Hawking radiation in a *d*-dimensional static spherically symmetric black hole surrounded by quintessence

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We present a solution of Einstein equations with quintessential matter surrounding a *d*-dimensional black hole, whose asymptotic structures are determined by the state of the quintessential matter. We examine the thermodynamics of this black hole and find that the mass of the black hole depends on the equation of state of the quintessence, while the first law is universal. Investigating the Hawking radiation in this black hole background, we observe that the Hawking radiation dominates on the brane in the low-energy regime. For different asymptotic structures caused by the equation of state of the quintessential matter surrounding the black hole, we learn that the influences by the state parameter of the quintessence on Hawking radiation are different.

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I. INTRODUCTION

String theory predicts the existence of extra dimensions. This inspired a lot of interest to study whether extra dimensions can be observed, which can present the signature of string and the correctness of string theory. A great deal of effort has been expended for the detection of extra dimensions. One among them is the study of perturbations around braneworld black holes. It has been argued that the extra dimension can imprint in the wave dynamics in the braneworld black hole background [1–4]. Another chief possibility to observe the extra dimension is the spectrum of Hawking radiation which is expected to be detected in particle accelerator experiments [5–17]. Recently through the study of Hawking radiation from squashed Kaluza-Klein black holes [14–16], it was argued that the luminosity of Hawking radiation can tell us the size of the extra dimension which opens a window to observe extra dimensions.

Recent astronomical observations strongly suggest that our universe is currently undergoing a phase of accelerated expansion, likely driven by some exotic component called dark energy. Despite the mounting observational evidence, the nature and origin of dark energy remains elusive and it has become a source of vivid debate. Quintessence is one candidate for the dark energy, which has negative pressure. If quintessence exists everywhere in the universe, it can cause the universe to accelerate. Besides with quintessence around a black hole, the black hole spacetime will be deformed. The Einstein equations for the static spherically symmetric quintessence surrounding a black hole in four dimensions were studied in [18]. It was found that the condition of additivity and linearity in the energy-momentum tensor allows one to get a correct limit to the known solutions for the electromagnetic static field and for the case of the cosmological constant.

In this work, we first extend [18] to the solution of Einstein equations with quintessential matter surrounding a *d*-dimensional black hole by assuming that quintessence is not only on the brane but full in the bulk. We get a new *d*-dimensional black hole, whose asymptotic structures are determined by the state of the quintessential matter surrounding the black hole. We examine the thermodynamics of this black hole and find that the mass of the black hole depends on the equation of state of the quintessence, while the first law keeps the same form independent of the dimensions and the state of the quintessence. Investigating Hawking radiation in this black hole background, we observe that Hawking radiation dominates on the brane. For different asymptotic structures caused by the equation of state of the quintessential matter surrounding the black hole, we learn that the influences by the state

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parameter of the quintessence on Hawking radiation are different. The signature of the dimension in Hawking radiation is also presented.

II. *d*-DIMENSIONAL STATIC SPHERICALLY SYMMETRIC BLACK HOLES SURROUNDED BY QUINTESSENCE

We study the Einstein equation for a static spherically symmetric black hole surrounded by quintessence in d-dimensions. The d-dimensional static black hole is described by

$$ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2 d\theta_1^2 - r^2\sin^2\theta_1 d\theta_2^2 - \cdots - r^2\sin^2\theta_1 \cdots \sin^2\theta_{d-3} d\theta_{d-2}^2, \quad (1)$$

where ν and λ are functions of radial coordinate r. The energy-momentum tensor of the quintessence in the static spherically symmetric state can be written as [18]

$$T_t{}^t = A(r), \qquad T_t{}^j = 0, \qquad T_i{}^j = C(r) r_i r^j + B(r)\delta_i{}^j. \quad (2)$$

After averaging over the angles of the isotropic state we get

$$\langle T_i{}^j \rangle = D(r)\delta_i{}^j, \qquad D(r) = -\frac{1}{d-1}C(r)r^2 + B(r). \quad (3)$$

For quintessence, we have

$$D(r) = -\omega_q A(r). \quad (4)$$

Thus, in terms of density $A(r)$, we can get the expression of $D(r)$ for fixed state parameter ω_q. As in Ref. [18], the appropriate constant coefficient $C(r)/B(r)$ is defined by the condition of additivity and linearity.

The Einstein equations of metric (1) have the form

$$2T_t{}^t = \frac{d-2}{2}\left[-e^{-\lambda}\left(\frac{d-3}{r^2} - \frac{\lambda'}{r}\right) + \frac{d-3}{r^2}\right], \quad (5)$$

$$2T_r{}^r = \frac{d-2}{2}\left[-e^{-\lambda}\left(\frac{d-3}{r^2} + \frac{\nu'}{r}\right) + \frac{d-3}{r^2}\right], \quad (6)$$

$$2T_{\theta_1}{}^{\theta_1} = 2T_{\theta_2}{}^{\theta_2} = \cdots 2T_{\theta_{d-2}}{}^{\theta_{d-2}} = -\frac{e^{-\lambda}}{2}\left[\nu'' + \frac{\nu'^2}{2} - \frac{\lambda'\nu'}{2} + \frac{(d-3)(\nu' - \lambda')}{r} + \frac{(d-3)(d-4)}{r^2}\right] + \frac{(d-3)(d-4)}{2r^2}, \quad (7)$$

where the prime denotes the derivative with respect to r.

The appropriate general expression of the energy-momentum tensor of quintessence in the d-dimensional spherically symmetric spacetime is given by

$$T_t{}^t = \rho_q(r), \qquad T_i{}^j = \rho_q(r)\alpha\left\{-[1 + (d-1)B(r)]\frac{r_i r^j}{r_n r^n} + B\delta_i{}^j\right\}. \quad (8)$$

This leads the spatial part of the energy-momentum tensor in proportion to the time component with the arbitrary parameter $B(r)$ which depends on the internal structure of quintessence. After taking isotropic average over the angles, we obtain

$$\langle r_i r^j \rangle = \frac{1}{d-1} r_n r^n \delta_i{}^j, \quad (9)$$

$$\langle T_i{}^j \rangle = -\rho_q(r)\frac{\alpha}{d-1}\delta_i{}^j = -p_q\delta_i{}^j. \quad (10)$$

From the state equation $p_q = \omega_q \rho_q$, it is easy to see

$$\omega_q = \frac{\alpha}{d-1}. \quad (11)$$

For quintessence, we have $-1 < \omega_q < 0$ and $-(d-1) < \alpha < 0$.

As in Ref. [18], we can define a principle of additivity and linearity by the equality

$$T_t{}^t = T_r{}^r \Rightarrow \lambda + \nu = 0. \quad (12)$$

And then substituting

$$\lambda = -\ln f, \quad (13)$$

we can obtain the linear differential equations in f

$$T_t{}^t = T_r{}^r = -\frac{d-2}{4r^2}[rf' + (d-3)(f-1)], \quad (14)$$

$$T_{\theta_1}{}^{\theta_1} = T_{\theta_2}{}^{\theta_2} = \cdots T_{\theta_{d-2}}{}^{\theta_{d-2}} = -\frac{1}{4r^2}[r^2 f'' + 2(d-3)rf' + (d-4)(d-3)(f-1)]. \quad (15)$$

From Eqs. (8) and (14), we can fix the free parameter B in the energy-momentum tensor for the matter

$$B = -\frac{(d-1)\omega_q + 1}{(d-1)(d-2)\omega_q}. \quad (16)$$

Thus the energy-momentum tensor (8) has the form

$$T_t{}^t = T_r{}^r = \rho_q, \quad (17)$$

$$T_{\theta_1}{}^{\theta_1} = T_{\theta_2}{}^{\theta_2} = \cdots T_{\theta_{d-2}}{}^{\theta_{d-2}} = -\frac{1}{d-2}\rho_q[(d-1)\omega_q + 1]. \quad (18)$$

Making use of Eqs. (14), (15), (17), and (18), we obtain a differential equation for f

$$r^2 f'' + [(d-1)\omega_q + 2d - 5]rf' + (d-3)[(d-1)\omega_q + d - 3](f-1) = 0. \quad (19)$$

The general solution of the above equation has the form

$$f = 1 - \frac{r_g}{r^{d-3}} + \frac{c_1}{r^{(d-1)\omega_q + d - 3}}, \quad (20)$$

where r_g and c_1 are normalization factors. When $c_1 = 0$, the function f describes the usual d-dimensional Schwarzschild black hole. Moreover, we also note that in the case $\omega_q = 0$, the second and the third term in f have the same order of r.

The energy density ρ_q for quintessence can be described by

$$\rho_q = \frac{c_1 \omega_q (d-1)(d-2)}{4r^{(d-1)(\omega_q+1)}}, \quad (21)$$

which should be positive. Since $\omega_q \leq 0$, it requires the normalization constant c_1 for quintessence to be negative. If we take $r_g = 2M$ and $c = -c_1$, the metric of the d-dimensional spherically symmetric black hole surrounded by quintessence reads

$$ds^2 = \left[1 - \frac{2M}{r^{d-3}} - \frac{c}{r^{(d-1)\omega_q + d - 3}}\right] dt^2 - \left[1 - \frac{2M}{r^{d-3}} - \frac{c}{r^{(d-1)\omega_q + d - 3}}\right]^{-1} dr^2 - r^2 d\Omega_{d-2}. \quad (22)$$

This spacetime depends not only on the dimension d, but also on the state parameter ω_q of quintessence. When $d = 4$, our result reduces to that obtained in [18]. In the limit $\omega_q = -1$, the metric (22) becomes

$$ds^2 = \left[1 - \frac{2M}{r^{d-3}} - cr^2\right] dt^2 - \left[1 - \frac{2M}{r^{d-3}} - cr^2\right]^{-1} dr^2 - r^2 d\Omega_{d-2}, \quad (23)$$

which reduces to the d-dimensional Schwarzschild-de Sitter black hole. We also note that the metric (22) can reduce to the d-dimensional Reissner-Nordström black hole if we take

$$\omega_q = \frac{d-3}{d-1}. \quad (24)$$

This implies that the state parameter ω_q of the electromagnetic field is a function of the dimension d of the spacetime, so that we might fix the number of extra dimensions of the spacetime by measuring the relation between the pressure p_q and the energy density ρ_q.

III. THERMODYNAMICS OF THE d-DIMENSIONAL STATIC SPHERICALLY SYMMETRIC BLACK HOLE SURROUNDED BY QUINTESSENCE

We now study the thermodynamical property at the black hole event horizon in the background (22). We write the mass E of a d-dimensional black hole as a product

$$E = F(d)M, \quad (25)$$

where $F(d)$ is a function of dimension d. We will see that the first law of thermodynamics at the black hole event horizon does not depend on this function. The entropy S, mass E, and Hawking temperature T of the black hole (22) can be described by

$$S = \frac{A_h}{4} = \frac{(d-1)\pi^{(d-1)/2}}{4\Gamma[\frac{d+1}{2}]} r_h^{d-2} = \frac{r_h^{d-2}}{G(d)}, \quad (26)$$

$$E = \frac{F(d)}{2}[G(d)S]^{(d-3)/(d-2)} - \frac{F(d)c}{2} \times [G(d)S]^{-(\omega_q(d-1))/(d-2)}, \quad (27)$$

$$T = \frac{F(d)G(d)}{2(d-2)}[(d-3)[G(d)S]^{-1/(d-2)} + \omega_q c(d-1)[G(d)S]^{-(\omega_q(d-1)+d-2)/(d-2)}], \quad (28)$$

respectively. As we did in [19], we treat the constant c as a variable, and have the generalized force

$$\Theta_c = \left(\frac{\partial E}{\partial c}\right)_S = -\frac{F(d)}{2}[G(d)S]^{-(\omega_q(d-1))/(d-2)}. \quad (29)$$

We find that the first law takes the form

$$\frac{d-3}{d-2}E = TS + \frac{\omega_q(d-1) + d - 3}{d-2}\Theta_c c. \quad (30)$$

It is clear that the mass depends on the state parameter ω_q of quintessence. In the limit $\omega_q \to -1$, the second term in the right-hand side of Eq. (30) becomes $-\frac{2}{d-2}\Theta_c c$. Setting $c = \frac{1}{l^2}$, where l^2 is defined as the cosmological constant Λ through $l^2 = \frac{(d-1)(d-2)}{2\Lambda}$, we have $\Theta_l = -\frac{2}{l^3}\Theta_c$ and $-\frac{2}{d-2}\Theta_c c = \frac{1}{d-2}\Theta_l l$. Then Eq. (30) reduces to the first law in the d-dimensional de Sitter (dS) black hole spacetimes [19]. In the case $\omega_q = \frac{d-3}{d-1}$, setting $c = -Q^2$, we can obtain $\Theta_Q = -2Q\Theta_c$, and $\frac{2(d-3)}{d-2}\Theta_c c = \frac{d-3}{d-2}\Theta_Q Q$. The first law returns to that in the d-dimensional Reissner-Nordström black hole[20].

From Eqs. (26)–(28), it is easy to obtain that

$$T = \left(\frac{\partial E}{\partial S}\right)_c. \quad (31)$$

Combining it with Eq. (29), we have

$$dE = \left(\frac{\partial E}{\partial S}\right)_c dS + \left(\frac{\partial E}{\partial c}\right)_S dc = TdS + \Theta_c dc, \quad (32)$$

which is the differential form of the first law of thermodynamics in the background of (22). Obviously, in the case of a d-dimensional static black hole surrounded by spherically symmetric quintessence, the differential form of the first law does not depend on the state parameter ω_q.

IV. GREYBODY FACTOR FOR A d-DIMENSIONAL STATIC SPHERICALLY SYMMETRIC BLACK HOLE SURROUNDED BY QUINTESSENCE

In this section, we study the greybody factors for the emission of scalar field in the low-energy limit on the brane and into the bulk from the d-dimensional static spherically symmetric black hole surrounded by quintessence (22). The greybody factors of the scalar field in the d-dimensional Schwarzschild and Schwarzschild-dS black holes have been investigated [21,22].

The equation of motion for a massless scalar particle propagating in the curved spacetime is described by

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\Phi(t, r, \Omega) = 0, \quad (33)$$

where $\Phi(t, r, \Omega)$ denotes the scalar field. Separating $\Phi(t, r, \Omega) = e^{-i\omega t}\Psi_{\rm bulk}(r)Y_{lm}(\Omega)$, we can obtain the radial equation for the scalar field propagating into the bulk

$$\frac{1}{r^{d-2}}\frac{d}{dr}\left[r^{d-2}f\frac{d\Psi_{\rm bulk}(r)}{dr}\right] + \left[\frac{\omega^2}{f} - \frac{l(l+d-3)}{r^2}\right]\Psi_{\rm bulk}(r) = 0, \quad (34)$$

with $f = 1 - \frac{2M}{r^{d-3}} - \frac{c}{r^{(d-1)\omega_q+d-3}}$. Similarly, we can also obtain the radial equation for scalar field propagating on the brane

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2 f\frac{d\Psi_{\rm brane}(r)}{dr}\right] + \left[\frac{\omega^2}{f} - \frac{l(l+1)}{r^2}\right]\Psi_{\rm brane}(r) = 0. \quad (35)$$

Adopting the tortoise coordinate $x = \int\frac{dr}{f}$, radial equations (34) and (35) can be further written as

$$\left[\frac{d^2}{dx^2} + \omega^2 - V_{\rm bulk}(r)\right][r^{(d-2)/2}\Psi_{\rm bulk}(r)] = 0, \quad (36)$$

and

$$\left[\frac{d^2}{dx^2} + \omega^2 - V_{\rm brane}(r)\right][r\Psi_{\rm brane}(r)] = 0, \quad (37)$$

with the effective potentials

$$V_{\rm bulk}(r) = f\left[\frac{(d-2)(d-4)f}{4r^2} + \frac{(d-2)f'}{2r} + \frac{l(l+d-3)}{r^2}\right], \quad (38)$$

and

$$V_{\rm brane}(r) = f\left[\frac{f'}{r} + \frac{l(l+1)}{r^2}\right]. \quad (39)$$

Here we only consider the greybody factor for the mode $l = 0$ which dominates in the low-energy regime $\omega \ll T_H$ and $\omega R_H \ll 1$. From the expression of f, we find that the spacetime (22) is asymptotically flat if $0 > \omega_q > -(d-3)/(d-1)$ and is asymptotically dS-like when $-1 \le \omega_q < -(d-3)/(d-1)$. As in [21], we write $f = f_a(r) + f_h(r)$. The function $f_a(r)$ is the asymptotic part of f and the function $f_h(r)$ contains physics which is specific for the black hole. We can define the asymptotic region to be $f_a(r) \gg f_h(r)$.

Near the black hole horizon $r \sim R_H$, taking into account the ingoing boundary condition, we obtain the solution of the radial equations (34) and (35) in the same form

$$\Psi(r)_{RH} = A_I e^{i\omega x}. \quad (40)$$

Near the black hole horizon $x \sim \frac{1}{2\kappa_H}\log\frac{r-R_H}{R_H}$, the solution (40) can be written as

$$\Psi(r)_{RH} = A_I\left[1 + \frac{i\omega}{2\kappa_H}\log\frac{r-R_H}{R_H}\right]. \quad (41)$$

In the intermediate region where the effective potentials $V_{\rm bulk}(r) \gg \omega^2$ and $V_{\rm brane}(r) \gg \omega^2$, the radial equations (34) and (35) can be reduced to

$$\frac{1}{r^{d-2}}\frac{d}{dr}\left[r^{d-2}f\frac{d\Psi_{\rm bulk}(r)}{dr}\right] = 0, \quad (42)$$

and

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2 f\frac{d\Psi_{\rm brane}(r)}{dr}\right] = 0, \quad (43)$$

respectively. The general solutions for Eqs. (42) and (43) can be determined as

$$\Psi_{\rm bulk}(r) = A_{II} + B_{II}G(r),$$
$$\text{and} \quad \Psi_{\rm brane}(r) = A_{II} + B'_{II}G'(r), \quad (44)$$

where

$$G(r) = \int_\infty^r\frac{dr}{r^{d-2}f}, \quad \text{and} \quad G'(r) = \int_\infty^r\frac{dr}{r^2 f}. \quad (45)$$

For $r \sim R_H$, we have

$$G(r) = \frac{1}{2R_H^{d-2}\kappa_H}\log(r - R_H), \quad \text{and}$$
$$G'(r) = \frac{1}{2R_H^2\kappa_H}\log(r - R_H). \tag{46}$$

Matching this solution to the wave function (41) of the near black hole horizon region, we obtain

$$A_{II} = A_I, \qquad B_{II} = i\omega R_H^{d-2}A_I, \qquad B'_{II} = i\omega R_H^2 A_I. \tag{47}$$

The asymptotic expressions of wave functions (44) in the limit $r \gg R_H$ with $V(r) \gg \omega^2$ read

$$\Psi_{\text{bulk}}(r) = A_I\left(1 + i\omega R_H^{d-2}\int_\infty^r \frac{dr}{r^{d-2}f_a(r)}\right),$$
$$\Psi_{\text{brane}}(r) = A_I\left(1 + i\omega R_H^2\int_\infty^r \frac{dr}{r^2 f_a(r)}\right). \tag{48}$$

We shall use these expressions to match the general solution for the scalar wave equation in the asymptotic region.

Until now, we just concentrated on the black hole. In the following, we will do the matching in the asymptotic region for the case of asymptotically flat spacetime and asymptotically dS spacetime, respectively.

Let us first consider the asymptotically flat spacetime where $0 > \omega_q > -(d-3)/(d-1)$. In this case, we take $f_a(r) = 1$.

The general solutions of the wave equations (34) and (35) in asymptotically flat spacetime are given by

$$\Psi_{\text{bulk}}(r) = \rho^{(3-d)/2}[C_1 H^{(1)}_{(d-3)/2}(\rho) + C_2 H^{(2)}_{(d-3)/2}(\rho)],$$
$$\Psi_{\text{brane}}(r) = \rho^{-(1/2)}[C_1 H^{(1)}_{1/2}(\rho) + C_2 H^{(2)}_{1/2}(\rho)], \tag{49}$$

where $\rho = r\omega$, $H^{(1)}_\nu(\rho) = J_\nu(\rho) + iN_\nu(\rho)$ and $H^{(2)}_\nu(\rho) = J_\nu(\rho) - iN_\nu(\rho)$ are the Hankel functions defined by the Bessel functions $J_\nu(\rho)$ and $N_\nu(\rho)$. In the limit $\rho \ll 1$, we have

$$\Psi_{\text{bulk}}(r) \sim \frac{C_1 + C_2}{\Gamma(\frac{d-1}{2})2^{(d-3)/2}} - i(C_1 - C_2)\frac{\Gamma(\frac{d-3}{2})2^{(d-3)/2}}{\pi\rho^{(d-3)/2}},$$
$$\Psi_{\text{brane}}(r) \sim \frac{\sqrt{2}(C_1 + C_2)}{\sqrt{\pi}} - \frac{i\sqrt{2}(C_1 - C_2)}{\sqrt{\pi}\rho^{(d-3)/2}}. \tag{50}$$

Matching the wave function (50) in the asymptotic region to that in the intermediate region, we get the relationship between the coefficients C_1 and C_2

$$C_1 + C_2 = \Gamma\left(\frac{d-1}{2}\right)2^{(d-3)/2}A_I,$$
$$C_1 - C_2 = \frac{\pi\omega^{d-2}R_H^{d-2}}{(d-3)\Gamma(\frac{d-3}{2})2^{(d-3)/2}}A_I, \tag{51}$$

in the bulk and

$$C_1 + C_2 = \sqrt{\frac{\pi}{2}}A_I, \qquad C_1 - C_2 = \sqrt{\frac{\pi}{2}}\omega^2 R_H^2 A_I, \tag{52}$$

on the brane. From the definition of greybody factor in the low-energy limit $\omega R_H \ll 1$,

$$\gamma(\omega) = 1 - \frac{|C_2|^2}{|C_1|^2} \simeq 4\frac{C_1 - C_2}{C_1 + C_2}, \tag{53}$$

we obtain the greybody factor

$$\gamma(\omega) = \frac{4\pi\omega^{d-2}R_H^{d-2}}{2^{d-2}\Gamma(\frac{d-1}{2})^2}, \tag{54}$$

in the bulk and

$$\gamma(\omega) = 4\omega^2 R_H^2, \tag{55}$$

on the brane. Obviously, the greybody factors in the bulk and on the brane depend on the black hole horizon radius. The changes of R_H and R_H^{d-2} with the state parameter ω_q and dimension d are listed in Table I and the factor $2^{d-2}\Gamma(\frac{d-1}{2})^2$ in (54) is only a monotonically increased function of the dimension numbers d, thus the greybody factors (54) and (55) increase with the increase of the absolute value of ω_q and decrease with the increase of d.

In the d-dimensional black hole spacetime, the luminosity of the black hole Hawking radiation for the mode $l = 0$ in the bulk and on the brane is given by

$$L_{\text{bulk}} = \int_0^\infty \frac{d\omega\,\omega^{d-1}R_H^{d-2}}{2^{d-3}\Gamma(\frac{d-1}{2})^2}\frac{1}{e^{\omega/T_H} - 1}, \tag{56}$$

$$L_{\text{brane}} = \int_0^\infty \frac{d\omega}{2\pi}\frac{4\omega^3 R_H^2}{e^{\omega/T_H} - 1}. \tag{57}$$

The integral expressions above are just for the sake of

TABLE I. The changes of R_H and R_H^{d-2} with different state parameter ω_q and dimension numbers d in the asymptotically flat case. Here $M = 1$ and $c = 0.01$.

	R_H			R_H^{d-2}		
	$\omega_q = -0.1$	$\omega_q = -0.2$	$\omega_q = -0.3$	$\omega_q = -0.1$	$\omega_q = -0.2$	$\omega_q = -0.3$
$d = 4$	2.0122	2.0151	2.0188	4.0481	4.0602	4.0756
$d = 5$	1.4183	1.4189	1.4200	2.8530	2.8566	2.8633
$d = 6$	1.2623	1.2626	1.2629	2.5389	2.5413	2.5438
$d = 7$	1.1909	1.1910	1.1912	2.3954	2.3964	2.3984

TABLE II. The change of T_H with different state parameter ω_q and dimension numbers d in the asymptotically flat case. Here $M = 1$ and $c = 0.01$.

	$\omega_q = -0.1$	$\omega_q = -0.2$	$\omega_q = -0.3$	$\omega_q = -0.4$	$\omega_q = -0.5$	$\omega_q = -0.6$
$d = 4$	0.039 47	0.039 31	0.039 09	...	...	...
$d = 5$	0.112 08	0.111 87	0.111 61	0.111 27	0.110 86	...
$d = 6$	0.188 95	0.188 69	0.188 37	0.187 98	0.187 52	0.186 96
$d = 7$	0.267 07	0.266 76	0.266 39	0.265 95	0.265 42	0.264 80

completeness by writing the integral range from 0 to infinity. However, as our analysis has focused only in the low-energy regime of the spectrum, an upper cutoff will be imposed on the energy parameter so that the low-energy conditions $\omega \ll T_H$ and $\omega R \ll 1$ are satisfied. The values derived for the luminosities of the black hole on the brane and in the bulk will therefore be based on the lower part of the spectrum and modifications may appear when the high-energy part of the spectrum is included in the calculation. The Hawking temperature T_H of the black hole in the asymptotically flat spacetime is listed in Table II. It is shown that T_H increases with of the dimension number d and decreases with the increase of the absolute value of ω_q. Table III tells us that both the luminosity of Hawking radiation in the bulk and on the brane decrease with the increase of the absolute ω_q and increase with the dimension number d. We observe that Hawking radiation dominates on the brane and the ratio $L_{\rm brane}/L_{\rm bulk}$ increases with the magnitude of ω_q and dimension d.

Now we start to consider asymptotically dS-like spacetime with $-1 \leq \omega_q < -(d-3)/(d-1)$. The function $f_a(r)$ is now given by

$$f_a(r) = 1 - \frac{c}{r^{\omega_q(d-1)+d-3}}. \tag{58}$$

The metric (22) now has a cosmological-like horizon located at $r = r_c = c^{1/[\omega_q(d-1)+d-3]}$. Assuming that $r_c \gg R_H$, we have that $f_h(r) \ll f_a(r)$ for $r \gg R_H$ and the $f_h(r)$ contribution to $f(r)$ is negligible. This allows us to define an intermediate region, $R_H \ll r \ll r_c$, in between the near horizon and the asymptotic region. Thus, for $r \gg R_H$, $r/r_c \ll 1$, and $r\omega \ll 1$, the wave functions (48) have the form

$$\Psi_{\rm bulk}(r) = A_I\left(1 - i\frac{\omega R_H^{d-2}}{(d-3)r^{d-3}}\right),$$
$$\Psi_{\rm brane}(r) = A_I\left(1 - i\frac{\omega R_H^2}{r}\right). \tag{59}$$

In the asymptotic region, $r \gg R_H$, we define the coordinate

$$z = \left(\frac{r}{r_c}\right)^{-n}, \tag{60}$$

with $n = \omega_q(d-1) + d - 3$, which is negative because in this dS-like spacetime $\omega_q < -(d-3)/(d-1)$. Then radial equations (34) and (35) can be approximated as

$$(1-z)z\frac{d^2P_{\rm bulk}}{dz^2} - \left[\frac{n+1}{n} - z\frac{2n+1}{n}\right]\frac{dP_{\rm bulk}}{dz} + \left\{\frac{\omega^2 r_c^2}{n^2(1-z)} - \frac{d-2}{4zn^2}[d-4-z(d-2n-4)]\right\}P_{\rm bulk} = 0, \tag{61}$$

and

$$(1-z)z\frac{d^2P_{\rm brane}}{dz^2} - \left[\frac{n+1}{n} - z\frac{2n+1}{n}\right]\frac{dP_{\rm brane}}{dz} + \left[\frac{\omega^2 r_c^2}{n^2(1-z)} - \frac{1}{n}\right]P_{\rm brane} = 0, \tag{62}$$

respectively. Here $P_{\rm bulk} = r^{(d-2)/2}\Psi_{\rm bulk}(r)$ and $P_{\rm brane} = r\Psi_{\rm brane}(r)$.

The general solution to the Eq. (61) is

TABLE III. The changes of $L_{\rm bulk}$, $L_{\rm brane}$ and $L_{\rm brane}/L_{\rm bulk}$ with different state parameter ω_q and dimension numbers d in the asymptotically flat case. Here $M = 1$ and $c = 0.01$.

	$L_{\rm bulk}(10^{-5})$			$L_{\rm brane}(10^{-5})$			$L_{\rm brane}/L_{\rm bulk}$		
	$\omega_q = -0.1$	$\omega_q = -0.2$	$\omega_q = -0.3$	$\omega_q = -0.1$	$\omega_q = -0.2$	$\omega_q = -0.3$	$\omega_q = -0.1$	$\omega_q = -0.2$	$\omega_q = -0.3$
$d = 4$	4.064 00	4.0087	3.9330	...	...	...	...	...	...
$d = 5$	31.405	31.145	30.818	131.27	130.39	129.26	4.180	4.186	4.194
$d = 6$	99.774	99.036	98.135	839.67	835.39	830.17	8.416	8.435	8.460
$d = 7$	263.32	261.38	259.06	2982.9	2969.9	2954.4	11.33	11.36	11.40

$$P_{\rm bulk} = C_1 z^{-(d-2)/2n}(1-z)^{i\omega r_c/n}\, {}_2F_1\bigg[\frac{i\omega r_c}{n}, \frac{3-d+n}{n} + \frac{i\omega r_c}{n}, \frac{3-d+n}{n}; z\bigg] + C_2 z^{(d-4)/2n}(1-z)^{i\omega r_c/n}\, {}_2F_1\bigg[1+\frac{i\omega r_c}{n}, \frac{d-3}{n} + \frac{i\omega r_c}{n}, \frac{d-3+n}{n}; z\bigg], \quad (63)$$

where $2F_1[a, b, \tilde{c}; z]$ is the standard hypergeometric function. Since, $n < 0$, for $z \to 0$, or $r/r_c \ll 1$, we have

$$\Psi_{\rm bulk}(r) = C_1 r_c^{(2-d)/2} + \frac{C_2 r_c^{(d-4)/2}}{r^{d-3}}. \quad (64)$$

Matching this wave function to the behavior (59) in the intermediate region, we can fix the coefficients C_1 and C_2

$$C_1 = r_c^{(d-2)/2} A_I, \qquad C_2 = -i r_c^{(4-d)/2}\frac{\omega R_H^{d-2}}{(d-3)} A_I. \quad (65)$$

In order to find the behavior of the wave function for $z \to 1$, we change the argument of the hypergeometric function of the solution (63) from z to $1-z$ and find that it has the form

$$\begin{aligned} P_{\rm bulk} &= C_1 b_{11} z^{-(d-2)/2n}(1-z)^{i\omega r_c/n}\, {}_2F_1\bigg[\frac{i\omega r_c}{n}, \frac{3-d+n+i\omega r_c}{n}, 1+\frac{i2\omega r_c}{n}; 1-z\bigg] \\ &+ C_1 b_{21} z^{-(d-2)/2n}(1-z)^{-i\omega r_c/n}\, {}_2F_1\bigg[\frac{3-d+n-i\omega r_c}{n}, -\frac{i\omega r_c}{n}, 1-\frac{i2\omega r_c}{n}; 1-z\bigg] \\ &+ C_2 b_{12} z^{(d-4)/2n}(1-z)^{i\omega r_c/n}\, {}_2F_1\bigg[1+\frac{i\omega r_c}{n}, \frac{d-3+i\omega r_c}{n}, 1+\frac{i2\omega r_c}{n}; 1-z\bigg] \\ &+ C_2 b_{22} z^{(d-4)/2n}(1-z)^{-i\omega r_c/n}\, {}_2F_1\bigg[\frac{d-3-i\omega r_c}{n}, 1-\frac{i\omega r_c}{n}, 1-\frac{i2\omega r_c}{n}; 1-z\bigg], \end{aligned} \quad (66)$$

with

$$b_{11} = \frac{\Gamma(\frac{3-d+n}{n})\Gamma(-\frac{i2\omega r_c}{n})}{\Gamma(\frac{3-d+n-i\omega r_c}{n})\Gamma(-\frac{i\omega r_c}{n})}, \qquad b_{12} = \frac{\Gamma(\frac{d-3+n}{n})\Gamma(-\frac{i2\omega r_c}{n})}{\Gamma(\frac{d-3-i\omega r_c}{n})\Gamma(1-\frac{i\omega r_c}{n})}, \quad (67)$$

$$b_{21} = \frac{\Gamma(\frac{3-d+n}{n})\Gamma(\frac{i2\omega r_c}{n})}{\Gamma(\frac{i\omega r_c}{n})\Gamma(\frac{n+3-d+i\omega r_c}{n})}, \qquad b_{22} = \frac{\Gamma(\frac{d-3+n}{n})\Gamma(\frac{i2\omega r_c}{n})}{\Gamma(\frac{d-3+i\omega r_c}{n})\Gamma(1+\frac{i\omega r_c}{n})}. \quad (68)$$

Thus, in the limit $z \to 1$, the wave function becomes

$$\Psi_{\rm bulk}(r) = \tilde{C}_1 r_c^{(2-d)/2} e^{-i\omega r_c \delta/n} e^{i\omega x} + \tilde{C}_2 r_c^{(2-d)/2} e^{i\omega r_c \delta/n} e^{-i\omega x}, \quad (69)$$

where $\delta = -\frac{\Gamma(1-1/n)}{\Gamma(-1/n)}$[EulerGamma + PolyGamma(0, $-1/n$)]. The relations between $\tilde{C}_1$, $\tilde{C}_2$ and C_1, C_2 can be expressed as

$$\begin{pmatrix} \tilde{C}_1 \\ \tilde{C}_2 \end{pmatrix} = \begin{pmatrix} b_{11} b_{12} \\ b_{21} b_{22} \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix}. \quad (70)$$

Then the greybody factor is given by

$$\begin{aligned} \gamma(\omega)_{\rm bulk} &= 1 - \frac{|\tilde{C}_2|^2}{|\tilde{C}_1|^2} \\ &= \bigg|\frac{b_{21}}{b_{11}}\bigg|^2 \bigg|1 - \frac{b_{11}b_{22} - b_{12}b_{21}}{b_{11}b_{21}}\frac{C2}{C1}\bigg|^2 \\ &= 4h(\omega r_c)\bigg(\frac{R_H}{r_c}\bigg)^{d-2}, \end{aligned} \quad (71)$$

where the function $h(\omega r_c)$ is defined by

$$h(\omega r_c) = \frac{1}{4|b_{11}|^2}. \quad (72)$$

Similarly, the greybody factor for the scalar emission on the brane is

$$\gamma(\omega)_{\rm brane} = \frac{1}{|b'_{11}|^2}\bigg(\frac{R_H}{r_c}\bigg)^2, \quad (73)$$

with

$$b'_{11} = \frac{\Gamma(\frac{n-1}{n})\Gamma(-\frac{i2\omega r_c}{n})}{\Gamma(\frac{n-1-i\omega r_c}{n})\Gamma(-\frac{i\omega r_c}{n})}. \quad (74)$$

The greybody factors $\gamma(\omega)_{\rm bulk}$ and $\gamma(\omega)_{\rm brane}$ depend on ω and the ratio R_H/r_c. The changes of R_H and r_c in the case $\omega_q(d-1) + d - 3 < 0$ with ω_q and d are listed in Table IV. One can find that the ratio R_H/r_c increases with the increase of the absolute value of ω_q and decreases with the increase of the dimension d. In the low-energy limit $\omega r_c < 1$, we have the quantities $|b_{11}|^2 \sim 1/4$ and

TABLE IV. The changes of R_H, r_c and R_H/r_c with different state parameter ω_q and dimension numbers d in the asymptotically dS-like case. Here $M = 1$ and $c = 0.01$.

	R_H			r_c			R_H/r_c		
	$\omega_q = -0.8$	$\omega_q = -0.9$	$\omega_q = -1$	$\omega_q = -0.8$	$\omega_q = -0.9$	$\omega_q = -1$	$\omega_q = -0.8$	$\omega_q = -0.9$	$\omega_q = -1$
$d = 4$	2.0564	2.0714	2.0915	25.294	13.680	8.789	0.0813	0.1514	0.2380
$d = 5$	1.4252	1.4269	1.4289	46.380	17.712	9.897	0.0307	0.0806	0.1444
$d = 6$	1.2653	1.2660	1.2667	100.00	21.542	9.990	0.0127	0.0588	0.1268
$d = 7$	1.1927	1.1930	1.1935	316.23	26.827	9.999	0.0038	0.0445	0.1194

$|b'_{11}|^2 \sim 1/4$; then the greybody factors $\gamma(\omega)_{\rm bulk}$ and $\gamma(\omega)_{\rm brane}$ also increase with ω_q and decrease with d.

The luminosity of Hawking radiation for the mode $l = 0$ in the bulk and on the brane can be given by

$$L_{\rm bulk} = \int_0^\infty \frac{d\omega\,\omega\gamma(\omega)_{\rm bulk}}{e^{\omega/T_H} - 1}, \tag{75}$$

$$L_{\rm brane} = \int_0^\infty \frac{d\omega}{2\pi} \frac{\omega\gamma(\omega)_{\rm brane}}{e^{\omega/T_H} - 1}. \tag{76}$$

As we did in (56) and (57), here we also focus only in the low-energy regime of the spectrum and impose an upper cutoff on the energy parameter so that the low-energy conditions $\omega \ll T_H$ and $\omega R \ll 1$ are satisfied. The values obtained for the luminosities of the black hole on the brane and in the bulk are based on the lower part of the spectrum. When the high-energy part of the spectrum is included, the results may significantly change.

Although the black hole Hawking temperature T_H (which is listed in Table V) decreases with the increase of the absolute value of ω_q, Table VI tells us that the luminosity of the black hole Hawking radiation increases with the increase of the magnitude of ω_q. This is different from that in the asymptotically flat case with $\omega_q(d-1) + d - 3 > 0$. Moreover comparing with the asymptotically flat case, although the ratio $L_{\rm brane}/L_{\rm bulk}$ tells us that the black hole Hawking radiation still dominates on the brane in the dS-like spacetime, its dependence on $|\omega_q|$ is different in the dS-like situation from that in the asymptotically flat spacetime. These differences can be understood from the behavior of the ratio $R_H/r_c \ll 1$, which increases with $|\omega_q|$. This means that when $|\omega_q|$ becomes bigger, the black hole horizon and the cosmological horizon will come closer, so that Hawking radiation on the black hole event horizon will be enhanced by the contribution from Hawking radiation from the cosmological horizon.

V. CONCLUSIONS AND DISCUSSIONS

In this paper, we obtain an exact solution of Einstein equations for the static spherically symmetric quintessential matter surrounding a black hole in d-dimensional spacetimes. For different state parameters ω_q of quintes-

TABLE V. The change of T_H with different state parameter ω_q and dimensional numbers d in the asymptotically dS-like case. Here $M = 1$ and $c = 0.01$.

	$\omega_q = -0.4$	$\omega_q = -0.5$	$\omega_q = -0.6$	$\omega_q = -0.7$	$\omega_q = -0.8$	$\omega_q = -0.9$	$\omega_q = -1.0$
$d = 4$	0.038 79	0.038 38	0.037 84	0.037 12	0.036 15	0.034 84	0.033 06
$d = 5$	⋯	⋯	0.110 34	0.109 71	0.108 94	0.108 00	0.106 84
$d = 6$	⋯	⋯	⋯	0.186 29	0.185 50	0.184 55	0.183 42
$d = 7$	⋯	⋯	⋯	0.264 07	0.263 20	0.262 19	0.261 01

TABLE VI. The changes of $L_{\rm bulk}$, $L_{\rm brane}$ and $L_{\rm brane}/L_{\rm bulk}$ with different state parameter ω_q and dimensional numbers d in the asymptotically dS-like case. Here $M = 1$ and $c = 0.01$.

	$L_{\rm bulk}(10^{-5})$			$L_{\rm brane}(10^{-5})$			$L_{\rm brane}/L_{\rm bulk}$		
	$\omega_q = -0.8$	$\omega_q = -0.9$	$\omega_q = -1.0$	$\omega_q = -0.8$	$\omega_q = -0.9$	$\omega_q = -1.0$	$\omega_q = -0.8$	$\omega_q = -0.9$	$\omega_q = -1.0$
$d = 4$	0.9045	2.9143	6.4797	⋯	⋯	⋯	⋯	⋯	⋯
$d = 5$	0.0361	0.6386	3.5966	1.1734	7.9265	24.912	32.544	12.413	6.9267
$d = 6$	9.2×10^{-5}	0.0425	0.9108	0.5769	12.318	56.647	6246.3	289.54	62.195
$d = 7$	5.5×10^{-9}	0.0013	0.1728	0.1032	14.238	101.64	1.864×10^7	11 369.5	588.06

sence, our solution can lead to different limits, such as the Schwarzschild, Reissner-Nordström, and de Sitter black holes in d-dimensions. We study the thermodynamics in this d-dimensional black hole spacetime and find that the first law is universal for the arbitrary state parameter ω_q of the quintessence.

We investigate the greybody factors and Hawking radiations of a scalar field in the bulk and on the brane, in the low-energy regime, in this d-dimensional black hole surrounded by quintessence. We observe that Hawking ra diation dominates on the brane. For the case $0 > \omega_q > -(d-3)/(d-1)$, the black hole is asymptotically flat; the luminosity of Hawking radiation both in the bulk and on the brane decreases with the increase of $|\omega_q|$. But for the case $-(d-3)/(d-1) > \omega_q > -1$, the black hole is in the asymptotically dS spacetime; Hawking radiation increases with the magnitude of $|\omega_q|$. The difference can be attributed to the different asymptotic structures of the spacetimes. In the asymptotic dS spacetime, besides the black hole event horizon, there also exists the cosmological horizon. When the absolute value of ω_q becomes bigger, these two horizons come closer. The contribution of Hawking radiation from the cosmological horizon enhances the Hawking radiation near the black hole event horizon.

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