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Gravity in two-time physics

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The field theoretic action for gravitational interactions in $d + 2$ dimensions is constructed in the formalism of two-time (2T) physics. General relativity in d dimensions emerges as a shadow of this theory with one less time and one less space dimensions. The gravitational constant turns out to be a shadow of a dilaton field in $d + 2$ dimensions that appears as a constant to observers stuck in d dimensions. If elementary scalar fields play a role in the fundamental theory (such as Higgs fields in the standard model coupled to gravity), then their shadows in d dimensions must necessarily be *conformal* scalars. This has the physical consequence that the gravitational constant changes at each phase transition (inflation, grand unification, electroweak, etc.), implying interesting new scenarios in cosmological applications. The fundamental action for pure gravity, which includes the spacetime metric $G_{MN}(X)$, the dilaton $\Omega(X)$, and an additional auxiliary scalar field $W(X)$, all in $d + 2$ dimensions with two times, has a mix of gauge symmetries to produce appropriate constraints that remove all ghosts or redundant degrees of freedom. The action produces on-shell classical field equations of motion in $d + 2$ dimensions, with enough constraints for the theory to be in agreement with classical general relativity in d dimensions. Therefore this action describes the correct classical gravitational physics directly in $d + 2$ dimensions. Taken together with previous similar work on the standard model of particles and forces, the present paper shows that 2T physics is a general consistent framework for a physical theory. Furthermore, the 2T-physics approach reveals more physical information for observers stuck in the shadow in d dimensions in the form of hidden symmetries and dualities, that are largely concealed in the usual one-time formulation of physics.

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I. GRAVITATIONAL BACKGROUND FIELDS IN TWO-TIME PHYSICS

Previous discussions on gravitational interactions in the context of two-time (2T) physics appeared in [1–3]. There it was shown how to formulate the motion of a particle in background fields (including gravity, electromagnetism, and high spin fields) with a target spacetime in $d + 2$ dimensions with two times. The previous approach was a worldline formalism in which consistency with an $\mathrm{Sp}(2, R)$ gauge symmetry produced some constraints on the backgrounds. Those restrictions should be regarded as gauge symmetry *kinematical* constraints on the background fields, which can be used to eliminate ghosts and redundant degrees of freedom by choosing a unitary gauge if one wishes to do so. Consistent with the notion of backgrounds, the $\mathrm{Sp}(2, R)$ constraints by themselves did not impose any conditions on the *dynamics* of the physical background fields that survive after choosing a unitary gauge.

In the present paper we construct the *off-shell* field theoretic action for gravity in $d + 2$ dimensions, which not only reproduces the correct $\mathrm{Sp}(2, R)$ gauge symmetry kinematical constraints mentioned above when the fields are on shell, but also yields the on-shell or off-shell *dynamics* of gravitational interactions. This $d + 2$ formulation of gravity is in full agreement with classical general relativity in $(d - 1) + 1$ dimensions, with one time, as described in the Abstract.

We will use the brief notation GR_d to refer to the emergent form of general relativity, which is the usual GR with some additional constraints that are explained below, while the notation GR_{d+2} is reserved for the parent theory from which GR_d is derived by solving the kinematic constraints. So GR_d can be regarded as a lower dimensional holographic shadow of GR_{d+2} which captures the gauge invariant physical sector that satisfies the $\mathrm{Sp}(2, R)$ kinematic constraints. There are, however, other holographic shadows of the same GR_{d+2} that need not look like GR_d but are related to it by duality transformations. These shadows, and the relations among them, provide additional information about the nature of gravity that is not captured by the usual one-time formulation of physics.

The key element of 2T physics is a worldline $\mathrm{Sp}(2, R)$ gauge symmetry which acts in *phase space* and makes position and momentum $[X^M(\tau), P_M(\tau)]$ indistinguishable at any worldline instant τ [3]. This $\mathrm{Sp}(2, R)$ gauge symmetry is an upgrade of worldline τ reparametrization to a higher gauge symmetry. It cannot be realized if the target spacetime has only one time dimension. It yields nontrivial physical content only if the target spacetime X^M includes two time dimensions. Simultaneously, this larger worldline gauge symmetry plays a crucial role to remove all unphysical degrees of freedom in a 2T spacetime, just as worldline reparametrization removes unphysical degrees of freedom in a one-time (1T) spacetime. Furthermore, more than two times cannot be permitted because the $\mathrm{Sp}(2,R)$ gauge symmetry cannot remove the ghosts of more than two timelike dimensions.

We could discuss the field theory for gravity directly, but it is useful to recall some aspects of the worldline $\mathrm{Sp}(2,R)$ formalism that motivates this construction. The general 2T-physics worldline action for a spin zero particle moving in any background field is given by [1]

$$S = \int d\tau\Big(\partial_\tau X^M P_M(\tau) - \frac{1}{2}A^{ij}(\tau)Q_{ij}(X(\tau),P(\tau))\Big). \tag{1.1}$$

This action has local $\mathrm{Sp}(2,R)$ symmetry on the worldline [1]. The three generators of $\mathrm{Sp}(2,R)$ are described by the symmetric tensor $Q_{ij} = Q_{ji}$ with $i = 1, 2$, and the gauge field is $A^{ij}(\tau)$. The background fields as functions of spacetime X^M are the coefficients in the expansion of $Q_{ij}(X,P)$ in powers of momentum, $Q_{ij}(X,P) = Q^0_{ij}(X) + Q^M_{ij}(X)P_M + Q^{MN}_{ij}(X)P_M P_N + \cdots$.

In the current paper we wish to describe only the gravitational background. Therefore, specializing to a simplified version of [1], we take just the following form of $Q_{ij}(X,P)$:

$$Q_{11} = W(X), \qquad Q_{12} = V^M(X)P_M, \qquad Q_{22} = G^{MN}(X)P_M P_N, \tag{1.2}$$

which includes the gravitational metric $G^{MN}(X)$, together with an auxiliary scalar field $W(X)$ and a vector field $V^M(X)$. A basic requirement for the $\mathrm{Sp}(2,R)$ gauge symmetry of the worldline action is that the generators $Q_{ij}(X,P)$ must satisfy the $\mathrm{Sp}(2,R)$ Lie algebra under Poisson brackets. This requirement turns into certain kinematical constraints on the background fields $[W(X), V^M(X), G^{MN}(X)]$, which are obtained by demanding closure of $\mathrm{Sp}(2,R)$ under Poisson brackets $\{A,B\} \equiv \frac{\partial A}{\partial X^M} \times \frac{\partial B}{\partial P_M} + \frac{\partial A}{\partial P_M}\frac{\partial B}{\partial X^M}$ as follows [1,2]:

$$\{Q_{11}, Q_{22}\} = 4Q_{12} \to V^M = \tfrac{1}{2}G^{MN}\partial_N W, \tag{1.3}$$

$$\{Q_{11}, Q_{12}\} = 2Q_{11} \to V^M \partial_M W = 2W, \tag{1.4}$$

$$\{Q_{22}, Q_{12}\} = -2Q_{22} \to \mathcal{L}_V G^{MN} = -2G^{MN}. \tag{1.5}$$

In the last line $\mathcal{L}_V G^{MN}$ is the Lie derivative of the metric, which is a general coordinate transformation of the metric using the vector $V^M(X)$ as the parameter of transformation,

$$-2G^{MN} = V^K\partial_K G^{MN} - \partial_K V^M G^{KN} - \partial_K V^N G^{MK} \tag{1.6}$$

$$= -\nabla^M V^N - \nabla^N V^M \equiv \mathcal{L}_V G^{MN}. \tag{1.7}$$

The equivalence of the expressions in (1.6) and (1.7) is seen by replacing every derivative in (1.6) by covariant derivatives using the Christoffel connection Γ^P_{MN}, such as $\nabla_P V^N = \partial_P V^N + \Gamma^N_{PQ}V^Q$, and recalling that the covariant derivative of the metric vanishes, $\nabla_K G^{MN} = 0$:

$$\nabla_K G^{MN} = 0 \leftrightarrow \Gamma^P_{MN} = \tfrac{1}{2}G^{PQ}(-\partial_Q G_{MN} + \partial_M G_{NQ} + \partial_N G_{MQ}). \tag{1.8}$$

We can deduce that the above relations imply that G_{MN} can be written as

$$G_{MN} = \nabla_M V_N = \tfrac{1}{2}\nabla_M \partial_N W. \tag{1.9}$$

This is proven by inserting the expression for the Christoffel connection in $G_{MN} = \nabla_M V_N = \partial_M V_N - \Gamma^P_{MN}V_P$ and using (1.3), (1.4), (1.5), and (1.6).

There are an infinite number of solutions [1] that satisfy (1.3), (1.4), (1.5), (1.6), (1.7), (1.8), and (1.9). An example is flat spacetime,

$$W_{\text{flat}}(X) = X\cdot X, \qquad V^M_{\text{flat}}(X) = X^M, \qquad G^{MN}_{\text{flat}}(X) = \eta^{MN}. \tag{1.10}$$

This satisfies the $\mathrm{Sp}(2,R)$ relations (1.3), (1.4), (1.5), (1.6), (1.7), (1.8), and (1.9). In this case the $\mathrm{Sp}(2,R)$ generators are simply

$$Q^{\text{flat}}_{11} = X\cdot X, \qquad Q^{\text{flat}}_{12} = X\cdot P, \qquad Q^{\text{flat}}_{22} = P\cdot P. \tag{1.11}$$

This flat background has an $\mathrm{SO}(d,2)$ global symmetry (Killing vectors of the flat metric η_{MN}) whose generators $L^{MN} = X^M P^N - X^N P^M$ commute with the dot products in (1.11).

The phase space (X^M, P_M) and the background fields $W(X)$, $V^M(X)$, $G^{MN}(X)$ are restricted by the $\mathrm{Sp}(2,R)$ relations (1.3), (1.4), (1.5), (1.6), (1.7), (1.8), and (1.9) as well as by the requirement of $\mathrm{Sp}(2,R)$ gauge invariance $Q_{ij}(X,P) = 0$ in the physical subspace. The latter is derived from the action (1.1) as the equation of motion for the gauge field A^{ij}. This combination of constraints is just the right amount to remove ghosts from a 2T spacetime and end up with a shadow sub-phase-space (x^μ, p_μ) with a 1T spacetime which describes the gauge fixed physical sector. There are no nontrivial solutions if the higher spacetime has fewer than two timelike dimensions. This is easy to verify for the flat example (1.10). Furthermore, if the higher spacetime has more than two timelike dimensions, there are always ghosts. Hence the Sp(2, R) gauge symmetry demands precisely two timelike dimensions, no less and no more.[1]

The solution of (1.3), (1.4), (1.5), (1.6), (1.7), (1.8), and (1.9) at the classical level was obtained in [1,2], where it was shown that the worldline action (1.1) reduces (as one of the shadows) to the well-known one-time worldline action of a particle moving in an arbitrary gravitational background field $g_{\mu\nu}(x^\mu)$ in d dimensions,

$$S = \int d\tau\Big(\partial_\tau x^\mu p_\mu(\tau) - \frac{1}{2}A^{22}(\tau)g^{\mu\nu}(x(\tau))p_\mu(\tau)p_\nu(\tau)\Big). \tag{1.12}$$

This 1T action has enough well-known gauge symmetry to remove ghosts in 1T physics. This remaining gauge symmetry is part of the original Sp(2, R).

This fixing of gauges to a unitary gauge demonstrates that the Sp(2, R) relations (1.3), (1.4), (1.5), (1.6), (1.7), (1.8), and (1.9) have the right amount of gauge symmetry to remove ghosts. Hence the 2T-physics approach provides a physical theory for gravity formulated directly in the higher spacetime X^M in $d+2$ dimensions with two times in the form of the action (1.1), as long as the background fields $W(X)$, $V^M(X)$, $G^{MN}(X)$ satisfy the Sp(2, R) kinematic constraints (1.3), (1.4), (1.5), (1.6), (1.7), (1.8), and (1.9) that are also formulated directly in $d+2$ dimensions.

Note, however, that the Sp(2, R) constraints are not enough to give the dynamical equations that the gravitational metric $g^{\mu\nu}(x)$ in $(d-1)+1$ dimensions should satisfy. To do this we must build a field theoretic action in $d+2$ dimensions that not only gives correctly the Sp(2, R) kinematic constraints (1.3), (1.4), (1.5), (1.6), (1.7), (1.8), and (1.9), but also gives dynamical equations in $d+2$ dimensions for the metric $G_{MN}(X)$, and auxiliary fields $W(X)$, $V^M(X)$, which in turn correctly reproduce the equations of general relativity for the metric $g_{\mu\nu}(x)$. This is what we will present in the rest of this paper.

II. GRAVITATIONAL ACTION

The first kinematic equation (1.3) will be imposed from the start, so the auxiliary field $V^M(X)$ will not be included as a fundamental one in the action, but instead will be replaced by $V_M = \frac{1}{2}\partial_M W$ consistent with (1.3). Recall that $Q_{11} = W(X) = 0$ is one of the Sp(2, R) constraints of the worldline theory. To implement this constraint covariantly in $d+2$ dimensions, we follow the methods that were successful in flat space [4,5]; namely, we include a delta function as part of the volume element $\delta(W(X))d^{d+2}X$ in the definition of the action of 2T field theory.[2] The field W will appear in other parts of the action as well. In flat space $W(X)$ is a fixed background $W_{\rm flat}(X) = X\cdot X$, but in the present case it is a field that will be allowed to vary as any other. In addition to $W(X)$ and $G_{MN}(X)$, we will need also the dilaton field $\Omega(X)$ in order to impose consistency with the kinematic constraints (1.3), (1.4), (1.5), (1.6), (1.7), (1.8), and (1.9) required by the underlying Sp(2, R). The dilaton plays a similar role even in flat 2T field theory, especially when $d \neq 4$ [5]. Our proposed action for the 2T gravity triplet G_{MN}, Ω, W is

$$S = S_G + S_\Omega + S_W, \tag{2.1}$$

$$S_G \equiv \gamma \int d^{d+2}X\delta(W)\sqrt{G}\Omega^2 R(G), \tag{2.2}$$

$$S_\Omega \equiv \gamma \int d^{d+2}X\delta(W)\sqrt{G}\Big\{\frac{1}{2a}\partial\Omega\cdot\partial\Omega - V(\Omega)\Big\}, \tag{2.3}$$

$$S_W = \gamma \int d^{d+2}X\delta'(W)\sqrt{G}\{\Omega^2(4-\nabla^2 W) + \partial W\cdot\partial\Omega^2\}. \tag{2.4}$$

Note that the last term in the action S_W contains $\delta'(W)$ rather than $\delta(W)$. The overall constant γ is a volume renormalization constant that also appears in flat 2T field theory [5,18,19], and is specified after Eq. (7.19). Demanding consistency with the Sp(2, R) kinematic constraints (1.3), (1.4), (1.5), (1.6), (1.7), (1.8), and (1.9) will fix the constant a uniquely to

$$a = \frac{(d-2)}{8(d-1)}. \tag{2.5}$$

As will be explained below, for this special value of a, the "conformal shadow" in d dimensions has an accidental

[1] A more general argument that applies to all backgrounds is the following. By canonical transformations that do not change the signature, the first two constraints Q_{11}, Q_{12} can always be brought to the flat form, while Q_{22} has the backgrounds (second reference in [1]). Then nontrivial solutions require two times. Another point is that the signature of the Sp(2, R) parameters, which is the same as SO(1, 2) with one space and two times, determines the signature of the constraints and of the removable degrees of freedom from (X^M, P_M).

[2] Some studies for conformal gravity in $4+2$ dimensions using Dirac's approach to conformal symmetry [6–17] also use fields in $4+2$ dimensions and include a delta function [15,17] (see also [11]). Their focus is conformal gravity aiming for and constructing a totally different action. While we have some overlap of methods with [15,17], we have important differences right from the start. They impose kinematic constraints as additional conditions that do not follow from the action, as we did also in our older work [2]. These are related to the conceptually more general Sp(2, R) constraints in 2T physics. The new progress in 2T field theory since [4,5] is to derive the constraints as well the dynamics from the action, without imposing them externally. In our present work, the unusual pieces of the action S_W, with W a field varied like any other, are the new crucial ingredients in curved space that allow us to derive all Sp(2, R) constraints from the action, and lead to the new physical consequences.

local Weyl symmetry (even though the $d+2$ theory does not have it).

The action above is a no-scale theory. The dimensionful gravitational constant will develop spontaneously from a vacuum expectation value of the dilaton $\langle\Omega\rangle \neq 0$. The corresponding Goldstone boson as seen by observers in d dimensions is gauge freedom that is removable by the accidental Weyl gauge symmetry.

The various factors in the action involving powers of Ω are determined as follows. We assign engineering dimensions for X^M, G_{MN}, Ω, W, which are consistent with their flat counterparts in (1.10), as follows:

$$\dim(X^M) = 1, \qquad \dim(G_{MN}) = 0, \qquad \dim(W) = 2, \qquad \dim\Omega = -\frac{d-2}{2}. \tag{2.6}$$

Accordingly, powers of the dilaton Ω are inserted as shown to insure that the action is dimensionless, $\dim(S) = 0$. The underlying reason for this is a gauge symmetry that we called the 2T gauge symmetry in field theory [5], which becomes valid when the factors of Ω are included. The dimensions (2.6) will appear in the $Sp(2, R)$ kinematic equations that follow from the action, and coincide precisely with the kinematic constraints (1.4) and (1.5) that are required by the worldline $Sp(2, R)$ gauge symmetry. These turn into homogeneity constraints in flat space, when $V^M_{\rm flat} = X^M$ and $X\cdot\partial W_{\rm flat} = 2W_{\rm flat}$ and $X\cdot\partial G^{MN}_{\rm flat} = 0$, which are consistent with $\dim(W) = 2$, $\dim(G_{MN}) = 0$, respectively, as given in (2.6). The consistency of the kinematic equations with each other (equivalently, the gauge symmetry) restricts the form of self-interactions of the scalar to the form

$$V(\Omega) = \frac{\lambda(d-2)}{2d}\Omega^{2d/(d-2)} \tag{2.7}$$

where the arbitrary constant λ is dimensionless.

III. EQUATIONS OF MOTION FOR G_{MN}

We first concentrate on S_G. Using the variational formulas

$$\delta\sqrt{G} = -\tfrac{1}{2}\sqrt{G}G_{MN}\delta G^{MN}, \qquad \delta R(G) = \{R_{MN} + (G_{MN}\nabla^2 - \nabla_M\nabla_N)\}\delta G^{MN}, \tag{3.1}$$

and doing integration by parts as needed, we obtain the following variation of S_G with respect to the metric:

$$\delta_G(S_G) = \gamma\int d^{d+2}X\,\delta(W)\Omega^2\delta_G(\sqrt{G}R(G)) = \gamma\int d^{d+2}X\sqrt{G}\,\delta G^{MN}(V^G_{MN}), \tag{3.2}$$

$$V^G_{MN} \equiv \delta(W)\Omega^2(R_{MN} - \tfrac{1}{2}G_{MN}R) + (G_{MN}\nabla^2 - \nabla_M\nabla_N)(\delta(W)\Omega^2). \tag{3.3}$$

The last term will generate terms proportional to $\delta(W)$, $\delta'(W)$, $\delta''(W)$ as follows:

$$\begin{aligned}(G_{MN}\nabla^2 - \nabla_M\nabla_N)(\delta(W)\Omega^2) &= \{\delta(W)[G_{MN}\nabla^2\Omega^2 - \nabla_M\partial_N\Omega^2] \\ &\quad + \delta'(W)[2G_{MN}\partial W\cdot\partial\Omega^2 - 2\partial_M W\partial_N\Omega^2 \\ &\quad + \Omega^2(G_{MN}\nabla^2 W - \nabla_M\partial_N W)] \\ &\quad + \delta''(W)\Omega^2[G_{MN}\partial W\cdot\partial W - \partial_M W\partial_N W]\}.\end{aligned} \tag{3.4}$$

Additional terms in the action are needed to modify the expressions proportional to $\delta'(W)$, $\delta''(W)$ because requiring $\delta_G(S_G)$ to vanish on its own would put severe and inconsistent constraints on G_{MN} and Ω that are incompatible with the $Sp(2, R)$ kinematic conditions in (1.3), (1.4), (1.5), (1.6), (1.7), (1.8), and (1.9). This is the first reason for introducing the additional term S_W which miraculously produces just the right structure of variational terms that make the $Sp(2, R)$ constraints (1.3), (1.4), (1.5), (1.6), (1.7), (1.8), and (1.9) compatible with the equations of motion derived from the action. Actually, S_W performs a few more miracles involving the variations of Ω and W as well, as we will see below.

Thus let us study the variation of S_W with respect to δG^{MN},

$$\begin{aligned}\delta_G(S_W) = \gamma\int d^{d+2}X\,\delta'(W)\{[4\delta_G\sqrt{G} &- \partial_M(\delta_G(\sqrt{G}G^{MN})\partial_N W)]\Omega^2 \\ &+ \delta_G(\sqrt{G}G^{MN})\partial_M W\partial_N\Omega^2\}.\end{aligned} \tag{3.5}$$

After an integration by parts this gives $\delta_G(S_W) = \gamma\int d^{d+2}X\sqrt{G}\,\delta G^{MN}(V^W_{MN})$ with

$$\begin{aligned}V^W_{MN} \equiv a\{&+\delta'(W)[2\partial_M W\partial_N\Omega^2 - G_{MN}(2\Omega^2 + \partial W\cdot\partial\Omega^2)] \\ &+ \delta''(W)\Omega^2[\partial_M W\partial_N W - \tfrac{1}{2}G_{MN}\partial W\cdot\partial W]\}.\end{aligned} \tag{3.6}$$

We will also need the variation of S_Ω with respect to δG^{MN}, but this contains only $\delta(W)$,

$$\delta_G(S_\Omega) = \gamma\int d^{d+2}X\sqrt{G}\,\delta G^{MN}(V^\Omega_{MN}), \tag{3.7}$$

$$V^\Omega_{MN} \equiv \delta(W)\left[\frac{1}{2a}\partial_M\Omega\partial_N\Omega + G_{MN}\left(-\frac{1}{4a}\partial\Omega\cdot\partial\Omega + \frac{1}{2}V(\Omega)\right)\right]. \tag{3.8}$$

The vanishing of the total variation $\delta_G(S_G + S_W + S_\Omega) = \gamma\int d^{d+2}X\sqrt{G}\,\delta G^{MN}(V_{MN}) = 0$ gives

$$V_{MN} = \delta(W)V_{MN}^{(0)} + \delta'(W)V_{MN}^{(1)} + \delta''(W)V_{MN}^{(2)} = 0, \tag{3.9}$$

$$V_{MN}^{(0)} \equiv \Big[\Omega^2\Big(R_{MN} - \frac{1}{2}G_{MN}R\Big) + (G_{MN}\nabla^2\Omega^2 - \nabla_M\partial_N\Omega^2)\frac{1}{2a}\partial_M\Omega\partial_N\Omega + G_{MN}\Big(-\frac{1}{4a}\partial\Omega\cdot\partial\Omega + \frac{1}{2}V(\Omega)\Big)\Big], \tag{3.10}$$

$$V_{MN}^{(1)} \equiv \Omega^2[G_{MN}(-6 + \nabla^2W + \partial W\cdot\partial\ln\Omega^2) - \nabla_M\partial_N W], \tag{3.11}$$

$$V_{MN}^{(2)} \equiv \tfrac{1}{2}\Omega^2 G_{MN}(\partial W\cdot\partial W - 4W). \tag{3.12}$$

The vanishing expression $\frac{1}{2}\Omega^2 G_{MN}[-8\delta'(W) - 4W\delta''(W)] = 0$, that follows from the identity $w\delta''(w) = -2\delta'(w)$, has been added to V_{MN} to obtain the forms of $V_{MN}^{(1)}$, $V_{MN}^{(2)}$ as shown.

Next, taking into account the remarks in the footnote,[3] we refine the three equations of motion implied by Eq. (3.9). Each field is expanded in powers of $W(X)$. For this, imagine parametrizing X^M in terms of some convenient set of coordinates such that $w \equiv W(X)$ is one of the independent coordinates. Denoting the remaining $d+1$ coordinates collectively as u, schematically we can write $G_{MN}(X) = G_{MN}(u, w)$, $\Omega(X) = \Omega(u, w)$, and $W(X) = w$. Then we may expand

$$G_{MN}(u, w) = G_{MN}(u, 0) + wG'_{MN}(u, 0) + \tfrac{1}{2}w^2 G''_{MN}(u, 0) + \cdots \tag{3.13}$$

and similarly for $\Omega(u, w) = \Omega(u, 0) + \cdots$. In 2T field theory in flat space, the zeroth order terms analogous to $G_{MN}(u, 0)$ and $\Omega(u, 0)$ were the physical part of the field, while the rest, which we called the "remainders," was gauge freedom, and could be set to zero. In this paper we will assume that there is a similar justification for setting the *remainders* to zero (or some other convenient gauge choice) *after* the variation of the action has been performed as in (3.9), (3.10), (3.11), and (3.12). A procedure for dealing with the remainders in this fashion could be justified in the case of 2T field theory in flat space.[4] In any case, setting all the remainders to zero is a legitimate solution of the classical equations of interest in this paper. Proceeding under this assumption, we keep only the zeroth order terms in the expansions (3.13). Then, in view of footnote 3, the three classical equations of motion implied by Eq. (3.9) are

$$[V_{MN}^{(0)}(X)]_{W(X)=0} = 0, \qquad [V_{MN}^{(1)}(X)]_{W(X)=0} = 0, \qquad [V_{MN}^{(2)}(X)]_{W(X)=0} = 0. \tag{3.14}$$

We see immediately from Eq. (3.12) that the equation of motion $V_{MN}^{(2)}(u, 0) = 0$,

$$\partial W\cdot\partial W = 4W, \tag{3.15}$$

reproduces the second Sp$(2, R)$ kinematic constraint (1.4), noting that we have already incorporated the first Sp$(2, R)$ kinematic constraint (1.3) in the form $V_M = \frac{1}{2}\partial_M W$ as stated in the beginning of Sec. II. We now turn to the equation of motion (3.11), $V_{MN}^{(1)}(u, 0) = 0$,

$$[G_{MN}(-6 + \nabla^2 W + \partial W\cdot\partial\ln\Omega^2) - \nabla_M\partial_N W]_{W(X)=0} = 0. \tag{3.16}$$

If we can show that $(-6 + \nabla^2 W + \partial W\cdot\partial\ln\Omega^2) = 2$, then (3.16) reproduces the third Sp$(2, R)$ constraint (1.5), (1.6), (1.8), and (1.9). This is proven as follows. The variation of the action with respect to Ω produces on-shell conditions for Ω; among these, Eq. (4.6), $F^{(1)} = 0$, is solved by $\partial W\cdot\partial\ln\Omega^2 = 8a(6 - \nabla^2 W)$. We insert this in (3.16) and then contract Eq. (3.16) with G^{MN} to obtain an equation for only $\nabla^2 W$, whose solution is a constant $\nabla^2 W = 6(d+2)(8a-1)[(8a-1)(d+2)+1]^{-1}$. Therefore $\partial W\cdot\partial\ln\Omega^2 = 48a[(8a-1)(d+2)+1]^{-1}$ is also a constant. These lead to the on-shell value $(-6 + \nabla^2 W + \partial W\cdot\partial\ln\Omega^2) = 6(8a-1)[(8a-1)(d+2)+1]^{-1}$, which takes the desired value of 2 provided $a = \frac{d-2}{8(d-1)}$ as given by Eq. (2.5). With this unique a we obtain the on-shell

[3]An expression of the form $A(w)\delta(w) + B(w)\delta'(w) + C(w)\delta''(w) = 0$, as in (3.9), is equivalent to three equations since $\delta(w)$, $\delta'(w)$, $\delta''(w)$ are three separate distributions. To carefully separate the equations one considers the Taylor expansion in powers of w, such as $C(w) = C(0) + C'(0)w + \frac{1}{2}C''(0)w^2 + \cdots$, and similarly for $B(w)$ and $A(w)$. Then by using the properties of the delta function as a distribution (i.e. under integration with smooth functions) $w\delta'(w) = -\delta(w)$, $w\delta''(w) = -2\delta'(w)$, and $w^2\delta''(w) = 2\delta(w)$, we obtain the following three equations: $C(0) = 0$, $B(0) - 2C'(0) = 0$, and $A(0) - B'(0) + C''(0) = 0$.

[4]This was justified in [5] by the fact that there is a more symmetric starting point for 2T field theory in the form of a BRST gauge field theory [4] analogous to string field theory. It is after gauge fixing and simplifying the BRST field theory that one obtains the simpler and more intuitive form of 2T field theory used in [5]. Then the working procedure for the simpler form was to first allow all the remainders as part of the simplified action and, only *after varying the action*, set the remainders to zero (or nonzero but homogeneous). This is the correct procedure in any gauge theory, i.e. do not forget the variation with respect to the gauge degrees of freedom. It agrees with the consequences of the original, fully gauge invariant, BRST gauge field theory, as well as the covariantly first quantized worldline theory, at the level of the classical field equations of motion. Possible consequences of the remainders, if any, at the second quantization level (path integral) were not fully clarified, and this is part of ongoing research. We do not know yet if the remainder could play a physically relevant role.

values as follows,

$$[\partial W\cdot\partial\ln\Omega^2]_{W(X)=0}=-2(d-2),$$
$$\nabla^2W=2(d+2),\qquad [G_{MN}=\tfrac{1}{2}\nabla_M\partial_NW]_{W(X)=0},\tag{3.17}$$

which is precisely the third Sp(2, R) kinematic constraint (1.5), (1.6), (1.7), (1.8), and (1.9).

Hence, we have constructed an action consistent with the Sp(2, R) conditions (1.3), (1.4), (1.5), (1.6), (1.7), (1.8), and (1.9), and the condition $Q_{11}=W(X)=0$. These were the necessary kinematic constraints to remove all the ghosts in the two-time theory for gravity. They produce a shadow that describes gravity in $(d-1)+1$ dimensions as in Eq. (1.12) in the worldline formalism, and also in the field theory formalism as discussed before [2] and which will be further explained below.

The remaining field equation $V^{(0)}_{MN}(u,0)=0$ in Eq. (3.10) now gives the desired dynamical equation that has the form of Einstein's equation in $d+2$ dimensions,

$$[R_{MN}(G)-\tfrac{1}{2}G_{MN}R(G)]_{W(X)=0}=[T_{MN}(\Omega,G)]_{W(X)=0},\tag{3.18}$$

with an energy-momentum source $T_{MN}(\Omega,G)$ provided by the dilaton field

$$T_{MN}=\bigg[-\frac{1}{2a}(\partial_M\ln\Omega)(\partial_N\ln\Omega)+\frac{1}{2}G_{MN}\Big(\frac{1}{2a}\partial\ln\Omega\cdot\partial\ln\Omega-\frac{V(\Omega)}{\Omega^2}\Big)-\frac{1}{\Omega^2}(G_{MN}\nabla^2\Omega^2-\nabla_M\partial_N\Omega^2)\bigg].\tag{3.19}$$

The unique value of the constant a (2.5) will be required also by additional Sp(2, R) relations as will be seen below. Under the assumption that the dilaton field Ω is invertible (certainly so if it has a nonzero vacuum expectation value), we have divided by the field Ω to extract T_{MN}. Once all the kinematic constraints obtained above and below are taken into account, this correctly reduces to general relativity in d dimensions as a shadow (see below). So, $S=S_G+S_\Omega+S_W$ is a consistent action that produces the correct gravitational classical field equations directly in $d+2$ dimensions.

IV. EQUATIONS OF MOTION FOR Ω

We now turn to the variation of the action with respect to the dilaton Ω to extract its equations of motion. After integration by parts that produce $\delta'(W)$, $\delta''(W)$ terms, we obtain

$$\delta_\Omega(S_\Omega)=\gamma\int d^{d+2}X\sqrt{G}\delta\Omega\Big\{\delta(W)\Big(-\frac{1}{a}\nabla^2\Omega-V'(\Omega)\Big)-\frac{1}{a}\delta'(W)\partial W\cdot\partial\Omega\Big\},\tag{4.1}$$

$$\delta_\Omega(S_W)=\gamma\int d^{d+2}X\sqrt{G}\delta\Omega^2\{\delta'(W)(4-\nabla^2W)-\nabla\cdot(\partial W\delta'(W))\}\tag{4.2}$$

$$=\gamma\int d^{d+2}X\sqrt{G}\delta\Omega\{\delta'(W)\Omega(24-4\nabla^2W)+\delta''(W)\Omega(-2\partial W\cdot\partial W+8W)\},\tag{4.3}$$

where we have added the vanishing expression $\Omega[16\delta'(W)+8W\delta''(W)]=0$ to obtain a convenient form. Including $\delta_\Omega(S_G)$, which contains only $\delta(W)$, we obtain the total variation $\delta_\Omega(S_\Omega+S_W+S_G)=\gamma\int d^{d+2}X\sqrt{G}\delta\Omega F(X)$, which gives the equation of motion $F=0$,

$$F\equiv\delta(W)F^{(0)}+\delta'(W)F^{(1)}+\delta''(W)F^{(2)}=0,\tag{4.4}$$

$$F^{(0)}\equiv 2R\Omega-\frac{1}{a}\nabla^2\Omega-V'(\Omega),\tag{4.5}$$

$$F^{(1)}\equiv-\frac{1}{a}\partial W\cdot\partial\Omega+4\Omega(6-\nabla^2W),\tag{4.6}$$

$$F^{(2)}\equiv-2\Omega[\partial W\cdot\partial W-4W].\tag{4.7}$$

As in the discussion before, we seek a solution when the remainders of the fields vanish. Then the three on-shell equations are $F^{(0)}=F^{(1)}=F^{(2)}=0$. The expression $F^{(2)}=0$ is satisfied since it is identical to Eq. (3.15) which amounts to the Sp(2, R) kinematic constraints (1.3) and (1.4). The condition $F^{(1)}=0$ produces a kinematic constraint $\partial W\cdot\partial\ln\Omega^2=8a(6-\nabla^2W)$ for the field Ω as used in the derivation of Eq. (3.17). After inserting the on-shell value $\nabla^2W=2(d+2)$ from Eq. (3.17) for the special value of a, the constraint becomes

$$F^{(1)}=[\partial W\cdot\partial\Omega+(d-2)\Omega]_{W(X)=0}=0.\tag{4.8}$$

In the flat limit of Eq. (1.10) this reduces to $F^{(1)}_{\rm flat}=[2X\cdot\partial+(d-2)]\Omega=0$, which is a homogeneity constraint on Ω consistent with the assigned dimension of the field Ω in Eq. (2.6). Therefore, this is another consistency condition that requires the value of a in Eq. (2.5). We will see below, when we study variations with respect to the field W, that there is a stronger, independent gauge symmetry argument that fixes uniquely the same value of a.

The dynamical equation for Ω is now determined by setting $F^{(0)}=0$ with the special a,

$$\left[\nabla^2\Omega + \frac{d-2}{8(d-1)}(V'(\Omega) - 2\Omega R(G))\right]_{W(X)=0} = 0. \tag{4.9}$$

Here there is an interesting point to be emphasized. The precise coefficient of ΩR (which is $2a$) is the one that would normally appear for the conformal scalar in d dimensions, but note that the Laplacian and the curvature $R(G)$ in our case are in $d+2$ dimensions not in d dimensions. If the coefficient had been the one appropriate for $d+2$ dimensions, namely, $-\frac{d}{4(d+1)}$, then there would have been a local Weyl symmetry that could eliminate $\Omega(X)$ from the theory by a local Weyl rescaling. However, this is not the case presently. Nevertheless, we will identify later an accidental local Weyl symmetry for the "conformal shadow" in d dimensions (that is, not Weyl in the full $d+2$ dimensions). This partially local "accidental" Weyl symmetry will indeed eliminate the fluctuations of $\Omega(X)$ in the shadow subspace, but still keeping some dependence of Ω in the extra dimensions. In this way, the special value of a will allow us to eliminate the massless Goldstone boson that arises due to the spontaneous breakdown of scale invariance in the shadow subspace.

V. EQUATIONS OF MOTION FOR W

The part of the action $S_G + S_\Omega$ contains W only in the delta function, so its variation is proportional to $\delta'(W)$,

$$\delta_W(S_G + S_\Omega) = \gamma \int d^{d+2}X\sqrt{G}(\delta W)\delta'(W) \times\left[\Omega^2 R(G) + \frac{1}{2a}\partial\Omega\cdot\partial\Omega - V(\Omega)\right]. \tag{5.1}$$

Varying W in S_W produces terms proportional to $\delta'(W)$, $\delta''(W)$, and $\delta'''(W)$ as follows:

$$\delta_W(S_W) = \gamma \int d^{d+2}X\sqrt{G}\delta W\{\delta''(W)[\Omega^2(4-\nabla^2 W) + \partial W\cdot\partial\Omega^2] - \nabla\cdot\partial[\Omega^2\delta'(W)] - \nabla\cdot[\delta'(W)\partial\Omega^2]\} \tag{5.2}$$

$$= \gamma \int d^{d+2}X\sqrt{G}\delta W\{\delta'(W)[-2\nabla^2\Omega^2] + \delta''(W)[\Omega^2(16-2\nabla^2 W) - 2\partial W\cdot\partial\Omega^2] + \delta'''(W)\Omega^2[-\partial W\cdot\partial W + 4W]\}. \tag{5.3}$$

We have added the vanishing expression $\Omega^2[12\delta''(W) + 4W\delta'''(W)] = 0$ to obtain a convenient form. Thus the δ_W variation of the total action has the form $\delta_W(S_G + S_\Omega + S_W) = \gamma\int d^{d+2}X\sqrt{G}\delta W Z(X)$, which leads to the equation of motion $Z(X) = 0$,

$$Z \equiv \delta'(W)Z^{(1)} + \delta''(W)Z^{(2)} + \delta'''(W)Z^{(3)} = 0, \tag{5.4}$$

$$Z^{(1)} \equiv \Omega^2 R(G) - 2\nabla^2\Omega^2 + \frac{1}{2a}\partial\Omega\cdot\partial\Omega - V(\Omega), \tag{5.5}$$

$$Z^{(2)} \equiv \Omega^2(16 - 2\nabla^2 W) - 2\partial W\cdot\partial\Omega^2, \tag{5.6}$$

$$Z^{(3)} \equiv -\Omega^2[\partial W\cdot\partial W - 4W]. \tag{5.7}$$

It is remarkable that, if we use the on-shell *kinematic* equations of motion for W and Ω (3.15), (3.17), and (4.8), we get $[Z^{(2)}]_{W=0} = Z^{(3)} = 0$. Then, if we also use the *dynamical* equations for both G_{MN} and Ω (3.18) and (4.9), we also obtain $[Z^{(1)}]_{W=0} = 0$. These remarkable identities are possible *only if a has precisely the special value in Eq.* (2.5).

Therefore minimizing the action with respect to W does not produce any new kinematic or dynamical on-shell conditions for the fields. Hence, the on-shell value of $W(X)$ is arbitrary, indicating the presence of a gauge symmetry only for the special value of $a = \frac{d-2}{8(d-1)}$.

VI. OFF-SHELL GAUGE SYMMETRY

Let us now prove that indeed there is an off-shell gauge symmetry without using any of the kinematic or the dynamical equations of motion. A gauge transformation of the total action has the form $\delta_\Lambda S = \gamma\int d^{d+2}X\sqrt{G}(V_{MN}\delta_\Lambda G^{MN} + F\delta_\Lambda\Omega + Z\delta_\Lambda W)$, where V_{MN}, F, Z are given in Eqs. (3.9), (4.4), and (5.4), respectively, but taken off shell. We explore a gauge transformation of the form

$$\delta_\Lambda G^{MN} = \alpha G^{MN}, \qquad \delta_\Lambda\Omega = \beta\Omega, \qquad \delta_\Lambda W = \Lambda W, \tag{6.1}$$

with local functions $\alpha(X)$, $\beta(X)$ that will be determined below in terms of $\Lambda(X)$. We collect the coefficients of $\delta(W)$, $\delta'(W)$, $\delta''(W)$ in the gauge transformation $\delta_\Lambda S$ after using the delta function identities $w\delta'(w) = -\delta(w)$, $w\delta''(w) = -2\delta'(w)$, and $w\delta'''(w) = -3\delta''(w)$. This gives

$$\begin{aligned} V_{MN}\delta_\Lambda G^{MN} + F\delta_\Lambda\Omega + Z\delta_\Lambda W &= \{\delta(W)[\alpha G^{MN}V^{(0)}_{MN} + \beta\Omega F^{(0)} - \Lambda Z^{(1)}] \\ &\quad + \delta'(W)[\alpha G^{MN}V^{(1)}_{MN} + \beta\Omega F^{(1)} - 2\Lambda Z^{(2)}] \\ &\quad + \delta''(W)[\alpha G^{MN}V^{(2)}_{MN} + \beta\Omega F^{(2)} - 3\Lambda Z^{(3)}]\}. \end{aligned} \tag{6.2}$$

We first analyze the term proportional to $\delta''(W)$. After inserting the off-shell quantities $V^{(2)}_{MN}$, $F^{(2)}$, $Z^{(3)}$ in Eqs. (3.12), (4.7), and (5.7), we see that the $\delta''(W)$ term can be written as a total divergence[5] plus a term proportional to $\delta'(W)$:

[5]Use the identity $\nabla\cdot[\partial W\delta'(W)A\Phi^2] = \delta''(W)\times(\partial W\cdot\partial W - 4W)A\Phi^2 + \delta'(W)[\nabla\cdot(\partial WA\Phi^2) - 8A\Phi^2]$.

$$\delta''(W)[\alpha G^{MN}V^{(2)}_{MN} + \beta\Omega F^{(2)} - 3\Lambda Z^{(3)}] = \delta''(W)\Omega^2(\partial W\cdot\partial W - 4W)\left(\frac{\alpha}{2}(d+2) - 2\beta + 3\Lambda\right) \tag{6.3}$$

$$= \nabla\cdot\left[\partial W\delta'(W)\left(\frac{\alpha}{2}(d+2) - 2\beta + 3\Lambda\right)\Omega^2\right] + U^{(1)}\delta'(W) \tag{6.4}$$

where

$$U^{(1)}(\alpha,\beta,\Lambda) = \left\{\Omega^2\left(\frac{\alpha}{2}(d+2) - 2\beta + 3\Lambda\right)(8 - \nabla^2 W) - \partial W\cdot\partial\left[\Omega^2\left(\frac{\alpha}{2}(d+2) - 2\beta + 3\Lambda\right)\right]\right\}.$$

The total divergence can be dropped in $\delta_\Lambda S$ since $\int d^{d+2}X\sqrt{G}(\nabla\cdot Q) = \int d^{d+2}X\partial_M(\sqrt{G}G^{MN}Q_N) \to 0$. Therefore, in the gauge transformation (6.2) the part proportional to $\delta''(W)$ can be eliminated at the expense of adding $U^{(1)}\delta'(W)$ to the part proportional to $\delta'(W)$. Now we have 3 functions (α, β, Λ) at our disposal to fix to zero the 2 remaining terms of the gauge transformation (6.2), namely,

$$0 = \alpha G^{MN}V^{(0)}_{MN} + \beta\Omega F^{(0)} - \Lambda Z^{(1)}, \tag{6.5}$$

$$0 = \alpha G^{MN}V^{(1)}_{MN} + \beta\Omega F^{(1)} - 2\Lambda Z^{(2)} + U^{(1)}(\alpha,\beta,\Lambda). \tag{6.6}$$

Clearly there is freedom to fix α, β in terms of an arbitrary Λ to insure the *off-shell gauge symmetry* of the action $\delta_\Lambda S = 0$.

The analysis of the equations of motion in the previous section had indicated that $W(X)$ was arbitrary on shell. The discussion in this section shows that this freedom also extends to off shell, since according to (6.1), we can use the gauge freedom $\Lambda(X)$ to choose $W(X)$ arbitrarily as a function of X.

VII. GENERAL RELATIVITY AS A SHADOW

From the gauge transformations (6.1) we see that the gauge symmetry indicates that $W(X)$ is gauge freedom, so it can be chosen arbitrarily as a function of X^M before restricting spacetime by the condition $W(X) = 0$ in $d+2$ dimensions. This freedom is related to the production of multiple d dimensional shadows of the same $d+2$ dimensional system.

Our action is also manifestly invariant under general coordinate transformations in $d+2$ dimensions, which can be used to fix components of the metric $G_{MN}(X)$. This freedom will also be used in the production of shadows.

To proceed to generate a shadow of our theory in d dimensions, it is useful to choose a parametrization of the coordinates X^M in $d+2$ dimensions in such a way as to embed a d dimensional subspace x^μ in the higher space X^M. There are many ways of doing this, to create various shadows with different meanings of "time" as perceived by observers that live in the fixed shadow x^μ. This was discussed in the past for the particle level of 2T physics and recently for the field theory level [18,19]. A particular parametrization which is useful to explain massless particles and conformal symmetry in flat space [6–8] as a shadow of Lorentz symmetry in flat $(d+2)$ dimensions was commonly used in our past work. We will call this the "conformal shadow." The parametrization in this section, which should be understood to correspond to one particular shadow, is a generalization of the conformal shadow to curved space.

We choose a parametrization of X^M in terms of $d+2$ coordinates named (w, u, x^μ). In the new curved space (w, u, x^μ), where the basis is specified by $\partial_M = (\partial_w, \partial_u, \partial_\mu)$, we use general coordinate transformations to gauge fix $d+2$ functions among the $G^{MN}(w, u, x^\mu)$, namely, $G^{wu} = 1$, $G^{uu} = G^{w\mu} = 0$, so that the metric takes the following form:

$$G^{MN} = \begin{array}{c} M\backslash N \\ w \\ u \\ \mu \end{array} \begin{array}{c} \quad w \quad\; u \quad\; \nu \\ \begin{pmatrix} G^{ww} & -1 & 0 \\ -1 & 0 & G^{u\nu} \\ 0 & G^{\mu u} & G^{\mu\nu} \end{pmatrix} \end{array}. \tag{7.1}$$

In this basis we make a choice for $W(X)$ which specifies the conformal shadow. Namely, we take $W(X) = w$ as one of the coordinates,

$$W(X) = W(w, u, x^\mu) = w. \tag{7.2}$$

We compute $\partial_M W(X)$ in this basis and find

$$\partial_M W = (1, 0, 0)_M. \tag{7.3}$$

Now we apply the Sp$(2, R)$ *kinematical* constraint $4W = \partial W\cdot\partial W$, derived from field theory in Eq. (3.15) or from the worldline theory in (1.3) and (1.4),

$$4W = G^{MN}\partial_M W\partial_N W = G^{MN}(1,0,0)_M(1,0,0)_N = G^{ww}. \tag{7.4}$$

This determines

$$G^{ww}(w, u, x^\mu) = 4w. \tag{7.5}$$

Next we apply the Sp$(2, R)$ *kinematical* constraint (1.9) which was also derived in field theory in Eq. (3.17). We will use the equivalent form in (1.6), $-2G^{MN} = V^K\partial_K G^{MN} - \partial_K V^M G^{KN} - \partial_K V^N G^{MK}$, where we insert V^M as obtained from (1.3),

$$V^M(w, u, x^\mu) = \tfrac{1}{2}G^{MN}\partial_N W = (2w, -\tfrac{1}{2}, 0)^M. \tag{7.6}$$

Then we get $V^M\partial_M = (2w\partial_w - \frac{1}{2}\partial_u)$, and the kinematic constraint (1.6) takes the form

$$-2G^{MN} = (2w\partial_w - \tfrac{1}{2}\partial_u)G^{MN} - 2\delta_w^M G^{wN} - 2\delta_w^N G^{Mw}. \tag{7.7}$$

We check that $G^{ww} = 4w$, $G^{wu} = 1$, $G^{uu} = G^{w\mu} = 0$ all satisfy these kinematical conditions automatically, while the remaining components, $G^{\mu u}$, $G^{\mu\nu}$, must depend on u, x, and w only in the following specific form:

$$G^{\mu\nu}(w, u, x^\mu) = e^{4u}\hat{g}^{\mu\nu}(x, e^{4u}w), \tag{7.8}$$

$$G^{\mu u}(w, u, x^\mu) = e^{4u}\gamma^\mu(x, e^{4u}w). \tag{7.9}$$

As explained following Eq. (3.13), in an expansion in powers of w only the zeroth order term is kept in our solution. So, for our purposes here, $G^{\mu\nu}(w, u, x^\mu) = e^{4u}g^{\mu\nu}(x)$ and $G^{\mu u}(w, u, x^\mu) = e^{4u}\gamma^\mu(x)$ are independent of w. Even though we have already used up all of the gauge freedom of general coordinate transformations to fix $d+2$ functions of (w, u, x^μ) as in Eq. (7.1), there still remains general coordinate symmetry to reparametrize arbitrarily the subspace (u, x^μ) in such a way that the form of the metric in Eq. (7.1) remains unchanged. This allows us to fix d functions of (u, x^μ) arbitrarily as gauge choices. Therefore, for the w independent components of the metric at $w = 0$, we can make the gauge choice

$$G^{\mu u}(0, u, x^\mu) = 0 \rightarrow \gamma^\mu(x) = 0. \tag{7.10}$$

We are left with only the degrees of freedom of the metric $g^{\mu\nu}(x)$ in d dimensions given by

$$G^{\mu\nu}(0, u, x^\mu) = e^{4u}g^{\mu\nu}(x). \tag{7.11}$$

There still remains gauge symmetry for general coordinate transformations in the x^μ subspace. In this form it is easy to compute the determinant of G^{MN}, given in (7.1). This gives $\det(G^{-1}) = -e^{4du}\det(g^{-1}(x))$, or

$$\sqrt{G(w, u, x^\mu)} = e^{-2du}\sqrt{-g(x)}. \tag{7.12}$$

As a final check we compute that $\nabla^2 W = 2(d+2)$ is also satisfied as required by Eq. (3.17), as follows:

$$\nabla^2 W = \frac{1}{\sqrt{G}}\partial_M(\sqrt{G}G^{MN}\partial_N W) = \frac{1}{\sqrt{G}}\partial_M(\sqrt{G}G^{Mw}\partial_w w) \tag{7.13}$$

$$= \frac{1}{\sqrt{G}}\partial_w(\sqrt{G}G^{ww}) + \frac{1}{\sqrt{G}}\partial_u(\sqrt{G}G^{uw}) \tag{7.14}$$

$$= \partial_w(4w) - e^{2du}\partial_u e^{-2du} = 4 + 2d. \tag{7.15}$$

The metric $G^{MN}(X)$ given in Eqs. (7.1), (7.5), (7.10), and (7.11) shows that, after imposing the kinematic constraints at the classical level, the conformal shadow is described only in terms of the degrees of freedom $g_{\mu\nu}(x)$ in d dimensions.

We now go through similar arguments to impose the kinematic constraint (4.8) for Ω. This takes the form

$$0 = \left(V^M\partial_M + \frac{d-2}{2}\right)\Omega$$
$$= \left(2w\partial_w - \frac{1}{2}\partial_u + \frac{d-2}{2}\right)\Omega(w, u, x). \tag{7.16}$$

The solution is $\Omega(w, u, x) = e^{-(d-2)u}\hat{\phi}(x, e^{-4u}w)$, in which the zeroth order term in the expansion in powers of w is identified as the physical field $\phi(x)$ in d dimensions,

$$\Omega(0, u, x) = e^{(d-2)u}\phi(x). \tag{7.17}$$

After solving the kinematic constraints we have arrived at the conformal shadow with only the degrees of freedom $g^{\mu\nu}(x)$, $\phi(x)$. We can now evaluate the full action for the shadow. The volume element becomes

$$d^{d+2}X\sqrt{G}\delta(W(X)) = dwdu(d^dx)\sqrt{-g(x)}e^{-2du}\delta(w). \tag{7.18}$$

Every term in the Lagrangian density is now independent of w and has the same overall factor e^{2du} as the only possible dependence on u. Specifically, Ω^2 is proportional to $e^{2(d-2)u}$ and $R(G)$ is proportional to e^{4u}, so $\Omega^2 R(G)$ is proportional to e^{2du}, etc. Both the w and u dependences are explicit. So the action in $d+2$ dimensions produces the following shadow action in d dimensions,

$$S_G + S_\Omega + S_W = \left(\gamma\int du\right)\int(d^dx)\sqrt{-g(x)}L_d(x), \tag{7.19}$$

where the overall renormalization constant γ is chosen so that $(\gamma\int du) = 1$. The factor of γ can be interpreted as a renormalization of Planck's constant $\hbar$ since in the path integral $\hbar$ appears only in the form $S/\hbar$.

The shadow Lagrangian in d dimensions $L_d(x)$ takes the form

$$S(g, \phi) = \int d^dx\sqrt{-g}\left(\frac{1}{2a}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + R\phi^2 - V(\phi)\right). \tag{7.20}$$

Recall that the special value of a was *required* to generate consistently all of the Sp(2, R) kinematic constraints. Then $\phi(x)$ is the *conformal* scalar in d dimensions. As discussed earlier following Eq. (4.9), this action has an accidental local Weyl symmetry given by $S(\tilde{g}, \tilde{\phi}) = S(g, \phi)$ under the gauge transformation

$$\tilde{g}_{\mu\nu}(x) = e^{2\lambda(x)}g_{\mu\nu}(x), \qquad \tilde{\phi}(x) = e^{-((d-2)/2)\lambda(x)}\phi(x). \tag{7.21}$$

This gauge freedom can be used to gauge fix $\phi(x)$ except for an overall constant that absorbs dimensions. Assuming $\phi(x)$ has a nonzero vacuum expectation value ϕ_0, we may write $\phi^2(x) = \phi_0^2 e^{(d-2)\sigma(x)}$ and gauge fix the fluctuation $\sigma(x) = 0$. Note that $\sigma(x)$ would have been the Goldstone boson for dilatations, but in the present theory it is not a physical degree of freedom.

We can try to trace back the origin of this accidental Weyl symmetry. It is related to the gauge symmetry discussed in Sec. VI. That symmetry was already used to gauge fix $W(X) = w$. There remains leftover gauge symmetry that does not change w, but can change the w independent parts of the fields Ω, G_{MN} which describe the shadow. So, the conformal shadow ends up having the accidental Weyl symmetry.

It is important to emphasize that the action in $d+2$ dimensions does not have a Weyl symmetry; therefore Ω could not be removed locally. In fact, as seen from (7.17), even after gauge fixing $\phi(x)$, as well putting the theory on shell, the original field becomes $\Omega(w,u,x) = e^{(d-2)u}\hat{\phi}(x, e^{4u}w) = e^{(d-2)u}\phi_0 + O(w)$, so even on shell it still depends on the spacetime coordinate u in $d+2$ dimensions (also on w before setting $w=0$). Thus, the full Ω is not a trivial pure gauge freedom in our theory.

The shadow that emerged with a constant ϕ_0 has exactly the form of general relativity with a possible cosmological constant contributed by $\phi_0^{-2}V(\phi_0)$, if this quantity is nonvanishing,

$$S(g,\phi_0) = \int d^dx\sqrt{-g}(\phi_0^2R(g) - V(\phi_0)). \tag{7.22}$$

What is left behind from $\phi(x)$ in the shadow is only the constant ϕ_0 of mass scale $M^{(d-2)/2}$. This constant cannot be determined within the theory we have outlined so far. With our potential $V(\phi)$ in Eq. (2.7), minimizing the action with respect to $\phi(x)$, and then gauge fixing to $\phi(x) = \phi_0$, does not produce a new equation for ϕ_0 other than the one obtained by minimizing the action with respect to the metric $g_{\mu\nu}$, namely, $R(g) = \frac{1}{2\phi_0}V'(\phi_0) = \lambda\phi_0^{4/(d-2)}$. An effective potential $V(\phi)$ with a nontrivial minimum could determine ϕ_0. We assume that a nontrivial minimum arises self-consistently from either quantum fluctuations (dimensional transmutation [20]) or from the completion of our theory into string theory or M theory (with two times). Although we could not determine $\phi_0 \sim M^{(d-2)/2}$ within the classical considerations here, this ϕ_0 that appears as a constant shadow of $\Omega(X)$ to observers in x space is evidently related to Newton's constant G_d, the Planck constant κ_d, or the Planck scale l_p in d dimensions

$$\phi_0^2 = \frac{1}{16\pi G_d} = \frac{1}{2\kappa_d^2} = \frac{2\pi}{(2\pi l_p)^{d-2}} \sim M^{d-2}. \tag{7.23}$$

VIII. GRAVITATIONAL NONCONSTANT, NEW COSMOLOGY?

We now outline the coupling of our gravity triplet (W, Ω, G^{MN}) to matter fields of the Klein-Gordon $[S_i(X)]$, Dirac $[\Psi(X)]$, and Yang-Mills $[A_M(X)]$ types. In flat 2T field theory these must have the following engineering dimensions [5]:

$$\dim(X^M) = 1, \qquad \dim(S_i) = -\frac{d-2}{2}, \qquad \dim(\Psi) = -\frac{d}{2}, \qquad \dim(A_M) = -1. \tag{8.1}$$

The general 2T field theory of these fields in flat space in $d+2$ dimensions was given in [5]. The matter part of the theory in curved space follows from the flat theory in [5] by making the substitutions indicated in Table I.

The dilaton Ω couples to Yang-Mills fields and fermions only as follows:

$$S(A) = -\frac{1}{4}\int(d^{d+2}X) \times \sqrt{G}\delta(W)\Omega^{2(d-4)/(d-2)}\,\mathrm{Tr}(F_{MN}F^{MN}), \tag{8.2}$$

$$S_{\mathrm{Yukawa}}(\Psi, S_i, \Omega) = g_i\int(d^{d+2}X)\delta(W)\Omega^{-((d-4)/(d-2))} \times V^M(\bar{\Psi}^L\Gamma_M\Psi^R S_i + \mathrm{H.c.}). \tag{8.3}$$

The dilaton disappears in these expressions when $d+2=6$. In addition, even when $d+2=6$, the dilaton can also couple to other scalars $S_i(X)$ in the potential energy $V(\Omega, S)$ with the only condition that $V(\Omega, S_i)$ has length dimension $(-d)$ when $\dim(\Omega) = \dim(S_i) = -(d-2)/2$. This is the only place the extra field Φ appeared in flat space in the standard model [5], so that field may or may not be the dilaton[6] $\Phi = \Omega$.

We now emphasize an important property of the scalars S_i (including the Higgs field in the standard model). It turns out that, for consistency with the Sp(2, R) conditions (1.3), (1.4), (1.5), (1.6), (1.7), (1.8), and (1.9), the *quadratic* part of the Lagrangian for any real scalar $S_i(X)$ must have exactly the same structure as the one for the dilaton field Ω. So, the *quadratic* part of the action for any scalar must have the form of the dilaton action $S(\Omega) = S_G(\Omega) + S_\Omega(\Omega) + S_W(\Omega)$ in Eqs. (2.1), (2.2), (2.3), and (2.4), except for substituting $\Omega \to S_i$, and except for an overall normalization constant.[7] This structure has been indicated in the table above, where the piece symbolically written as $L(W, S_i^2)$ or $L(W, \Omega^2)$ is the piece that contributes to the action S_W in Eq. (2.4), which appears with a $\delta'(W)$ rather than $\delta(W)$,

[6] An important additional field that was required when $d+2 \neq 6$ even in flat space was a "dilaton." It was named Φ in [5] and had dimension $\dim(\Phi) = -\frac{d-2}{2}$ like any other scalar field Ω, S_i. A natural as well as economical assumption (although not necessary) is to identify the scalar field Φ that appeared in the $4+2$ dimensional standard model with the dilaton field $\Omega = \Phi$ that now appears as part of the gravity triplet (W, Ω, G^{MN}).

[7] A complex scalar would be constructed from two real scalars, $\varphi = (S_1 + iS_2)/\sqrt{2}$.

TABLE I. Matter in curved space. The dilaton is normalized with an extra $(-a)^{-1}$.

Quantity	Flat	Curved
Metric	η^{MN}	$G^{MN}(X)$
Volume element	$(d^{d+2}X)\delta(X^2)$	$(d^{d+2}X)\sqrt{G}\delta(W(X))$
Explicit X	X^M	$V^M = \frac{1}{2}G^{MN}\partial_N W$
Gamma matrix, vielbein	Γ_M	$E_M^a(X)\Gamma_a$
Spin connection	$\Gamma^M\partial_M\Psi$	$E^{Mc}\Gamma_c(\partial_M + \frac{1}{4}\Gamma_{ab}\omega_M^{ab}(X))\Psi$
Real scalar field S_i	$-\frac{1}{2}\partial_M S_i \partial^M S_i$	$-\frac{1}{2}G^{MN}\partial_M S_i\partial_N S_i - aS_i^2 R(G) - aL(W, S_i^2)$
Dilaton Ω	(extra $-\frac{1}{a}$ factor)	$+\frac{1}{2a}G^{MN}\partial_M\Omega\partial_N\Omega + \Omega^2 R(G) + L(W, \Omega^2)$

$$\delta(W)L(W, S_i^2) = \delta'(W)\{S_i^2(4 - \nabla^2 W) + \partial W\cdot\partial S_i^2\}. \quad (8.4)$$

Furthermore the same special $a = (d-2)/8(d-1)$ must appear in the action of any scalar.

This last requirement is related to the underlying Sp$(2, R)$, and is most directly understood by analyzing the consistency of the equations of motion for the fields G^{MN}, S_i, and W in the same footsteps as Secs. III, IV, and V. The Sp$(2, R)$ constraint is that we must always obtain the same kinematic equations of motion, in particular, $G_{MN} = \frac{1}{2}\nabla_M\partial_M W$ in Eqs. (1.9) and (3.17), independent of the field content in the action. This is a strong condition that demands the stated structure for the Lagrangian for any scalar field S_i. Of course, in flat space this is immaterial since $R(G)$ is zero, but it has an important physical effect on the meaning of the gravitational constant, as perceived by observers in the shadow worlds in d dimensions, as we will see below.

There remains, however, the freedom of an overall normalization which, for physical reasons, must be taken as specified in the table above. Namely, for the dilaton, the sign of the term $\Omega^2 R(G)$ must be positive since this is required by the positivity condition of gravitational energy in the conformal shadow as seen from Eq. (7.22). Since the dilaton is gauge freedom in the conformal shadow, the sign or normalization of the term $\frac{1}{2a}G^{MN}\partial_M\Omega\partial_N\Omega$ is not crucial. However, for the remaining scalar fields the sign and normalization of the kinetic term $-\frac{1}{2}G^{MN}\partial_M S_i\partial_N S_i$ must be fixed by the requirements of unitarity (no negative norm fluctuations) and a conventional definition of the norm.

It is interesting that there is a physical consequence. We consider again the conformal shadow and try to interpret the physical structure for observers in the smaller d dimensional space. The conformal shadow is obtained by the same steps as before by taking $W(X) = w$. We concentrate only on the scalars and the metric. These fields have the following shadows:

$$G^{MN}(w, u, x) = \begin{array}{c} M\backslash N \\ w \\ u \\ \mu \end{array} \begin{array}{c} \quad w \quad\quad u \quad\quad \nu \\ \begin{pmatrix} 4w & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & e^{4u}g^{\mu\nu}(x) \end{pmatrix} \end{array}, \quad (8.5)$$

$$\Omega(w, u, x) = e^{(d-2)u}\phi(x), \qquad S_i(w, u, x) = e^{(d-2)u}s_i(x). \quad (8.6)$$

The action in the conformal shadow at $w = 0$ is then[8]

$$S(g, \phi, s_i) = \int d^d x\sqrt{-g}\bigg(\frac{1}{2a}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu s_i\partial_\nu s_i + (\phi^2 - as_i^2)R - V(\phi, s_i)\bigg). \quad (8.7)$$

Because of the special value of a, there is *one* overall local Weyl symmetry which can be used to fix the gauge

$$\phi(x) = \phi_0 \quad (8.8)$$

as discussed above. So, $\phi(x)$ disappears, while the remaining scalar fields $s_i(x)$ are correctly normalized and are physical. The modified Einstein equation that follows from this action is

[8]We must be careful that the equations of motion derived from this action are consistent with the original equations of motion in $d+2$ dimensions. In fact, this is not trivial. The shadow extends to w, u space through first and second order terms in the expansion in powers of w, such as

$$g_{\mu\nu}(x, we^{4u}) = g_{\mu\nu}(x) + we^{4u}\tilde{g}_{\mu\nu}(x) + w^2 e^{8u}\tilde{\tilde{g}}_{\mu\nu}(x) + \cdots,$$
$$\Omega(x, we^{4u}) = \phi(x) + we^{4u}\tilde{\phi}(x) + w^2 e^{8u}\tilde{\tilde{\phi}}(x) + \cdots,$$
$$S_i(x, we^{4u}) = s_i(x) + we^{4u}\tilde{s}_i(x) + w^2 e^{8u}\tilde{\tilde{s}}_i(x) + \cdots.$$

The Riemann tensor $R_{MNPQ}(G)$ constructed from $G_{MN}(w, u, x)$ contains the modes $\tilde{g}_{\mu\nu}$, $\tilde{\tilde{g}}_{\mu\nu}$ even after setting $w = 0$ because there are derivatives with respect to w. Thus, we emphasize that $R_{\mu\nu\lambda\sigma}(G)$ at $w = 0$ depends on $g_{\mu\nu}$, $\tilde{g}_{\mu\nu}$, and $\tilde{\tilde{g}}_{\mu\nu}$, so it is not the same as $R_{\mu\nu\lambda\sigma}(g)$, and similarly for other components. Consistency with the full set of equations of motion given above requires also the modes $\tilde{\phi}$, $\tilde{\tilde{\phi}}$, $\tilde{s}_i$, $\tilde{\tilde{s}}_i$. However, all extra modes get determined in terms of only $g_{\mu\nu}$, ϕ, s_i self-consistently through the full set of equations of motion in $d+2$ dimensions. The self-consistent dynamics in shadow space x^μ is then determined only by $g_{\mu\nu}(x)$, and the interactions among fields involve only $\phi(x)$ and $s_i(x)$. Their consistent interactions, as derived from the original equations of motion, are then described by the shadow action given here. These technical details will be given in a separate paper.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}(\phi_0, s_i), \tag{8.9}$$

with the energy-momentum tensor given by

$$T_{\mu\nu} = \frac{1}{(\phi_0^2 - a\sum_i s_i^2)}\Big[\sum_i\Big(\frac{1}{2}\partial_\mu s_i \partial_\nu s_i - \frac{1}{4}g_{\mu\nu}\partial s_i \cdot \partial s_i\Big) - \frac{1}{2}g_{\mu\nu}V(\phi_0, s_i) + a\sum_i(g_{\mu\nu}\nabla^2 s_i^2 - \nabla_\mu\partial_\nu s_i^2)\Big]. \tag{8.10}$$

The trace of this energy-momentum tensor is

$$g^{\mu\nu}T_{\mu\nu} = \frac{(d-2)}{8(\phi_0^2 - a\sum_i s_i^2)}\Big[-\frac{4d}{d-2}V(\phi_0, s_i) + 2\sum_i s_i\nabla^2 s_i\Big]. \tag{8.11}$$

After using the equations of motion $\nabla^2 s_i = \partial V/\partial s_i + 2as_iR$, the special value of a, and the homogeneity of the potential $(\phi\frac{\partial V}{\partial\phi} + \sum_i s_i\frac{\partial V}{\partial s_i}) = \frac{2d}{d-2}V$, we compare to the trace of Eq. (8.9), $(1 - d/2)R = g^{\mu\nu}T_{\mu\nu}$, and solve for R. We obtain

$$R(g) = \frac{1}{2\phi_0}\frac{\partial V(\phi_0, s_i)}{\partial\phi_0}. \tag{8.12}$$

The same result is obtained if one starts with the equation of motion for $\phi(x)$ and then chooses the gauge $\phi(x) \to \phi_0$. Therefore the ϕ equation of motion is recovered from the equations of motion of the other fields, showing consistency.

When the s_i are small fluctuations, ϕ_0^{-2} approximates the overall factor in $T_{\mu\nu}$. Then the gravitational constant is determined approximately by ϕ_0, as specified in Eq. (7.23).

However, if $V(\phi_0, s_i)$ has nontrivial minima that lead to nontrivial vacuum expectation values for some of the $\langle s_i\rangle = v_i$, then in that vacuum the gravitational constant is determined by

$$16\pi G_d = (\phi_0^2 - av_i^2)^{-1} \tag{8.13}$$

rather than only ϕ_0^{-2}. The massless Goldstone boson, which is removed by the Weyl symmetry, is then a combination of ϕ and the scalars s_i that developed vacuum expectation values.

Such phase transitions of the vacuum can occur in the history of the universe as it expands and cools down. This is represented by an effective $V(\phi, s_i)$ that changes with temperature. So, the various v_i may turn on as a function of temperature $v_i(T)$ or, equivalently, as a function of time. Among the phase transitions to be considered are inflation, possible grand unification symmetry breaking, electroweak symmetry breaking, as well as some others that are possible in the context of string theory to determine how we end up in 4 dimensions with a string vacuum state compatible with the standard model.

It would be interesting to pursue the possibility of a changing effective gravitational constant, as above, since this cosmological scenario is now well motivated by 2T physics. This scenario may not have been investigated before.

IX. COMMENTS

As naively expected, extra timelike dimensions potentially introduce ghosts (negative probabilities) as well as the possibility of causality violation, leading to interpretational problems. However, 2T physics overcomes these problems by introducing the right set of gauge symmetries, thus correctly describing the physical world, including the physics of the standard model of particles and forces [5,21], and now general relativity.

At the same time, 2T physics also gives additional physical information which is not encoded in 1T physics. This is because according to 2T physics there is a larger spacetime in $d + 2$ dimensions X^M, where the fundamental rules of physics are encoded. These rules include a complete symmetry of position-momentum X^M, P_M according to the principles of a local Sp$(2, R)$ with generators $Q_{ij}(X, P)$. This leads effectively to gauge symmetries in $d + 2$ dimensions that can remove degrees of freedom and create a holographic shadow of the $d + 2$ universe in d dimensions x^μ. There are many such shadows, and since observers in different shadows use different definitions of time, they interpret their observations as different 1T dynamics. However, the shadows are related since they represent the same higher dimensional universe. These predicted relations would be interpreted as dualities by observers that live in the lower dimension x^μ who use 1T-physics rules. With hard work, observers in the smaller x^μ space could discover enough of these dualities among the shadows to reconstruct the $d + 2$ dimensional, highly symmetric universe. Two-time physics provides a road map for this reconstruction by predicting the properties of the shadows.

Examples of some simple dualities in d dimensions, which arise from flat $d + 2$ dimensional spacetime, in the context of field theory such as the standard model, were discussed in [18,19]. In the flat case, each shadow has SO$(d, 2)$ global symmetry as hidden symmetry, where this SO$(d, 2)$ is the shadow of the global Lorentz symmetry in $d + 2$ dimensions as identified in Eq. (1.11). So clues of the higher spacetime can also appear within each shadow in the form of hidden symmetries. Examples of these in field theory were also discussed in [18,19].

In curved spacetime, the details of the shadow, as seen by observers stuck in the smaller spacetime x^μ, depends partially on the choice of W as a function of (w, u, x^μ). In this paper we discussed the "conformal shadow" defined by $W(w, u, x^\mu) = w$ in Eq. (7.2) and the gauge fixed form of the metric (7.1). Together, these define the timeline in the shadow space x^μ as some curve embedded in the two-time spacetime in $d + 2$ dimensions. A different choice of gauges leads to a different shadow space with a different timeline. The same dynamics in $d + 2$ dimensions X^M tracked as a function of one timeline can appear to be quite different from one-time dynamics relative to another timeline. Evidently, there are many choices that correspond to many embeddings of d dimensional spacetime x^μ (with one time) into $d + 2$ dimensional spacetime X^M (with two times), and these are expected to lead to dualities that relate the different-looking one-time dynamics. Depending on the nature of the higher curved space X^M, there could be hidden symmetries that would be seen in each smaller x^μ space as clues of the extra space and time.

The kinds of predictions above can be used to generate multiple tests of 2T physics. This line of investigation is at its infancy and is worth pursuing vigorously.

In addition to the above, the emergent 1T-physics conformal shadow seems to come with certain natural constraints, which remarkably are not in contradiction with known phenomenology so far. On the contrary, they lead to some new guidance for phenomenology:

(i) The standard model is correctly reproduced as a shadow,[9] but in addition, the Higgs sector is *required* to interact with an additional scalar Φ that induces the electroweak phase transition as discussed in [5] (Φ could be the dilaton Ω, but not necessarily; see footnote 6). This leads to interesting physics scenarios at CERN LHC energy scales (an additional new neutral scalar) or cosmological scales (inflaton candidate, dark matter candidate) as suggested in [5].[10] The supersymmetric[11] version [21] of this 2T-physics feature with extra required scalars leads to richer, phenomenologically interesting possibilities.

(ii) The gravitational constant could be time dependent as described in the previous section. This is because according to 2T physics, if there are any fundamental scalars $s_i(x)$ at all, they all must be conformal scalars coupled to the curvature term R with the special coefficient $(-a)$ as in the last line of the table above. It would be interesting to study the effects of this scenario in the context of cosmology.

There are many open questions. In particular, quantization in the path integral formalism is still awaiting clarification of the gauge symmetries so that Faddeev-Popov techniques can be correctly applied. Other issues include the question of whether there might be some physical role, either at the classical or quantum levels, for the remainders in the expansion of the fields in powers of W, as in Eq. (3.13).

Having accomplished a formulation of gravity as well as supersymmetry in 2T field theory [21], it is natural to next try supergravity. In particular, the 2T generalization of 11-dimensional supergravity is quite intriguing and worth a few speculative comments. If constructed, such a theory will provide a low energy 2T-physics corner of M theory. This would be a theory in $11 + 2$ dimensions whose global supersymmetry can only be OSp(1|64), so it should be related to S theory [30]. We remind the reader that S theory gives an algebraic BPS-type setting based on OSp(1|64) for the usual M-theory dualities among its corners, with 11 dimensions, or 10 dimensions with type IIA, IIB, heterotic, type-I supersymmetries. A corresponding 2T-physics theory would provide a dynamical basis that could give shadows-type meaning to these famous dualities, as outlined in [31].

Finally, let us emphasize that the fundamental concept behind 2T physics is the momentum-position symmetry based on $Sp(2, R)$. Despite the fact that the worldline approach in Eq. (1.1) treats position and momentum on an equal footing, the field theoretic approach that we have discussed blurs this symmetry, although the constraints implied by the $Sp(2, R)$ symmetry in the form of the kinematic constraints were still maintained. There should be a more fundamental approach with a more manifest position-momentum symmetry, perhaps with fields that depend both on X^M and P_M, and in that case, perhaps based on noncommutative field theory. Basic progress along this line that included fields of all integer spins was reported in [32]. If this avenue could be developed to a comparable level as the current field theory formalism, it is likely that it will go a lot farther than our current approach.

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[9] The theta term $\theta F * F$ can be reproduced as a shadow in $3 + 1$ dimensions from 2T field theory in $4 + 2$ dimensions (to appear). So a previous claim of the resolution of strong *CP* violation without an axion [5] is retracted.

[10] Scenarios that include such a scalar field in both theoretical and phenomenological contexts have been discussed independently in recent papers [22–27] that mainly appeared after [5].

[11] It was suggested in the second reference in [5] that a conformal scalar of the type Φ, with the required SO(4,2), could provide an alternative to supersymmetry as a mechanism that could address the mass hierarchy problem. This possibility has been more recently discussed in [28,29].

[1] I. Bars, Phys. Rev. D **62**, 085015 (2000); I. Bars and C. Deliduman, Phys. Rev. D **64**, 045004 (2001); Classical Quantum Gravity **18**, 3113 (2001).
[2] I. Bars, Phys. Rev. D **62**, 046007 (2000).
[3] I. Bars, C. Deliduman, and O. Andreev, Phys. Rev. D **58**, 066004 (1998); For reviews of subsequent work, see I. Bars, in *Proceedings of the 22nd International Colloquium on Group Theoretical Methods in Physics*, edited by S. Corney (World Scientific, Singapore, 1999), arXiv:hep-th/9809034; Classical Quantum Gravity **18**, 3113 (2001); AIP Conf. Proc. **589**, 18 (2001); **607**, 17 (2001); arXiv:hep-th/0601091.
[4] I. Bars and Y.-C. Kuo, Phys. Rev. D **74**, 085020 (2006).
[5] I. Bars, Phys. Rev. D **74**, 085019 (2006); AIP Conf. Proc. **903**, 550 (2007).
[6] P. A. M. Dirac, Ann. Math. **37**, 429 (1936).
[7] H. A. Kastrup, Phys. Rev. **150**, 1183 (1966).
[8] G. Mack and A. Salam, Ann. Phys. (N.Y.) **53**, 174 (1969).
[9] S. Adler, Phys. Rev. D **6**, 3445 (1972); **8**, 2400 (1973).
[10] R. Marnelius, Phys. Rev. D **20**, 2091 (1979).
[11] R. Marnelius and B. Nilsson, Phys. Rev. D **22**, 830 (1980).
[12] F. Bayen, M. Flato, C. Fronsdal, and A. Haidari, Phys. Rev. D **32**, 2673 (1985).
[13] W. Siegel, Int. J. Mod. Phys. A **3**, 2713 (1988); Int. J. Mod. Phys. A **4**, 2015 (1989).
[14] S. Ferrara, Nucl. Phys. **B77**, 73 (1974).
[15] C. R. Preitschopf and M. A. Vasiliev, Nucl. Phys. **B549**, 450 (1999).
[16] M. A. Vasiliev, J. High Energy Phys. 12 (2004) 046.
[17] P. Arvidsson and R. Marnelius, arXiv:hep-th/061206.
[18] I. Bars, S. H. Chen, and G. Quelin, Phys. Rev. D **76**, 065016 (2007).
[19] I. Bars and G. Quelin, arXiv:0802.1947.
[20] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).
[21] I. Bars and Y. C. Kuo, Phys. Rev. Lett. **99**, 041801 (2007); Phys. Rev. D **76**, 105028 (2007).
[22] M. Shaposhnikov and I. Tkachev, Phys. Lett. B **639**, 414 (2006).
[23] [19] M. Shaposhnikov, arXiv:0708.3550.
[24] W. D. Goldberger, B. Grinstein, and W. Skiba, arXiv:0708.1463.
[25] J. Ramón Espinosa and M. Quirós, Phys. Rev. D **76**, 076004 (2007).
[26] K. Meissner and H. Nicolai, Phys. Lett. B **648**, 312 (2007).
[27] R. Foot, A. Kobakhidze, and R. R. Volkas, Phys. Lett. B **655**, 156 (2007).
[28] K. Meissner and H. Nicolai, Phys. Lett. B **660**, 260 (2008).
[29] R. Foot, A. Kobakhidze, K. L. McDonald, and R. R. Volkas, Phys. Rev. D **77**, 035006 (2008).
[30] I. Bars, Phys. Rev. D **55**, 2373 (1997); **54**, 5203 (1996); **55**, 3633 (1997); **56**, 7954 (1997).
[31] I. Bars, C. Deliduman, and D. Minic, Phys. Rev. D **59**, 125004 (1999); Phys. Lett. B **457**, 275 (1999).
[32] I. Bars, Phys. Rev. D **64**, 126001 (2001); I. Bars and S. J. Rey, Phys. Rev. D **64**, 046005 (2001).