{3\rho}\Big(-\frac{1}{4}F^2 - \frac{\lambda_{\rm GB}}{4}L^2[[R^B]^{abcd}F_{ab}F_{cd} + 4([R^B]^{\hat y a\hat y b} - [R^B]^{ab})F_{ac}F_b^c + ([R^B] - 2[R^B]^{\hat y\hat y})F^2]\Big). \tag{D12}$$

3. Shear viscosity via membrane paradigm

Next we need to extend the analysis of [6] to accommodate the generalized vector action (D12) which arises in Gauss-Bonnet gravity. In particular, we can write the latter in the form

$$I_{\rm vec} \simeq \int d^4x\sqrt{-\tilde g}\Big(-\frac{1}{4}F_{ab}X^{abcd}F_{cd}\Big), \tag{D13}$$

where the background tensor X^{abcd} necessarily has the following symmetries:

$$X^{abcd} = X^{[ab][cd]} = X^{cdab}. \tag{D14}$$

Though this generalized action will not accommodate the contributions of an arbitrary higher curvature term (e.g., compare with (D7)), this special form would also apply for generalized Lovelock theories of gravity. Further, in the present case where we are studying a static AdS black brane background, this tensor satisfies the two important properties: First, X is diagonal in the index pairs $[ab]$ and $[cd]$, e.g., $X^{\hat{t}\hat{x}\hat{z}\hat{x}} = 0$. Second, all of the components of X are nonsingular at the horizon $r = r_+$ when described with frame indices. Alternatively, if the tensor carries coordinate indices, the latter can be phrased as saying that all of the components of $X_{\mu\nu}{}^{\rho\sigma}$ are nonsingular at the horizon.

Given the above framework, one easily extends the analysis of [6]. After defining a stretched horizon at $r = r_H$ (with $r_H > r_+$ and $r_H - r_+ \ll r_+$), the natural conserved current to consider is

$$j^a = \tfrac{1}{2} n_b X^{abcd} F_{cd}|_{r=r_H}, \tag{D15}$$

where n_a is an outward-pointing radial unit vector. One then simply follows each of the steps appearing in [6] to arrive at the following simple result for the effective diffusion constant:

$$\begin{aligned} D &= \sqrt{-\tilde{g}}\tilde{g}^{xx}\sqrt{-\tilde{g}^{tt}\tilde{g}^{rr}}\sqrt{X_{xt}{}^{xt}X_{xr}{}^{xr}}|_{r=r_+}\int_{r_+}^{\infty}\frac{(-)dr}{\sqrt{-\tilde{g}}\tilde{g}^{tt}\tilde{g}^{rr}X_{tr}{}^{tr}} \\ &= \sqrt{-\tilde{g}}\sqrt{-X^{xtxt}X^{xrxr}}|_{r=r_+}\int_{r_+}^{\infty}\frac{(-)dr}{\sqrt{-\tilde{g}}X^{trtr}}. \end{aligned} \tag{D16}$$

For the standard effective Maxwell action with Lagrangian $-\frac{1}{4g_{\rm eff}^2}F^2$,

$$X_{xt}{}^{xt} = \frac{1}{2g_{\rm eff}^2} = X_{xr}{}^{xr} = X_{tr}{}^{tr}$$

and then the first expression above reduces to the usual result, first derived in [6].

Now we apply this analysis to the specific action (D12) arising from Gauss-Bonnet gravity. First we must extract expressions for the background tensor, which is simplified if we divide up X into three contributions

$$X_{ab}{}^{cd} = X^{(0)}{}_{ab}{}^{cd} + X^{(1)}{}_{ab}{}^{cd} + X^{(2)}{}_{ab}{}^{cd}, \tag{D17}$$

where $X^{(0)}$ and $X^{(1)}$ correspond to the contributions coming from the terms proportional to F^2 and $F_{ac}F_b{}^c$, respectively. $X^{(2)}$ captures the remaining contributions. For the action (D12), one finds

$$\begin{aligned} X^{(0)}{}_{ab}{}^{cd} &= \delta_{[a}{}^{c}\delta_{b]}{}^{d}e^{3\rho}(1 + \lambda_{\rm GB}L^2([R^B] - 2[R^B]^{\hat{y}\hat{y}})), \\ X^{(1)}{}_{ab}{}^{cd} &= Y_{[a}{}^{[c}\delta_{b]}{}^{d]}e^{3\rho} \quad \text{with} \\ Y_a{}^b &= 4\lambda_{\rm GB}L^2([R^B]_{\hat{y}a}{}^{\hat{y}b} - [R^B]_a{}^b), \\ X^{(2)}{}_{ab}{}^{cd} &= \lambda_{\rm GB}L^2 e^{3\rho}[R^B]_{ab}{}^{cd}. \end{aligned} \tag{D18}$$

The expression (D16) for the diffusion constant requires three of the components of X in particular. Using the background metric (D6) and the expressions (D18), one finds that

$$\begin{aligned} X_{tr}{}^{tr} &= \frac{1}{2}e^{3\rho}\left(1 - 2\lambda_{\rm GB}\left(\frac{L^2}{r^2}f\right)\right), \\ X_{xr}{}^{xr} = X_{xt}{}^{xt} &= \frac{1}{2}e^{3\rho}\left(1 - \lambda_{\rm GB}\frac{L^2}{r}\partial_r f\right). \end{aligned} \tag{D19}$$

To proceed further, we must explicitly introduce the solution (3.2)

$$\frac{L^2}{r^2}f(r) = \frac{1}{2\lambda_{\rm GB}}\left[1 - \sqrt{1 - 4\lambda_{\rm GB}\left(1 - \frac{r_+^4}{r^4}\right)}\right] \tag{D20}$$

for the black brane in the Gauss-Bonnet theory. Recall that the temperature (3.4) is given by $T = N_\sharp r_+/\pi L^2$. Further implementing the KK reduction (D1) on this background (D6) yields

$$\sqrt{-\tilde{g}} = N_\sharp\frac{r^2}{L^2}, \qquad e^{3\rho} = \frac{r^3}{L^3}. \tag{D21}$$

Given these results, the prefactor in (D16) reduces to

$$\begin{aligned} &\sqrt{-\tilde{g}}\tilde{g}^{xx}\sqrt{-\tilde{g}^{tt}\tilde{g}^{rr}}\sqrt{X_{xt}{}^{xt}X_{xr}{}^{xr}}|_{r=r_+} = X_{xt}{}^{xt}|_{r=r_+} \\ &= \frac{1}{2}\frac{r_+^3}{L^3}(1 - 4\lambda_{\rm GB}). \end{aligned} \tag{D22}$$

We note that the second factor in $X_{tr}{}^{tr}$ has a particularly simple form: $1 - 2\lambda_{\rm GB}\frac{L^2}{r^2}f = \sqrt{1 - 4\lambda_{\rm GB}(1 - \frac{r_+^4}{r^4})}$. Then the integral in (D16) is evaluated as

$$\begin{aligned} \int_{r_+}^{\infty}\frac{(-)dr}{\sqrt{-\tilde{g}}\tilde{g}^{tt}\tilde{g}^{rr}X_{tr}{}^{tr}} &= 2L^5N_\sharp\int_{r_+}^{\infty}\frac{dr/r^5}{\sqrt{1 - 4\lambda_{\rm GB}(1 - \frac{r_+^4}{r^4})}} \\ &= \frac{L^5}{2r_+^4}N_\sharp\frac{1 - \sqrt{1 - 4\lambda_{\rm GB}}}{2\lambda_{\rm GB}}. \end{aligned} \tag{D23}$$

Combining the results in (D22) and (D23) then yields

$$\begin{aligned} D &= \frac{L^2}{4r_+}(1 - 4\lambda_{\rm GB})N_\sharp\frac{1 - \sqrt{1 - 4\lambda_{\rm GB}}}{2\lambda_{\rm GB}} \\ &= \frac{c^2}{4\pi T}(1 - 4\lambda_{\rm GB}), \end{aligned} \tag{D24}$$

where $c = N_\sharp/a$ is the boundary speed of light, with a defined in (3.3). Hence we recover the expected result for the Gauss-Bonnet theory:

$$\frac{\eta}{s} = \frac{DT}{c^2} = \frac{1}{4\pi}(1 - 4\lambda_{\rm GB}). \tag{D25}$$

4. Graviton cone revisited

Given that the background tensor X is expressed in terms of curvatures of the background spacetime (see (D17) and (D18)), we should be able to express the effective "null" cone of the gravitons in terms of these curvatures.

In Sec. IVA, we considered the scalar channel which corresponds to a perturbation $h_x{}^y$ with dependence on t, r, and z. In the present notation, this is precisely an excitation of the vector component A_x. In a high-frequency or WKB limit, we write

$$A_x = e^{ik\cdot x}\phi_{\rm en}, \tag{D26}$$

where the first factor is the rapidly varying phase and $\phi_{\rm en}$ is the slowly modulated envelope function. The coordinate dependence of the scalar channel also requires that $k_x = 0 = k_y$. For these modes, the effective action (D13) reduces to

$$I_{\rm vec} \simeq \int d^4x\sqrt{-\tilde{g}}(-F_{a\hat{x}}X^{a\hat{x}b\hat{x}}F_{b\hat{x}}). \tag{D27}$$

From this action, we can readily derive the full equations of motion, however, we do not need these here. In the high-frequency limit, the graviton cone is given by

$$0 = X^{a\hat{x}b\hat{x}}k_a k_b. \tag{D28}$$

Now as indicated above, we use (D17) and (D18) to express this result in terms of the background curvatures. Hence the effective metric defining the graviton cone can be written as

$$\begin{aligned} 2e^{-3\rho}X_{a\hat{x}}{}^{b\hat{x}} = {} & \delta_a{}^b - 2\lambda_{\rm GB}L^2([R^B]_a{}^b - \tfrac{1}{2}[R^B]\delta_a{}^b) \\ & + 2\lambda_{\rm GB}L^2([R^B]_{a\hat{y}}{}^{b\hat{y}} + [R^B]_{a\hat{x}}{}^{b\hat{x}} \\ & - 2[R^B]_{\hat{y}\hat{x}}{}^{\hat{y}\hat{x}}\delta_a{}^b), \end{aligned} \tag{D29}$$

where we have implicitly assumed that a and b only take values in $\{\hat{t}, \hat{r}, \hat{z}\}$. We have also canceled certain terms using $R_{\hat{x}}{}^{\hat{x}} = R_{\hat{y}}{}^{\hat{y}}$ for the backgrounds of interest here. In the first line, the correction term is proportional to the Einstein tensor, a result that is reminiscent of that in [52,53]. Their results for the characteristic hypersurfaces of Gauss-Bonnet gravity do not include the nontrivial contribution in the second line above. We do not entirely understand the source of this discrepancy but note that the analysis of [52,53] uses complementary techniques to ours. At least in the context of small $\lambda_{\rm GB}$, the additional terms in (D29) have an important consequence. That is, using the equations of motion for the background geometry, the results of [52] would have predicted that the deviation of the graviton cone from the standard light cone only occurs at $O(\lambda_{\rm GB}^2)$. However, our results in Sec. IV indicate that there is the nontrivial result at $O(\lambda_{\rm GB})$. The additional terms appearing in the second line of (D29) must be responsible for this effect.

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); Int. J. Theor. Phys. **38**, 1113 (1999).
[2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998).
[3] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
[4] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
[5] G. Policastro, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. **87**, 081601 (2001).
[6] P. Kovtun, D. T. Son, and A. O. Starinets, J. High Energy Phys. 10 (2003) 064.
[7] A. Buchel and J. T. Liu, Phys. Rev. Lett. **93**, 090602 (2004).
[8] P. Kovtun, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. **94**, 111601 (2005).
[9] D. Teaney, Phys. Rev. C **68**, 034913 (2003).
[10] P. Romatschke and U. Romatschke, Phys. Rev. Lett. **99**, 172301 (2007).
[11] H. Song and U. W. Heinz, Phys. Lett. B **658**, 279 (2008).
[12] A. Adare *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **98**, 172301 (2007).
[13] P. Romatschke, arXiv:0710.0016.
[14] K. Dusling and D. Teaney, Phys. Rev. C **77**, 034905 (2008).
[15] T. Schafer, Prog. Theor. Phys. Suppl. **168**, 303 (2007).
[16] H. B. Meyer, Phys. Rev. D **76**, 101701 (2007).
[17] S. Sakai and A. Nakamura, Proc. Sci., LAT2007 (2006) 221 [arXiv:0710.3625].
[18] T. D. Cohen, Phys. Rev. Lett. **99**, 021602 (2007).
[19] A. Cherman, T. D. Cohen, and P. M. Hohler, J. High Energy Phys. 02 (2008) 026.
[20] J. W. Chen, M. Huang, Y. H. Li, E. Nakano, and D. L. Yang, arXiv:0709.3434.
[21] D. T. Son, Phys. Rev. Lett. **100**, 029101 (2008).
[22] I. Fouxon, G. Betschart, and J. D. Bekenstein, Phys. Rev. D **77**, 024016 (2008).
[23] P. Benincasa and A. Buchel, J. High Energy Phys. 01 (2006) 103.
[24] A. Buchel, J. T. Liu, and A. O. Starinets, Nucl. Phys. **B707**, 56 (2005).
[25] For a review, see M. R. Douglas and S. Kachru, Rev. Mod. Phys. **79**, 733 (2007).
[26] S. M. Carroll, *Spacetime and Geometry: An Introduction to General Relativity* (Addison-Wesley, San Francisco, 2004), p. 513.
[27] Yevgeny Kats and Pavel Petrov, arXiv:0712.0743.
[28] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, and R. Rattazzi, J. High Energy Phys. 10 (2006) 014.
[29] Y. Aharonov, A. Komar, and L. Susskind, Phys. Rev. **182**, 1400 (1969).
[30] B. Zwiebach, Phys. Lett. B **156**, 315 (1985).
[31] D. G. Boulware and S. Deser, Phys. Rev. Lett. **55**, 2656 (1985).
[32] R. G. Cai, Phys. Rev. D **65**, 084014 (2002).
[33] R. C. Myers, Phys. Rev. D **36**, 392 (1987).
[34] G. Policastro, D. T. Son, and A. O. Starinets, J. High Energy Phys. 09 (2002) 043.

[35] G. Policastro, D. T. Son, and A. O. Starinets, J. High Energy Phys. 12 (2002) 054.
[36] P. K. Kovtun and A. O. Starinets, Phys. Rev. D **72**, 086009 (2005).
[37] D. T. Son and A. O. Starinets, Annu. Rev. Nucl. Part. Sci. **57**, 95 (2007).
[38] D. T. Son and A. O. Starinets, J. High Energy Phys. 09 (2002) 042.
[39] C. P. Herzog and D. T. Son, J. High Energy Phys. 03 (2003) 046.
[40] S. Nojiri and S. D. Odintsov, Phys. Lett. B **521**, 87 (2001); **542**, 301 (2002).
[41] Y. M. Cho and I. P. Neupane, Phys. Rev. D **66**, 024044 (2002).
[42] I. P. Neupane, Phys. Rev. D **67**, 061501 (2003).
[43] I. P. Neupane, Phys. Rev. D **69**, 084011 (2004).
[44] T. Jacobson and R. C. Myers, Phys. Rev. Lett. **70**, 3684 (1993).
[45] M. Brigante, H. Liu, R. C. Myers, S. Shenker, and S. Yaida, Phys. Rev. Lett. **100**, 191601 (2008).
[46] S. L. Dubovsky and S. M. Sibiryakov, Phys. Lett. B **638**, 509 (2006).
[47] C. Eling, B. Z. Foster, T. Jacobson, and A. C. Wall, Phys. Rev. D **75**, 101502 (2007).
[48] R. C. Myers, A. O. Starinets, and R. M. Thomson, J. High Energy Phys. 11 (2007) 091.
[49] G. Festuccia and H. Liu (to be published).
[50] M. Henningson and K. Skenderis, J. High Energy Phys. 07 (1998) 023.
[51] S. Nojiri and S. D. Odintsov, Int. J. Mod. Phys. A **15**, 413 (2000).
[52] C. Aragone, in *SILARG VI: Proceedings on Stringy Characteristics of Effective Gravity*, edited by M. Novello (World Scientific, Singapore, 1988).
[53] Y. Choquet-Bruhat, J. Math. Phys. (N.Y.) **29**, 1891 (1988).