Infrared finite ghost propagator in the Feynman gauge

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(Received 14 March 2008; revised manuscript received 18 April 2008; published 20 June 2008)

We demonstrate how to obtain from the Schwinger-Dyson equations of QCD an infrared finite ghost propagator in the Feynman gauge. The key ingredient in this construction is the longitudinal form factor of the nonperturbative gluon-ghost vertex, which, contrary to what happens in the Landau gauge, contributes nontrivially to the gap equation of the ghost. The detailed study of the corresponding vertex equation reveals that in the presence of a dynamical infrared cutoff this form factor remains finite in the limit of vanishing ghost momentum. This, in turn, allows the ghost self-energy to reach a finite value in the infrared, without having to assume any additional properties for the gluon-ghost vertex, such as the presence of massless poles. The implications of this result and possible future directions are briefly outlined.

DOI: 10.1103/PhysRevD.77.125022 PACS numbers: 12.38.Lg, 12.38.Aw

I. INTRODUCTION

The nonperturbative properties of the basic Green's functions of QCD have been the focal point of intensive scrutiny in recent years, with particular emphasis on the propagators of the fundamental degrees of freedom, gluons, quarks, and ghosts. Even though it is well known that these quantities are not physical, since they depend on the gauge-fixing scheme and parameters used to quantize the theory, it is generally accepted that reliable information on their nonperturbative structure is essential for unraveling the infrared (IR) dynamics of QCD.

There are two main tools usually employed in this search: the lattice, where spacetime is discretized and the quantities of interest are evaluated numerically [1–3], and the intrinsically nonperturbative equations governing the dynamics of the Green's functions, known as Schwinger-Dyson equations (SDE) [4–7]. In principle, the lattice includes all nonperturbative features, and no approximations are employed at the level of the theory. In practice, the main limitations appear when attempting to extrapolate the results obtained with finite lattice volume to the continuous spacetime limit. On the other hand, the main difficulty with the SDE has to do with the need to devise a self-consistent truncation scheme that preserves crucial field-theoretic properties, such as the transversality of the gluon self-energy, known to be valid both perturbatively and nonperturbatively, as a consequence of the Becchi-Rouet-Stora-Tyutin symmetry [8].

Significant progress has been accomplished on this last issue due to the development of the truncation scheme that is based on the all-order correspondence [9] between the pinch technique (PT) [10,11] and the Feynman gauge of the background field method (BFM) [12]. One of its most powerful features is the special way in which the transversality of the gluon self-energy is realized. Specifically, by virtue of the Abelian-like Ward identities satisfied by the vertices involved, gluonic and ghost contributions are *separately* transverse, within *each* order in the "dressed-loop" expansion of the SDE [13] for the gluon propagator. This property, in turn, allows for a systematic truncation of the full SDE, preserving at every step the crucial property of gauge invariance.

The first approximation to the SDE of the gluon propagator involves the one-loop dressed gluonic graphs only, since in this scheme the ghost loops may be omitted without compromising the transversality of the answer. As is well known, the Feynman gauge of the BFM is particularly privileged, being dynamically singled out as the gauge that directly encompasses the relevant gauge cancellations of the PT [9]. Therefore, the aforementioned one-loop dressed graphs have been considered in this particular gauge. The detailed study of the resulting integral equation for the gluon propagator gave rise to solutions that reach a *finite* value in the deep IR [13,14]. Following Cornwall's original idea [10,15] of describing the IR sector of QCD in terms of an effective gluon mass [16,17], these solutions have been fitted using "massive" propagators of the form $\Delta^{-1}(q^2) = q^2 + m^2(q^2)$, with $m^2(0) > 0$, and the crucial characteristic that $m^2(q^2)$ is not "hard," but depends nontrivially on the momentum transfer q^2. In addition, finite solutions for the gluon propagator in the Landau gauge have been reported in various lattice studies [18] and were recently confirmed using lattices with significantly larger volumes [19].

Even though the omission of the ghost loops within this formulation does not introduce any artifacts, such as the loss of transversality, the actual behavior of the ghosts may change the initial prediction for the gluon propagator, not just quantitatively but also qualitatively. For example, an IR divergent solution for the ghost propagator could destabilize the finite solutions found for the gluon propagator.

Therefore, a detailed study of the ghost sector constitutes the next challenge in this approach. In the present work, we will consider the SDE for the ghost sector in the (BFM) Feynman gauge, in order to complement the corresponding analysis presented in [13,14] in the same gauge. The BFM Feynman rules are in general different to those of the covariant renormalizable gauges [12]; in the former, for example, in addition to the bare gluon propagator, the bare three- and four-gluon vertices involving background and quantum gluons depend on the (quantum) gauge-fixing parameter. Notice, however, that, since there are no background ghosts, the Feynman rules relevant for the ghost sector are identical to both the covariant gauges and the BFM. Therefore, the analysis and the results presented in this article carries over directly to the conventional Feynman gauge.

In this article we demonstrate that the ghost propagator in the Feynman gauge can be made finite in the IR, through the self-consistent treatment of the gluon-ghost vertex and the ghost gap equations. The key ingredient that makes this possible is the "longitudinal" form factor in the tensorial decomposition of the gluon-ghost vertex $\mathbf{\Gamma}_{\mu}^{bcd}(p,q,k)$, i.e. the cofactor of k_{μ}, where k is the four momentum of the gluon; evidently this term gets annihilated when contracted with the usual transverse projection operator. As we will explain in detail, this component acquires a special role for all values of the gauge-fixing parameter, with the very characteristic exception of the Landau gauge. The reason is simply that in the Landau gauge the entire gluon propagator is transverse, both its self-energy and its free part, whereas for any other value of the gauge-fixing parameter the free part is not transverse. As a result, when the gluon-ghost vertex is inserted into the SDE for the ghost propagator $D(p^2)$, its part proportional to k_{μ} dies when contracted with the gluon propagator in the Landau gauge; however, in any other gauge it survives due to the free part of the gluon propagator. The resulting contribution has the additional crucial property of not vanishing as the external momentum of the ghost goes to zero. Therefore, contrary to what happens in the Landau gauge where only the part of the vertex proportional to p_{μ} survives, one does *not* need to assume the presence of massless pole terms of the form $1/p^2$ in order to obtain a nonvanishing value for $D^{-1}(0)$. Instead, the only requirement is that the longitudinal form factor simply does not vanish in that limit.

The paper is organized as follows: In Sec. II, we set up the SDE for the ghost propagator, assuming the most general Lorentz structure for the fully dressed gluon-ghost vertex $\mathbf{\Gamma}_{\mu}^{bcd}(p,q,k)$. We then discuss under what condition the resulting expression may yield a finite value for $D^{-1}(0)$, and analyze the profound differences between the Landau- and the Feynman-type of gauges. In Sec. III, we first derive the gluon-ghost vertex under certain simplifying assumptions, and discuss in detail the approximations employed. Next, we study its nonperturbative solutions employing various physically motivated, IR-finite Ansätze for the gluon and ghost propagators. In Sec. IV, we combine the results of the previous two sections, deriving the self-consistency condition necessary for the system of equations to be simultaneously satisfied. Finally, in Sec. V, we discuss our results and present our conclusions.

II. GENERAL CONSIDERATIONS ON THE IR BEHAVIOR OF THE GHOST

In this section, we derive the SDE for the ghost propagator $D(p^2)$ in a general covariant gauge, and study qualitatively its predictions for $D(0)$ for various gauge choices. In particular, we establish that away from the Landau gauge the ghost propagator may acquire a finite value at the origin, without the need to assume a singular IR behavior for the form factors of the fully dressed ghost-gluon vertex entering into the SDE. Our attention will eventually focus on the Feynman gauge, which, as mentioned in the Introduction, is singled out within the PT-BFM scheme.

The full ghost propagator $D^{ab}(p)$ is usually written in the form

$$D^{ab}(p) = i\delta^{ab}D(p), \tag{2.1}$$

and the SDE satisfied by $D(p^2)$, depicted diagrammatically in Fig. 1, reads

$$D^{-1}(p^2) = p^2 + iC_{\rm A}g^2\int[dk]\Gamma^{\nu}\Delta_{\mu\nu}(k)\mathbf{\Gamma}^{\mu}(p,p+k,k) \times D(p+k). \tag{2.2}$$

We have used $f^{acd}f^{bcd} = \delta^{ab}C_{\rm A}$, with $C_{\rm A}$ the Casimir eigenvalue in the adjoint representation [$C_{\rm A} = N$ for $SU(N)$], and have introduced the shorthand notation $[dk] = d^dk/(2\pi)^d$, where $d = 4 - \epsilon$ is the dimension of spacetime used in dimensional regularization. $\Delta_{\mu\nu}(k)$ is the fully dressed gluon propagator, whereas $\mathbf{\Gamma}$ denotes the fully dressed gluon-ghost vertex, and Γ its tree-level value.

Specifically, in the covariant gauges the full gluon propagator $\Delta_{\mu\nu}^{df}(k) = -i\delta^{df}\Delta_{\mu\nu}(k)$ has the general form

$$\Delta_{\mu\nu}(k) = \left[{\rm P}_{\mu\nu}(k)\Delta(k^2) + \xi\frac{k_{\mu}k_{\nu}}{k^4}\right], \tag{2.3}$$

where

$${\rm P}_{\mu\nu}(k) = g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2}, \tag{2.4}$$

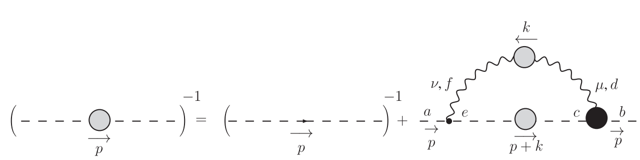

FIG. 1. The SDE of the ghost propagator.

is the transverse projector, and ξ is the gauge-fixing parameter; $\xi = 1$ corresponds to the Feynman gauge and $\xi = 0$ to the Landau gauge. The scalar function $\Delta(k^2)$ is related to the all-order gluon self-energy $\Pi_{\mu\nu}(k)$

$$\Pi_{\mu\nu}(k) = \mathrm{P}_{\mu\nu}(k)\Pi(k^2) \tag{2.5}$$

through

$$\Delta^{-1}(k^2) = k^2 + i\Pi(k^2). \tag{2.6}$$

The bare gluon-ghost vertex appearing in (2.2) is given by $\Gamma_\mu^{eaf} = -gf^{eaf}q_\mu$, with $(q = p + k)$. Choosing p_μ and k_μ as the two linearly independent four vectors, the most general decomposition for the fully dressed gluon-ghost vertex $\mathbf{\Gamma}_\nu^{bcd}(p, q, k)$ is expressed as [20]

$$\begin{aligned}\mathbf{\Gamma}_\mu^{bcd}(p, q, k) &= -gf^{bcd}\mathbf{\Gamma}_\mu(p, q, k),\\ \mathbf{\Gamma}_\mu(p, q, k) &= A(p^2, q^2, k^2)p_\mu + B(p^2, q^2, k^2)k_\mu,\end{aligned} \tag{2.7}$$

where k is the outgoing gluon momentum, and p, q the outgoing and incoming ghost momenta, respectively. The dimensionless scalar functions $A(p^2, q^2, k^2)$ and $B(p^2, q^2, k^2)$ are the form factors of the gluon-ghost vertex. In particular, notice that the tree-level result is recovered when we set $A(p^2, q^2, k^2) = 1$ and $B(p^2, q^2, k^2) = 0$. Finally, it is important to emphasize that all fully dressed scalar quantities (D, Δ, A, and B) depend explicitly (and nontrivially) on the value of the gauge-fixing parameter ξ already at the level of one-loop perturbation theory.

It is then straightforward to derive the Euclidean version of Eq. (2.2); to that end, we set $p^2 = -p_\mathrm{E}^2$, define $\Delta_\mathrm{E}(p_\mathrm{E}^2) = -\Delta(-p_\mathrm{E}^2)$, and $D_\mathrm{E}(p_\mathrm{E}^2) = -D(-p_\mathrm{E}^2)$, and for the integration measure we have $[dk] = i[dk]_\mathrm{E} = i\, d^4k_\mathrm{E}/(2\pi)^4$. Suppressing the subscript "E" everywhere except in the integration measure, and without any assumptions on the functional form of $A(p^2, q^2, k^2)$ and $B(p^2, q^2, k^2)$, the ghost SDE of Eq. (2.2) becomes

$$\begin{aligned}D^{-1}(p^2) = p^2 - C_\mathrm{A}g^2 \int [dk]_\mathrm{E}\bigg[p^2 - \frac{(p\cdot k)^2}{k^2}\bigg]\\ \times A(p^2, q^2, k^2)\Delta(k)D(p+k) - C_\mathrm{A}g^2\xi\\ \times \int [dk]_\mathrm{E}\frac{p\cdot k}{k^2}\bigg[A(p^2, q^2, k^2) + B(p^2, q^2, k^2)\\ + \frac{p\cdot k}{k^2}A(p^2, q^2, k^2)\bigg]D(p+k) - C_\mathrm{A}g^2\xi\\ \times \int [dk]_\mathrm{E}B(p^2, q^2, k^2)D(p+k),\end{aligned} \tag{2.8}$$

As a check, we can recover from (2.8) the one-loop result for the ghost propagator in the Feynman gauge ($\xi = 1$) by substituting the tree-level expressions for the ghost and gluon propagators and setting $A(p^2, q^2, k^2) = 1$ and $B(p^2, q^2, k^2) = 0$; specifically,

$$D^{-1}(p^2) = p^2\bigg[1 + \frac{C_A g^2}{32\pi^2}\ln\bigg(\frac{p^2}{\mu^2}\bigg)\bigg]. \tag{2.9}$$

In order to obtain from (2.8) the behavior of $D(p^2)$ for the full range of the momentum p^2, one needs to provide additional information for the forms factors $A(p^2, q^2, k^2)$ and $B(p^2, q^2, k^2)$, obtained from the corresponding SDE satisfied by the gluon-ghost vertex. Thus, the complete treatment of this problem would require the solution of a complicated system of coupled SDE. However, several interesting conclusions about the IR behavior of $D(p^2)$ may be drawn by considering the qualitative behavior of the forms factors $A(p^2, q^2, k^2)$ and $B(p^2, q^2, k^2)$ as $p \to 0$.

We start by considering what happens in the Landau gauge. First of all, let us assume that the various quantities appearing on the right-hand side of (2.8) are regular functions of ξ [21]. Then, if we set $\xi = 0$, only the first integral on the right-hand side of (2.8) survives; thus, $D^{-1}(p^2)$ is only affected by the functional form of $A(p^2, q^2, k^2)$. In particular, the behavior of $D(p^2)$ as $p \to 0$ will depend on whether $A(p^2, q^2, k^2)$ is divergent or finite in that limit, i.e. on whether or not $A(p^2, q^2, k^2)$ contains $(1/p^2)$ terms. Evidently, if $A(p^2, q^2, k^2)$ does not contain poles, one has that $\lim_{p\to 0}D^{-1}(0) = 0$, and therefore the ghost propagator will be divergent in the IR. On the other hand, if $A(p^2, q^2, k^2)$ contains $(1/p^2)$ terms, $\lim_{p\to 0}D^{-1}(0) \neq 0$ allowing for finite solutions for the ghost propagator.

According to this general argument, the only way for getting an IR-finite propagator in the Landau gauge is by assuming that $A(p^2, q^2, k^2)$ contains poles [22,23]. However, lattice simulations in the Landau gauge seem to favor a IR-finite $A(p^2, q^2, k^2)$; specifically, it was found that deviations of the gluon-ghost vertex from its tree-level value are very small in the IR, i.e. $A(p^2, q^2, k^2) \approx 1$ [24]. In addition, a detailed study of the SDE equation for $\mathbf{\Gamma}$ in the same gauge shows no singular behavior for $A(p^2, q^2, k^2)$ [25]. These findings appear to be consistent with recent lattice results on the nonperturbative structure of the ghost propagator, which indicate that $D^{-1}(p^2)$ in the Landau gauge diverges at a rate that deviates only mildly from the tree-level expectation of $1/p^2$ [19].

Evidently, the picture for $\xi \neq 0$ is drastically different. Indeed, away from the Landau gauge the right-hand side of (2.8) involves both form factors, $A(p^2, q^2, k^2)$ and $B(p^2, q^2, k^2)$. Moreover, unlike the first two terms, the third one does not contain any kinematic factors proportional to p. Thus, in order for it not to vanish as $p \to 0$ one does not need to assume any singular structure for $B(p^2, q^2, k^2)$; instead, it is sufficient to simply have that $B(0, k^2, k^2) \neq 0$.

After this key observation, we will take the limit of Eq. (2.8) as $p \to 0$, assuming that $A(p^2, q^2, k^2)$ does not contain $(1/p^2)$ terms. Focusing for concreteness on the physically relevant case of $\xi = 1$, we find that in the aforementioned kinematic limit Eq. (2.8) reduces to

$$D^{-1}(0) = -C_A g^2 \int [dk]_{\rm E} B(0, k^2, k^2) D(k). \tag{2.10}$$

Of course, if the assumption that $A(p^2, q^2, k^2)$ is regular as $p \to 0$ does not hold, then the other integrals will also contribute to the right-hand side of (2.10). However, modulo the rather contrived scenario of fine-tuned cancellations, the right-hand side will still be different from zero. Evidently, from (2.10) we deduce that if $B(0, k^2, k^2) = 0$ then $D^{-1}(0) = 0$. On the other hand, if $B(0, k^2, k^2) \neq 0$, i.e. if it does not vanish identically, then one may have a nonvanishing $D^{-1}(0)$. Of course, having a nonvanishing $B(0, k^2, k^2)$ is not a sufficient condition for $D^{-1}(0) \neq 0$; one has to assume in addition that (i) the integral on the right-hand side of (2.10). is convergent, or it can be made convergent through proper regularization, and (ii) that the integral is not zero due to some other, rather contrived circumstances [for instance, if $B(0, k^2, k^2)$ turned out not to be a monotonic function, the various contributions from different integration regions could cancel against each other].

An explicit calculation may confirm that $B(0, k^2, k^2)$ vanishes at one loop [26], and it is reasonable to expect this to persist to all orders in perturbation theory. Therefore, in what follows we will examine the possibility that $B(0, k^2, k^2)$ may not vanish nonperturbatively. In particular, we will study the SDE determining $B(p^2, q^2, k^2)$ for the special kinematic configuration appearing in (2.10), namely, where the outgoing ghost momentum p is set equal to zero (i.e. $p = 0$ and $q = k$). In the context of the linearized approximation that we employ in the next section this kinematic configuration offers the particular technical advantage of dealing with a function of only one variable instead of two.

III. THE GLUON-GHOST VERTEX

In this section, we set up and solve, after certain simplifying approximations, the SDE governing the behavior of the form factor $B(0, k^2, k^2)$. This can be done by taking the following limit of the gluon-ghost vertex $\Gamma_\mu(p, q, k)$

$$B(0, k^2, k^2) = \lim_{p\to 0}\left[\frac{1}{k^2} k^\mu \Gamma_\mu(p, q, k)\right], \tag{3.1}$$

where $\Gamma_\mu(p, q, k)$ obeys the SDE [7] represented in Fig. 2.

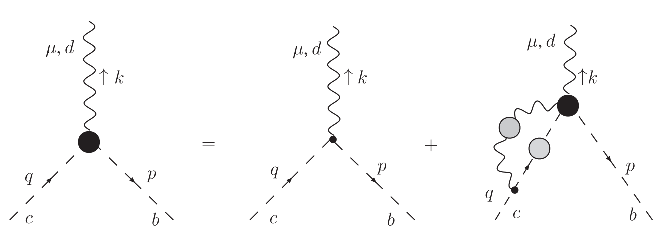

FIG. 2. SDE for the gluon-ghost vertex.

We next introduce some approximations regarding the form of the two-ghost-two-gluon scattering kernel, appearing on the right-hand side of Fig. 2. The first approximation is to keep only the lowest-order contributions in its skeleton expansion, i.e. we expand the aforementioned kernel in terms of the 1PI fully dressed three-particle vertices of the theory, neglecting diagrams that contain four-point functions.

We then arrive at the truncated SDE shown in Fig. 3, which reads

$$\Gamma_\mu^{bcd}(p, q, k) = \Gamma_\mu^{bcd} + \Gamma_\mu^{bcd}(p, q, k)|_{a_1} + \Gamma_\mu^{bcd}(p, q, k)|_{a_2}, \tag{3.2}$$

where the closed expressions corresponding to the diagrams (a_1) and (a_2) are given by

$$\begin{aligned}\Gamma_\mu^{bcd}|_{a_1} &= \int [dl] \Gamma_\mu^{emd}(l+p, l+q, k) D_{ee'}(l+p) \\ &\quad \times \Gamma_{\nu'}^{be'n'}(p, l+p, l) \Delta_{nn'}^{\nu\nu'}(l) \Gamma_\nu^{m'cn} D_{mm'}(l+q), \\ \Gamma_\mu^{bcd}|_{a_2} &= \int [dl] \Gamma_{\mu\nu\sigma}^{dem}(-k, q-l, l-p) \Delta_{mm'}^{\sigma\sigma'}(l-p) \\ &\quad \times \Gamma_{\sigma'}^{bn'm'}(p, l, l-p) D_{nn'}(l) \Gamma_{\nu'}^{nce'} \Delta_{ee'}^{\nu\nu'}(l-q),\end{aligned} \tag{3.3}$$

with the momentum routing as given in Fig. 3.

Our next approximation is to linearize the equation by substituting in (3.3) $\Gamma_\mu^{emd}(l+p, l+q, k)$ and $\Gamma_{\mu\nu\sigma}^{dem}(-k, q-l, l-p)$ by their bare, tree-level expressions. Since we are eventually interested in the limit of the equation as $p \to 0$, this amounts finally to the replacement

$$\begin{aligned}\Gamma_\mu^{emd}(l+p, l+q, k) &\to -g f^{emd} l_\mu, \\ \Gamma_{\mu\nu\sigma}^{dem}(-k, q-l, l-p) &\to g f^{dem}[(2l-k)_\mu g_{\nu\sigma} \\ &\quad - (k+l)_\nu g_{\mu\sigma} + (2k-l)_\sigma g_{\mu\nu}]\end{aligned} \tag{3.4}$$

in diagrams (a_1) and (a_2), respectively. The diagrammatic representation of the resulting contributions at $p \to 0$ is given in Fig. 4.

Factoring out the color structure by using the standard identity $f^{axm} f^{bmn} f^{cnx} = \frac{1}{2} C_A f^{abc}$, it is easy to verify that in the limit $p \to 0$ the linearized version of Eq. (3.3) reads

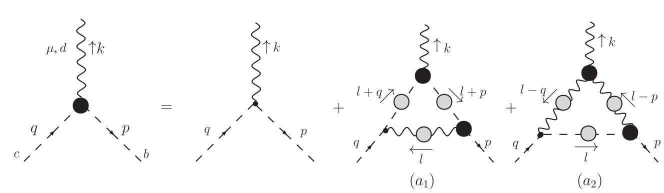

FIG. 3. Truncated version of the SDE for the gluon-ghost vertex.

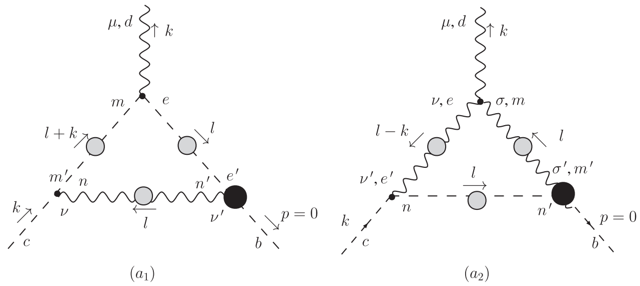

FIG. 4. Contributions for the gluon-ghost vertex equation in the limit of $p \to 0$.

$$
\begin{aligned}
\mathbf{\Gamma}_\mu^{bcd}(0,k,k)|_{a1} &= if^{bcd}\frac{C_A g^3}{2} \\
&\quad\times \int [dl] l_\mu (l+k)_{\nu'} l_\nu \Delta^{\nu\nu'}(l) B(0,l^2,l^2) \\
&\quad\times D(l)D(l+k), \\
\mathbf{\Gamma}_\mu^{bcd}(0,k,k)|_{a2} &= -if^{bcd}\frac{C_A g^3}{2} \\
&\quad\times \int [dl] \Gamma_{\mu\nu\sigma} l_{\nu'} l_{\sigma'} \Delta^{\sigma\sigma'}(l)\Delta^{\nu\nu'}(l-k) \\
&\quad\times B(0,l^2,l^2)D(l).
\end{aligned}
\tag{3.5}
$$

Since the bare gluon ghost is proportional to p_μ, it follows immediately from Eqs. (3.1), (3.2), and (3.5), that

$$
\begin{aligned}
B(0,k^2,k^2) &= \frac{k^\mu}{k^2}[\mathbf{\Gamma}_\mu(0,k,k)|_{a_1} + \mathbf{\Gamma}_\mu(0,k,k)|_{a_2}], \\
k^\mu \mathbf{\Gamma}_\mu(0,k,k)|_{a_1} &= -\frac{i}{2}C_A g^2 \int [dl]\left[k\cdot l + \frac{(k\cdot l)^2}{l^2}\right] \\
&\quad\times B(0,l^2,l^2)D(l)D(l+k), \\
k^\mu \mathbf{\Gamma}_\mu(0,k,k)|_{a_2} &= +\frac{i}{2}C_A g^2 \int [dl]\left[\frac{(k\cdot l)^2}{l^2} - k^2\right] \\
&\quad\times B(0,l^2,l^2)D(l)\Delta(l+k).
\end{aligned}
\tag{3.6}
$$

The Euclidean version of (3.6) can be easily derived using the same rules as before, leading to

$$
\begin{aligned}
B(0,k^2,k^2) = -\frac{C_A g^2}{32\pi^4}\bigg\{ & \frac{1}{k^2}\int d^4 l \frac{(k\cdot l)^2}{l^2} B(0,l^2,l^2)D(l) \\
&\times [D(l+k) - \Delta(l+k)] \\
&+ \int d^4 l B(0,l^2,l^2)D(l)\Delta(l+k) + \frac{1}{k^2} \\
&\times \int d^4 l (k\cdot l) B(0,l^2,l^2)D(l)D(l+k)\bigg\}.
\end{aligned}
\tag{3.7}
$$

It is convenient to express the measure in spherical coordinates

$$
\int d^4 l = 2\pi \int_0^\pi d\chi \sin^2\chi \int_0^\infty dy y; \tag{3.8}
$$

and rewrite (3.7) in terms of the new variables $x \equiv k^2$, $y \equiv l^2$, and $z \equiv (l+k)^2$. In order to convert Eq. (3.7) into a one-dimensional integral equation, we resort to the standard angular approximation defined as

$$
\int_0^\pi d\chi \sin^2\chi f(z) \approx \frac{\pi}{2}[\theta(x-y)f(x) + \theta(y-x)f(y)], \tag{3.9}
$$

where $\theta(x)$ is the Heaviside step function.

Then, introducing the above change of variables and using Eqs. (3.8) and (3.9) in (3.7), we arrive at the following linear and homogeneous equation:

$$
\begin{aligned}
B(0,x,x) = \frac{C_A g^2}{128\pi^2}\bigg\{ & \frac{1}{x}[D(x) - \Delta(x)]\int_0^x dy y^2 B(0,y,y)D(y) \\
&+ \int_x^\infty dy (x-2y) B(0,y,y)D(y)[D(y) - \Delta(y)] \\
&+ 2\int_x^\infty dy y B(0,y,y)D(y)\Delta(y) - \frac{2}{x}D(x) \\
&\times \int_0^x dy y^2 B(0,y,y)D(y) + 4\Delta(x) \\
&\times \int_0^x dy y B(0,y,y)D(y)\bigg\}.
\end{aligned}
\tag{3.10}
$$

Because of the linear nature of (3.10) it is evident that if B is one solution then the entire family of functions cB, generated by multiplying B by an arbitrary constant c, are also solutions.

Before embarking into the numerical treatment of (3.10), it is useful to study the asymptotic solution that this equation furnishes for $x \to \infty$. In this limit one can safely replace the various propagators appearing on the right-hand side of (3.10) by their tree-level values, i.e. $\Delta(t) \to 1/t$ and $D(t) \to 1/t$ with $(t = x, y)$. Then, the first and second terms vanish, and the leading contribution comes from the third term of (3.10). Specifically, the asymptotic behavior of $B(0,x,x)$ is determined from the integral equation

$$
B(0,x,x) = \lambda \int_x^\infty dy \frac{B(0,y,y)}{y}, \tag{3.11}
$$

where $\lambda = C_A g^2/64\pi^2$. Equation (3.11) can be solved easily by converting it into a first-order differential equation, which leads to the following asymptotic behavior

$$
B(0,x,x) = \sigma x^{-\lambda}, \tag{3.12}
$$

with σ as an arbitrary parameter, with dimension $[M^2]^\lambda$, where M is an arbitrary mass-scale. As we will see in what follows, σ will be treated as an adjustable parameter, whose dimensionality will be eventually saturated by that of the effective gluon mass, or, equivalently, by the QCD mass scale Λ.

With the asymptotic behavior (3.12) at hand, we can solve numerically the integral equation given in (3.10). To do so, we start by specifying the expressions we will use for the gluon and ghost propagators.

As has been advocated in a series of studies based on a variety of approaches, the gluon propagator reaches a finite value in the deep IR [27,28]. This type of behavior has been observed in Landau gauge in previous lattice studies [18], and more recently in new, large-volume simulations [19]. Within the gauge-invariant truncation scheme implemented by the PT, the gluon propagator (effectively in the background Feynman gauge) was shown to saturate in the deep IR [13,14]. The numerical solutions may be fitted very accurately by a propagator of the form

$$\Delta(k^2) = \frac{1}{k^2 + m^2(k^2)}, \tag{3.13}$$

where $m^2(k^2)$ acts as an effective gluon mass, presenting a nontrivial dependence on the momentum k^2. Specifically, the mass displays either a logarithmic running

$$m^2(k^2) = m_0^2\left[\ln\left(\frac{k^2+\rho m_0^2}{\Lambda^2}\right)\Big/\ln\left(\frac{\rho m_0^2}{\Lambda^2}\right)\right]^{-1-\gamma_1}, \tag{3.14}$$

where $\gamma_1 > 0$ is the anomalous dimension of the effective mass, or power-law running of the form

$$m^2(k^2) = \frac{m_0^4}{k^2+m_0^2}\left[\ln\left(\frac{k^2+\rho m_0^2}{\Lambda^2}\right)\Big/\ln\left(\frac{\rho m_0^2}{\Lambda^2}\right)\right]^{\gamma_2-1}, \tag{3.15}$$

with $\gamma_2 > 1$. Which of these two behaviors will be realized is a delicate dynamical problem, and depends, among other things, on the specific form of the full three-gluon vertex employed in the SDE for the gluon propagator (for a detailed discussion see [14]). Here, we will employ both functional forms and study the numerical impact they may have on the solutions of (3.10). A plethora of phenomenological studies favor values of m_0 in the range of 0.5–0.7 GeV.

In addition, when solving (3.10) an appropriate Ansatz for the ghost propagator $D(k^2)$ must also be furnished, given that we are in no position to solve the ghost SDE of (2.8) for arbitrary values of the momentum, since this would require the solution of a coupled system of several integral equations involving D, A, and B, for arbitrary values of the four momenta. Given that our aim is to study the self-consistent realization of an IR-finite ghost propagator, it is natural to employ an Ansatz in close analogy to (3.13), namely,

$$D(k^2) = \frac{1}{k^2 + M^2(k^2)}, \tag{3.16}$$

where $M^2(k^2)$ stands for a dynamically generated, effective "ghost mass." Evidently, $D^{-1}(0) = M^2(0)$, and $D^{-1}(0) \neq 0$ provided that $M^2(0) \neq 0$. Of course, once the corresponding solutions for $B(0, x, x)$ have been obtained the self-consistency of the Ansatz for $M^2(k^2)$ must be verified. The way this will be done in the next section is by substituting $B(0, x, x)$ into the (properly regularized) integral on the right-hand side of Eq. (2.10), and then demanding that its value is equal to the $M^2(0)$ appearing on the left-hand side.

For the actual momentum dependence of the effective ghost mass $M(k^2)$ we will assume three different characteristic behaviors and will analyze the sensitivity of $B(0, x, x)$ on them.

We will employ the following three types of $M(k^2)$:

(i) "hard mass," i.e. a constant mass with no running

$$M^2(k^2) = M_0^2, \tag{3.17}$$

(ii) logarithmic running of the form

$$M^2(k^2) = M_0^2\left[\ln\left(\frac{k^2+\rho M_0^2}{\Lambda^2}\right)\Big/\ln\left(\frac{\rho M_0^2}{\Lambda^2}\right)\right]^{-1-\kappa_1}, \tag{3.18}$$

(iii) power-law running, given by

$$M^2(k^2) = \frac{M_0^4}{k^2+M_0^2} \times\left[\ln\left(\frac{k^2+\rho M_0^2}{\Lambda^2}\right)\Big/\ln\left(\frac{\rho M_0^2}{\Lambda^2}\right)\right]^{\kappa_2-1}. \tag{3.19}$$

Clearly, the last two possibilities (3.18) and (3.19) are exactly analogous to the corresponding two types of running of the gluon mass (3.14) and (3.15), respectively.

We then solve numerically Eq. (3.10) using the gluon and ghost propagators given by Eqs. (3.13) and (3.16), respectively, supplemented by the various types of running for $m^2(k^2)$ and $M^2(k^2)$. The integration range is split in two regions, $[0, s]$ and $(s, \infty]$, where $s \gg \Lambda^2$. For the second interval, we impose the asymptotic behavior of (3.12), choosing a value for σ.

It turns out that the numerical solution obtained for $B(0, x, x)$ is rather insensitive to the form of the gluon mass employed, and it mainly depends on the form of the ghost propagator. More specifically, we can fit the numerical solution with an impressive accuracy by means of the simple, physically motivated function

$$B(0, x, x) = \frac{\sigma}{[x + M^2(x)]^{\lambda}}, \tag{3.20}$$

regardless of the form of momentum dependence employed for $M^2(x)$. Evidently, for large values of x, the above expression goes over the asymptotic solution of Eq. (3.12). In Fig. 5, we present a typical solution for $B(0, x, x)$ together with the fit given by (3.20).

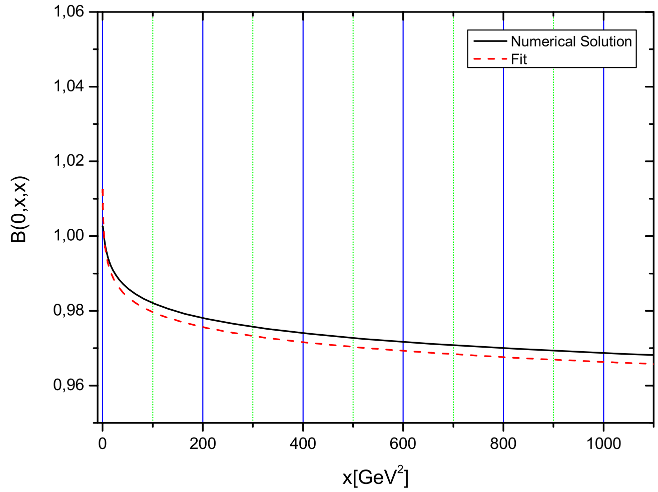

FIG. 5 (color online). The black solid line is the numerical solution of Eq. (3.10), assuming logarithmic type of running for $m^2(k^2)$ and $M^2(k^2)$, with $\gamma_1 = \kappa_1 = 0.6$, $m_0^2 = 0.35$ GeV^2, $M_0^2 = 0.4$ GeV^2, $\rho = 4$, and $\sigma^{1/\lambda} = 1.000\,04$ GeV^2. The red dashed line represents the fit of Eq. (3.20); the relative difference between the two curves is less than 1% and the reduced χ^2 is $\chi^2/\text{dof} = 2 \times 10^{-5}$ (note the fine spacing of the y axis).

IV. INFRARED-FINITE GHOST PROPAGATOR

In the previous section, we have obtained the general solutions for $B(0, x, x)$, under the assumption that the ghost propagator was finite in the IR, and more specifically, that it was given by the general form of (3.16). The next crucial step consists in substituting the solutions obtained for $B(0, x, x)$ into (2.10) and in examining under what conditions the two-hand sides of the equation can be made to be equal. As we will see, this procedure will eventually boil down to constraints on the values that one is allowed to choose for the free parameter σ.

Substituting Eqs. (3.16) and (3.20) into (2.10), we arrive at

$$D^{-1}(0) = -C_A g^2 \sigma \int [dk] \frac{1}{[k^2 + M^2(k^2)]^{1+\lambda}}. \qquad (4.21)$$

The left-hand side of (4.21) is simply given by

$$D^{-1}(0) = M_0^2 \qquad (4.22)$$

for any form of $M^2(k^2)$. Let us first verify the self-consistency of (4.21) for the case where the ghost mass vanishes identically, i.e. $M^2(k^2) = 0$. Then, (4.21) reduces to nothing but the standard dimensional regularization result [29]

$$\int [dk](k^2)^{-\alpha} = 0, \qquad (4.23)$$

valid for any value of α, for the special value $\alpha = 1 + \lambda$.

For nonvanishing $M^2(k^2)$, the integral on the right-hand side of (4.21) is UV divergent: at large k^2 it goes as $(\Lambda_{UV})^{1-\lambda}$, where Λ_{UV} is a UV-momentum cutoff. It turns out that the right-hand side can be made UV finite by simply subtracting from it its perturbative value, i.e. the vanishing integral of (4.23) [30].

Carrying out this regularization procedure explicitly, one obtains

$$\begin{aligned} M_0^2 &= -C_A g^2 \sigma \int [dk] \left(\frac{1}{[k^2 + M^2(k^2)]^{1+\lambda}} - \frac{1}{(k^2)^{1+\lambda}} \right) \\ &= -C_A g^2 \sigma \int \frac{[dk]}{[k^2 + M^2(k^2)]^{1+\lambda}} \\ &\quad \times \left(1 - \left[1 + \frac{M^2(k^2)}{k^2} \right]^{1+\lambda} \right). \end{aligned} \qquad (4.24)$$

It is now elementary to verify that the integral on the right-hand side of (4.24) converges. At large k^2, we can expand the second term in the parenthesis and neglecting in the denominator $M^2(k^2)$ next to k^2, we find that the resulting integral (apart of multiplicative factors) is given by

$$\int dy \frac{M^2(y)}{y^{1+\lambda}}. \qquad (4.25)$$

Notice that the above integral converges even for the less favorable case of a constant $M^2(y)$; then, (4.25) is proportional to $y^{-\lambda}$, and is therefore convergent, since $\lambda > 0$. Clearly, when $M^2(y)$ drops off in the UV, as described by (3.18) or (3.19), the integral converges even faster. Next, we will analyze separately what happens for each one of the three different Ansätze we have employed for $M^2(y)$, Eqs. (3.17), (3.18), and (3.19).

The case of a constant ghost mass can be easily worked out. Replacing $M^2(k^2) \to M_0^2$ in Eq. (4.24), keeping only the leading contribution to the integral, we arrive at (notice the cancellation of the coupling constant g^2 appearing in front of the integral)

$$M_0^2 = \frac{4\sigma}{1-\lambda} M_0^{2(1-\lambda)}. \qquad (4.26)$$

Then, in order to enforce the equality of both sides of (4.26) σ must satisfy

$$\sigma = \frac{(1-\lambda)}{4} M_0^{2\lambda}. \qquad (4.27)$$

Evidently, σ depends very weakly on M_0, and its value is practically fixed at $1/4$. Indeed, given that λ is a small number, of the order of $\mathcal{O}(10^{-2})$, Eq. (4.27) may be expanded as

$$\sigma \approx \frac{(1-\lambda)}{4} \Lambda^{2\lambda} \left[1 + \lambda \ln\left(\frac{M_0^2}{\Lambda^2} \right) \right], \qquad (4.28)$$

from where it is clear that σ can only assume values slightly different of $1/4$. In Fig. 6, we show this mild dependence of σ on M_0 for $\Lambda = 300$ MeV.

We next turn to the case where $M^2(y)$ displays the logarithmic or power-law dependence on the momentum, described by Eqs. (3.18) and (3.19), respectively. Now the

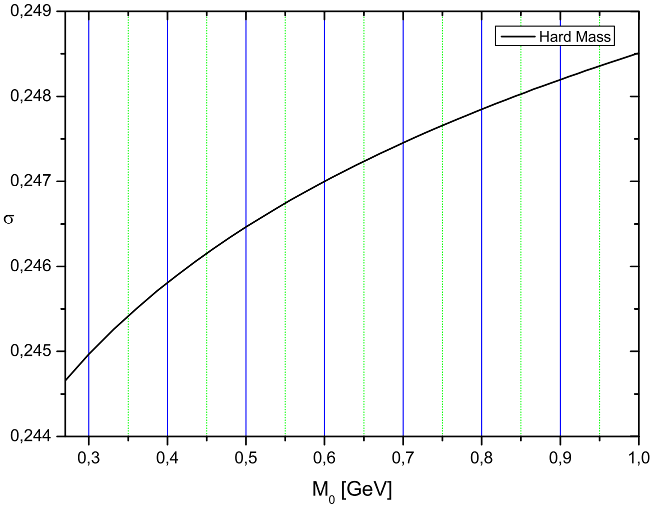

FIG. 6 (color online). σ as a function of the hard ghost mass M_0, obtained from Eq. (4.27).

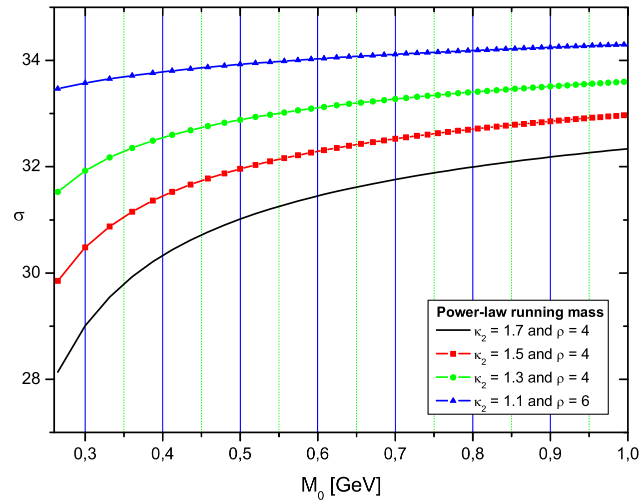

FIG. 8 (color online). σ as function of M_0, when the power-law running of Eq. (3.19) is assumed for $M^2(k^2)$.

integrals cannot be carried out analytically and have been computed numerically. Choosing different values for κ_1, κ_2, and ρ, we obtain the curves presented in Figs. 7 and 8, showing the dependence of σ on M_0.

Several observations are in order:

(i) For both types of running the results show a stronger dependence on M_0 than in the case of the hard mass.

(ii) The range of possible values for σ increases significantly. Whereas in the case of constant mass one was practically restricted to a unique value for σ, namely, $\sigma \approx 1/4$ (viz. Figure 6), now one may obtain self-consistent solutions choosing values for σ over a much wider interval.

(iii) There is a qualitative difference between the logarithmic and power-law running: in the former case, σ is a decreasing function of M_0, while in the latter it is increasing. This offers the particularly interesting possibility of finding values for σ that furnish self-consistent solutions for either types of running of $M^2(k^2)$.

A characteristic example where Eq. (4.24) is satisfied for the same value of M_0 for both types of running is shown in Fig. 9: for $\sigma \approx 20$, one may generate a ghost mass of $M_0 \approx 560$ MeV, assuming for $M^2(k^2)$ either the logarithmic running of Eq. (3.18), or the power-law running of Eq. (3.19).

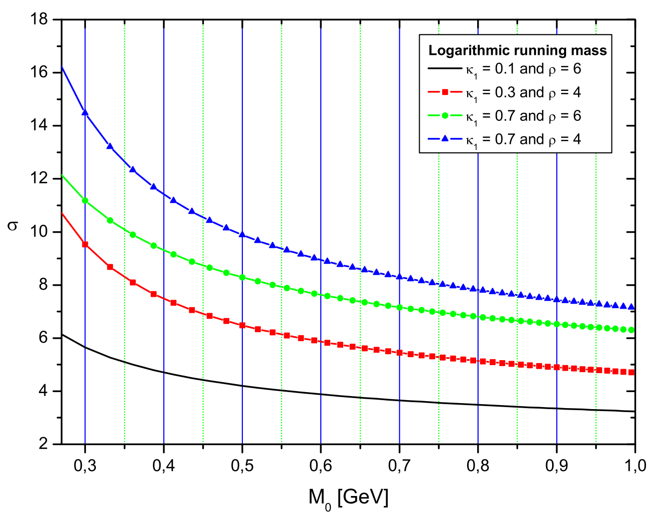

FIG. 7 (color online). σ as function of M_0, when $M^2(k^2)$ runs logarithmically, as in Eq. (3.18).

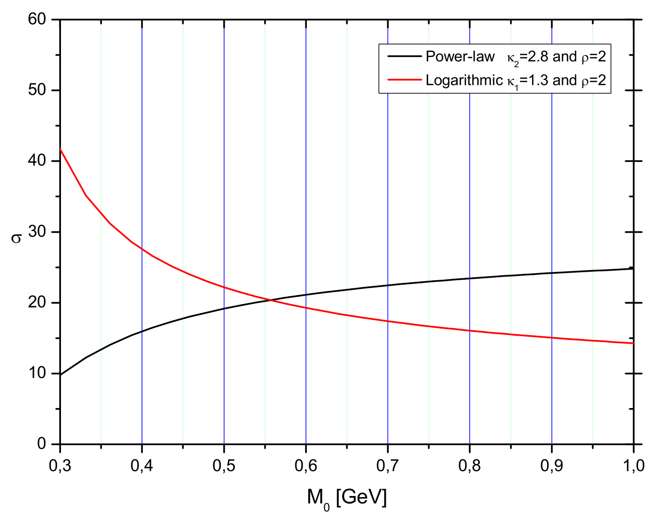

FIG. 9 (color online). For $\sigma = 20$, a ghost mass can be generated from Eq. (4.24) for either type of running.

V. DISCUSSION AND CONCLUSIONS

In this article we have demonstrated that it is possible to obtain from the SDEs of QCD an IR-finite ghost propagator in the Feynman gauge. In this construction, the longitudinal component of the gluon-ghost vertex, which is inert in the Landau gauge, assumes a central role, allowing for $D(0)$ to be finite. This is accomplished without having to assume any special properties of the form factor, other than a nonvanishing limit in the IR; in particular, we do not need to impose the presence of massless poles of the type $1/p^2$.

Our procedure may be summarized as follows: First of all, since we are interested in the possibility of obtaining $D^{-1}(0) \neq 0$, we have focused on the form of the ghost gap equation in the limit of vanishing external momentum $p \to 0$. Next, we have set up the SDE for the form factor $B(p^2, q^2, k^2)$ by considering the general SDE for the gluon-ghost vertex and projecting out the relevant tensorial structure. In order to finally arrive at a manageable equation for $B(p^2, q^2, k^2)$, we have made several approximations to the SDE for the gluon-ghost vertex. Specifically, (i) the four-point interactions have been omitted in the expansion of the gluon-ghost scattering kernel, and (ii) with the exception of the gluon-ghost vertex under study, all other fully dressed three-particle vertices have been replaced by their bare, tree-level expressions. Then, we have taken the $p \to 0$ limit of the integral equation, since this is the relevant limit for the ghost-gap equation. As a result, the final integral equation governing $B(p^2, q^2, k^2)$, namely, Eq. (3.7), is linearized, and, at the same time, $B(p^2, q^2, k^2)$ gets converted into a function of one variable only, $B(0, k^2, k^2)$, instead of two. The next approximation we use is the so-called angular approximation, given in Eq. (3.9), which allows us to reduce the momentum integration into a one-dimensional integral. The resulting integral equation involves, in addition to $B(0, k^2, k^2)$, the fully dressed gluon and ghost propagators. We have first considered the solution of the integral equation for asymptotically large values of the external momentum by setting all propagators on the right-hand side to their tree-level values, thus converting it to a simple differential equation. The family of asymptotic solutions is given in Eq. (3.12), parametrized by a dimensionful parameter σ. To obtain solutions for the entire range of momenta we employ IR-finite Ansätze, inspired by previous studies as well as recent lattice results. Specifically, we assume that the gluon and ghost propagators are effectively "massive," with masses that are generated dynamically, and depend therefore nontrivially on the momentum transfer. Employing two physically motivated types of running for the gluon and ghost masses (logarithmic and power-law running), we have solved the equation numerically, obtaining a solution for $B(0, p^2, p^2)$ from the deep IR to the deep UV. These solutions may be fitted by a particularly simple, physically motivated expression, given in Eq. (3.20).

After solving the integral equation for $B(0, p^2, p^2)$, we substitute the solutions into the ghost-gap equation of Eq. (2.10), arriving at Eq. (4.21). The integral on the right-hand side of Eq. (4.21) is then regularized, by resorting to a well-known result, valid in dimensional regularization. Then, we have computed the regularized integral numerically, thus obtaining the value for $D^{-1}(0) = M_0^2$ predicted by the ghost-gap equation; self-consistency requires that the value of M_0^2 should coincide with the one chosen in the corresponding Ansatz used for the ghost propagator when solving the integral equation for $B(0, p^2, p^2)$. Enforcing the self-consistency essentially boils down to relations between the free parameter σ and the values of $D^{-1}(0)$, or equivalently M_0^2, as captured in Figs. 6–8. These figures furnish the value of M_0 one obtains if a concrete value of σ is chosen, assuming certain characteristic types of running for the ghost-mass function $M^2(k^2)$. The results may be summarized as follows. For the (unphysical) case of a constant mass, the value for σ is practically fixed: unless $\sigma \approx 1/4$, one cannot obtain self-consistent solutions. Things change drastically when considering ghost masses with logarithmic or power-law running, and the range of acceptable values for σ increases significantly. This becomes possible, in part, due to the freedom in adjusting the running of the ghost and the gluon masses by freely choosing the corresponding anomalous dimensions. Notice also that both types of running may coexist for special values of σ, as shown characteristically in Fig. 9. The freedom in choosing the value of σ will be restricted, or completely eliminated, in the nonlinear version of the vertex equation. It would certainly be interesting to venture into such a study, because it is liable to pin down completely the value of $D^{-1}(0)$.

The most immediate physical implication of the results presented here is that the finite gluon propagator obtained in the previous SDE studies in the PT-BFM framework, with the ghost contributions gauge invariantly omitted, will not get destabilized by the inclusion of the ghost loops. Specifically, one would expect that the addition of the ghost loop into the corresponding SDE should not change the qualitative picture. The quantitative changes induced should also be small; mainly the correct coefficient of $11C_A/48\pi^2$ multiplying the renormalization group logarithms will be restored (without the ghosts it is $10C_A/48\pi^2$), and it might inflate or deflate slightly the corresponding solutions for the gluon propagator in the intermediate region between 0.1–1 GeV^2. Of course, a complete analysis of the coupled SDE system is needed in order to fully corroborate this general picture.

Given the complexity and importance of the problem at hand it would certainly be essential to confront these SDE results with lattice simulations of the ghost propagator in the Feynman gauge. In addition, since the formulation of the BFM on the lattice has been presented long ago by

Dashen and Gross (in the Feynman gauge) [31], and has already been used [32,33], one might also consider the possibility of simulating the gluon propagator within that particular gauge-fixing scheme, thus enabling a direct comparison with the SDE results predicting an IR-finite answer.

ACKNOWLEDGMENTS

This work was supported by the Spanish MEC under the Grants No. FPA 2005-01678 and No. FPA 2005-00711. The research of J. P. is funded by the Fundación General of the UV.

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$$\int \frac{[dk]}{k^2 + m^2} - \int \frac{[dk]}{k^2} = -m^2 \int \frac{[dk]}{k^2(k^2 + m^2)}. \qquad (5.1)$$

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