Transverse-momentum-dependent parton distribution function and jet transport in a nuclear medium

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We show that the gauge-invariant transverse-momentum-dependent (TMD) quark distribution function can be expressed as a sum of all higher-twist collinear parton matrix elements in terms of a transport operator. From such a general expression, we derive the nuclear broadening of the transverse-momentum distribution. Under the maximal two-gluon correlation approximation, in which all higher-twist nuclear multiple parton correlations with the leading nuclear enhancement are given by products of twist-two nucleon parton distributions, we find the nuclear transverse-momentum distribution as a convolution of a Gaussian distribution and the nucleon TMD quark distribution. The width of the Gaussian, or the mean total transverse-momentum broadening squared, is given by the path integral of the quark transport parameter $\hat{q}_F$ which can also be expressed in a gauge-invariant form and is given by the gluon distribution density in the nuclear medium. We further show that contributions from higher-twist nucleon gluon distributions can be resummed under the extended adjoint two-gluon correlation approximation and the nuclear transverse-momentum distribution can be expressed in terms of a transverse-scale-dependent quark transport parameter or gluon distribution density. We extend the study to hot medium and compare to dipole model approximation and $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory in the strong coupling limit. We find that multiple gluon correlations become important in the strongly coupled system such as $\mathcal{N} = 4$ SYM plasma.

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I. INTRODUCTION

The success of perturbative QCD (pQCD) in describing hard processes in hadronic interactions relies on the factorization theorem [1] that separates the coherent long distance interaction between projectile and target from the incoherent short distance interactions. The physical observables such as cross sections of deeply inelastic scattering (DIS) and Drell-Yan (DY) dilepton production can be expressed as a convolution of hard partonic scattering cross sections, parton distribution functions and parton fragmentation functions. The hard partonic parts are calculable in a perturbative expansion in the strong coupling constant $\alpha_s(Q^2)$ which becomes small for large momentum scale Q^2 of the hard processes [2,3]. Though the parton distribution and fragmentation functions are not calculable in pQCD since they involve long distance interaction, they are universal and independent of the specific partonic hard processes. Therefore, they can be measured in one hard process and then applied to another, therein lies the predictable power of pQCD.

The most practiced factorization scheme is collinear factorization in which one integrates out the transverse momentum of the initial (final) parton up to a factorization scale and the final observables will only depend on the transverse-momentum-integrated or collinear factorized parton distribution (fragmentation) functions. Such a proof of factorization has also been extended to semi-inclusive processes [4,5] that involve finite transverse momentum of the final hadron or dilepton with the introduction of transverse-momentum-dependent (TMD) parton distribution and fragmentation functions. The final observables can be expressed as a convolution of collinear hard parts (setting the initial parton transverse momenta to zero) and TMD parton distribution and fragmentation functions [6]. Such TMD parton distribution and fragmentation functions are important for the study of hadronic interactions with singly or doubly polarized beams, such as single-spin asymmetry in semi-inclusive processes in DIS (SIDIS) [7–9] and proton-proton scattering.

In the proof of factorization [10], in DIS off a nucleon or nucleus target, for example, one important step is to eikonalize all soft interactions between the struck quark and the target remnant as shown in Fig. 1. The summation of these soft gluon interactions gives rise to a definition of TMD quark distribution function in a nucleon or nucleus,

$$f_q^A(x, \vec{k}_\perp) = \int \frac{dy^-}{2\pi} \frac{d^2 y_\perp}{(2\pi)^2} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \langle A|\bar{\psi}(0, \vec{0}_\perp) \times \frac{\gamma^+}{2} \mathcal{L}_{\rm TMD}(0, y)\psi(y^-, \vec{y}_\perp)|A\rangle, \tag{1}$$

where

$$\mathcal{L}_{\rm TMD}(0, y) \equiv \mathcal{L}_\parallel^\dagger(-\infty, 0; \vec{0}_\perp)\mathcal{L}_\perp^\dagger(-\infty; \vec{y}_\perp, \vec{0}_\perp)\mathcal{L}_\parallel(-\infty, y^-; \vec{y}_\perp) \tag{2}$$

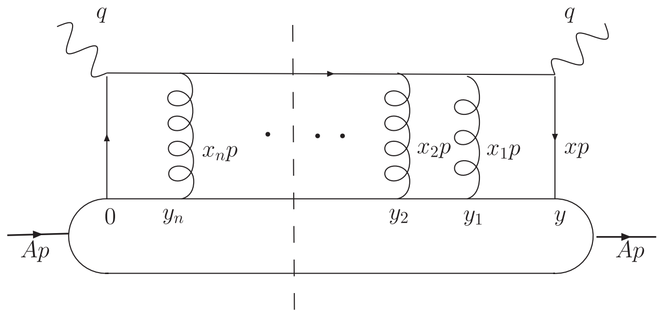

FIG. 1. Multiple soft gluon interaction between the struck quark and the remnant of the target nucleus in DIS.

is the complete gauge link in TMD quark distribution function that contains both the transverse [10]

$$\mathcal{L}_\perp(-\infty; \vec{y}_\perp, \vec{0}_\perp) \equiv P\exp\left[-ig\int_{\vec{0}_\perp}^{\vec{y}_\perp} d\vec{\xi}_\perp \cdot \vec{A}_\perp(-\infty, \vec{\xi}_\perp)\right] \quad (3)$$

and longitudinal gauge link [6]

$$\mathcal{L}_\parallel(-\infty, y^-; \vec{y}_\perp) \equiv P\exp\left[-ig\int_{y^-}^{-\infty} d\xi^- A_+(\xi^-, \vec{y}_\perp)\right]. \quad (4)$$

The above gauge links for quark propagation are defined in the fundamental representation, $A_\mu = A_\mu^a T^a (a = 1 - 8)$. Note that in our convention of the definition of the quark distribution function in Eq. (1), the quark is produced at $(y^-, \vec{y}_\perp)$ [or $(0, \vec{0}_\perp)$] and propagates toward $(-\infty, \vec{y}_\perp)$ $[(-\infty, \vec{0}_\perp)]$. The path ordering along the light cone is then defined from y^- to $-\infty$.

Even though the parton distribution in Eq. (1) describes the probability to find a quark with momentum fraction x and transverse momentum $\vec{k}_\perp$ in a nucleon or nucleus, it also contains information about final state interaction with the target remnant encoded through the gauge links. These gauge links are not only crucial to ensure the gauge invariance of the TMD parton distribution functions in both light-cone and covariant gauge but also lead to physical consequences such as single-spin asymmetry in semi-inclusive DIS and Drell-Yan process [8,10,11]. For DIS off a nucleus target, they should also contain information about transverse-momentum broadening of the struck quark due to multiple scattering inside the nucleus. The main purpose of this work is to study nuclear transverse-momentum broadening in DIS from the TMD quark distribution functions and extend the result to the case of quark or jet propagation in a thermal medium as in high-energy heavy-ion collisions.

In the study of nuclear matter and quark-gluon plasma in high-energy lepton-nucleus, hadron-nucleus and nucleus-nucleus collisions, jet transverse-momentum broadening plays a crucial role in unravelling the medium properties through modification of the final jet or hadron spectra (jet quenching) due to final state interaction between the energetic partons and the nuclear or hot medium [12,13]. Current phenomenological studies of experimental data [14–18] on jet quenching rely on pQCD calculations of the parton energy loss or modification of the parton fragmentation functions due to gluon radiation induced by multiple scattering during the parton propagation in the medium. One important parameter that controls parton energy loss or medium modification of the jet fragmentation function is the jet transport parameter $\hat{q}$ or transverse-momentum broadening squared per unit of propagation length [19–25]. Therefore, calculation and measurement of the jet transport parameter is an important step toward understanding the intrinsic properties of the QCD medium.

There are many calculations of the jet transverse-momentum broadening. Early studies dealt with the mean average transport parameter [21,26] under one particular gauge without apparent guaranty of gauge invariance. Transverse-momentum broadening in Drell-Yan pair production in proton-nucleus scattering [27] and a recent calculation of nuclear transverse-momentum broadening distribution in DIS [28] were obtained by a direct summation of multiple scattering in covariant gauge, again without apparent gauge invariance in the final result. One approach to the parton propagation in medium with a gauge (mostly) invariant framework is the Wilson line formulation of multiple parton scattering [23] that resembles the longitudinal gauge link in Eq. (1), but again in the covariant gauge. Since resummation of all soft gluon interactions is crucial for the gauge-invariant form of the TMD parton distribution function in Eq. (1), it must contain the nuclear transverse-momentum broadening due to multiple parton scattering in nuclei. One should be able to derive a completely gauge-invariant form of the nuclear transverse-momentum broadening from the nuclear TMD quark distribution function. This is what we will prove in this paper. We will derive the nuclear broadening of the transverse-momentum distribution simply from the gauge-invariant form of TMD quark distribution function in Eq. (1) with a given nuclear distribution function. The broadened distribution will have a Gaussian form as found in earlier studies, considering only two-gluon field correlations in the nucleons. However, the broadening parameter in our derivation will have an explicit gauge-invariant form. We will also show that contributions from higher-twist multigluon correlations in a nucleon can be resummed to give a nuclear TMD quark distribution that depends on the transverse-scale-dependent gluon distribution density inside the nucleus.

We will briefly summarize our main results here. To calculate the nuclear transverse-momentum broadening including all higher-twist nuclear parton matrix elements, we first express the gauge-invariant TMD quark distribution function in a nucleus as a sum of collinear higher-twist nuclear parton matrix elements which can be exponentiated to give

$$f_q^A(x_B, \vec{k}_\perp) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \langle A|\bar{\psi}(0, \vec{0}_\perp) \frac{\gamma^+}{2} \mathcal{L}_\parallel(0, y^-; \vec{0}_\perp) e^{\vec{W}_\perp(y^-,\vec{0}_\perp)\cdot\vec{\nabla}_{k_\perp}} \psi(y^-, \vec{0}_\perp)|A\rangle \delta^{(2)}(\vec{k}_\perp), \tag{5}$$

where the transport operator $\vec{W}_\perp(y^-, \vec{y}_\perp)$ is defined as

$$\vec{W}_\perp(y^-, \vec{y}_\perp) \equiv i\vec{D}_\perp(y^-, \vec{y}_\perp) + g \int_{-\infty}^{y^-} d\xi^- \mathcal{L}_\parallel^\dagger(\xi, y^-, \vec{y}_\perp) \vec{F}_{+\perp}(\xi^-, \vec{y}_\perp) \mathcal{L}_\parallel(\xi^-, y^-; \vec{y}_\perp), \tag{6}$$

and $\vec{D}_\perp(y^-, \vec{y}_\perp) = \vec{\partial}_\perp + ig\vec{A}_\perp(y^-, \vec{y}_\perp)$ is the covariant derivative. The transport operator is supposed to operate on both the quark field and the gluon fields within itself and the transverse coordinate is set to $\vec{y}_\perp = \vec{0}_\perp$ after the operation.

With a maximal two-gluon correlation approximation, the high-twist multiparton correlations in a large and weakly bound nucleus can be expressed as products, which have the maximum nuclear size (or medium length) dependence, of twist-two nucleon gluon distribution functions. The leading contribution to the nuclear TMD quark distribution is shown to have a simple form,

$$f_q^A(x_B, \vec{k}_\perp) = A \exp\left[\int d\xi_N^- \hat{q}_F(\xi_N) \frac{\nabla_{k_\perp}^2}{4}\right] f_q^N(x_B, \vec{k}_\perp) = \frac{A}{\pi\Delta_{2F}} \int d^2\ell_\perp e^{-(\vec{k}_\perp - \vec{\ell}_\perp)^2/\Delta_{2F}} f_q^N(x, \vec{\ell}_\perp), \tag{7}$$

with $\Delta_{2F} = \int d\xi_N^- \hat{q}_F(\xi_N)$ as the total transverse-momentum broadening squared. Such a distribution also satisfies a 2D diffusion equation in transverse momentum $\vec{k}_\perp$ with the diffusion constant given by $\hat{q}_F$, the so-called (twist-two) quark transport parameter,

$$\hat{q}_F(\xi_N) = \frac{2\pi^2\alpha_s}{N_c} \rho_N^A(\xi_N) \int \frac{d\xi^-}{2\pi p^+} (-2) \langle N|\mathrm{Tr}[F_{+\sigma}(0, \vec{0}_\perp) \times \mathcal{L}_\parallel(0, \xi^-; \vec{0}_\perp) F_+{}^\sigma(\xi^-, \vec{0}_\perp) \mathcal{L}_\parallel^\dagger(0, \xi^-; \vec{0}_\perp)]|N\rangle, \tag{8}$$

which is related to the twist-two collinear nucleon gluon distribution function. It is also the mean transverse-momentum broadening squared per unit length and is proportional to gluon distribution density inside the nucleus. Here $\rho_N^A(\xi_N)$ is the single nucleon density inside the nucleus and p^+ is the longitudinal momentum per nucleon. Inclusion of higher-twist nucleon gluon matrix elements under an extended adjoint two-gluon correlation approximation will give rise to a similar nuclear transverse-momentum distribution with an effective transverse-scale-dependent ($\xi_\perp^2 = -\nabla_{k_\perp}^2$) transport parameter,

$$\hat{q}_F(\xi_N, \xi_\perp^2) = \frac{2\pi^2\alpha_s}{N_c} \rho_N^A(\xi_N) \int \frac{d\xi^-}{2\pi p^+} (-2) \times \langle N|\mathrm{Tr}[F_{+\sigma}(0, \vec{0}_\perp) \mathcal{L}_{\mathrm{TMD}}(0, \xi) F_+{}^\sigma(\xi^-, \vec{\xi}_\perp) \times \mathcal{L}_{\mathrm{TMD}}^\dagger(0, \xi)]|N\rangle, \tag{9}$$

which is related to the transverse-scale-dependent gluon distribution function in a nucleon. Such transverse-scale-dependent transport parameter constituents power corrections to the Gaussian form of the final nuclear broadening distribution.

The rest of this paper is organized as follows. In the next section, we will first formulate the nuclear TMD quark distribution in terms of collinear nuclear high-twist parton matrix elements in the light-cone gauge and then derive the nuclear transverse-momentum broadening distribution under the maximal two-gluon correlation approximation. We then include the higher-twist nucleon quark matrix elements to consider the effect of nucleon intrinsic transverse momentum. The derivation of nuclear broadening of the TMD quark distribution is generalized to any arbitrary gauge with the final results expressed in an explicit gauge-invariant form. Effects of higher-twist gluon distribution functions are also discussed and are shown to lead to a transverse-scale-dependent quark transport parameter. In Sec. III, we will extend the results on nuclear transverse-momentum broadening to the case of quark propagation in hot medium and compare our results under the maximal two-gluon approximation to the dipole model approximation in the Wilson line approach to multiple parton scattering. We will also compare to the result from $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) calculation [29] of the transverse-momentum broadening and discuss the importance of multiple gluon correlation in a strongly coupled system. Finally we will give a summary in Sec. IV.

II. NUCLEAR TRANSVERSE-MOMENTUM BROADENING

We will consider DIS off a large nucleus as depicted in Fig. 1. In the infinite momentum frame, the nucleus has a longitudinal momentum $p = [p^+, 0, \vec{0}_\perp]$ per nucleon and a quark with fractional longitudinal momentum $x_B = Q^2/2p^+q^-$ is knocked out by a virtual photon with four momentum $q = [-Q^2/2q^-, q^-, \vec{0}_\perp]$. The differential cross section for $e^-(l_1) + A(Ap) \to e^-(l_2) + q(k) + X$ can be written as

$$E_{l_2}E_k\frac{d\sigma}{d^3l_2d^3k} = \frac{\alpha_{\rm EM}^2}{2\pi s}\frac{1}{Q^4}L^{\mu\nu}(l_1, l_2)E_k\frac{dW_{\mu\nu}}{d^3k} \qquad (10)$$

where $s = (p + l_1)^2$ and $\alpha_{\rm EM}$ is the fine structure constant in electrodynamics. The SIDIS hadronic tensor $W_{\mu\nu}$ is defined as

$$W_{\mu\nu} = \frac{1}{2}\sum_X\langle A|J_\mu(0)|k, X\rangle\langle k, X|J_\nu(0)|A\rangle 2\pi\delta^4 \times (p + q - k - p_X), \qquad (11)$$

and $J_\mu(y) = e_q\bar{\psi}(y)\gamma_\mu\psi(y)$ is the hadronic electromagnetic current. The leptonic tensor $L^{\mu\nu}$ is defined as usual and is given by

$$L^{\mu\nu}(l_1, l_2) = 4[l_1^\mu l_2{}^\nu + l_1^\nu l_2{}^\mu - (l_1 \cdot l_2)g^{\mu\nu}]. \qquad (12)$$

The struck quark carrying large negative longitudinal momentum q^- will suffer multiple soft scattering with the rest of the nucleus before hadronization into hadrons. We assume that the virtuality of the photon Q^2 is very large and consider now only the lowest order of the hard partonic part. The soft interaction as shown in Fig. 1 can be resummed and the final leading-twist SIDIS tensor [6],

$$\frac{dW_{\mu\nu}}{d^2k_\perp} = H_{\mu\nu}^{(0)}(x_Bp, q)f_q^A(x_B, \vec{k}_\perp), \qquad (13)$$

can be factorized as the product of the lowest hard partonic part $H_{\mu\nu}^{(0)}(x_Bp, q)$,

$$H_{\mu\nu}^{(0)}(x_Bp, q) = e_q^2\,{\rm Tr}[\not{p}\gamma_\mu(x_B\not{p} + \not{q})\gamma_\nu]\frac{\pi}{2p\cdot q}. \qquad (14)$$

and the nuclear TMD quark distribution function $f_q^A(x_B, \vec{k}_\perp)$ as defined in Eq. (1). Since the final state interactions are already included in the nuclear TMD quark distribution function as the gauge links, one should be able to derive the nuclear transverse-momentum broadening.

A. Nuclear TMD quark distribution function in light-cone gauge

One important feature of the complete and gauge-invariant nuclear TMD quark distribution function in Eq. (1) is the transverse gauge link $\mathcal{L}_\perp(-\infty; \vec{y}_\perp, \vec{0}_\perp)$ which depends on the transverse gauge potential $\vec{A}_\perp(-\infty, \vec{\xi}_\perp)$ at $-\infty$ along the light cone. Without it, the gauge links in the TMD quark distribution would completely vanish in the light-cone gauge and one will be misled to assume that the final state interactions become absent. Furthermore the TMD quark distribution without the transverse gauge link is no longer gauge invariant under residual gauge transformation since the transverse gauge potential at infinity does not vanish in the light-cone gauge [30] and is closely related to the singularity of the gluon propagator in the light-cone gauge which has to be properly regularized. Therefore, the effects of final state interaction are actually encoded in the transverse gauge link in the light-cone gauge and cannot be casually discarded. For this reason, we will first derive the nuclear transverse momentum broadening in light-cone gauge and repeat the derivation later in an arbitrary gauge.

Let us first consider the nuclear TMD quark distribution function in the light-cone gauge $A_+ = 0$. In this gauge all the longitudinal gauge links vanish and we are left with only the transverse gauge link in the TMD quark distribution function. We first insert a δ function into the TMD quark distribution function,

$$f_q^A(x, \vec{k}_\perp) = \int d^2\ell_\perp f_q^A(x, \vec{\ell}_\perp)\delta^{(2)}(\vec{k}_\perp - \vec{\ell}_\perp). \qquad (15)$$

Using a Taylor expansion of the δ function,

$$\delta^{(2)}(\vec{k}_\perp - \vec{\ell}_\perp) = e^{-\vec{\ell}_\perp\cdot\vec{\nabla}_{k_\perp}}\delta^{(2)}(\vec{k}_\perp), \qquad (16)$$

the quark transverse-momentum distribution can be written as

$$f_q^A(x, \vec{k}_\perp) = \int\frac{dy^-}{2\pi}\frac{d^2y_\perp}{(2\pi)^2}d^2\ell_\perp e^{ixp^+y^- - i\vec{\ell}_\perp\cdot\vec{y}_\perp}\langle A|\bar{\psi}(0, \vec{0}_\perp)\frac{\gamma^+}{2}\mathcal{L}_\perp^\dagger(-\infty; \vec{y}_\perp, \vec{0}_\perp)\psi(y^-, \vec{y}_\perp)|A\rangle e^{-\vec{\ell}_\perp\cdot\vec{\nabla}_{k_\perp}}\delta^{(2)}(\vec{k}_\perp)$$
$$= \int\frac{dy^-}{2\pi}\frac{d^2y_\perp}{(2\pi)^2}d^2\ell_\perp e^{ixp^+y^- - i\vec{\ell}_\perp\cdot\vec{y}_\perp}\langle A|\bar{\psi}(0, \vec{0}_\perp)\frac{\gamma^+}{2}e^{i\vec{\partial}_{y_\perp}\cdot\vec{\nabla}_{k_\perp}}\mathcal{L}_\perp^\dagger(-\infty; \vec{y}_\perp, \vec{0}_\perp)\psi(y^-, \vec{y}_\perp)|A\rangle\delta^{(2)}(\vec{k}_\perp), \qquad (17)$$

after partial integration in the transverse coordinate $\vec{y}_\perp$. Since both the quark field and the transverse gauge link depend on $\vec{y}_\perp$, we have

$$i\vec{\partial}_{y_\perp}\mathcal{L}_\perp^\dagger(-\infty;\vec{y}_\perp,\vec{0}_\perp) = \mathcal{L}_\perp^\dagger(-\infty;\vec{y}_\perp,\vec{0}_\perp)[-g\vec{A}_\perp(-\infty,\vec{y}_\perp) + i\vec{\partial}_{y_\perp}]$$
$$= \mathcal{L}_\perp^\dagger(-\infty;\vec{y}_\perp,\vec{0}_\perp)\Big[i\vec{D}_\perp(y^-,\vec{y}_\perp) + g\int_{-\infty}^{y^-} d\xi^- \vec{F}_{+\perp}(\xi^-,\vec{y}_\perp)\Big], \quad (18)$$

where $\vec{D}_\perp(y^-,\vec{y}_\perp) = \vec{\partial}_{y_\perp} + ig\vec{A}_\perp(y^-,\vec{y}_\perp)$ is the covariant derivative. In the last step of the above equation we used the following identity,

$$\vec{A}_\perp(-\infty^-, y_\perp) = \vec{A}_\perp(y^-, y_\perp) - \int_{-\infty}^{y^-} d\xi^- \partial_+ \vec{A}_\perp(\xi^-, y_\perp), \quad (19)$$

and $\partial_+\vec{A}_\perp = \vec{F}_{+\perp}$ in the light-cone gauge. Completing the integration over the transverse momentum $\vec{\ell}_\perp$ in Eq. (17) will now produce a δ-function $\delta^{(2)}(\vec{y}_\perp)$ which will set transverse coordinate $\vec{y}_\perp = \vec{0}_\perp$ at which the transverse gauge link will disappear. We have then

$$f_q^A(x,\vec{k}_\perp) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-}\langle A|\bar{\psi}(0) \times \frac{\gamma^+}{2} e^{\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}}\psi(y^-)|A\rangle\delta^{(2)}(\vec{k}_\perp). \quad (20)$$

Here we define the transport operator $\vec{W}_\perp(y^-,\vec{y}_\perp)$ in the light-cone gauge as

$$\vec{W}_\perp(y^-,\vec{y}_\perp) \equiv i\vec{D}_\perp(y^-,\vec{y}_\perp) + g\int_{-\infty}^{y^-} d\xi^- \vec{F}_{+\perp}(\xi^-,\vec{y}_\perp). \quad (21)$$

Note that in the light-cone gauge, the transport operator is translational invariant,

$$\vec{W}_\perp(y^-,\vec{y}_\perp) = \vec{W}_\perp(0,\vec{y}_\perp), \quad (22)$$

along the light cone. For brevity in notation we will suppress the transverse coordinates whenever they are set to zero in the field operators,

$$\mathcal{O}(y^-,\vec{0}_\perp) \equiv \mathcal{O}(y^-). \quad (23)$$

Equation (20) is a general result for the transverse-momentum quark distribution function in the light-cone gauge. In the case of a large nucleus target, we can make further simplifications under the assumption of a weakly bound nucleus. We first expand the exponential factor in Eq. (20) in power of the transport operator $\vec{W}_\perp(0)$. The expectation value of any odd power of the operator under any unpolarized nuclear state should vanish under the parity invariance. We therefore are left only with the even-power terms of the expansion,

$$\mathcal{M}_{2n} \equiv \frac{1}{(2n)!}\langle A|\bar{\psi}(0)\frac{\gamma^+}{2}[\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}]^{2n}\psi(y^-)|A\rangle. \quad (24)$$

We will neglect the covariant derivative first in the transport operator and consider first the twist-four nuclear matrix elements ($n = 1$),

$$\mathcal{M}_2 = \frac{g^2}{2}\int dy^- e^{ixp^+y^-}\int_{-\infty}^{y^-} d\xi_1^- \int_{-\infty}^{y^-} d\xi_2^- \langle A|\bar{\psi}(0) \times \frac{\gamma^+}{2}F_{+i}(\xi_1^-)F_{+j}(\xi_2^-)\psi(y^-)|A\rangle. \quad (25)$$

Because a nucleus consists of nucleons which are color singlet states, the quark and gluon fields could either be all attached to a single nucleon or to two separate nucleons. In the first case, all four parton fields in the above correlation matrix elements are confined to the size of a nucleon y^-, ξ_1^-, $\xi_2^- \sim r_N$. On the other hand, if quark and gluon fields are confined to two separate nucleons, $y^-, |\xi^-| = |\xi_1^- - \xi_2^-| \sim r_N$, the overall position of the gluon field $\xi_N^- = (\xi_1^- + \xi_2^-)/2$ will follow the second nucleon and are only confined to the size of the nucleus R_A. Therefore, the quark-gluon correlation function in this case will have a nuclear enhancement of the order $R_A/r_N \sim A^{1/3}$ as compared to the first case where both quark and gluon fields are confined to a single nucleon. As a two-parton correlation approximation for a large nucleus target we will only keep the matrix elements with the nuclear enhancement. We will also neglect the correlation between different nucleons and assume the large nucleus as a weakly bound. The leading contribution to the above quark-gluon correlation function will be then,

$$\mathcal{M}_2 \approx A\int dy^- e^{ixp^+y^-}\langle N|\bar{\psi}(0)\frac{\gamma^+}{2}\psi(y^-)|N\rangle\frac{g^2}{2} \times \int_{-\infty}^{0} d\xi_1^- \int_{-\infty}^{0} d\xi_2^- \frac{1}{N_c} \times \langle\langle \mathrm{Tr}[F_{+i}(\xi_1^-)F_{+j}(\xi_2^-)]\rangle\rangle_A. \quad (26)$$

If we further assume the large and weakly bound nucleus as a homogenous system of nucleons,

$$\langle\langle \mathrm{Tr}[F_{+i}(\xi_1^-)F_{+j}(\xi_2^-)]\rangle\rangle_A = \langle\langle \mathrm{Tr}[F_{+i}(0)F_{+j}(\xi_2^- - \xi_1^-)]\rangle\rangle_A \quad (27)$$

and the nuclear length is much larger than nucleon size due to confinement, $|\xi^-| \ll \xi_N^-$, we can approximate the quark-gluon correlation as [31,32]

$$\mathcal{M}_2 \approx Af_q^N(x)\frac{-g^2}{2}\int_{-\infty}^{0} d\xi_N^-\rho_N^A(\xi_N)\int\frac{d\xi^-}{2p^+} \times \langle N|F_{+\sigma}(0)F_+^\sigma(\xi^-)|N\rangle\frac{\delta_{ij}}{2}\frac{1}{2N_c} = Af_q^N(x)\frac{\delta_{ij}}{4}\int d\xi_N^-\hat{q}_F(\xi_N), \quad (28)$$

where

$$\langle\langle \cdots \rangle\rangle_A = \int \frac{d^3 p_N}{(2\pi)^3 2p^+} f_A(p_N, \xi_N)\langle N| \cdots |N\rangle = \frac{1}{2p^+}\rho_N^A(\xi_N)\langle N|\cdots|N\rangle, \quad (29)$$

denotes the ensemble (medium) average and $\rho_N^A(\xi_N)$ is the spatial nucleon density inside the nucleus normalized to the atomic number A. The quark transport parameter $\hat{q}_F(\xi_N)$ is defined as

$$\hat{q}_F(\xi_N) = -\frac{g^2}{2N_c}\rho_N^A(\xi_N)\int\frac{d\xi^-}{2p^+}\langle N|F_{+\sigma}(0)F_+^\sigma(\xi^-)|N\rangle = \frac{2\pi^2\alpha_s}{N_c}\rho_N^A(\xi_N)[xf_N^g(x)]_{x=0}, \quad (30)$$

and the gluon distribution function in a nucleon is

$$xf_g^N(x) = -\int_{-\infty}^{\infty}\frac{d\xi^-}{2\pi p^+}e^{ixp^+\xi^-}\langle N|F_{+\sigma}(0)F_+{}^\sigma(\xi^-)|N\rangle, \quad (31)$$

where the summation over the gluon's color index in the matrix element is implied in the definition of the gluon distribution function.

The TMD quark distribution function in Eq. (1) is a leading-twist result in terms of the momentum scale (Q^2) dependence of the DIS process. Among higher-twist corrections, one has neglected those from the transverse phase factors such as

$$e^{ix_\perp p^+(\xi_1^- - \xi_2^-)}; \qquad x_\perp = \frac{k_\perp^2}{2p^+q^-} = x_B\frac{k_\perp^2}{Q^2}, \quad (32)$$

that the propagating quark accumulates in the above two-gluon correlation matrix element. They generally lead to contributions that are proportional to higher-twist nuclear parton matrix elements and are power suppressed $\mathcal{O}(1/Q^{2n})$, $n \geq 1$. One can resurrect these higher-twist contributions by substituting $k_\perp^2$ with its average value and setting the fractional momentum $x = x_\perp$ in the nucleon gluon distribution function in the quark transport parameter in Eq. (30). This will introduce the Q^2 or energy dependence of the quark transport parameter [32]. For the rest of this paper we will focus our attention to the leading-twist TMD nuclear quark distribution.

In the above approximation of the twist-four nuclear quark-gluon matrix we have neglected multiple nucleon correlation in a large nucleus. Such an approximation is violated for small x where quark-gluon and gluon-gluon fusion from different nucleons become important and can lead to modification of the quark distribution function and gluon saturation in a large nucleus [33–35]. We also neglected the real part of the nucleon gluon matrix elements which is responsible for Pomeron-like elastic (or diffractive) scattering and the nuclear shadowing of the quark and gluon distribution functions [36–39]. One can effectively take into account these effects by using a nuclear modified quark distribution function $f_q^A(x_B) \neq Af_q^N(x_B)$ and saturated gluon distribution function in the transport parameter $\hat{q}_F$ which could lead a nontrivial nuclear and energy dependence [32].

For other higher-twist nuclear matrix elements, we similarly consider only the case where quark and gluon fields are attached to different nucleons inside the nucleus.

$$\mathcal{M}_{2n} \approx Af_q^N(x)\frac{1}{(2n)!N_c}\langle\langle \mathrm{Tr}[\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}]^{2n}\rangle\rangle_A = Af_q^N(x)\frac{1}{(2n)!N_c}\langle\langle \mathrm{Tr}[\vec{W}_\perp(0)\cdot\vec{\nabla}_{k_\perp}]^{2n}\rangle\rangle_A. \quad (33)$$

Again we will only keep the dominant terms that have the maximum nuclear enhancement for each given power $2n$ (or twist) of the transport operator. Such contributions come from contracting one pair of the gluonic fields with one nucleon inside the large nucleus. Because of color confinement, the relative longitudinal coordinate of the gluon pair is limited to the size of the nucleon while the average coordinate is set by the position of the nucleon which can be anywhere inside the nucleus. Therefore, each pair of the gluon fields will give rise to one power of nuclear enhancement factor $R_A \sim A^{1/3}$.

We will also neglect all terms that contain any power of the covariant derivative $\vec{D}_\perp(0)$ in $\vec{W}_\perp(0)$ since they are subleading in the nuclear enhancement comparing to the same twist nuclear matrix elements without any covariant derivatives. We call the above approximation *maximal two-gluon correlation approximation* since we reduce the multiple gluon correlations in the nucleus to products of two-gluon correlations that have the maximum nuclear size enhancement. The leading contribution to the $2n$ gluon correlation function is then

$$\frac{1}{N_c}\langle\langle \mathrm{Tr}[\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}]^{2n}\rangle\rangle_A \approx \frac{(2n)!}{2^n n!}\left[\frac{g^2}{2N_c}\frac{-1}{2p^+}\int d\xi_N^-\rho_N^A(\xi_N)d\xi^- \times\langle N|F_{+\sigma}(0)F_+^\sigma(\xi^-)|N\rangle\frac{\nabla_{k_\perp}^2}{2}\right]^n. \quad (34)$$

This is essentially the extension of the approximation for twist-four quark-gluon correlation in a large nucleus to the case of quark-n-gluon correlation in which we assume the correlation of $2n$ gluon fields is approximately the product of n two-gluon correlators (or gluon distribution functions). In the above equation, the combinatorial factor for grouping $2n$ number of gluon field operators into n pairs,

$$(2n-1)!! = \frac{(2n)!}{2^n n!},$$

and the color factor for $2n$ gluon insertions,

$$\frac{1}{N_c(N_c^2-1)^n}\,\mathrm{Tr}[T^{a_1}\cdots T^{a_n}T^{a_n}\cdots T^{a_1}] = \frac{C_F^n}{(N_c^2-1)^n} = \frac{1}{(2N_c)^n},$$

are used. Summation over polarization and color indices in the matrix elements for quark and gluon distributions are implied. For gluon propagation, the above color factor should be $C_A^n/(N_c^2-1)^n$ instead.

Using the definition of the quark transport parameter in nuclear matter $\hat{q}_F(\xi_N)$ as defined in Eq. (30), we can now express the power expansion of the matrix elements as

$$\mathcal{M}_{2n} \approx \frac{1}{(2n)!}\int dy^- e^{ixp^+y^-}\langle A|\bar{\psi}(0)\frac{\gamma^+}{2} \times \left[g\int_{-\infty}^{y^-} d\xi^- \vec{F}_{+\perp}(\xi^-)\cdot\vec{\nabla}_{k_\perp}\right]^{2n}\psi(y^-)|A\rangle \approx Af_q^N(x)\frac{1}{n!}\left[\int d\xi_N^-\hat{q}_F(\xi_N)\frac{\nabla_{k_\perp}^2}{4}\right]^n, \quad (35)$$

With the above simplification of the dominant contributions to the nuclear matrix elements, we obtain the transverse-momentum distribution of the struck quark in DIS off a large nucleus from Eq. (20),

$$f_q^A(x,\vec{k}_\perp) \approx Af_q^N(x)\exp\left[\int d\xi_N^-\hat{q}_F(\xi_N)\frac{\nabla_{k_\perp}^2}{4}\right]\delta^{(2)}(\vec{k}_\perp) \quad (36)$$

in terms of collinear factorized (or transverse-momentum integrated) quark distribution functions and the quark transport parameter $\hat{q}_F$ which in turn is related to the gluon distribution density inside the nucleus. This result is also recently obtained by Majumder and Müller [28] via direct resummation of all twist diagrams in a covariant gauge calculation.

From Eq. (37) one can then calculate the total transverse-momentum broadening of the struck quark due to multiple scattering inside the nuclear matter,

$$\Delta_{2F} = \frac{1}{Af_q^N(x)}\int d^2k_\perp k_\perp^2 f_q^A(x,\vec{k}_\perp) = \int d\xi_N^-\hat{q}_F(\xi_N), \quad (37)$$

which is the same as the twist-four contribution [40]. Though other multiple parton scatterings contribute to the modified transverse-momentum distribution, they do not affect the broadening of the mean transverse momentum squared within the maximal two-gluon correlation approximation.

If we define transverse (coordinate) distribution as

$$f_q^A(x,\vec{y}_\perp) = \int d^2k_\perp e^{i\vec{k}_\perp\cdot\vec{y}_\perp} f_q^A(x,\vec{k}_\perp), \quad (38)$$

the corresponding nuclear quark transverse distribution is

$$f_q^A(x,\vec{y}_\perp) = \int\frac{dy^-}{2\pi}e^{ixp^+y^-}\langle A|\bar{\psi}(0) \times\frac{\gamma^+}{2}e^{-i\vec{y}_\perp\cdot\vec{W}_\perp(y^-)}\psi(y^-)|A\rangle \approx Af_q^N(x)\langle\langle e^{-i\vec{y}_\perp\cdot\vec{W}_\perp(y^-)}\rangle\rangle_A \approx \exp\left[-\int d\xi_N^-\hat{q}_F(\xi_N)\frac{y_\perp^2}{4}\right]Af_q^N(x), \quad (39)$$

which has a Gaussian form in the transverse coordinate $\vec{y}_\perp$. It is then easy to obtain nuclear TMD quark distribution function as

$$f_q^A(x,\vec{k}_\perp) \approx Af_q^N(x)\frac{1}{\pi\Delta_{2F}}\exp[-k_\perp^2/\Delta_{2F}], \quad (40)$$

which is again a Gaussian with width given by the total transverse-momentum broadening squared Δ_{2F}.

B. Effect of nucleon TMD quark distribution

In terms of twist expansion in the collinear factorization, one can consider the nuclear modified transverse-momentum distribution in Eq. (36) as the summation of all twist contributions. However, it contains only contributions with the dominant nuclear enhancement $A^{n/3}$ in the $2(n+1)$-twist multiple parton correlation inside a large nucleus. Such dominant multiparton correlations in a large nucleus are shown to be made up of the products of leading-twist nucleon parton distributions. We have neglected higher-twist contributions to the nucleon parton distribution, for example, the intrinsic transverse momentum of quarks inside a nucleon.

Since the expression for the TMD nuclear parton distribution function in Eq. (20) is general, it should contain these higher-twist effects. To isolate the contributions of the intrinsic quark transverse momentum inside a nucleon, we will make the following expansion of the matrix element in Eq. (20):

$$\begin{aligned}\langle A|\bar{\psi}(0)\frac{\gamma^+}{2}e^{\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}}\psi(y^-)|A\rangle &= \sum_{n=0}^{\infty}\frac{1}{(2n)!}\langle A|\bar{\psi}(0)\frac{\gamma^+}{2}[\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}]^{2n}\psi(y^-)|A\rangle \\ &\approx \sum_{n=0}^{\infty}\frac{1}{(2n)!}\sum_{m=0}^{n}\frac{(2n)!}{(2m)!(2n-2m)!}\frac{1}{N_c}\langle\langle \mathrm{Tr}[\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}]^{2m}\rangle\rangle_A A\langle N|\bar{\psi}(0)\frac{\gamma^+}{2} \\ &\quad\times[\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}]^{2n-2m}\psi(y^-)|N\rangle \\ &\approx \sum_{m=0}^{\infty}\frac{1}{m!}\left[\frac{g^2}{2N_c}\frac{1}{2p^+}\int d\xi_N^- d\xi\rho_N^A(\xi_N)\langle N|F_{+\sigma}(0)F_+^\sigma(\xi^-)|N\rangle\frac{\nabla_{k_\perp}^2}{4}\right]^m\sum_{n=m}^{\infty}\frac{A}{(2n-2m)!} \\ &\quad\times\langle N|\bar{\psi}(0)\frac{\gamma^+}{2}[\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}]^{2n-2m}\psi(y^-)|N\rangle \\ &= \exp\left[\int d\xi_N^-\hat{q}_F(\xi_N)\frac{\nabla_{k_\perp}^2}{4}\right]A\langle N|\bar{\psi}(0)\frac{\gamma^+}{2}e^{\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}}\psi(y^-)|N\rangle, \end{aligned} \tag{41}$$

where the approximation for the expectation value of m pair of gluon fields in a nucleus in Eq. (34) is used. Substituting the above matrix element into Eq. (20), we obtain the nuclear TMD parton distribution function,

$$f_q^A(x,\vec{k}_\perp) = A\exp\left[\int d\xi_N^-\hat{q}_F(\xi_N)\frac{\nabla_{k_\perp}^2}{4}\right]f_q^N(x,\vec{k}_\perp), \tag{42}$$

which is now related to the TMD parton distribution function of the nucleon, $f_q^N(x,\vec{k}_\perp)$. Comparing to the Eq. (36), in which the nucleon TMD quark distribution is assumed to be just a δ function, the nucleon intrinsic transverse momentum in the above equation is the result of the inclusion of a subset of nonleading (in nuclear enhancement) higher-twist contributions.

From the above nuclear TMD quark distribution one can derive a diffusion equation [21,28] for the evolution of the quark transverse-momentum distribution with the nuclear size (or propagation length),

$$\frac{\partial f_q^A(x,\vec{k}_\perp)}{\partial \xi_N^-} = \frac{1}{4}\hat{q}_F(\xi_N)\nabla_{k_\perp}^2 f_q^A(x,\vec{k}_\perp), \tag{43}$$

with the diffusion constant given by the quark transport parameter $\hat{q}_F(\xi_N)$. Apparently, this is the reason why $\hat{q}_F(\xi_N)$ is often referred to as quark transport coefficient [21].

In coordinate space, nuclear quark transverse distribution can be obtained from Eqs. (20) and (42) by partial integration,

$$\begin{aligned} f_q^A(x,\vec{y}_\perp) &= \int\frac{dy^-}{2\pi}e^{ixp^+y^-}\langle A|\bar{\psi}(0) \\ &\quad\times\frac{\gamma^+}{2}e^{-i\vec{W}_\perp(y^-)\cdot\vec{y}_\perp}\psi(y^-)|A\rangle \\ &\approx Af_q^N(x,\vec{y}_\perp)\langle\langle e^{-i\vec{y}_\perp\cdot\vec{W}_\perp(y^-)}\rangle\rangle_A \\ &\approx \exp\left[-\int d\xi_N^-\hat{q}_F(\xi_N)\frac{y_\perp^2}{4}\right]Af_q^N(x,\vec{y}_\perp), \end{aligned} \tag{44}$$

as the product of the nucleon transverse distribution and a Gaussian. The final quark transverse-momentum distribution can then be obtained from the Fourier transform of the above,

$$f_q^A(x,\vec{k}_\perp) = \frac{A}{\pi\Delta_{2F}}\int d^2\ell_\perp e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}}f_q^N(x,\vec{\ell}_\perp), \tag{45}$$

as a convolution of the nucleon TMD quark distribution and a Gaussian with a width Δ_{2F} given by the path integral of the quark transport parameter or the total transverse-momentum broadening squared [Eq. (37)]. This is also a solution to the diffusion equation with an initial condition at $\xi_N^- = 0$ given by the nucleon TMD quark distribution function, $Af_q^N(x,\vec{k}_\perp)$.

C. Arbitrary gauge

In an arbitrary gauge, one has to include both the longitudinal and transverse gauge links in the gauge-invariant definition of the TMD parton distribution function in Eq. (1). Following the same procedure as in the light-cone gauge, one can rewrite the nuclear TMD quark distribution function as

$$\begin{aligned} f_q^A(x,\vec{k}_\perp) &= \int\frac{dy^-}{2\pi}\frac{d^2y_\perp}{(2\pi)^2}d^2\ell_\perp e^{ixp^+y^- - i\vec{\ell}_\perp\cdot\vec{y}_\perp}\langle A|\bar{\psi}(0,\vec{0}_\perp) \\ &\quad\times\frac{\gamma^+}{2}\mathcal{L}_{\mathrm{TMD}}(0,y)\psi(y^-,\vec{y}_\perp)|A\rangle e^{-\vec{\ell}_\perp\cdot\vec{\nabla}_{k_\perp}}\delta^{(2)}(\vec{k}_\perp) \\ &= \int\frac{dy^-}{2\pi}\frac{d^2y_\perp}{(2\pi)^2}d^2\ell_\perp e^{ixp^+y^- - i\vec{\ell}_\perp\cdot\vec{y}_\perp}\langle A|\bar{\psi}(0,\vec{0}_\perp) \\ &\quad\times\frac{\gamma^+}{2}e^{i\vec{\partial}_{y_\perp}\cdot\vec{\nabla}_{k_\perp}}\mathcal{L}_{\mathrm{TMD}}(0,y)\psi(y^-,\vec{y}_\perp)|A\rangle\delta^{(2)}(\vec{k}_\perp). \end{aligned} \tag{46}$$

The transverse differentiation should act on both the quark field operator $\psi(y^-,\vec{y}_\perp)$ and the gauge link $\mathcal{L}_{\mathrm{TMD}}(0,y)$. We will use the following identity [Eq. (A27) in Appendix A],

$$i\vec{\partial}_{y_\perp}\mathcal{L}_{\rm TMD}(0,y)=\mathcal{L}_{\rm TMD}(0,y)\vec{W}_\perp(y^-,\vec{y}_\perp) \tag{47}$$

with the gauge covariant form of the transport operator $\vec{W}_\perp(y^-,\vec{y}_\perp)$ given by

$$\vec{W}_\perp(y^-,\vec{y}_\perp)\equiv i\vec{D}_\perp(y^-,\vec{y}_\perp)+g\int_{-\infty}^{y^-}d\xi^-\mathcal{L}_\parallel^\dagger(\xi^-,y^-;\vec{y}_\perp)\vec{F}_{+\perp}(\xi^-,\vec{y}_\perp)\times\mathcal{L}_\parallel(\xi^-,y^-;\vec{y}_\perp), \tag{48}$$

which transforms like a covariant derivative $\vec{D}_\perp(y^-,\vec{y}_\perp)$ under any gauge transformation. One can recover from the above the equivalent identity in the case of light-cone gauge in Eq. (18) by setting $A_+=0$. These identities have been used to relate the T-odd and spin-dependent part of the quark distribution function to twist-three parton matrix elements [41,42]. These twist-three matrix elements are related to the first moments in transverse momentum of the TMD parton distribution function and the twist-three contribution to DIS process.

Completing the integration over the transverse momentum $\vec{k}_\perp$ and coordinate $\vec{y}_\perp$, we can recast the nuclear TMD parton distribution function as

$$f_q^A(x,\vec{k}_\perp)=\int\frac{dy^-}{2\pi}e^{ixp^+y^-}\langle A|\bar{\psi}(0)\times\frac{\gamma^+}{2}\mathcal{L}_\parallel(0,y^-)e^{\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}}\psi(y^-)|A\rangle\delta^{(2)}(\vec{k}_\perp). \tag{49}$$

It is in exactly the same form as the expression in the light-cone gauge in Eq. (20) except that there is the longitudinal gauge link which is necessary to ensure the gauge invariance of the above form of nuclear TMD parton distribution function. The transport operator $\vec{W}_\perp(y^-)\equiv\vec{W}_\perp(y^-,\vec{0}_\perp)$ is now given by its more general form in Eq. (48). Integrating over the transverse momentum, one obtains the collinear factorized (or transverse-momentum integrated) quark distribution function,

$$f_q^A(x)=\int\frac{dy^-}{2\pi}e^{ixp^+y^-}\langle A|\bar{\psi}(0)\frac{\gamma^+}{2}\mathcal{L}_\parallel(0,y^-)\psi(y^-)|A\rangle, \tag{50}$$

that is also gauge invariant under any arbitrary gauge transformation. In the explicit calculation of the transverse-momentum broadening via cut diagrams as in Ref. [28], these gauge links arise from the resummation of an extra number of collinear gluons on either side of the cut in addition to the soft gluons with transverse momentum that contribute to the transverse momentum of the final quark.

Following the same steps as in the case of light-cone gauge, we will be able to derive from Eq. (49) the nuclear modified transverse-momentum distribution function in an arbitrary gauge as given in Eqs. (42) and (45). The corresponding quark transport parameter $\hat{q}_F$ is simply replaced by a gauge-invariant form

$$\hat{q}_F(\xi_N)=\frac{2\pi^2\alpha_s}{N_c}\rho_N^A(\xi_N)[xf_g^N(x)]_{x=0};$$
$$xf_g^N(x)=-2\int\frac{d\xi^-}{2\pi p^+}e^{ixp^+\xi^-}\langle N|{\rm Tr}[F_{+\sigma}(0)\times\mathcal{L}_\parallel(0,\xi^-)F_+^\sigma(\xi^-)\mathcal{L}_\parallel(\xi^-,0)]|N\rangle, \tag{51}$$

where the gluon field is expressed in the fundamental representation $F_{+\sigma}=F_{+\sigma}^aT^a$. Note that the above definition of the gauge-invariant gluon distribution function in the fundamental color representation is equivalent to the definition in the adjoint representation [43,44],

$$xf_g^N(x)=-\int\frac{d\xi^-}{2\pi p^+}e^{ixp^+\xi^-}\langle N|F_{+\sigma}^a(0)\times\mathcal{L}_{\parallel ab}^A(0,\xi^-)F_+^{b\sigma}(\xi^-)|N\rangle, \tag{52}$$

where

$$\mathcal{L}_\parallel^A(0,\xi^-)\equiv\exp\left[-ig\int_{\xi^-}^0d\zeta^-A_+^c(\zeta^-,\vec{0}_\perp)t_A^c\right], \tag{53}$$

with $(t_A^c)_{ab}=-if_{abc}$, is the longitudinal gauge link in the adjoint representation. One can similarly introduce the transverse gauge link in the adjoint representation

$$\mathcal{L}_\perp^A(-\infty;\vec{\xi}_\perp,\vec{0}_\perp)\equiv\exp\left[-ig\int_{\vec{0}_\perp}^{\vec{\xi}_\perp}d\vec{\xi}_\perp\cdot\vec{A}_\perp^c(-\infty,\vec{\xi}_\perp)t_A^c\right], \tag{54}$$

and the TMD gluon distribution function,

$$\begin{aligned}xf_g^N(x,\vec{k}_\perp)&=-\int\frac{d^2\xi_\perp}{(2\pi)^2}\times\frac{d\xi^-}{2\pi p^+}e^{ixp^+\xi^--i\vec{\xi}_\perp\cdot\vec{k}_\perp}\langle N|F_{+\sigma}^a(0,\vec{0}_\perp)\times\mathcal{L}_{{\rm TMD}ab}^A(0,\xi)F_+^{b\sigma}(\xi^-,\vec{\xi}_\perp)|N\rangle,\\&=-\int\frac{d\xi^-}{2\pi p^+}e^{ixp^+\xi^-}\langle N|F_{+\sigma}^a(0)\times[e^{\vec{W}_\perp^A(\xi^-)\cdot\vec{\nabla}_{k_\perp}}]_{ab}F_+^{b\sigma}(\xi^-)|N\rangle\delta^{(2)}(\vec{k}_\perp),\end{aligned} \tag{55}$$

where

$$\mathcal{L}_{{\rm TMD}ab}^A(0,\xi)=\mathcal{L}_{\parallel ac}^{A\dagger}(-\infty,\xi^-;\vec{0}_\perp)\mathcal{L}_{\perp cd}^{A\dagger}(-\infty;\vec{\xi}_\perp,\vec{0}_\perp)\times\mathcal{L}_{\parallel db}^A(-\infty,\xi^-;\vec{\xi}_\perp), \tag{56}$$

and

$$\vec{W}_{\perp ab}^A(\xi^-,\vec{\xi}_\perp)\equiv i\vec{D}_{\perp ab}^A(\xi^-,\vec{\xi}_\perp)+g\int_{-\infty}^{\xi^-}d\zeta^-\mathcal{L}_{\parallel ac}^{A\dagger}(\zeta^-,\xi^-;\vec{\xi}_\perp)\times\vec{F}_{+\perp ce}^A(\zeta^-,\vec{\xi}_\perp)\mathcal{L}_{\parallel eb}^A(\zeta^-,\xi^-;\vec{\xi}_\perp), \tag{57}$$

is the transport operator in the adjoint representation. The covariant derivative $D_\perp^A$ and gluon field strength $\vec{F}_{+\perp}^A$ in the adjoint representation are defined as

$$\vec{D}^A_{\perp ab}(\xi^-, \vec{\xi}_\perp) = \delta_{ab}\vec{\partial}_\perp + ig\vec{A}^c_\perp(\xi^-, \xi_\perp)(t^c_A)_{ab},\quad \vec{F}^A_{+\perp ab} \equiv \vec{F}^c_{+\perp}(t^c_A)_{ab}, \tag{58}$$

respectively. The corresponding transverse coordinate gluon distribution is then

$$xf^N_g(x, \vec{y}_\perp) = -\int \frac{d\xi^-}{2\pi p^+} e^{ixp^+\xi^-}\langle N|F^a_{+\sigma}(0) \times [e^{-i\vec{y}_\perp\cdot\vec{W}^A_\perp(\xi^-)}]_{ab}F^{b\sigma}_+(\xi^-)|N\rangle. \tag{59}$$

D. Effect of nucleon TMD gluon distribution

In the maximal two-gluon correlation approximation, we approximate the higher-twist nuclear parton matrix elements with products of twist-two nucleon parton matrix elements [see Eqs. (33) and (34)]. Higher-twist nucleon quark matrix elements and quark-gluon correlations lead to intrinsic transverse-momentum distribution inside the nucleon. However, we have so far neglected contributions from higher-twist nucleon gluon matrix elements that involve covariant derivative $\vec{D}_\perp$ or multigluon correlation within a nucleon. These matrix elements have nonleading nuclear length dependence as compared to the products of twist-two gluon distributions. In order to consider the effect of these higher-twist nucleon gluon matrix elements, we separate the covariant derivative from the transport operator in light-cone gauge (to simplify notations),

$$\vec{W}_\perp(y^-) = \vec{\mathcal{F}}_\perp(y^-) + i\vec{D}_\perp(y^-);\quad \vec{\mathcal{F}}_\perp(y^-) \equiv g\int_{-\infty}^{y^-} d\xi^-\vec{F}_{+\perp}(\xi^-). \tag{60}$$

Note that [see Eq. (A29) for arbitrary gauge in Appendix A]

$$\begin{aligned} D_{\perp i}(y^-)\vec{\mathcal{F}}_\perp(y^-) &= g\int_{-\infty}^{y^-} d\xi^- D^A_{\perp i}(\xi^-)\vec{F}_{+\perp}(\xi^-) + \vec{\mathcal{F}}_\perp(y^-)D_{\perp i}(y^-) \\ &\equiv D^A_{\perp i}\vec{\mathcal{F}}_\perp(y^-) + \vec{\mathcal{F}}_\perp(y^-)D_{\perp i}(y^-), \end{aligned} \tag{61}$$

where $\vec{D}^A_\perp F = \vec{\partial}_\perp F + ig[\vec{A}_\perp, F]$ is the covariant derivative in the adjoint representation. Since one can factor out the covariant derivatives of the quark field together with other higher-twist nucleon quark-gluon matrix elements into the TMD nucleon quark distribution function, one can effectively replace the covariant derivative $\vec{D}_\perp$ with its adjoint form $\vec{D}^A_\perp$ in the Taylor expansion of the nuclear gluon matrix element

$$f(y_\perp) = \frac{1}{N_c}\langle\langle \mathrm{Tr} e^{-i\vec{y}_\perp\cdot\vec{W}_\perp(y^-)}\rangle\rangle_A, \tag{62}$$

which should be a function of $y^2_\perp$ because of the parity invariance of the unpolarized nucleus state. We will again neglect nucleon correlations and assume homogeneity in the nucleus. However, we now relax the maximal two-gluon correlation approximation to include higher-twist nucleon matrix elements that contain covariant derivatives and multiple gluon correlations. They are considered subleading in the nuclear length dependence in the above Taylor expansion. We denote $f_n(y_\perp)$ as the nth term in the Taylor expansion of the matrix elements. Therefore, the linear term in $y^2_\perp$ is

$$\begin{aligned} f_1(y_\perp) &= -\frac{1}{2}\frac{1}{N_c}\langle\langle \mathrm{Tr}[\vec{y}_\perp\cdot\vec{W}_\perp(y^-)]^2\rangle\rangle_A \\ &= -\frac{y^2_\perp}{4}\frac{1}{2N_c}\langle\langle \vec{W}^a_\perp(y^-)\cdot\vec{W}^a_\perp(y^-)\rangle\rangle_A \\ &= -\frac{y^2_\perp}{4}\Delta_{2F}. \end{aligned} \tag{63}$$

Note that the medium averaged value of matrix elements linear in $\mathcal{F}_\perp(y^-)$ should vanish.

For the quadratic term in $y^2_\perp$, we first separate the gluon correlation into connected and disconnected parts,

$$\begin{aligned} f_2(y_\perp) &= \frac{1}{4!N_c}\langle\langle \mathrm{Tr}[\vec{y}_\perp\cdot\vec{W}_\perp(y^-)]^4\rangle\rangle_A \\ &= \left(\frac{y^2_\perp}{4}\right)^2\frac{1}{2!N_c}\langle\langle \mathrm{Tr}[\vec{W}_\perp(y^-)]^4\rangle\rangle_A \\ &= \left(\frac{y^2_\perp}{4}\right)^2\frac{1}{2!}\left\{\Delta^2_{2F} + \frac{1}{N_c}\langle\langle \mathrm{Tr}[\vec{W}_\perp(y^-)]^4\rangle\rangle_{AC}\right\} \\ &\equiv \left(\frac{y^2_\perp}{4}\right)^2\left[\frac{1}{2!}\Delta^2_{2F} + \Delta_{4F}\right], \end{aligned} \tag{64}$$

using the identity for generators of the fundamental representation,

$$T^aT^b = \frac{1}{2N_c}\delta_{ab} + \frac{1}{2}d_{abc}T^c + \frac{i}{2}f_{abc}T^c, \tag{65}$$

where the connected parts of the matrix elements $\langle\langle\cdots\rangle\rangle_{AC}$ exclude the singlet contribution in the above color decomposition. We call Δ_{4F} twist-four quark transport parameter which contains all the twist-four nucleon gluon matrix elements in the connected part of the nuclear gluon matrix elements.

In evaluating the connected parts of the nuclear gluon matrix elements, we will now adopt what we call *extended two-gluon* correlation approximation, in which we separate two-gluon fields out of the nuclear matrix elements,

$$\Delta_{4F} = \frac{1}{2N_c}\langle\langle \mathrm{Tr}[\vec{W}_\perp(y^-)]^4\rangle\rangle_{AC} \approx \frac{2}{2N_c}\langle\langle \mathrm{Tr}[\vec{\mathcal{F}}_\perp(y^-)\cdot W_\perp^2(y^-)\vec{\mathcal{F}}_\perp(y^-)]\rangle\rangle_{AC}$$
$$= \frac{1}{N_c}\langle\langle \mathrm{Tr}\{\vec{\mathcal{F}}_\perp\cdot[\mathcal{F}_\perp^2 + 2\vec{\mathcal{F}}_\perp\cdot i\vec{D}_\perp^A + (i\vec{D}_\perp^A\cdot\vec{\mathcal{F}}_\perp) + (iD_\perp^A)^2]\vec{\mathcal{F}}_\perp\}\rangle\rangle_{AC}$$
$$= \frac{1}{2N_c}\langle\langle \vec{\mathcal{F}}_\perp^a\cdot[(\mathcal{F}^O)^2_{ab} + 2(i\vec{D}_\perp^A\cdot\vec{\mathcal{F}}_\perp^O)_{ab} + (i\vec{D}_\perp^A\cdot\vec{\mathcal{F}}_\perp^O)_{ab} + (iD_\perp^A)^2_{ab}]\vec{\mathcal{F}}_\perp^b\rangle\rangle_A \equiv \frac{1}{2N_c}\langle\langle \vec{\mathcal{F}}_\perp^a\cdot[W_\perp^O]^2_{ab}\vec{\mathcal{F}}_\perp^b\rangle\rangle_A, \quad (66)$$

or

$$\Delta_{4F} = \int \frac{d\xi_1^- d\xi_2^-}{2\pi p^+}\rho_N^A(\xi_N)\frac{\pi g^2}{2N_c} \times \langle N|\vec{F}^a_{+\perp}(\xi_1^-)\cdot[W_\perp^O]^2_{ab}\vec{F}^b_{+\perp}(\xi_2^-)|N\rangle. \quad (67)$$

Note that there are 2 pairs of gluons for the extended two-gluon correlation approximation and we have excluded the disconnected contribution (singlet) from the trace operation. We have defined the *octect* gluon field strength

$$\vec{\mathcal{F}}^{\,O}_{\perp ab} \equiv \tfrac{1}{2}\vec{\mathcal{F}}^c_\perp(d_{cab} - if_{cab}) \quad (68)$$

and the corresponding transport operator

$$\vec{W}^{\,O}_{\perp ab} \equiv \vec{\mathcal{F}}^O_{\perp ab} + i\vec{D}^A_{\perp ab}. \quad (69)$$

Again we have dropped terms linear in $\mathcal{F}_\perp(y^-)$.

Similarly, one can also get the third term in the Taylor expansion of the transverse distribution,

$$f_3(y_\perp^2) = \frac{-1}{6!N_c}\langle\langle \mathrm{Tr}[\vec{y}_\perp\cdot\vec{W}_\perp(y^-)]^6\rangle\rangle_A$$
$$= -\left(\frac{y_\perp^2}{4}\right)^3\frac{1}{3!N_c}\langle\langle \mathrm{Tr}[\vec{W}_\perp(y^-)]^6\rangle\rangle_A$$
$$= -\left(\frac{y_\perp^2}{4}\right)^3\frac{1}{3!}\left\{\Delta_{2F}^3 + 3\Delta_{2F}\frac{1}{N_c}\langle\langle \mathrm{Tr}[\vec{W}_\perp(y^-)]^4\rangle\rangle_{AC} + \frac{1}{N_c}\langle\langle \mathrm{Tr}[\vec{W}_\perp(y^-)]^6\rangle\rangle_{AC}\right\}$$
$$\equiv \left(\frac{y_\perp^2}{4}\right)^3\left[\frac{1}{3!}\Delta_{2F}^3 + \Delta_{2F}\Delta_{4F} + \frac{1}{2!}\Delta_{6F}\right]. \quad (70)$$

We again assume the extended two-gluon correlation approximation,

$$\Delta_{6F} \equiv \frac{2}{3!N_c}\langle\langle \mathrm{Tr}[\vec{W}_\perp(y^-)]^6\rangle\rangle_{AC}$$
$$\approx \frac{6}{3!N_c}\langle\langle \mathrm{Tr}[\vec{\mathcal{F}}_\perp(y^-)\cdot\vec{W}_\perp^4(y^-)\vec{\mathcal{F}}_\perp(y^-)]\rangle\rangle_{AC}$$
$$= \frac{1}{2N_c}\langle\langle \vec{\mathcal{F}}^a_\perp\cdot[\vec{W}^O_\perp]^4_{ab}\vec{\mathcal{F}}^b_\perp\rangle\rangle_A$$
$$= \int\frac{d\xi_1^- d\xi_2^-}{2\pi p^+}\rho_N^A(\xi_N)\frac{\pi g^2}{2N_c} \times \langle N|\vec{F}^a_{+\perp}(\xi_1^-)\cdot[W^O_\perp]^4_{ab}\vec{F}^b_{+\perp}(\xi_2^-)|N\rangle. \quad (71)$$

Following the same procedure and the extended two-gluon approximation, we can obtain other terms in the Taylor expansion and the final form of the transverse expansion,

$$f(y_\perp^2) \approx 1 - \frac{y_\perp^2}{4}\Delta_{2F} + \left(\frac{y_\perp^2}{4}\right)^2\left[\frac{1}{2!}\Delta_{2F}^2 + \Delta_{4F}\right] - \left(\frac{y_\perp^2}{4}\right)^3 \times\left[\frac{1}{3!}\Delta_{2F}^3 + \Delta_{2F}\Delta_{4F} + \frac{1}{2!}\Delta_{6F}\right] + \mathcal{O}(y_\perp^8)$$
$$= \exp\left\{-\frac{y_\perp^2}{4}\left[\Delta_{2F} - \frac{y_\perp^2}{4}\Delta_{4F} + \left(\frac{y_\perp^2}{4}\right)^2 \times\frac{1}{2!}\Delta_{6F} + \cdots\right]\right\}$$
$$\equiv \exp\left[-\frac{y_\perp^2}{4}\Delta_F(y_\perp^2)\right]. \quad (72)$$

As one can observe, inclusion of higher-twist nucleon gluon matrix elements in the medium averaged products of the transport operator will give rise to a transverse-distance-dependent (TDD) quark transport parameter,

$$\Delta_F(y_\perp^2) \equiv \int d\xi_N^-\hat{q}_F(\xi_N, y_\perp^2)$$
$$= \int\frac{d\xi_1^- d\xi_2^-}{2\pi p^+}\rho_N^A(\xi_N)\frac{\pi g^2}{2N_c}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\left(\frac{y_\perp^2}{4}\right)^n \times\langle N|\vec{F}^a_{+\perp}(\xi_1^-)\cdot[W^O_\perp]^{2n}_{ab}\vec{F}^b_{+\perp}(\xi_2^-)|N\rangle, \quad (73)$$

that depends on higher-twist nucleon gluon matrix elements. Such TDD quark transport parameter effectively contributes to power corrections to the nuclear transverse-momentum distribution and renders it a non-Gaussian form, especially in the small transverse momentum or large transverse coordinate region. However, these contributions have subleading nuclear length dependence as compared to those associated with the leading-twist quark transport parameter Δ_{2F}. Since the leading-twist (twist-two) transport parameter is proportional to the collinear nucleon gluon distribution function, such TTD transport parameter might be related to the TTD gluon distribution function.

It is helpful, therefore, to make a similar Taylor expansion of the TDD gluon distribution function from Eq. (59),

$$\begin{aligned}[xf_g^N(x, y_\perp^2)]_{x=0} &= \int \frac{d\xi^-}{2\pi p^+} \langle N|\vec{F}^a_{+\perp}(0) \cdot [e^{-i\vec{y}_\perp \cdot \vec{W}^A_\perp(\xi^-)}]_{ab} \vec{F}^b_{+\perp}(\xi^-)|N\rangle \\ &= \int \frac{d\xi^-}{2\pi p^+} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \langle N|\vec{F}^a_{+\perp}(0) \cdot [\vec{y}_\perp \cdot \vec{W}^A_\perp(\xi^-)]^{2n}_{ab} \vec{F}^b_{+\perp}(\xi^-)|N\rangle \\ &= \int \frac{d\xi^-}{2\pi p^+} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left(\frac{y_\perp^2}{4}\right)^n \langle N|\vec{F}^a_{+\perp}(0) \cdot [W^A_\perp(\xi^-)]^{2n}_{ab} \vec{F}^b_{+\perp}(\xi^-)|N\rangle. \end{aligned} \quad (74)$$

Comparing the above Taylor expansion of the TDD gluon distribution function to the TDD quark transport parameter in Eq. (73), one can indeed relate the two,

$$\hat{q}_F(\xi_N, y_\perp^2) \approx \rho_N^A(\xi_N) \frac{\pi g^2}{2N_c} [xf_g^N(x, y_\perp^2)]_{x=0}, \quad (75)$$

if we approximate the octet gluon field strength with its adjoint value

$$\vec{\mathcal{F}}^{\,O}_{\perp ab} = \tfrac{1}{2}\vec{\mathcal{F}}^c_\perp (d_{abc} - if_{abc}) \approx -i\vec{\mathcal{F}}^c_\perp f_{abc} = \vec{\mathcal{F}}^A_{\perp ab}. \quad (76)$$

We will refer to the above approximation and the extended two-gluon correlation approximation together as *extended adjoint two-gluon* correlation approximation. Under such approximation, we can resume contributions associated with the higher-twist nucleon gluon matrix elements to a TDD quark transport parameter which is related to TDD gluon distribution function. We would like to point out that under the extended adjoint two-gluon correlation approximation, one actually only includes a subset of higher-twist nucleon gluon matrix elements (see Appendix B). This is similar to the approximation we made in order to fact out the nucleon TMD quark distribution from the nuclear TMD quark distribution in Sec. II B.

Note that even though the higher-twist contributions we have considered so far lead to a non-Gaussian nuclear transverse-momentum broadening, they do not contribute to the averaged transverse-momentum broadening squared which is still given by the twist-two transport parameter Δ_{2F}. These higher-twist nucleon gluon matrix elements only contribute to higher moments of the transverse-momentum broadening.

One should also keep in mind that we have only considered the leading order in α_s of the hard scattering. Higher-order contributions should also lead to leading-twist non-Gaussian components of the transverse-momentum distribution [5].

Finally, in a finite nucleus, one has to take into account the finite number of nucleons A in the nucleus when factorizing the nuclear parton matrix elements into products of nucleon parton matrix elements. Such consideration will lead to a quark distribution function in the coordinate space [31],

$$\begin{aligned} f(y_\perp^2) &= 1 - (A-1)\frac{y_\perp^2}{4} \int \frac{d\xi_N}{A} \hat{q}(\xi_N, y_\perp^2) \\ &\quad + \frac{(A-1)(A-2)}{2!} \\ &\quad \times \left[\frac{y_\perp^2}{4} \int \frac{d\xi_N}{A} \hat{q}(\xi_N, y_\perp^2)\right]^2 + \cdots \\ &= \left[1 - \frac{y_\perp^2}{4} \int \frac{d\xi_N}{A} \hat{q}(\xi_N, y_\perp^2)\right]^{A-1}, \end{aligned} \quad (77)$$

which can be approximated as that in Eq. (72) for a large nucleus $A \gg 1$.

III. QUARK PROPAGATION IN A THERMAL MEDIUM

We can generalize our study of nuclear transverse-momentum broadening to quark propagation in a hot medium such as the quark-gluon plasma produced in high-energy heavy-ion collisions. In this case, the initial quark production cross section is assumed to be factorized from the quark propagation in medium.

The thermal medium can be considered as an interacting gas of colored constituents with a local density $\rho_N^A(\xi_N)$, with N now referring to the color constituents. The correlation lengths among these constituents are determined by the screening scale $1/\mu$ which could be longer than the interconstituent distance as given by the temperature $1/T$ in the weak coupling limit. Under such scenario, one can still apply the maximal two-gluon correlation approximation to the medium averaged multiple gluon matrix elements as we have used in the cold nuclear medium.

With an initial condition of longitudinal momentum p^+ and zero transverse momentum, the final quark transverse-momentum distribution can be easily read from Eq. (42),

$$\begin{aligned} f_q^A(x, \vec{k}_\perp) &= f(\vec{k}_\perp)\delta(x-1) f(\vec{k}_\perp) \\ &= \exp\left[\int d\xi_N^- \hat{q}_F(\xi_N) \frac{\nabla^2_{k_\perp}}{4}\right] \delta^{(2)}(\vec{k}_\perp) \\ &= \frac{1}{\pi\Delta_{2F}} \exp\left[-\frac{k_\perp^2}{\Delta_{2F}}\right], \end{aligned} \quad (78)$$

or in terms of the transverse coordinate distribution

$$f(\vec{y}_\perp) = \exp\left[-\int d\xi_N^- \hat{q}_F(\xi_N) \frac{y_\perp^2}{4}\right] \quad (79)$$

for the final quark. From the general form of the TMD quark distribution function in Eq. (46), one can replace the nuclear state with a quark with momentum $[p^+, \vec{0}_\perp]$ and the medium A. Averaging over the initial state of both quark and the medium, one obtains the transverse coordinate distribution for a propagating quark as given by the medium expectation value of a pure gauge link,

$$f(\vec{y}_\perp) = \frac{1}{N_c}\langle\langle \mathrm{Tr}[\mathcal{L}_\parallel^\dagger(-\infty,\infty;\vec{0}_\perp) \times \mathcal{L}_\perp(-\infty;\vec{0}_\perp,\vec{y}_\perp)\mathcal{L}_\parallel(-\infty,\infty;\vec{y}_\perp)]\rangle\rangle, \quad (80)$$

where we have assumed the quark is produced at ∞ and propagates toward $-\infty$ along the light cone according to the convention used in this paper.

A. Dipole model approximation

In a covariant gauge (where the transverse gauge link becomes unity), the above becomes the Wilson line formulation of multiple scattering [23,45]. Under a dipole model, the medium averaged Wilson line can be approximated [46] as

$$\frac{1}{N_c}\langle \mathrm{Tr}[\mathcal{L}_\parallel^\dagger(-\infty,\infty;\vec{0}_\perp)\mathcal{L}_\parallel(-\infty,\infty;\vec{y}_\perp)]\rangle \approx \exp\left[-\frac{1}{2}\int d\xi_N^- \rho_N^A(\xi_N)\sigma(\vec{y}_\perp)\right] \quad (81)$$

in terms of the dipole cross section $\sigma(\vec{y}_\perp)$ and the medium density $\rho_N^A(\xi_N)$. Using the short distance form or the leading logarithmic approximation of the dipole cross section [20],

$$\rho_N^A(\xi_N)\sigma(\vec{y}_\perp) \approx \tfrac{1}{2}\hat{q}_F(\xi_N)y_\perp^2,$$

one can obtain the expression in Eq. (79) for the transverse coordinate distribution $f(\vec{y}_\perp)$. One can easily identify the first coefficient in the power expansion of the dipole expansion $\hat{q}_F(\xi_N)$ with the quark transport parameter as we have defined in our twist expansion approach [Eq. (51)]. Therefore, the maximal two-gluon correlation approximation for the dominant multiple gluon correlation in nuclear medium in our study here is equivalent to the dipole model approximation of the Wilson line approach [23,45] when the short distance form of the dipole cross section is used.

To relate our twist expansion result to the dipole model approximation beyond the maximal two-gluon correlation, we can use the identity in Eq. (A27) to recast the transverse coordinate distribution,

$$\begin{aligned} f(\vec{y}_\perp) &= \frac{1}{N_c}\langle\langle \mathrm{Tr}[e^{\vec{y}_\perp\cdot\partial_{\xi_\perp}}\mathcal{L}_\parallel^\dagger(-\infty,\infty;\vec{0}_\perp) \\ &\quad \times \mathcal{L}_\perp(-\infty;\vec{0}_\perp,\vec{\xi}_\perp)\mathcal{L}_\parallel(-\infty,\infty;\vec{\xi}_\perp)]\rangle\rangle_{\xi_\perp=0} \\ &= \frac{1}{N_c}\langle\langle \mathrm{Tr}e^{-i\vec{W}_\perp(\infty)\cdot\vec{y}_\perp}\rangle\rangle, \end{aligned} \quad (82)$$

in terms of the transport operator $\vec{W}_\perp(\infty)$ [Eq. (48)]. A Taylor expansion of the above distribution in $\vec{y}_\perp$ in the extended adjoint two-gluon correlation approximation will lead to a transverse distribution as in Eq. (72),

$$\begin{aligned} f(\vec{y}_\perp) &\approx \exp\left\{-\frac{y_\perp^2}{4}\Delta_F(y_\perp^2)\right\} \\ &= \exp\left\{-\frac{y_\perp^2}{4}\sum_{n=1}^\infty\left(\frac{y_\perp^2}{4}\right)^{n-1}\frac{(-1)^{n-1}}{(n-1)!}\Delta_{2nF}\right\}. \end{aligned} \quad (83)$$

Comparing the above distribution to the dipole model approximation in Eq. (81), one can relate the dipole cross section to the nucleon TDD gluon distribution function,

$$\sigma(\vec{y}_\perp) \equiv y_\perp^2\frac{\pi^2\alpha_s}{N_c}[xf_g^N(x,y_\perp^2)]_{x=0}. \quad (84)$$

This is exactly the cross section between a nucleon and a quark-antiquark pair in a dipole configuration with transverse separation $\vec{y}_\perp$ [38,47].

Our calculation can also be extended to a gluon propagation. The results are the same and one only has to change the color factor to get the definition of the gluon transport parameter,

$$\hat{q}_A(\xi_N, y_\perp^2) = \frac{4\pi^2\alpha_s C_A}{N_c^2-1}\rho_N^A(\xi_N)[xf_g^N(x,y_\perp^2)]_{x=0}. \quad (85)$$

Comparison to the approximation of the averaged Wilson line,

$$\frac{1}{N_c^2-1}\langle \mathrm{Tr}[\mathcal{L}_\parallel^\dagger(-\infty,0;\vec{0}_\perp)\mathcal{L}_\parallel(-\infty,y^-;\vec{y}_\perp)]\rangle \approx \exp\left[-\frac{1}{4}Q_{\mathrm{sat}}^2(x,y_\perp^2)y_\perp^2\right], \quad (86)$$

in the study of gluon saturation in large nuclei [48] will relate the saturation scale $Q_{\mathrm{sat}}^2(y_\perp^2)$ with the path-integrated gluon transport parameter $\hat{q}_A(\xi_N, y_\perp^2)$,

$$\begin{aligned} Q_{\mathrm{sat}}^2(y_\perp^2) &= \int d\xi_N^-\hat{q}_A(\xi_N,y_\perp^2) \\ &= \frac{4\pi^2\alpha_s C_A}{N_c^2-1}\int d\xi_N^-\rho_N^A(\xi_N)xf_g^N(x,y_\perp^2). \end{aligned} \quad (87)$$

The transverse scale dependence of the transport parameter, the dipole cross section or the saturation scale in our calculation come from contributions of higher-twist nucleon gluon matrix elements and therefore are nonperturbative in nature. At very short transverse distance scale, radiative corrections will become important at the leading twist and they will give rise to a transverse scale dependence of the transport parameter that is governed by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [49–51].

B. Multiple gluon correlations in $\mathcal{N} = 4$ SYM

In the Taylor expansion of the TDD quark transport parameter $\Delta_F(y_\perp^2)$ in Eq. (83), the coefficients are higher-twist nucleon gluon matrix elements which generally involve multigluon correlations of the medium. Therefore studying these power corrections to the transport parameter will shed light on multigluon correlations in a medium, especially a strongly coupled system when they become important.

Nonperturbative calculation of the medium averaged Wilson line in Eq. (82) is difficult for a strongly coupled system. The developed technique of lattice QCD is not applicable because it is formulated in the Euclidean space and is only suited for the study of static thermodynamic observables. However, many transport coefficients, such as the shear viscosity to entropy density ratio η/s [52] and transverse-momentum broadening of a heavy quark [53,54], have been studied for $\mathcal{N} = 4$ SYM theory in the large 't Hooft coupling ($\lambda \equiv g_{\rm SYM}^2 N_c$) limit, employing the AdS/CFT correspondence [55]. Though SYM is not exactly QCD, its study might provide indicative information on the properties of a strongly coupled system.

Recently, the thermal averaged Wilson loop along the light cone with longitudinal distance L^- and transverse separation $y_\perp$ was calculated [29] in the strong coupling limit of $\mathcal{N} = 4$ SYM theory. The corresponding transverse distribution from the calculated Wilson loop in the fundamental representation is

$$f(\vec{y}_\perp) = \exp\left\{-a\sqrt{\lambda}L^- T\left[\sqrt{\left(\frac{\pi T y_\perp}{2a}\right)^2 + 1} - 1\right]\right\}, \quad (88)$$

where T is the temperature and $a = \sqrt{\pi}\Gamma(5/4)/\Gamma(3/4) \approx 1.311$. Note a factor of $1/\sqrt{2}$ is missing here because of our definition of light-cone variables. Expanding the exponent in a Taylor series of the transverse distance $y_\perp^2$, one has

$$f(\vec{y}_\perp) = \exp\left[-\frac{y_\perp^2}{4}L^-\frac{\sqrt{\lambda}\pi^2 T^3}{2a}\sum_{n=1}^{\infty}\left(\frac{y_\perp^2}{4}\right)^{n-1}\left(\frac{\pi^2 T^2}{2a^2}\right)^{n-1} \times \frac{(-1)^{n-1}|(2n-3)!!|}{n!}\right]. \quad (89)$$

Comparing to Eq. (83), one can extract the leading-twist quark transport parameter in $\mathcal{N} = 4$ SYM theory,

$$\hat{q}_F^{\rm SYM} = \frac{\sqrt{\lambda}\pi^2 T^3}{2a}, \quad (90)$$

which is half (versus 4/9 in QCD) of the gluon transport parameter as obtained in Ref. [29].

Comparing the power corrections, one can also extract higher-twist quark transport parameters in SYM ($\mathcal{N} = 4$),

$$\frac{\Delta_{2nF}^{\rm SYM}}{\Delta_{2F}^{\rm SYM}} = \left(\frac{\pi^2 T^2}{2a^2}\right)^{n-1}\frac{|(2n-3)|!!}{n}(n \geq 1). \quad (91)$$

It is interesting to note that all higher-twist gluon matrix elements that include multigluon correlations are proportional to the leading-twist gluon matrix elements or two-gluon correlation. The coefficients are set only by the temperature of the medium and are independent of the coupling $\sqrt{\alpha_{\rm SYM}}$ and number of colors N_c. This implies that multiple gluon correlations in the strong coupling limit of SYM are as important as the two-gluon correlation.

Phenomenologically, one can use the relationship between the transport parameter and the gluon distribution function in Eq. (75) to obtain the TDD gluon distribution density in a $\mathcal{N} = 4$ SYM plasma,

$$\rho_N[xf_g(x, y_\perp^2)]_{x=0}^{\rm SYM} = \frac{8aN_c^2}{\pi\sqrt{\lambda}}\frac{T}{y_\perp^2}\left[\sqrt{\left(\frac{\pi T y_\perp}{2a}\right)^2 + 1} - 1\right], \quad (92)$$

which is proportional to N_c^2 for fixed t' Hooft coupling constant λ.

IV. SUMMARY AND DISCUSSION

In this paper, we have derived a gauge-invariant form of nuclear transverse-momentum broadening distribution, utilizing the gauge-invariant TMD quark distribution function from Ref. [10]. We first express such TMD quark distribution function in terms of a sum of all higher-twist and gauge-invariant collinear parton matrix elements. These higher-twist parton matrix elements are expectation values of the moments of a transport operator $\vec{W}_\perp(y)$ which generates the transverse momentum in a nucleon or nucleus when it acts on the parton field $\psi(y)$. The defined transport operator $\vec{W}_\perp(y)$ transforms like a covariant derivative and the final expression is explicitly gauge invariant. With this general form of the TMD parton distribution function, one can then calculate any moment of the parton's transverse momentum in terms of the higher-twist parton matrix elements.

To calculate the nuclear broadening of transverse-momentum distribution, we approximate the nuclear matrix elements of n pair of parton field operators as a product of n nucleon parton distributions, neglecting nuclear bounding effect and multiple nucleon correlations which have nonleading nuclear size dependence in a large nuclear. In other words, multiple gluon correlations are assumed to be given as products of two-gluon correlations, which we called maximal two-gluon correlation approximation. With such approximated nuclear matrix elements that have the dominant nuclear size dependence of $A^{n/3}$ for fixed dimension of the multiparton operators, we can express the final nuclear TMD quark distribution function in terms of the nucleon TMD quark distribution. This form also obeys a 2D diffusion equation whose solution is a convolution of a Gaussian distribution function and the nucleon TMD quark distribution. The width of the Gaussian, or the mean total transverse-momentum broadening squared, is just the path integral of the quark transport parameter $\hat{q}_F$, which is also defined in an explicitly gauge-invariant form and is related to the local gluon distribution density.

Under an extended adjoint two-gluon correlation approximation, one can resum some of the higher-twist nucleon gluon matrix elements to obtain a transverse-distance-dependent quark transverse parameter which is given by the TDD gluon distribution function. Such a TDD quark transport parameter will give rise to power corrections to the Gaussian form of nuclear transverse-momentum distribution function.

We compared our final results with that of the Wilson line approach to multiple parton scattering [23,45] in the dipole model approximation. The two results are identical for short distance approximation of the dipole cross section, which is equivalent to the maximal two-gluon correlation approximation employed in our calculation. If we relax the maximal two-gluon correlation to include non-leading length-dependent contributions involving higher-twist gluon distribution functions, one can then relate the dipole cross section to the TDD gluon distribution function in the medium.

We also compared our results with the AdS/CFT calculation [29] of the transverse distribution of a Wilson line for a propagating quark in the $\mathcal{N} = 4$ SYM theory, in particular, the power corrections to the leading Gaussian distribution. We found that the SYM result indicates the importance of multiple gluon correlations in a strongly coupled system.

Though our final result for the nuclear modified transverse-momentum distribution contains all higher-twist collinear nuclear parton matrix elements, it is still only the leading-twist contribution in terms of power suppression $\mathcal{O}(1/Q^n)$. One can follow the procedure as outlined in Ref. [6] to compute higher-twist contributions to the momentum broadening which has the same nuclear length dependence but are power suppressed in the momentum scale of the hard processes as compared to the leading-twist result obtained in this paper.

The transverse-momentum broadening we calculated in this paper is only valid in a small transverse-momentum region. At large transverse momentum radiative corrections become important and will lead to large logarithmic corrections to the transverse scale dependence of the transport parameter [32]. The form of nuclear broadening will also have large power corrections to the Gaussian form.

The Gaussian form of the transverse-momentum broadening in nuclear medium we discussed in this paper will also have phenomenological implications in the hard processes in hadron-nucleus and nucleus-nucleus collisions. Our study here justifies the phenomenological approach to the initial transverse-momentum broadening [56,57] in $p + A$ and $A + A$ collisions in which a Gaussian form of the broadening is often used. Convoluted together with the power-law-like transverse-momentum spectra due to hard scattering, the Gaussian broadening will naturally lead to Cronin enhancement of the final hadron spectra at small and intermediate transverse momentum.

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APPENDIX A: TRANSVERSE DERIVATIVE OF A GAUGE LINK

Consider a caplike gauge link,

$$\mathcal{L}_{\sqcap} \equiv \mathcal{L}^{\dagger}_{d\perp}(-\infty)\mathcal{L}_{\parallel}(-\infty, y^-; \vec{y}_\perp + d\vec{y}_\perp)\mathcal{L}_{d\perp}(y^-) \tag{A1}$$

with

$$\mathcal{L}_{d\perp}(y^-) \equiv \mathcal{L}_{\perp}(y^-; \vec{y}_\perp + d\vec{y}_\perp, y_\perp,) = 1 - igd\vec{y}_\perp \cdot \vec{A}_\perp(y^-, \vec{y}_\perp) \tag{A2}$$

and an infinitesimal transverse displacement $d\vec{y}_\perp$. One can break the longitudinal gauge link into a product of many small ones each with an infinitesimal length $d\xi^-$,

$$\mathcal{L}_{\parallel}(-\infty, y^-; \vec{y}_\perp + d\vec{y}_\perp) = \cdots \mathcal{L}_{d\parallel}(i+1, i)\cdots \times \mathcal{L}_{d\parallel}(3,2)\mathcal{L}_{d\parallel}(2,1), \tag{A3}$$

$$\mathcal{L}_{\parallel}(-\infty, y^-; \vec{y}_\perp) = \cdots \mathcal{L}_{\parallel}(i+1, i)\cdots \mathcal{L}_{\parallel}(3,2)\mathcal{L}_{\parallel}(2,1), \tag{A4}$$

where $\mathcal{L}_{d\parallel}(i+1, i)$ and $\mathcal{L}_{\parallel}(i+1, i)$ are defined as

$$\mathcal{L}_{d\parallel}(i+1, i) \equiv \mathcal{L}_{\parallel}(\xi^-_{i+1}, \xi^-_i; \vec{y}_\perp + d\vec{y}_\perp), \tag{A5}$$

$$\mathcal{L}_{\parallel}(i+1, i) \equiv \mathcal{L}_{\parallel}(\xi^-_{i+1}, \xi^-_i; \vec{y}_\perp), \tag{A6}$$

and $\xi^-_i = y^- - (i-1)d\xi^-$. Inserting unit matrices

$$1 = \mathcal{L}_{d\perp}(\xi_i)\mathcal{L}_{\parallel}(i+1, i)\mathcal{L}^{\dagger}_{\parallel}(i+1, i)\mathcal{L}^{\dagger}_{d\perp}(\xi_i) \tag{A7}$$

between all neighboring links $\mathcal{L}_{d\parallel}(i+1, i)$ and $\mathcal{L}_{d\parallel}(i, i-1)$ in $\mathcal{L}_{\parallel}(-\infty, y^-, \vec{y}_\perp + d\vec{y}_\perp)$ [Eq. (A3)], as illustrated in Fig. 2, except the last point where one instead inserts the unit matrix

$$1 = \mathcal{L}_{\parallel}(-\infty, -\infty + d\xi^-)\mathcal{L}^{\dagger}_{\parallel}(-\infty, -\infty + d\xi^-) \tag{A8}$$

after $\mathcal{L}^{\dagger}_{d\perp}(-\infty)$. One can then recast the caplike gauge link,

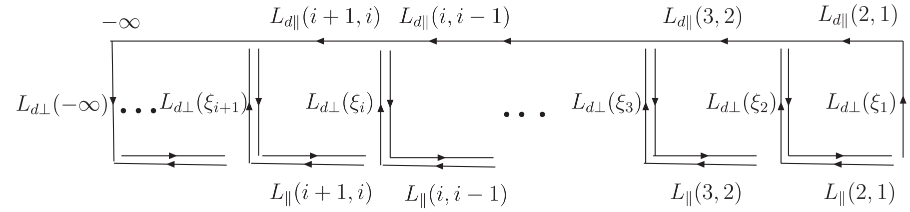

FIG. 2. Splitting the caplike gauge link into chains of closed plaquette linked by short Wilson lines.

$$\begin{aligned}\mathcal{L}_{\sqcap} = {} & \mathcal{L}_{\|}(-\infty,-\infty+d\xi^-)\cdots\mathcal{L}_{\|}(i+1,i)\mathcal{L}_{\square}(\xi_i^-)\\ & \times\mathcal{L}_{\|}(i,i-1)\mathcal{L}_{\square}(\xi_{i-1}^-)\cdots\\ & \times\mathcal{L}_{\square}(\xi_2^-)\mathcal{L}_{\|}(2,1)\mathcal{L}_{\square}(\xi_1^-),\end{aligned}\tag{A9}$$

as a product of closed plaquette,

$$\mathcal{L}_{\square}(\xi_i^-)\equiv\mathcal{L}_{\|}^{\dagger}(i+1,i)\mathcal{L}_{d\perp}^{\dagger}(\xi_{i+1})\mathcal{L}_{d\|}(i+1,i)\mathcal{L}_{d\perp}(\xi_i),\tag{A10}$$

that are linked by short Wilson lines $\mathcal{L}_{\|}(i+1,i)$.

Using the expansion of the closed plaquette [58] up to the linear term in $d\xi^-d\vec{y}_\perp$,

$$\mathcal{L}_{\square}(\xi_i^-)=1-igd\xi^-d\vec{y}_\perp\cdot\vec{F}_{+\perp}(\xi_i^-,\vec{y}_\perp)\tag{A11}$$

one can expand $\mathcal{L}_{\sqcap}$ up to the term linear in $d\vec{y}_\perp$,

$$\begin{aligned}\mathcal{L}_{\sqcap} &= \mathcal{L}_{\|}(-\infty,y^-;\vec{y}_\perp)-igd\vec{y}_\perp\cdot\sum_{i=1}^{\infty}d\xi^-\mathcal{L}_{\|}(-\infty,\xi_i^-;\vec{y}_\perp)\\ &\quad\times\vec{F}_{+\perp}(\xi_i^-,\vec{y}_\perp)\mathcal{L}_{\|}(\xi_i^-,y^-;\vec{y}_\perp)\\ &= \mathcal{L}_{\|}(-\infty,y^-;\vec{y}_\perp)-igd\vec{y}_\perp\cdot\int_{-\infty}^{y^-}d\xi^-\mathcal{L}_{\|}(-\infty,\xi^-;\vec{y}_\perp)\\ &\quad\vec{F}_{+\perp}(\xi^-,\vec{y}_\perp)\mathcal{L}_{\|}(\xi^-,y^-;\vec{y}_\perp).\end{aligned}\tag{A12}$$

Comparing to the direct expansion of $\mathcal{L}_{\sqcap}$ in Eq. (A1) in $d\vec{y}_\perp$,

$$\begin{aligned}\mathcal{L}_{\sqcap} = {} & \mathcal{L}_{\|}(-\infty,y^-;\vec{y}_\perp)+d\vec{y}_\perp\cdot[\vec{\partial}_{y_\perp}\mathcal{L}_{\|}(-\infty,y^-;\vec{y}_\perp)\\ & -ig\mathcal{L}_{\|}(-\infty,y^-;\vec{y}_\perp)\vec{A}_\perp(y^-,\vec{y}_\perp)\\ & +ig\vec{A}_\perp(-\infty,\vec{y}_\perp)\mathcal{L}_{\|}(-\infty,y^-;\vec{y}_\perp)],\end{aligned}\tag{A13}$$

one obtains

$$\begin{aligned}\vec{\partial}_{y_\perp}\mathcal{L}_{\|}(-\infty,y^-;\vec{y}_\perp) = {} & -ig\vec{A}_\perp(-\infty,\vec{y}_\perp)\mathcal{L}_{\|}(-\infty,y^-;\vec{y}_\perp)\\ & +\mathcal{L}_{\|}(-\infty,y^-;\vec{y}_\perp)\Big[ig\vec{A}_\perp(y^-,\vec{y}_\perp)\\ & -ig\int_{-\infty}^{y^-}d\xi^-\mathcal{L}_{\|}^{\dagger}(\xi^-,y^-;\vec{y}_\perp)\vec{F}_{+\perp}\\ & \times(\xi^-,\vec{y}_\perp)\mathcal{L}_{\|}(\xi^-,y^-;\vec{y}_\perp)\Big].\end{aligned}\tag{A14}$$

One can also obtain the above transverse derivative via the direct expansion of the path-ordered longitudinal gauge link,

$$\begin{aligned}\vec{\partial}_{y_\perp}\mathcal{L}_{\|}(-\infty,y^-;\vec{y}_\perp) &= \sum_{n=1}^{\infty}(-ig)^n\sum_{i=1}^{n}\int_{y^-}^{-\infty}[d\xi]_1^nA_+(\xi_1^-,\vec{y}_\perp)\\ &\quad\times\cdots\vec{\partial}_\perp A_+(\xi_i^-,\vec{y}_\perp)\cdots A_+(\xi_n^-,\vec{y}_\perp)\\ &= \textstyle\sum_{n=1}^{\infty}(-ig)^n\sum_{i=1}^{n}\int_{y^-}^{-\infty}[d\xi]_1^nA_+(\xi_1^-,\vec{y}_\perp)\\ &\quad\times\cdots[\partial_+\vec{A}_\perp(\xi_i^-,\vec{y}_\perp)\\ &\quad-\vec{F}_{+\perp}(\xi_i^-,\vec{y}_\perp)]\cdots A_+(\xi_n^-,\vec{y}_\perp),\end{aligned}\tag{A15}$$

where

$$\int_{y^-}^{-\infty}[d\xi]_1^n\equiv\int_{y^-}^{-\infty}d\xi_1\int_{y^-}^{\xi_1^-}d\xi_2^-\cdots\int_{y^-}^{\xi_{n-1}^-}d\xi_n^-,\tag{A16}$$

$$\vec{F}_{+\perp}(\xi_i^-,\vec{y}_\perp)\equiv\partial_+\vec{A}_\perp(\xi_i^-,\vec{y}_\perp)-\vec{\partial}_\perp A_+(\xi_i^-,\vec{y}_\perp).\tag{A17}$$

One can complete the integration of the terms with $\partial_+\vec{A}_\perp(\xi_i^-,\vec{y}_\perp)$ by changing the order of integration,

$$\begin{aligned}&\int_{y^-}^{\xi_{i-1}^-}d\xi_i^-\int_{y^-}^{\xi_i^-}d\xi_{i+1}^-\partial_+\vec{A}_\perp(\xi_i^-,\vec{y}_\perp)\\ &\quad=\int_{y^-}^{\xi_{i-1}^-}d\xi_{i+1}^-\int_{\xi_{i+1}}^{\xi_{i-1}^-}d\xi_i^-\partial_+\vec{A}_\perp(\xi_i^-,\vec{y}_\perp)\\ &\quad=\int_{y^-}^{\xi_{i-1}^-}d\xi_{i+1}^-[\vec{A}_\perp(\xi_{i-1}^-,\vec{y}_\perp)-\vec{A}_\perp(\xi_{i+1}^-,\vec{y}_\perp)],\end{aligned}\tag{A18}$$

for $i=1,2,\ldots,n-1$, with $\xi_0^-=-\infty$ and

$$\int_{y^-}^{\xi_{n-1}^-}d\xi_n^-\partial_+\vec{A}_\perp(\xi_n^-,\vec{y}_\perp)=\vec{A}_\perp(\xi_{n-1}^-,\vec{y}_\perp)-\vec{A}_\perp(y^-,\vec{y}_\perp).\tag{A19}$$

We can rearrange the sum of terms associated with $\partial_+\vec{A}_\perp(\xi_i^-,\vec{y}_\perp)$ in the nth order (in the coupling g) of the expansion as

$$\sum_{i=1}^{n}\int_{y^-}^{-\infty}[d\xi]_1^n A_+(\xi_1^-,\vec{y}_\perp)\cdots\partial_+\vec{A}_\perp(\xi_i^-,\vec{y}_\perp)\cdots A_+(\xi_n^-,\vec{y}_\perp),$$

$$= \int_{y^-}^{-\infty}[d\xi]_1^{n-1}\Big[\prod_{j=1}^{n-2}A_+(\xi_j^-,\vec{y}_\perp)\Big]A_+(\xi_{n-1}^-,\vec{y}_\perp)[\vec{A}_\perp(\xi_{n-1}^-,\vec{y}_\perp)-\vec{A}_\perp(y^-,\vec{y}_\perp)]+\sum_{i=2}^{n-1}\int_{y^-}^{-\infty}[d\xi]_1^{i-1}\Big[\prod_{j=1}^{i-2}A_+(\xi_j^-,\vec{y}_\perp)\Big]$$
$$\times\int_{y^-}^{\xi_{i-1}^-}d\xi_{i+1}^-A_+(\xi_{i-1}^-,\vec{y}_\perp)[\vec{A}_\perp(\xi_{i-1}^-,\vec{y}_\perp)-\vec{A}_\perp(\xi_{i+1}^-,\vec{y}_\perp)]A_+(\xi_{i+1}^-,\vec{y}_\perp)\int_{y^-}^{\xi_{i+1}}[d\xi]_{i+2}^n\prod_{j=i+2}^{n}A_+(\xi_j^-,\vec{y}_\perp)$$
$$+\int_{y^-}^{-\infty}[d\xi]_2^n[\vec{A}_\perp(-\infty,\vec{y}_\perp)-\vec{A}_\perp(\xi_2^-,\vec{y}_\perp)]A_+(\xi_2^-,\vec{y}_\perp)\prod_{j=3}^{n}A_+(\xi_j^-,\vec{y}_\perp)$$
$$=-\int_{y^-}^{-\infty}[d\xi]_1^{n-1}\Big[\prod_{j=1}^{n-1}A_+(\xi_j^-,\vec{y}_\perp)\Big]\vec{A}_\perp(y^-,\vec{y}_\perp)+\vec{A}_\perp(-\infty,\vec{y}_\perp)\int_{y^-}^{-\infty}[d\xi]_2^n\prod_{j=2}^{n}A_+(\xi_j^-,\vec{y}_\perp)$$
$$+\sum_{i=1}^{n-1}\int_{y^-}^{-\infty}[d\xi]_1^{i-1}\Big[\prod_{j=1}^{i-1}A_+(\xi_j^-,\vec{y}_\perp)\Big]\int_{y^-}^{\xi_{i-1}^-}d\xi_i^-[A_+(\xi_i^-,\vec{y}_\perp),\vec{A}_\perp(\xi_i^-,\vec{y}_\perp)]\int_{y^-}^{\xi_i}[d\xi]_{i+1}^n\prod_{j=i+1}^{n-1}A_+(\xi_j^-,\vec{y}_\perp). \quad \text{(A20)}$$

The terms containing the commutator, $[A_+(\xi_i^-,\vec{y}_\perp), \vec{A}_\perp(\xi_i^-,\vec{y}_\perp)]$, can be combined with $\vec{\tilde{F}}_{+\perp}(\xi_i^-,\vec{y}_\perp)$ in the $(n-1)$th order of the expansion to give the gluon field strength tensor,

$$\vec{F}_{+\perp}(\xi_i^-,\vec{y}_\perp)=\vec{\tilde{F}}_{+\perp}(\xi_i^-,\vec{y}_\perp)+ig[A_+(\xi_i^-,\vec{y}_\perp),\vec{A}_\perp(\xi_i^-,\vec{y}_\perp)]. \quad \text{(A21)}$$

Note that

$$\sum_{n=1}^{\infty}(-ig)^n\int_{y^-}^{\xi_i}[d\xi]_{i+1}^n\prod_{j=i+1}^{n-1}A_+(\xi_j^-,\vec{y}_\perp)=\mathcal{L}_\parallel(\xi_i,y^-,\vec{y}_\perp), \quad \text{(A22)}$$

and

$$\sum_{i=1}^{\infty}(-ig)^{i-1}\int_{y^-}^{-\infty}[d\xi]_1^{i-1}\Big[\prod_{j=1}^{i-1}A_+(\xi_j^-,\vec{y}_\perp)\Big]\int_{y^-}^{\xi_{i-1}^-}d\xi_i^-$$
$$=\sum_{i=1}^{\infty}(ig)^{i-1}\int_{y^-}^{-\infty}d\xi_i^-\int_{-\infty}^{\xi_i^-}d\xi_{i-1}^-\cdots\times\int_{-\infty}^{\xi_2^-}d\xi_1^-A_+(\xi_1^-,\vec{y}_\perp)\cdots A_+(\xi_{i-1}^-,\vec{y}_\perp)$$
$$=\int_{y^-}^{-\infty}d\xi_i^-\mathcal{L}_\parallel^\dagger(\xi_i,-\infty,\vec{y}_\perp). \quad \text{(A23)}$$

One obtains now the transverse derivative of the longitudinal gauge link,

$$\vec{\partial}_{y_\perp}\mathcal{L}_\parallel(-\infty,y^-;\vec{y}_\perp)=-ig\int_{-\infty}^{y^-}d\xi\,\mathcal{L}_\parallel^\dagger(\xi^-,-\infty)\times\vec{F}_{+\perp}(\xi^-,\vec{y}_\perp)\mathcal{L}_\parallel(\xi^-,y^-;\vec{y}_\perp)+ig\mathcal{L}_\parallel(-\infty,y^-;\vec{y}_\perp)\vec{A}_\perp(y^-,\vec{y}_\perp)-ig\vec{A}_\perp(-\infty,\vec{y}_\perp)\mathcal{L}_\parallel(-\infty,y^-;\vec{y}_\perp). \quad \text{(A24)}$$

Another general form of the above identity is

$$\vec{D}_\perp(y_1^-,\vec{y}_\perp)\mathcal{L}_\parallel(y_1^-,y^-;\vec{y}_\perp)=-ig\int_{y_1^-}^{y^-}d\xi\,\mathcal{L}_\parallel^\dagger(\xi^-,y_1^-)\times\vec{F}_{+\perp}(\xi^-,\vec{y}_\perp)\mathcal{L}_\parallel(\xi^-,y^-;\vec{y}_\perp)+\mathcal{L}_\parallel(y_1^-,y^-;\vec{y}_\perp)\vec{D}_\perp(y^-,\vec{y}_\perp). \quad \text{(A25)}$$

Using the above identity, the derivative operation on the gauge link in the TMD quark distribution function,

$$\mathcal{L}_{\rm TMD}(0,y)\equiv\mathcal{L}_\parallel^\dagger(-\infty,0;\vec{0}_\perp)\mathcal{L}_\perp^\dagger(-\infty;\vec{y}_\perp,\vec{0}_\perp)\times\mathcal{L}_\parallel(-\infty,y^-;\vec{y}_\perp), \quad \text{(A26)}$$

will yield

$$\vec{\partial}_{y_\perp}\mathcal{L}_{\rm TMD}(0,y)=\mathcal{L}_{\rm TMD}(0,y)\vec{\partial}_{y_\perp}+\mathcal{L}_\parallel^\dagger(-\infty,0;\vec{0}_\perp)\mathcal{L}_\perp^\dagger(-\infty;\vec{y}_\perp,\vec{0}_\perp)\vec{\partial}_{y_\perp}\mathcal{L}_\parallel(-\infty,y^-;\vec{y}_\perp)$$
$$+ig\mathcal{L}_\parallel^\dagger(-\infty,0;\vec{0}_\perp)\mathcal{L}_\perp^\dagger(-\infty;\vec{y}_\perp,\vec{0}_\perp)\vec{A}_\perp(-\infty,\vec{y}_\perp)\mathcal{L}_\parallel(-\infty,y^-;\vec{y}_\perp)$$
$$=\mathcal{L}_{\rm TMD}(0,y)\Big[\vec{D}_\perp(y^-,\vec{y}_\perp)-ig\int_{-\infty}^{y^-}d\xi^-\mathcal{L}_\parallel^\dagger(\xi^-,y^-;\vec{y}_\perp)\vec{F}_{+\perp}(\xi^-,\vec{y}_\perp)\mathcal{L}_\parallel(\xi^-,y^-;\vec{y}_\perp)\Big]. \quad \text{(A27)}$$

If we define

$$\vec{\mathcal{F}}_\perp(y^-,\vec{y}_\perp) \equiv g\int_{-\infty}^{y^-} d\xi^- \mathcal{L}_\parallel^\dagger(\xi^-,y^-;\vec{y}_\perp)\vec{F}_{+\perp}(\xi^-,\vec{y}_\perp) \times \mathcal{L}_\parallel(\xi^-,y^-;\vec{y}_\perp), \tag{A28}$$

it is easy to use Eq. (A25) to show

$$D_{\perp i}(y,\vec{y}_\perp)\vec{\mathcal{F}}_\perp(y^-,\vec{y}_\perp) = \vec{\mathcal{F}}_\perp(y^-,\vec{y}_\perp)D_{\perp i}(y,\vec{y}_\perp) + g\int_{-\infty}^{y^-} d\xi^- \mathcal{L}_\parallel^\dagger(\xi^-,y^-;\vec{y}_\perp)D_{\perp i}^A \times \vec{F}_{+\perp}(\xi^-,\vec{y}_\perp)\mathcal{L}_\parallel(\xi^-,y^-;\vec{y}_\perp), \tag{A29}$$

where $\vec{D}_\perp^A F = \vec{\partial}_\perp + ig[\vec{A}_\perp, F]$ is the covariant derivative in the adjoint representation.

APPENDIX B: EXTENDED TWO-GLUON CORRELATION APPROXIMATION

In this Appendix we will examine higher-twist nucleon gluon matrix elements that are neglected in the extended two-gluon correlation approximation in Sec. II D.

In the connected part of the twist-four gluon matrix elements one can extend the product of transport operator without assuming extended two-gluon correlation approximation,

$$\begin{aligned}\Delta_{4F} &= \frac{1}{2N_c}\langle\langle \mathrm{Tr}[W_\perp(y^-)]^4\rangle\rangle_{AC} \\ &= \frac{1}{2N_c}\langle\langle \mathrm{Tr}[\mathcal{F}_\perp]^4\rangle\rangle_{AC} + \frac{2}{2N_c}\langle\langle \mathrm{Tr}[(i\vec{D}_\perp^A\cdot\vec{\mathcal{F}}_\perp)\mathcal{F}_\perp^2 + \vec{\mathcal{F}}_\perp\cdot(i\vec{D}_\perp^A\cdot\vec{\mathcal{F}}_\perp)\vec{\mathcal{F}}_\perp + \mathcal{F}_\perp^2(i\vec{D}_\perp^A\cdot\vec{\mathcal{F}}_\perp)]\rangle\rangle_A - \frac{1}{2N_c}\langle\langle \mathrm{Tr}[3\vec{\mathcal{F}}_\perp \\ &\quad \cdot(D_\perp^{A2}\vec{\mathcal{F}}_\perp) + (D_\perp^{A2}\vec{\mathcal{F}}_\perp)\cdot\vec{\mathcal{F}}_\perp + 3(D_\perp^A\cdot\vec{\mathcal{F}})(D_\perp^A\cdot\vec{\mathcal{F}}_\perp)]\rangle\rangle_A \\ &= \frac{1}{4N_c}\langle\langle \vec{\mathcal{F}}_\perp^a[(\mathcal{F}_\perp^O)^2_{ab} + (iD_\perp^A)^2_{ab} + 6(i\vec{D}_\perp^A\cdot\vec{\mathcal{F}}_\perp^O)_{ab}]\vec{\mathcal{F}}_\perp^b\rangle\rangle_A.\end{aligned} \tag{B1}$$

Terms linear in $\mathcal{F}_\perp(y^-)$ are dropped since they vanish after the medium average. The following identities along medium averaged matrix elements are also used,

$$\langle\langle \mathrm{Tr}[\vec{\mathcal{F}}_\perp\cdot(D_\perp^A)^2\vec{\mathcal{F}}_\perp]\rangle\rangle_A = \langle\langle \mathrm{Tr}[(D_\perp^A)^2\vec{\mathcal{F}}_\perp\cdot\vec{\mathcal{F}}_\perp]\rangle\rangle_A = -\langle\langle \mathrm{Tr}[(\vec{D}_\perp^A\cdot\vec{\mathcal{F}}_\perp)\cdot(\vec{D}_\perp^A\cdot\vec{\mathcal{F}}_\perp)]\rangle\rangle_A, \tag{B2}$$

$$\langle\langle \mathrm{Tr}[\mathcal{F}_\perp^2 i\vec{D}_\perp^A\cdot\mathcal{F}_\perp]\rangle\rangle_A = \langle\langle \mathrm{Tr}[\vec{\mathcal{F}}_\perp\cdot(i\vec{D}_\perp^A\cdot\mathcal{F}_\perp)\vec{\mathcal{F}}_\perp]\rangle\rangle_A = \langle\langle \mathrm{Tr}[i\vec{D}_\perp^A\cdot\vec{\mathcal{F}}_\perp\mathcal{F}_\perp^2]\rangle\rangle_A. \tag{B3}$$

One can see that there are differences between the higher-twist gluon matrix elements in the above expansion and that under extended two-gluon correlation approximation in Sec. II D which include some extra higher-twist gluon matrix elements.

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