Standard model on a domain-wall brane?

Rhys Davies,* Damien P. George,+ and Raymond R. Volkas‡

School of Physics, Research Centre for High Energy Physics, The University of Melbourne, Victoria 3010, Australia
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We propose a 4 + 1-dimensional action that is a candidate for realizing a standard-model-like effective theory for fields dynamically localized to a domain-wall brane. Our construction is in part based on the conjecture that the Dvali-Shifman mechanism for dynamically localizing gauge bosons to a domain wall works correctly in 4 + 1-d. Assuming this to be so, we require the gauge symmetry to be SU(5) in the bulk, spontaneously breaking to SU(3) ⊗ SU(2) ⊗ U(1) inside the domain wall, thus dynamically localizing the standard-model gauge bosons provided that the SU(5) theory in the bulk exhibits confinement. The wall is created jointly by a real singlet-Higgs field η configured as a kink, and an SU(5) adjoint-Higgs field χ that takes nonzero values inside the wall. Chiral 3 + 1-dimensional quarks and leptons are confined and split along the bulk direction via their Yukawa couplings to η and χ. The Higgs doublet and its color triplet SU(5) partner are similarly localized and split. The splittings can suppress colored-Higgs-induced proton decay and, because of the different localization profiles, the usual SU(5) mass relation $m_e = m_d$ does not arise. Localized gravity is generated via the Randall-Sundrum alternative to compactification.

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I. INTRODUCTION

There is no known fundamental principle requiring spacetime to be 3 + 1 dimensional, so extra dimensions of space might exist. If so, then the effective 3 + 1 dimensionality we observe in everyday life and in high-energy experiments has to be explained. It could be that the extra dimensions are topologically compact and small, as per the Kaluza-Klein idea. Alternatively, the extra dimensions could be large but as yet unobserved because standard-model (SM) fields are confined to a 3 + 1-d brane. Large extra dimensions might be compact, as proposed by Arkani-Hamed, Dimopoulos, and Dvali [1], or infinite, as shown in the second of the Randall-Sundrum papers of 1999 [2] (hereinafter RS2). See also Refs. [3–7].

The purpose of this paper is to propose a 4 + 1-dimensional action that is a candidate for realizing a SM-like effective theory for fields dynamically localized to a domain-wall (DW) brane. Like RS2, there is one extra dimension, and it is infinite. Unlike RS2, the brane is not a fundamental object but rather a solitonic solution of the theory, as per the Rubakov and Shaposhnikov [8] proposal that we might live on a DW. Our construction assembles a number of dynamical localization mechanisms into what we hope is a complete theory of a DW-localized SM. These mechanisms are

(i) the localization of 3 + 1-d chiral fermion zero modes through the Yukawa coupling of 4 + 1-d fermions to the background scalar fields;

(ii) the localization of a SM Higgs doublet to the DW through its Higgs potential couplings to the DW-forming scalar fields;

(iii) the localization of SM gauge bosons via the Dvali-Shifman (DS) mechanism, instituted through a bulk that respects SU(5) gauge invariance [9];

(iv) the DW generalization of the RS2 mechanism for localizing gravitons.

Three of these four mechanisms involve well-established phenomena. The DS gauge boson localization idea remains an interesting conjecture in the 4 + 1-d context, not as yet proven to work. What we shall show in this paper is that *if* one takes the DS mechanism to work in 4 + 1-d, *then* the construction of a DW-localized SM follows readily, and even elegantly. We hope that our model spurs rigorous studies of the DS mechanism in 4 + 1-d, to either confirm it or disprove it. Were it to be confirmed, then our model-building setup would provide a clear pathway to the construction of phenomenologically realistic effective theories of DW-localized fields. We shall review the DS mechanism below.

The main aesthetic motivation for our model is to treat all spatial dimensions on an equal footing in the action. In particular, all these dimensions are infinite, as in the RS2 setup. But "dimensional democracy" is taken further than in RS2, because that theory has translational invariance along the extra dimension explicitly broken through the introduction of an infinitely thin fundamental brane into the action. To achieve dimensional democracy we must have no explicit brane terms in the action, but replace the RS2 fundamental brane with a finite-thickness stable DW configuration of scalar fields.

*r.davies1@physics.ox.ac.uk
+d.george@physics.unimelb.edu.au
‡raymondv@unimelb.edu.au

We shall argue that our theory is likely to be the minimal way to get a purely field-theoretic realization of a DW-confined SM. It is interesting that in order to achieve this the DS mechanism immediately motivates an extension to SU(5). We are also encouraged by the fact that some of the usual problems of SU(5) grand unification have solutions automatically provided by the minimal theory, without "epicyclic" *ad hoc* fixes. As we shall explain below, the usual $m_d = m_e$ style SU(5) mass relations are simply absent, because the fermion localization realizes a modified version of the split-fermion idea of Arkani-Hamed and Schmaltz [10] (see also [11]). The down-type quarks necessarily have different bulk profile functions from the charged leptons, and because the 3 + 1-d masses are computed from overlap integrals of profile functions, the quark-lepton mass degeneracy just does not arise. The fermion splitting can also suppress colored-Higgs-induced proton decay. An important loose end is that we are not yet able to analyze gauge coupling constant unification in our unusual version of SU(5). We shall explain below why a full unification study is premature.

Our focus in this paper is on model building rather than detailed phenomenology. We wish to explain the logic of our construction, and provide evidence that it has good phenomenology without supplying absolute proof.

We review the DS mechanism in the next section, describe our model in the following section, and conclude in the last section.

II. DVALI-SHIFMAN MECHANISM

The most plausible mechanism for localizing gauge bosons to a DW in such a way as to preserve gauge invariance is that proposed by Dvali and Shifman [12]. This requires a confining non-Abelian gauge theory in the bulk, with the symmetry G broken to a subgroup H inside the DW. Massless gauge bosons corresponding to H are then localized to the wall. As we wish to localize the SM fields, the minimal choice is to take $G = \mathrm{SU}(5)$ and $H = \mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$.

The truth of the DS mechanism rests on quite a firm foundation for DWs residing in a background 3 + 1-d spacetime [13–15]. Following DS, let us consider the simple toy example of $G = \mathrm{SU}(2)$ and $H = \mathrm{U}(1)$. Place a U(1) source charge inside the wall. Because the SU(2)-respecting bulk is in confinement phase, the electric field lines of the source charge cannot penetrate into the bulk. Instead, the field lines are repelled from the DW-bulk interface, thus reducing the effective dimensionality of the Coulomb field by one. Adopting the 't Hooft-Mandelstam proposal that confinement arises from the magnetic dual of superconductivity, the repulsion of field lines from the interface is readily understood from the dual Meissner effect [14,15].

Now, place the source charge in the bulk. By confinement, which is tantamount to the expulsion of electric fields, the electric flux from the source must form a flux tube that ends on the DW [14,15]. Once inside the wall, the field lines are able to spread out in the plane of the wall. It is as if the charge was actually inside the wall: the electric field configuration is the same at large distances inside the wall irrespective of the position of the source. In the quantal situation where the position of a source charge is indefinite, it follows that the long-range Coulomb field is independent of how the wave function depends on the coordinate perpendicular to the wall (the "extra" dimension). We shall be using this result below when we assume that gauge universality for H holds *independently of the bulk profiles of the trapped fields*.

If H is non-Abelian, then these arguments generalize to the case of chromoelectric field line expulsion from the bulk.

Another perspective on the localization physics is provided by the mass gap [12]. In the bulk, because of confinement, the gauge bosons of H cannot themselves propagate but instead form constituents of propagating G glueballs. But the glueballs of G are massive. In the $G = \mathrm{SU}(2)$ and $H = \mathrm{U}(1)$ example, the U(1) gauge boson, which is both massless and free inside the wall, must somehow incorporate itself into a massive SU(2) glueball if it propagates into the bulk. But the mass gap implies an energy cost in doing so, thus any U(1) gauge boson inside the wall is dynamically constrained to remain there. If H has non-Abelian factors that are themselves in confinement phase inside the wall, then the mass gap suppression corresponds to the H glueballs inside the wall being less massive than the G glueballs in the bulk.

These arguments are rather convincing, because they rest on the well-established confinement property for asymptotically free non-Abelian gauge theories in 3 + 1-d. In the 4 + 1-d case, the DS mechanism is a conjecture, because 4 + 1-d confinement (or lack thereof) is not properly understood. The main issue is that pure Yang-Mills theory is not renormalizable in 4 + 1-d (or larger). At the level of lattice gauge calculations, this corresponds to the lack of a physical limit when taking the lattice spacing to zero. To expand on this point, it is known that 4 + 1-d SU(2) has a first-order phase transition for finite lattice spacing [16]. We have verified this conclusion for 4 + 1-d SU(5), and so presumably SU(5) has a confining phase for sufficiently large values of the gauge coupling constant. This analysis cannot be extended to the continuum limit, and so we must be content with 4 + 1-d SU(5) exhibiting confinement *below* a relevant cutoff of the theory. Thus, we consider 4 + 1-d DS to be an effective mechanism, valid below this cutoff, which does the job of confining gauge fields to the DW. As we remark below, any field-theoretic brane-world model is nonrenormalizable and hence must be defined with an ultraviolet cutoff, so in our context we do not need to take the continuum limit.

To the best of our knowledge, the DS mechanism has not been directly checked in 4 + 1-d, which would require more than just an analysis of the phase structure of pure Yang-Mills theory. But we are encouraged by lattice gauge calculations in 2 + 1-d [17], which do verify the mechanism. We shall assume that it works also in 4 + 1-d and show that realistic model building is then quite possible.

III. THE MODEL

We now describe our model. As stated above, the DS mechanism immediately motivates that the bulk should respect at least an SU(5) gauge symmetry. By one definition of "minimal," the bulk symmetry should be exactly SU(5), and it should also be the symmetry of the action; the model presented below has these features. (It is also interesting to consider models not adhering to these strictures. For example, Ref. [18] describes a theory where the symmetry of the action is larger than the symmetry of the bulk.)

The SU(5) 4 + 1-d field content is

$$\begin{aligned} &\text{scalars: } \eta \sim 1, \chi \sim 24, \Phi \sim 5^* \\ &\text{fermions: } \Psi_5 \sim 5^*, \Psi_{10} \sim 10, \end{aligned} \tag{1}$$

plus gauge fields. The field η is real, χ is conveniently represented as a 5×5 Hermitian traceless matrix, while Φ is a fivefold column vector of complex fields. Chirality does not exist in 4 + 1-d, so both the Ψ's are Dirac fields, with Ψ_{10} being a 5×5 antisymmetric matrix. The SU(5) transformations are $\chi \to U\chi U^\dagger$, $\Phi \to U^*\Phi$, $\Psi_5 \to U^*\Psi_5$, and $\Psi_{10} \to U\Psi_{10}U^T$. We shall, for simplicity, consider only one quark-lepton family here, though the generalization to three families is straightforward. The neutrino mass question is also deferred to later work.

Let us begin by ignoring gravity, to focus on the purely particle-physics aspects of the model. Later, we discuss what remains the same, and what changes, when RS2-style warped gravity is added. The action is

$$S = \int d^5x (T - Y_{\rm DW} - Y_5 - V), \tag{2}$$

where T contains the SU(5) gauge-covariant kinetic-energy terms, $Y_{\rm DW}$ has the Yukawa couplings of the fermions to η and χ,

$$\begin{aligned} Y_{\rm DW} = {}& h_{5\eta}\bar{\Psi}_5\Psi_5\eta + h_{5\chi}\bar{\Psi}_5\chi^T\Psi_5 + h_{10\eta}\,{\rm Tr}(\bar{\Psi}_{10}\Psi_{10})\eta \\ &- 2h_{10\chi}\,{\rm Tr}(\bar{\Psi}_{10}\chi\Psi_{10}), \end{aligned} \tag{3}$$

and Y_5 is the SU(5) Yukawa Lagrangian used to generate quark and lepton masses

$$Y_5 = h_- \overline{(\Psi_5)^c}\Psi_{10}\Phi + h_+(\epsilon\overline{(\Psi_{10})^c}\Psi_{10}\Phi^*) + {\rm H.c.} \tag{4}$$

The last term can only be written in SU(5) index notation: $\epsilon^{ijklm}\overline{(\Psi_{10})^c}_{ij}\Psi_{10kl}\Phi^*_m$.

The Higgs potential is $V = V_{\eta\chi} + V_{\rm rest}$, where

$$\begin{aligned} V_{\eta\chi} = {}& (c\eta^2 - \mu_\chi^2)\,{\rm Tr}(\chi^2) + a\eta\,{\rm Tr}(\chi^3) + \lambda_1[{\rm Tr}(\chi^2)]^2 \\ &+ \lambda_2\,{\rm Tr}(\chi^4) + l(\eta^2 - v^2)^2; \end{aligned} \tag{5}$$

$$\begin{aligned} V_{\rm rest} = {}& \mu_\Phi^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2 + \lambda_4\Phi^\dagger\Phi\eta^2 \\ &+ 2\lambda_5\Phi^\dagger\Phi\,{\rm Tr}(\chi^2) + \lambda_6\Phi^\dagger(\chi^T)^2\Phi \\ &+ \lambda_7\Phi^\dagger\chi^T\Phi\eta. \end{aligned} \tag{6}$$

The action is invariant under the reflection discrete symmetry $y \to -y$, $\eta \to -\eta$, $\chi \to -\chi$, and $\Psi_{5,10} \to i\Gamma^5\Psi_{5,10}$. The 4 + 1-d Dirac matrices are $\Gamma^M = (\gamma^\mu, -i\gamma^5)$, where $M, N = (0, 1, 2, 3, 5)$, $\mu, \nu = (0, 1, 2, 3)$, and $x^5 \equiv y$.

The theory is nonrenormalizable in 4 + 1-d. As usual in these kinds of models, there is an implicitly assumed ultraviolet cutoff $\Lambda_{\rm UV}$ and an ultraviolet completion above that scale. We shall adopt the agnostic stance for both the existence and nature of this UV completion. Our action is perhaps best considered as the set of lowest-dimensional operators, consistent with the stated symmetries, of a nonrenormalizable effective theory that is putatively to be derived from the UV completion.

The background DW is found by solving the (η, χ) Euler-Lagrange equations for an x^μ-independent configuration obeying the boundary conditions

$$\eta(y = \pm\infty) = \pm v, \qquad \chi(y = \pm\infty) = 0, \tag{7}$$

corresponding to degenerate global minima of $V_{\eta\chi}$. The spontaneously broken reflection symmetry ensures topological stability for the DW. Numerical solutions exist for a significant region of parameter space. Purely for the sake of giving a concrete example, we can impose the parameter conditions

$$2\mu_\chi^2(c - \tilde{\lambda}) + (2c\tilde{\lambda} - 4l\tilde{\lambda} - c^2)v^2 = 0, \quad a = 0, \tag{8}$$

with $\tilde{\lambda} \equiv \lambda_1 + 7\lambda_2/30$, permitting the analytic solution

$$\eta(y) = v\tanh(ky), \qquad \chi_1(y) = A{\rm sech}(ky), \tag{9}$$

where $k^2 = cv^2 - \mu_\chi^2$, $A^2 = (2\mu_\chi^2 - cv^2)/\tilde{\lambda}$, and χ_1 is the adjoint component associated with the weak-hypercharge generator ${\rm diag}(2/3, 2/3, 2/3, -1, -1) \times \sqrt{3}/2\sqrt{5}$. All other χ components vanish. The configuration η is the usual kink, while χ_1 induces SU(5) $\to$ SU(3) $\otimes$ SU(2) $\otimes$ U(1) within the DW, which has width $1/k$. This background solution creates the brane and simultaneously confines SM gauge fields to it provided the DS mechanism works with this kind of an SU(5) bulk in 4 + 1-d.

We have checked numerically that configurations such as Eq. (9) are perturbatively stable against the formation of additional nonzero χ components. The other fields, Φ, Ψ_5, and Ψ_{10} propagate in this background. Within the wall, the SU(5) confinement dynamics are suppressed, so we can analyze classical localization solutions for fermions and scalars in the usual way. Outside the wall, the nonperturbative SU(5) physics makes calculating impossible absent a dedicated lattice program. Since the localization takes place with a characteristic distance scale of $1/k$, ignoring the nonperturbative corrections is approximately valid.

It may be worthwhile to expand on this point. The computation of localized lowest-energy modes, such as fermion zero modes, is but the start of a systematic mode analysis, whereby 4 + 1-d fields are reinterpreted as infinite towers of 3 + 1-d fields (generalization of a Kaluza-Klein decomposition); see, for example, Refs. [19,20] for an introduction to this procedure. Schematically, one employs a mode decomposition of the form $\Psi(x, y) = \sum_n f_n(y)\psi_n(x)$ $(x \equiv x^\mu)$, where the sum is over suitable modes and includes an integration if the modes contain a continuum. The ψ_n are the 3 + 1-d fields, and the f_n are mode functions. The mode functions are usually chosen to obey certain suitable differential equations so that the ψ_n fields are those of definite mass in the effective 3 + 1-d theory (in the familiar Kaluza-Klein case of a circular extra dimension, the mode functions are chosen to be sinusoidal for precisely this reason). However, from a mathematical point of view, the set of mode functions is just some complete set of functions that permits the decomposition of $\Psi(x, y)$ without loss of generality, and so one has the usual freedom to change basis by changing the mode-function set. This is a pertinent observation for theories that employ the nonperturbative quantum-field-theoretic DS mechanism. In the bulk, the ψ_n component fields are subject to these dynamics unless $\Psi(x, y)$ is a gauge singlet, and thus the physical meaning ascribed to the mode functions has to take this into account. There is no problem in using the same mode decomposition one would use in the absence of the nonperturbative bulk, because that is simply a mathematically valid recasting of $\Psi(x, y)$ as an infinite set of ψ_n components. If the bulk is indeed in confinement phase, then the gauge nonsinglet ψ_n fields will not propagate as free particles, so their physical interpretation will be as constituent particles. This is conceptually no different from expressing the QCD Lagrangian in terms of quarks and gluons, even though the propagating states are hadrons. Fortunately, we are mainly interested in the lowest modes, whose mode functions are sharply peaked inside the DW, and so to a first approximation we need not be concerned with interpretive complications because of the nonperturbative bulk.

The 4 + 1-d fermions couple to the background y-dependent scalar fields as per Y_{DW}. A full mode-decomposition analysis would involve writing each 4 + 1-d fermion field as

$$\Psi(x, y) = \sum_m [f_L^m(y)\psi_L^m(x) + f_R^m(y)\psi_R^m(x)], \tag{10}$$

substituting this into 4 + 1-d Dirac equation

$$i\Gamma^M \partial_M \Psi - b(y)\Psi = 0, \tag{11}$$

where $b(y)$ is given by the relevant background DW scalar-field configuration (see below), and requiring that the ψ^m components satisfy the 3 + 1-d Dirac equations

$$i\gamma^\mu \partial_\mu \psi_{L,R}^m = m\psi_{R,L}^m. \tag{12}$$

The mode functions $f_{L,R}^m$ then obey the Schrödinger-like equations

$$-f_{L,R}^{m\prime\prime} + W_\mp f_{L,R}^m = m^2 f_{L,R}^m, \tag{13}$$

with effective potentials

$$W_\mp(y) = b(y)^2 \mp b'(y). \tag{14}$$

The nature of the mode functions is then readily deduced from the analogy with the equivalent quantum-mechanical problem. In particular, note that as $|y| \to \infty$ the potentials tend to the positive constant $b(\pm\infty)^2$. We shall return to this observation later on when we consider gravity.

To analyze the localization of the lowest mode (the $m = 0$ zero mode) for each fermion, the full-mode analysis above is unnecessary. Instead, it suffices to solve the Dirac equations with separated variable configurations $\Psi(x, y) = f(y)\psi_L(x)$, where the $\psi_L(x)$ are 3 + 1-d zero-mode left-chiral fields.[1] The existence of the χ Yukawa terms means that *different* background fields are felt by the various SM components of Ψ_5 and Ψ_{10}. The Dirac equations are

$$\left[i\Gamma^M \partial_M - h_{n\eta}\eta(y) - \sqrt{\frac{3}{5}}\frac{Y}{2}h_{n\chi}\chi_1(y)\right]\Psi_{nY}(x, y) = 0, \tag{15}$$

where $n = 5, 10$ and Y is the weak hypercharge of the SM components denoted Ψ_{5Y} and Ψ_{10Y}. *The SU(5) structure automatically gives different localization points and profiles to the different SM components—splitting* [10,11]*—depending on hypercharge and whether they are in the* 5^* *or the* 10. The zeroes of

$$b_{nY}(y) \equiv h_{n\eta}\eta(y) + \sqrt{\frac{3}{5}}\frac{Y}{2}h_{n\chi}\chi_1(y) \tag{16}$$

are the localization centers, with the bulk profiles

$$f_{nY}(y) \propto e^{-\int^y b_{nY}(y')dy'}. \tag{17}$$

To localize 3 + 1-d left-chiral fields, all the b_{nY} must pass through zero with positive slope. Examples of these split profiles are given in Fig. 1.

The Higgs doublet Φ_w and colored scalar Φ_c contained in Φ are similarly localized [19] and split by their interaction with the background fields, as given by the λ_{4-7} terms

[1]As pointed out by Dvali and Shifman [12], as well as localizing gauge bosons, the confining bulk can localize gauge nonsinglet fermions and scalar fields. However, for our application, we have to retain the seemingly redundant localization-to-a-kink mechanism. The DS mechanism on its own will not suffice, because it will localize vectorlike fermions, not massless chiral fermions. The kink configuration is necessary for the spontaneous generation of chirality in the 3 + 1-d effective theory.

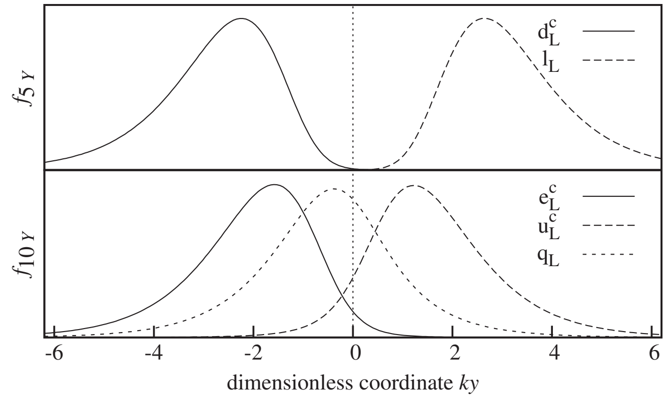

FIG. 1. Typical extra-dimensional profiles $f_{nY}(y)$ for the fermions contained in the 5^* (top) and the 10 (bottom). The fields η and χ are as per Eq. (9), and parameter choices are $v = A = 1$, $h_{n\eta} = 1$, $h_{5\chi} = 6$, $h_{10\chi} = 1$. The profiles are normalized such that $\int dy f_{nY}^2(y) = 1$.

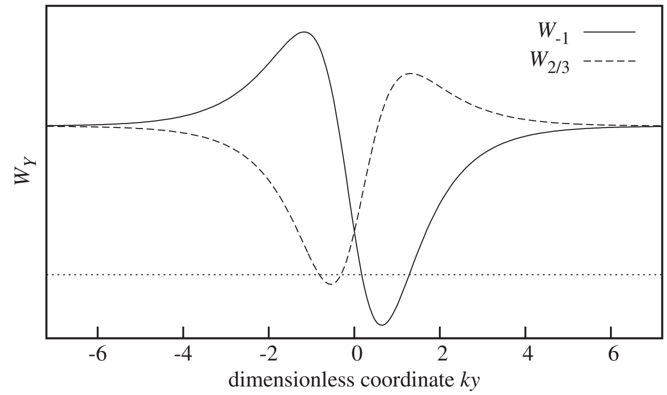

FIG. 2. Example potential profiles $W_Y(y)$, Eq. (19), which trap the Higgs doublet and colored scalar. The straight horizontal line is $W_Y = 0$. Parameters are chosen such that the lowest eigenstate of $W_{-1}(W_{2/3})$ has a negative (positive) eigenvalue. This gives the Higgs doublet a tachyonic mass on the brane while keeping the colored scalar heavy.

in Eq. (6). Writing $\Phi_{w,c}(x, y) = p_{w,c}(y)\phi_{w,c}(x)$, where $\phi_{w,c}$ are required to satisfy a massive 3 + 1-d Klein-Gordon equation with mass-squared parameters $m_{w,c}^2$, the profiles $p_{w,c}$ obey the Schrödinger-like equation,

$$-\frac{d^2}{dy^2}p_{w,c}(y) + W_Y(y)p_{w,c}(y) = m_{w,c}^2 p_{w,c}(y), \tag{18}$$

with a weak-hypercharge-dependent effective potential

$$W_Y(y) = \mu_\Phi^2 + \lambda_4\eta^2 + \lambda_5\chi_1^2 + \frac{3Y^2}{20}\lambda_6\chi_1^2 + \sqrt{\frac{3}{5}}\frac{Y}{2}\lambda_7\eta\chi_1. \tag{19}$$

The full spectrum of localized and delocalized Φ modes is obtained by solving these eigenvalue equations, but we are interested here in only the lowest-mass eigenstates. There is sufficient parameter freedom to allow $m_w^2 < 0$ while $m_c^2 > 0$, thus setting the stage for an effective Mexican-hat potential for ϕ_w and hence electroweak symmetry breakdown inside the wall. An example of the effective potentials $W_Y(y)$ are given in Fig. 2. (The scalar spectrum also contains the kink translational zero mode; Ref. [19] explains how this mode can be frozen out.)

We can now see how natural resolutions arise to some of the usual problems with an SU(5) grand unified theory. The mass relation $m_e = m_d$ is *not* obtained, because the 3 + 1-d Yukawa couplings depend on overlap integrals in the extra dimension, which will be different because of the different fermion localization profiles. The colored scalar ϕ_c induces $p \to \pi^0 e^+$ proton decay through the Yukawa terms $\bar{u}_R(e_R)^c\phi_c^*$ and $\bar{d}_R(u_R)^c\phi_c$, but this effect can be suppressed by making the relevant profile overlaps very small. For example, splitting u_R and d_R so that they overlap exponentially little would suffice [11].

For the one-family SM, it is obvious that we have enough parameters to fit the quark and lepton masses. For the three-family case, it is plausible that sufficient parameter freedom exists, though this has not been rigorously proven as yet. It is a complicated problem, because the physical observables depend on profile functions, which depend in complicated ways on the Lagrangian parameters (and corrections to the classical calculations due to the effect of the nonperturbative bulk will also exist at some level).

Gauge coupling constant evolution cannot be examined until a proper phenomenological parameter fitting is done, because the higher mass modes both depend on these parameters and affect the coupling constant evolution. Since the higher mass modes are split SU(5) multiplets, the running will be different from standard 3 + 1-d nonsupersymmetric SU(5), and successful unification may be possible. Note that coupling constants run logarithmically, not through a power law, in the effective 3 + 1-d theory of localized fields.

We now turn on gravity, with Eq. (2) modified to

$$S = \int d^5x\sqrt{G}(-2M^3R - \Lambda + T - Y_{\rm DW} - Y_5 - V), \tag{20}$$

where G is the determinant of the metric, M the 5D gravitational mass scale, R the scalar curvature, and Λ the bulk cosmological constant. The other terms now include minimal coupling to gravity. We first seek a background η-χ-metric configuration that will simultaneously localize gauge bosons and gravitons. For a significant parameter-space region, the Einstein-Klein-Gordon equations admit numerical solutions where η is a kink, χ_1 is an even function that asymptotes to zero at $|y| = \infty$, and the metric assumes the Minkowski-brane warped form

$$ds_5^2 = e^{-\rho(y)/6M^3}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \tag{21}$$

with $\rho(y) \sim |y|$ asymptotically. The usual Randall-Sundrum fine-tuning condition involving the bulk cosmological constant must be imposed to ensure a Minkowski brane. For the special parameter choices

$$0 = 2c - 4l - \tilde{\lambda}, \qquad a = 0,$$
$$\mu_\chi^2 = lv^2\left(\frac{6M^3}{6M^3 + v^2}\right) + \frac{\tilde{\lambda}v^2}{2}\left(\frac{9M^3 + 2v^2}{6M^3 + v^2}\right) \tag{22}$$

an analytic solution exists; this is useful because it serves as a concrete example

$$\eta(y) = v\tanh(ky), \tag{23}$$

$$\chi_1(y) = v\,\mathrm{sech}(ky), \tag{24}$$

$$\rho(y) = v^2\log[\cosh(ky)], \tag{25}$$

where $k^2 = 3M^3(cv^2 - \mu_\chi^2)/(3M^3 + v^2)$. As in RS2, the linearized graviton fluctuation equation has a confined zero mode that is identified as the usual graviton [21].

The 3 + 1-d fermion spectrum still contains a localized zero mode for each species. However, far from the brane the effective potentials that replace those in Eq. (14) are now driven asymptotically to zero by the exponentially decreasing warp factor [22], whereas in the gravity-free case they tended to the strictly positive constants $b_{nY}^2(y = \pm\infty)$. The gravity-case effective potentials are thus volcanolike and consequently support modes of arbitrarily small energy: continua starting at zero mass. This feature is quite analogous to the well-known graviton-mode situation in the RS2 model: there is no mass gap, but rather a continuum of modes starting immediately above the localized massless graviton mode [2]. The Φ spectrum similarly contains a localized SM Higgs doublet plus a continuum starting at zero mass [20]. We need to make sure that the absence of a mass gap does not spoil the existence of a low-energy effective theory displaying dimensional reduction down to 3 + 1-d.

Let us first for simplicity ignore the DS-like bulk physics, or focus, if you like, on the modes of the gauge-singlet scalar field η. Except for discrete resonant modes, corresponding to quasilocalized states [20,22], the lowest-mass continuum modes are suppressed on the brane, because they have to tunnel through the potential barrier of the volcanolike effective potential. As such, their integrated effects at low energies will be dominated by the zero modes, just as in the well-known graviton case [2]. Because of this phenomenon, the localized modes do indeed form a low-energy effective 3 + 1-d theory. A detailed discussion of these matters can be found in Ref. [20].

The analysis of the gauge nonsinglet fermion and scalar modes is affected by the DS phenomenon. Since the continuum modes penetrate into the bulk, they feel the full effects of the confinement-phase physics we assume holds there. We therefore expect the low-mass continuum modes to manifest physically as the constituents of massive "hadrons" in the bulk. But the lowest-mass hadrons still have to tunnel through the volcanolike potential barriers to get inside the DW, and since the nonperturbative effects switch off near the wall, the situation analyzed in the previous paragraph is regained and with some plausibility the same conclusions follow.

Having described the construction of the model, it is now worth surveying the various scales it contains and how they should relate to each other. Of the many scales in the model, four need careful consideration: the ultraviolet cutoff $\Lambda_{\rm UV}$, the SU(5) breaking scale on the brane $\Lambda_{\rm SU(5)} \sim [\chi_1(y=0)]^{2/3}$, the bulk SU(5) confinement scale $\Lambda_{\rm conf}$, and the DW inverse width $\Lambda_{\rm DW} \equiv k$. All of these scales must be well above the electroweak scale. Within the four, the required hierarchy is

$$\Lambda_{\rm UV} > \Lambda_{\rm SU(5)} > \Lambda_{\rm conf} > \Lambda_{\rm DW}. \tag{26}$$

For obvious reasons, the UV cutoff must be the highest scale in the theory. The SU(5) breaking scale on the brane must be higher than the SU(5) bulk confinement scale, because we need to suppress the SU(5) confinement dynamics on the brane. If the opposite were the case, then the dynamics of the field χ would be everywhere dominated by the strong SU(5) interactions, and our classical background scalar field configuration would have no physical relevance. Finally, the SU(5) bulk glueball radius scale must be smaller than the width of the DW in order for the DS effect to work, as discussed in the lattice gauge analysis of Ref. [17]. This translates into the confinement scale being higher than the inverse wall width. The UV, DW-width, and SU(5) breaking scales are governed by free parameters, so the required hierarchy amongst those three can always be achieved. The SU(5) confinement scale is in principle to be calculated from the UV-cutoff bulk SU(5) gauge theory, and will depend on $\Lambda_{\rm UV}$ and the dimensionful gauge coupling constant g. If the qualitative behavior of the pure Yang-Mills theory discussed in Sec. II also holds for the complete theory, then we expect there to be a critical coupling $g_c(\Lambda_{\rm UV})$ above which the theory is confining. The hypothetical lattice gauge theory calculation would have to allow values of $g > g_c$ to furnish a $\Lambda_{\rm conf}$ that obeyed Eq. (26). This calculation has not been performed.

IV. CONCLUSION

In summary, we have proposed a candidate 4 + 1-d action for realizing a SM-like theory plus gravity dynamically localized to a DW. The dynamical localization mechanisms for fermions, scalars, and gravitons are well understood, whereas gauge boson localization is postulated by way of the DS mechanism. The DS mechanism is at this stage a conjecture in the 4 + 1-d context because of an incomplete understanding of confinement. What we have shown is that it is quite straightforward to construct a DW-localized SM if confinement exists for an SU(5) gauge theory bulk.

The proposed model—a 4 + 1-d SU(5) gauge theory minimally coupled to gravity – enjoys some interesting qualitative features. Notably, the usual tree-level SU(5) relation $m_d = m_e$ is automatically absent and colored-Higgs-induced proton decay can be suppressed.

There are a number of open problems, including the following:

(i) the veracity of the DS mechanism in 4 + 1-d, as discussed above;

(ii) to understand the phenomenological implications, including for proton decay, of the gauge bosons that are massive inside the DW;

(iii) to see whether there is enough parameter freedom to fit the three-family SM masses and mixing angles while obeying experimental bounds on proton decay;

(iv) to study how the effective 3 + 1-d $SU(3) \otimes SU(2) \otimes U(1)$ gauge coupling constants unify into a 4 + 1-d SU(5) gauge coupling constant;

(v) to generate nonzero neutrino masses;

(vi) to understand the phenomenology of the kink translational zero mode in the gravity case [23].

The DS gauge boson localization mechanism appears to be a keystone. If it can work in 4 + 1-d, then a whole world of DW brane model building is opened up, of which the theory presented above is but an example. If it does not work, then it is not at all clear that realistic field-theoretic DW-brane models exist when the extra dimension is noncompact. We hope that our efforts lead to renewed interest in the issue of confinement in higher-dimensional gauge theories.

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