# Reduced time delay for gravitational waves with dark matter emulators

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We discuss the implications for gravitational wave detectors of a class of modified gravity theories which dispense with the need for dark matter. These models, which are known as *dark matter emulators*, have the property that weak gravitational waves couple to the metric that would follow from general relativity without dark matter whereas ordinary particles couple to a combination of the metric and other fields which reproduces the result of general relativity with dark matter. We show that there is an appreciable difference in the Shapiro delays of gravitational waves and photons or neutrinos from the same source, with the gravitational waves always arriving first. We compute the expected time lags for GRB 070201, for SN 1987a and for Sco-X1. We estimate the probable error by taking account of the uncertainty in position, and by using three different dark matter profiles.

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## I. INTRODUCTION

The direct detection of gravitational waves from astrophysical sources would enable us to open a new window into the Universe and get insights which are complementary to electromagnetic astronomy [1]. Many ground-based interferometric detectors such as LIGO, VIRGO, GEO600 and TAMA have been online for several years. In October 2007, LIGO completed a long science run to collect 1 yr of coincident data at design sensitivity [2], and the VIRGO detector also joined this science run in the last five months. During the latest LIGO science run, the sensitivity of the 4 km Hanford and Livingston LIGO detectors to detect binary neutron-star coalescence with mass $1.4M_\odot$ with signal to noise ratio greater than 8 (after averaging over all orientations and sky positions) was about 15 Mpc [2]. Analysis of the latest LIGO and VIRGO data for gravitational wave (GW) searches from a wide variety of sources is in progress [3].

An important science goal pursued is the search for impulsive transient GW signals from sources with electromagnetic and/or neutrino counterparts. Some examples of such sources include core-collapse supernovae, gamma-ray bursts (GRBs), soft gamma-ray repeaters (SGRs), pulsar glitches, low mass x-ray binaries, blazar flares, optical transients, etc. [4]. These ''triggered'' searches allow us to get better sensitivity for a given false alarm rate as compared to an all-sky search at all times and to design custom-made analysis algorithms taking into account our knowledge of the source astrophysics. Conversely, there has been a proposal to look for optical and infrared counterparts at the time of coincident GW burst candidates [5]. An overview and benefits of such triggered searches carried out by the current interferometric gravitational wave detectors are reviewed in Ref. [4]. There have been proposals to determine neutrino mass using simultaneous neutrino and GW observations from core-collapse supernovae [6]. Similar triggered GW searches will also be important for the future LISA experiment [7].

In all present and past triggered searches for gravitational waves, the analysis is done by looking at the data from GW detectors within a narrow time window (of about hundreds of seconds) around the time of the electromagnetic trigger. With this assumption, one can detect gravitational waves only if the propagation time of photons/neutrinos is the same as that of gravitational waves. In general relativity, photons, neutrinos and gravitational waves propagate on the same null geodesics. Hence the total time of propagation is the light travel time delay plus the Shapiro time delay due to intervening matter [8]. For electromagnetic waves, Shapiro delay has been detected in a wide variety of systems such as radar ranging to Venus, Doppler tracking of Cassini spacecraft and in binary pulsars [9]. From the relative arrival times of photons and neutrinos from SN 1987a, we also know that the Shapiro time delay for neutrinos is the same as that for photons to within 0.5% [10,11].

The conventional view is that general relativity describes gravity on cosmic scales. If this is so, the gravitation of stars and gas is not sufficient to account for the velocity dispersions in clusters [12], or for the rotation curves of spiral galaxies [13–15], or for the weak lensing in galactic clusters [16–21]. Big bang nucleosynthesis severely limits the extent to which the deficit can be made up of unseen but ordinary matter [22]; the remainder must consist of an exotic, nonrelativistic substance which has never been detected except gravitationally. This *dark matter* must vastly predominate over ordinary matter. For example, only about one-fifth of our galaxy's mass is made up of normal matter, with the rest being composed of dark

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matter [23]. Thus the dominant contribution to the Shapiro delay for photons from GRBs and other sources is due to the gravitational potential of the intervening dark matter.

None of the proposed dark matter candidates have been detected, either directly in a laboratory experiment or indirectly through their annihilation products [24–27]. This prompts the suspicion that perhaps it is gravity which must be modified, rather than the universe's inventory of nonrelativistic matter. Of course that would invalidate the assumption which is the basis for all current and proposed GW searches from sources seen in photons and neutrinos [4,6]. A previous study [28,29] has considered the consequences for gravitational wave detection of a certain class of modified gravity theories known as *dark matter emulators*. In this paper we correct a mistake in the original work [28] that led to the wrong sign for the effect, and we work out explicit results for three interesting sources.

Section II defines and motivates dark matter emulators. In Sec. III we review three popular dark matter profiles which these models are designed to obviate. Section IV computes the expected time lag between the arrival of the pulse of gravitational waves from some cosmic event and its optical or neutrino counterpart. In Sec. V we give explicit results for three sources of interest. Section VI gives a very brief discussion of other alternate gravity models, and our conclusions comprise Sec. VII.

## II. DARK MATTER EMULATORS

Certain regularities in cosmic structures suggest modified gravity. One of these is the Tully-Fisher relation, which states that the luminosity of a spiral galaxy is proportional to the fourth power of the peak velocity in its rotation curve [30]. If luminous matter is insignificant compared to dark matter, why should such a relation exist? Another regularity is Milgrom's Law, which states that the need for dark matter occurs at gravitational accelerations of $a_0 \simeq 10^{-10}$ m/s$^2$ [31]. A third regularity is that $a_0$ also seems to give the internal accelerations of pressure-supported objects ranging over 6 orders of magnitude in size—from massive molecular clouds within our own galaxy to x-ray clusters of galaxies [32].

A modification of Newtonian gravity which explains these regularities was proposed by Milgrom in 1983 [33]. His model, modified newtonian dynamics (MOND), was soon given a Lagrangian formulation in which conservation of energy, 3-momentum and angular momentum are manifest [34]. However, there was for years no successful relativistic generalization which could be employed to study cosmological evolution. Even in the context of static, spherically symmetric geometries,

$$ds^2 \equiv -B(r)c^2dt^2 + A(r)dr^2 + r^2d\Omega^2, \qquad (1)$$

the early formulation of MOND fixed only $B(r)$, not $A(r)$. It was therefore incapable of making definitive predictions about gravitational lensing.

A relativistic extension of MOND has recently been proposed by Bekenstein [35]. This model is known as TeVeS for "tensor-vector-scalar." In addition to reproducing the MOND force law at low accelerations, TeVeS has acceptable post-Newtonian parameters, and it gives a plausible amount of gravitational lensing [35]. When TeVeS is used in place of general relativity + dark matter to study cosmological evolution, the results are in better agreement with data than many thought possible [36–40]. The model does have problems with stability [41,42]. The Bullet Cluster is sometimes cited as a fatal blow for the model [43] but opinion on this differs [44–46], and this system in any case poses problems for dark matter [45,47].

What concerns us here is the curious property of TeVeS that small amplitude gravitational waves are governed, as in general relativity, by the metric $g_{\mu\nu}$, whereas matter couples to a "disformally transformed" metric which involves the vector and scalar fields,

$$\tilde{g}_{\mu\nu} = e^{-2\phi}(g_{\mu\nu} + A_\mu A_\nu) - e^{2\phi}A_\mu A_\nu. \qquad (2)$$

The scalar-vector-tensor gravity theory proposed by Moffat also has different metrics for matter and small amplitude gravitational waves [48,49]. The appearance of this feature in two very different models is the result of trying to reconcile solar system tests with modified gravity at ultralow accelerations. Solar system tests strongly predispose the Lagrangian to possess an Einstein-Hilbert term [50,51]. On the other hand, failed attempts to generalize MOND [52] have led to a theorem that one cannot get sufficient weak lensing from a stable, covariant and purely metric theory which reproduces the Tully-Fisher relation without dark matter [53]. Hence the MOND force must be carried by some other field, and it is a combination of this other field and the metric which determines the geodesics for ordinary matter. However, the dynamics of small amplitude gravitational waves are still set by the linearized Einstein equation. This simple observation makes for a sensitive and generic test.

We define a *dark matter emulator* as any modified gravity theory for which:

(1) Ordinary matter couples to the metric $\tilde{g}_{\mu\nu}$ that would be produced by general relativity + dark matter; and
(2) Small amplitude gravitational waves couple to the metric $g_{\mu\nu}$ produced by general relativity without dark matter.

Now consider a cosmic event such as a supernova which emits simultaneous pulses of gravitational waves and either neutrinos or photons. If physics is described by a dark matter emulator then the pulse of gravitational waves will reach us on a lightlike geodesic of $g_{\mu\nu}$, whereas neutrinos and photons travel along a lightlike geodesic of $\tilde{g}_{\mu\nu}$. If significant propagation occurs over regions that would be dark matter dominated in general relativity then there will be a measurable lag between arrival times.

Currently the only observational constraint on the speed $v_g$ of gravity relative to that of ordinary matter $v_m$ derives from the consequences of gravitational Cherenkov radiation from particles moving faster than gravity [54,55]. From observations of the highest energy cosmic rays Moore and Nelson infer the bound $v_m - v_g < 2 \times 10^{-15} c$ [55]. Although the original study of dark matter emulators [28] in fact violated this bound, that was the result of incorrectly choosing the dimensional constant in a certain logarithm. In the next section we show that the speed of gravity is always greater than that of light for dark matter emulators. A discussion of the Shapiro delay calculation in some other alternate gravity theories can be found in Refs. [56,57].

## III. THREE DARK MATTER PROFILES

We shall specialize to static, spherically symmetric distributions of dark matter, consistent with the invariant element (1). It is well to bear in mind that hierarchical structure formation will not necessarily result in spherically symmetric distributions [58]. There is even evidence that the dark matter halo conjectured to surround our own galaxy is not spherical [59].

For a pressureless, static, spherically symmetric system the Einstein equations take the form

$$\frac{B}{A}\left[\frac{A'}{rA} + \left(\frac{A-1}{r^2}\right)\right] = \frac{8\pi G}{c^2}\rho, \tag{3}$$

$$\frac{B'}{rB} - \left(\frac{A-1}{r^2}\right) = 0. \tag{4}$$

If the potential $B(r)$ goes to a constant at infinity we can choose the time units so that Eq. (4) has the exact solution

$$B(r) = \exp\left[-\int_r^\infty dr' \left(\frac{A(r')-1}{r'}\right)\right]. \tag{5}$$

However, our study requires only small corrections to $A(r)$ and $B(r)$,

$$A(r) \equiv 1 + \Delta A(r), \qquad B(r) \equiv 1 + \Delta B(r). \tag{6}$$

This not only simplifies (3) and (4), it also means we can dispense with the contribution of ordinary matter to the mass density $\rho(r)$. The reason is that we are computing the difference in propagation times along null geodesics between the same points in two different geometries. The geometry felt by gravitational waves is sourced only by ordinary matter, while the geometry felt by photons and neutrinos is sourced by the sum of ordinary matter and dark matter. At first order in the mass density, the effect of ordinary matter cancels out when computing the difference in propagation times between the two geometries. We will henceforth consider $\rho(r)$ to be the density of dark matter.

The potentials $\Delta A(r)$ and $\Delta B(r)$ can be given simple expressions in terms of the mass function,

$$M(r) \equiv 4\pi \int_0^r dr' \rho(r'). \tag{7}$$

The linearized solution of (3) is

$$\Delta A(r) = \frac{8\pi G}{c^2 r}\int_0^r dr' r'^2 \rho(r') = \frac{2G}{c^2}\frac{M(r)}{r}. \tag{8}$$

Note that $\Delta A(r)$ is positive semidefinite. From (8) we find $\Delta B(r)$,

$$\begin{aligned}\Delta B(r) &= -\int_r^\infty dr' \frac{\Delta A(r')}{r'} \\ &= -\Delta A(r) - \frac{2G}{c^2}\int_r^\infty dr' \frac{M(r')}{r'}.\end{aligned} \tag{9}$$

Note that $\Delta B(r)$ is negative semidefinite and in fact less than or equal to $-\Delta A(r)$. This guarantees that gravitational waves travel faster than photons or neutrinos, so there is no problem with the bound of Moore and Nelson [55].

Our study requires the dark matter density functions for our own galaxy and (for the most distant source) for the Andromeda galaxy. We took these in the form of fits to three popular density profiles whose analytic forms are presented at the end of this section. Given the current rough quality of the observational data, a dark matter emulator that reproduced the potentials $\Delta A(r)$ and $\Delta B(r)$ for any of these profiles would be judged successful. One can therefore regard the slightly different time delays that result as one measure of the theoretical uncertainty. The fits for our own galaxy appear in Table I and were taken from the study by Ascasibar, Jean, Boehm and Knödlseder of the positron annihilation line from the galactic center [60]. The fits for Andromeda appear in Table II and were done by Tempel, Tamm and Tenjes [61].

TABLE I. Dark matter profile parameters for the Milky Way Galaxy from Ascasibar *et al.* [60].

| Profile | $8\pi G\rho_0 r_0^3/c^3$ | $r_0$ | $r_c$ |
|---|---|---|---|
| Isothermal | 3.98 days | 4.00 kpc | 219 kpc |
| NFW | 60.8 days | 16.7 kpc | ... |
| Moore | 51.8 days | 29.5 kpc | ... |

TABLE II. Dark matter profile parameters for the Andromeda Galaxy from Tempel *et al.* [61].

| Profile | $8\pi G\rho_0 r_0^3/c^3$ | $r_0$ | $r_c$ |
|---|---|---|---|
| Isothermal | 1.88 days | 1.47 kpc | 117 kpc |
| NFW | 48.6 days | 12.5 kpc | ... |
| Moore | 45.8 days | 25.0 kpc | ... |

### A. Isothermal halo profile

We shall use a variant of the isothermal halo profile in which the density vanishes after a cutoff radius $r_c$ [62,63],

$$\rho(r) = \left[\frac{\rho_0}{1+(\frac{r}{r_0})^2} - \frac{\rho_0}{1+(\frac{r_c}{r_0})^2}\right]\theta(r_c - r). \tag{10}$$

Such a cutoff is inevitable, even in MOND, owing to the presence of other galaxies. Of course it is also necessary to make the potential $\Delta B(r)$ vanish at infinity.

For $r < r_c$ the mass function and potentials are

$$M(r) = 4\pi\rho_0 r_0^3\left\{\frac{r}{r_0} - \tan^{-1}\left(\frac{r}{r_0}\right) - \frac{r^3}{3r_0(r_0^2+r_c^2)}\right\}, \tag{11}$$

$$\Delta A(r) = \frac{8\pi G\rho_0 r_0^2}{c^2}\left\{1 - \frac{r_0}{r}\tan^{-1}\left(\frac{r}{r_0}\right) - \frac{r^2}{3(r_0^2+r_c^2)}\right\}, \tag{12}$$

$$\Delta B(r) = \frac{8\pi G\rho_0 r_0^2}{c^2}\left\{-1 + \frac{r_0}{r}\tan^{-1}\left(\frac{r}{r_0}\right) + \frac{3r_c^2 - r^2}{6(r_0^2+r_c^2)} - \frac{1}{2}\ln\left[\frac{r_c^2+r_0^2}{r^2+r_0^2}\right]\right\}. \tag{13}$$

For $r > r_c$ the mass is constant and the (equal and opposite) potentials fall off like $1/r$,

$$M(r) = 4\pi\rho_0 r_0^3\left\{\frac{r_c}{r_0} - \tan^{-1}\left(\frac{r_c}{r_0}\right) - \frac{r_c^3}{3r_0(r_0^2+r_c^2)}\right\}, \tag{14}$$

$$\Delta A(r) = \frac{8\pi G\rho_0 r_0^2}{c^2}\left\{\frac{r_c(2r_c^2+3r_0^2)}{3r(r_c^2+r_0^2)} - \frac{r_0}{r}\tan^{-1}\left(\frac{r_c}{r_0}\right)\right\}, \tag{15}$$

$$\Delta B(r) = \frac{8\pi G\rho_0 r_0^2}{c^2}\left\{-\frac{r_c(2r_c^2+3r_0^2)}{3r(r_c^2+r_0^2)} + \frac{r_0}{r}\tan^{-1}\left(\frac{r_c}{r_0}\right)\right\}. \tag{16}$$

It should be noted that Ascasibar, Jean, Boehm and Knödlseder used a simpler version of the isothermal profile without a cutoff radius $r_c$. This would cause the potential $\Delta B(r)$ to eventually become positive, which violates the bound of Moore and Nelson [55]. It also does not make any sense when one considers the effect of other galaxies. Because the isothermal profile is the most closely related to MOND we considered it important to include results for this profile, so we used the values of $\rho_0$ and $r_0$ given by Ascasibar, Jean, Boehm and Knödlseder [60], along with $r_c = 219$ kpc. This choice for $r_c$ causes the ratio of the total masses of the Milky Way and Andromeda galaxies for the isothermal profile to agree with that of the Navarro-Frenk-White (NFW) profile considered in the next subsection.

### B. NFW profile

The NFW profile was the result of studying the equilibrium density profiles of dark matter halos in numerical simulations of structure formation [64],

$$\rho(r) = \frac{\rho_0}{\frac{r}{r_0}[1+\frac{r}{r_0}]^2}. \tag{17}$$

The associated mass function and potentials are

$$M(r) = 4\pi\rho_0 r_0^3 \times \left\{\ln\left[1+\frac{r}{r_0}\right] - \frac{r}{r_0+r}\right\}, \tag{18}$$

$$\Delta A(r) = \frac{8\pi G\rho_0 r_0^2}{c^2} \times \left\{\frac{r_0}{r}\ln\left[1+\frac{r}{r_0}\right] - \frac{r_0}{r_0+r}\right\}, \tag{19}$$

$$\Delta B(r) = \frac{8\pi G\rho_0 r_0^2}{c^2} \times -\frac{r_0}{r}\ln\left[1+\frac{r}{r_0}\right]. \tag{20}$$

### C. Moore profile

A later effort along the same lines showed a better fit to a density function which is more sharply peaked at the center [65],

$$\rho(r) = \frac{\rho_0}{(\frac{r}{r_0})^{3/2}[1+\frac{r}{r_0}]^{3/2}}. \tag{21}$$

The associated mass function and potentials are

$$M(r) = 4\pi\rho_0 r_0^3 \times \frac{2}{3}\ln\left[1+\left(\frac{r}{r_0}\right)^{3/2}\right], \tag{22}$$

$$\Delta A(r) = \frac{8\pi G\rho_0 r_0^2}{c^2} \times \frac{2}{3}\frac{r_0}{r}\ln\left[1+\left(\frac{r}{r_0}\right)^{3/2}\right], \tag{23}$$

$$\Delta B(r) = \frac{8\pi G\rho_0 r_0^2}{c^2} \times \left\{-\frac{2}{3}\frac{r_0}{r}\ln\left[1+\left(\frac{r}{r_0}\right)^{3/2}\right] + \ln\left[1+\left(\frac{r_0}{r}\right)^{1/2}\right] - \frac{1}{3}\ln\left[1+\left(\frac{r_0}{r}\right)^{3/2}\right] - \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{\sqrt{3r_0}}{2\sqrt{r}-\sqrt{r_0}}\right]\right\}. \tag{24}$$

## IV. THE SHAPIRO DELAY FOR GIVEN $M(r)$

It is more convenient to convert the spatial coordinates from spherical $(r, \theta, \phi)$ to Cartesian $x^i$,

$$\vec{x} \equiv r(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \equiv r\hat{r}. \tag{25}$$

The invariant element (1) has a simple expression in terms of these coordinates,

$$ds^2 = -c^2dt^2 + d\vec{x}\cdot d\vec{x} - \Delta Bc^2dt^2 + \Delta A(\hat{r}\cdot d\vec{x})^2. \tag{26}$$

Before moving on we digress to note that specializing to

$ds^2 = 0$ and identifying the velocity of photons and neutrinos as $\vec{v}_m = d\vec{x}/dt$ results in an equation for the speed of effectively massless, ordinary matter,

$$0 = -(1 + \Delta B)c^2 + \vec{v}_m \cdot \vec{v}_m + \Delta A(\hat{r} \cdot \vec{v}_m)^2. \tag{27}$$

Now recall that we are ignoring the role of ordinary matter in both the metrics of gravity (where it is the only part of the mass density) and of ordinary matter. It follows that the speed of gravity is $v_g$ is $c$. Treating to first order in the potentials and assuming $\Delta A(r) \geq 0$ and $\Delta B(r) \leq 0$, we see that $v_m - v_g \leq 0$.

We can express the invariant element (26) as the flat space contribution plus a perturbation,

$$ds^2 \equiv (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu. \tag{28}$$

Comparing (26) and (28) allows us to read off the 3 + 1 decomposition of the graviton field $h_{\mu\nu}$,

$$h_{00} = -\Delta B, \qquad h_{0i} = 0 \quad \text{and} \quad h_{ij} = \Delta A \hat{r}^i \hat{r}^j. \tag{29}$$

One advantage of Cartesian coordinates is that the affine connection vanishes for the flat background. It is easy to give the first correction,

$$\Gamma^\mu{}_{\rho\sigma} = \Delta\Gamma^\mu{}_{\rho\sigma} + O(h^2), \tag{30}$$

$$\Delta\Gamma^\mu{}_{\rho\sigma} = \frac{1}{2}\eta^{\mu\nu}(h_{\nu\rho,\sigma} + h_{\sigma\nu,\rho} - h_{\rho\sigma,\nu}). \tag{31}$$

We need the null geodesic $\chi^\mu(\tau)$ that connects the spacetime points $x_1^\mu = (0, \vec{x}_1)$ and $x_2^\mu = (ct, \vec{x}_2)$. It obeys the geodesic equation,

$$\ddot{\chi}^\mu + \Gamma^\mu{}_{\rho\sigma}(\chi(\tau))\dot{\chi}^\rho\dot{\chi}^\sigma = 0, \tag{32}$$

subject to the conditions,

$$\chi^\mu(0) = x_1^\mu, \tag{33}$$

$$\chi^i(1) = x_2^i, \tag{34}$$

$$g_{\mu\nu}(x_1)\dot{\chi}^\mu(0)\dot{\chi}^\nu(0) = 0. \tag{35}$$

Of course we solve this perturbatively in the potentials. The zeroth order solution is of course the flat space result,

$$\chi_0^\mu(\tau) = x_1^\mu + \Delta x^\mu \tau. \tag{36}$$

Here the temporal and spatial components of the interval $\Delta x^\mu$ are

$$\Delta x^0 \equiv \|\vec{x}_2 - \vec{x}_1\| \quad \text{and} \quad \Delta x^i \equiv x_2^i - x_1^i. \tag{37}$$

The first order corrections to the spatial components of the geodesic are

$$\chi_1^i(\tau) = \tau\int_0^1 d\tau'(1-\tau')\Delta\Gamma^i_{\rho\sigma}(x + \Delta x\tau)\Delta x^\rho \Delta x^\sigma - \int_0^\tau d\tau'(\tau-\tau')\Delta\Gamma^i_{\rho\sigma}(x_1 + \Delta x\tau)\Delta x^\rho\Delta x^\sigma. \tag{38}$$

Of course it is from the first order temporal correction that we infer the time lag. This correction is more complicated,

$$\chi_1^0(\tau) = \frac{\tau}{2\Delta x}h_{\rho\sigma}(x_1)\Delta x^\rho\Delta x^\sigma + \frac{\tau}{\Delta x}\int_0^1 d\tau'(1-\tau')\Delta x^i\Delta\Gamma^i_{\rho\sigma}(x+\Delta x\tau)\Delta x^\rho\Delta x^\sigma - \int_0^\tau d\tau'(\tau-\tau')\Delta\Gamma^0_{\rho\sigma}(x_1+\Delta x\tau)\Delta x^\rho\Delta x^\sigma. \tag{39}$$

Ignoring ordinary matter makes the graviton geodesics identical to $\chi_0^\mu(\tau)$. Hence the time lag between the arrival of gravitational waves and the arrival of photons or neutrinos is (to first order in the potentials)

$$c\Delta t \equiv \chi_1^0(1) = \frac{1}{2\Delta x}\int_0^1 d\tau h_{\mu\nu}(x_1 + \Delta x\tau)\Delta x^\mu\Delta x^\nu. \tag{40}$$

Expression (40) can be simplified a great deal further. First expand out the potentials,

$$h_{\mu\nu}\Delta x^\mu\Delta x^\nu = -\Delta B\Delta x^2 + \Delta A(\hat{r}\cdot\Delta\vec{x})^2. \tag{41}$$

(Note that the time lag is positive semidefinite because $\Delta B \leq 0$ and $\Delta A \geq 0$.) Now use relation (9) for $\Delta B(r)$ in terms of $\Delta A(r)$ and partially integrate to reach the form

$$c\Delta t = -\frac{\Delta x}{2}\Delta B(r_2) + \frac{\Delta x}{2}\int_0^1 d\tau\Delta A(r)\left[\frac{\tau}{r}\frac{\partial r}{\partial\tau} + \left(\frac{\hat{r}\cdot\Delta\vec{x}}{\Delta x}\right)^2\right] \tag{42}$$

$$= \frac{\Delta\vec{x}\cdot\vec{x}_1}{2\Delta x}\Delta B(r_1) - \frac{\Delta\vec{x}\cdot\vec{x}_2}{2\Delta x}\Delta B(r_2) + \Delta x\int_0^1 d\tau\Delta A(r)\left[1 + \frac{(\vec{x}_1\cdot\Delta\vec{x})^2 - r_1^2\Delta x^2}{\Delta x^2 r^2}\right]. \tag{43}$$

It is useful to define the constant $C$,

$$C \equiv \frac{1}{\Delta x^2}\sqrt{r_1^2\Delta x^2 - (\vec{x}_1\cdot\Delta\vec{x})^2}. \tag{44}$$

Finally, we change variables from $\tau$ to $r$,

$$r(\tau) = \Delta x\sqrt{\tau + \left(\frac{\vec{x}_1\cdot\Delta\vec{x}}{\Delta x^2}\right)^2 + C^2}. \tag{45}$$

Assuming $r_2 < r_1$ the result is

$$c\Delta t = \frac{\Delta\vec{x}\cdot\vec{x}_1}{2\Delta x}\Delta B(r_1) - \frac{\Delta\vec{x}\cdot\vec{x}_2}{2\Delta x}\Delta B(r_2) + \int_{r_2}^{r_1} dr\frac{2GM(r)}{c^2 r}\sqrt{1 - \left(\frac{C\Delta x}{r}\right)^2}. \tag{46}$$

For $r_1 < r_2$ we take the other root of the solution for $\tau(r)$, which reverses the upper and lower limits in (46).

## V. RESULTS

We have worked out explicit results for three typical sources at vastly different distances: GRB 070201, SN 1987a and Sco-X1. Their celestial coordinates are given in Table III. Table III also gives the centers of the Milky Way and Andromeda dark matter halos.

GRB 070201 was a short hard gamma-ray burst whose angular error box corresponded to a 0.124° quadrilateral which overlapped with the Andromeda galaxy [66]. Short hard gamma-ray bursts are believed to be caused by the mergers of two neutron stars or a neutron star and a black hole [67]. If GRB 070201 derived from such a merger, with masses close to $1.4 M_\odot$ and a reasonable orientation, the GW signal should have been seen if its distance was 780 kpc [68]. It is however possible that the GRB did not originate in the Andromeda galaxy, and in that case the signal from a compact object merger may be inaccessible to LIGO.

No gravitational waves were found from a search done within $\pm 180$ s time window with the LIGO Hanford detectors around the time of this GRB [69,70]. One interpretation of this null result is that GRB 070201 was a SGR flare [69,71]. However, it is also possible that physics is described by a dark matter emulator, in which case the pulse of gravitational waves would have arrived long before the electromagnetic signal. Table IV gives our results for the time lag one would expect at the central position using each of the three dark matter density profiles. Although the time lags differ by as much as 69 days, none of the lags is less than 2 yr.

Table V considers another measure of the likely error by specializing to the isothermal profile and varying the angular position (at the fixed distance of 780 kpc) over the four vertices of the angular error box. In this case distinct results are reported for the contributions from the Milky Way and Andromeda halos, which are of course independent at linearized order. As expected, varying the position has no effect on the contribution from the Milky Way halo, but it can change the contribution from the Andromeda halo by as much as 15 days. We should however stress that this delay calculation was done by assuming that this GRB is at a distance of 780 kpc. If this GRB did not originate in Andromeda, then the calculated delay would be much larger.

SN 1987a was a core-collapse supernova in the Large Magellanic Cloud at a distance of 51.4 kpc [72]. Neutrinos were observed by the Kamiokande-II [73,74] and Irvine-Michigan-Brookhaven [75,76] detectors. The optical signal arrived several hours later because photons must traverse the optically dense stellar environment [72]. The total Shapiro delay for SN 1987A from the visible and dark matter distribution has also been independently estimated [10,77] to be between 0.29 to 0.36 yr. If the oblateness of SN 1987a was in relation to that of the Sun, the current gravitational wave detectors would probably not have seen anything had they been operating at the time [78]. However, advanced LIGO would detect such a supernova out to 0.8 Mpc [78]. This includes the Andromeda galaxy, which doubles the expected rate and also ensures that the signal passes through dark matter dominated regions. Of course the effective coverage from neutrino detectors will remain limited to our galaxy and its satellites [79,80].

Table IV gives our results for the expected time lag from a dark matter emulator which reproduces each of the three dark matter profiles. These results include only the effect of the Milky Way halo. In contrast to the much more distant GRB 070201, the scatter between the various models for SN 1987a is much smaller—a mere 2.7 days.

Sco-X1 (located at a distance of 2.8 kpc) is one of the brightest low mass x-ray binaries (LMXBs). LMXBs are potential sources of gravitational waves from $r$-modes getting excited due to accretion, or from a deformed crust [81–83]. One proposed search is to look for coincidences between the data from LIGO and Rossi X-Ray Timing satellite [4]. This search also assumes that gravitational waves and x-ray photons arrive at the same time.

TABLE III. Angular coordinates and distances for the Milky Way and Andromeda galaxies and for the three sources of this study.

| Object | Right Ascen. | Declination | Distance |
|---|---|---|---|
| Milky Way | 17 h 45 m 40 s | $-29°\,00'\,28''$ | 7.94 kpc |
| Andromeda | 00 h 42 m 44 s | $+41°\,16'\,09''$ | 778 kpc |
| GRB 070201 | 00 h 44 m 32 s | $+42°\,14'\,21''$ | 780 kpc |
| SN 1987a | 05 h 35 m 28 s | $-69°\,16'\,12''$ | 51.4 kpc |
| Sco-X1 | 16 h 19 m 55 s | $-15°\,38'\,24''$ | 2.80 kpc |

TABLE IV. Time delays from three dark matter profiles for each of the three sources of this study.

| Profile | GRB 070201 | SN 1987a | Sco-X1 |
|---|---|---|---|
| Isothermal | 742 days | 78.2 days | 4.98 days |
| NFW | 804 days | 74.8 days | 4.88 days |
| Moore | 811 days | 74.5 days | 4.97 days |

TABLE V. Shapiro delays for GRB 070201 from the isothermal profiles of the Milky Way ($\Delta t_{\rm MW}$) and Andromeda ($\Delta t_{\rm M31}$) at the central value of the angular position and at the four vertices of the error box. In all cases the distance to the burst was taken to be 780 kpc.

| Right Ascension | Declination | $\Delta t_{\rm MW}$ | $\Delta t_{\rm M31}$ |
|---|---|---|---|
| 00 h 44 m 32 s | $42°\,14'\,21''$ | 407 dy | 335 dy |
| 00 h 46 m 18 s | $41°\,56'\,42''$ | 407 dy | 337 dy |
| 00 h 41 m 51 s | $42°\,52'\,08''$ | 407 dy | 322 dy |
| 00 h 42 m 47 s | $42°\,31'\,41''$ | 407 dy | 330 dy |
| 00 h 47 m 14 s | $41°\,35'\,35''$ | 407 dy | 338 dy |

Table IV reports the expected time lag for each of the three dark matter profiles, again from the Milky Way halo. Although the time lag is still easily observable at $\sim$4.9 days, the agreement between the three models for this source is excellent. The largest discrepancy is just 0.1 day.

## VI. OTHER MODIFIED GRAVITY THEORIES

Since the 1970s, there have been various proposed tests of general relativity through gravitational wave observations (See Ref. [84] for a recent review). Most of these tests are in the strong field regime. In this section, we list some other non-GR gravity theories which also predict a nonzero time delay between photons and GWs and are not yet ruled out through other observations.

These are massive graviton theories and brane-world models. In massive graviton theories [85], the gravitational waves would arrive *after* the photons, with the delay being dependent on the graviton mass. However, the Moore-Nelson lower bound on the speed of gravitational waves imposes stringent constraints on the validity of massive graviton models. It should be noted that there is no consistent interacting theory for massive spin two particles which is limited to a finite number of fields [86].

In various brane-world models, gravitational waves propagate faster than photons or neutrinos, depending on the curvature of the bulk [87–89]. Therefore, it is not possible to calculate model-independent time delays for the three sources we considered in this paper.

## VII. CONCLUSIONS

The power and generality of our analysis derives from ignoring the details of how a dark matter emulator dispenses with the need for galactic dark matter. We merely assume that it does, which implies that ordinary matter must couple to the metric that general relativity would predict with dark matter. The special characteristic of dark matter emulators is that weak gravitational waves couple to the metric that general relativity would predict without dark matter. Both of these metrics can be inferred from observation, and all geometrical quantities worked out, without regard to the details of specific models. Although a dark matter emulator is not the only conceivable way of evading the no-go theorem [53] while preserving solar system tests [50], it is the only way that has so far been given a concrete realization.

If dark matter does not exist and the observed cosmic motions and lensing instead derive from a dark matter emulator then the assumption upon which all triggered gravitational wave searches are based breaks down. In this case the optical or neutrino identification of a plausible gravitational wave source would not imply a simultaneous pulse of gravitational waves but rather that such a pulse occurred *earlier*. Even for nearby sources such as Sco-X1 (at about 2.8 kpc) the gravitational waves would arrive almost five days earlier. For a source in the Andromeda galaxy the time difference would be over two years.

It is obviously premature to proclaim that the failure of triggered searches to reveal any coincident gravitational wave pulse implies that physics is described by a dark matter emulator. But if plausible sources continue to produce null results this possibility has to be considered. In that case the key question becomes the accuracy with which one can estimate the expected time lag. Some measure of this is given by the spread in Table IV for different reasonable dark matter profiles. Table V considers variations in the angular position, and there will be comparable results for varying the much less well-determined distances. Based on these analyses it seems unlikely that the uncertainty can be reduced below the level of a few percent. This has important implications for the way data needs to be kept and for the types of searches that should be contemplated.

Of course a gravitational wave signal might be loud enough to show up in all-sky untriggered searches [90]. In that case the gravitational wave signal would trigger electromagnetic and neutrino searches. A single gold-plated event in which the counterpart signal arrived after a plausible delay would be powerful evidence in support of dark matter emulators. Conversely, a single detection of coincident signals would rule out the entire class of dark matter emulators. This is a novel way of using gravitational wave detectors to test alternate gravity theories in the ultraweak field regime. Indeed, this is an ideal test because dark matter emulators do not change aspects of the tensor component of a gravitational wave signal such as the number of polarizations or, to any reasonable accuracy, the travel time between Earth-bound detectors. So there need be no change in the data analysis algorithms that would be used in any case.

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