Space-based gravitational-wave detectors can determine the thermal history of the early Universe

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It is shown that space-based gravitational-wave detectors such as DECIGO and/or the Big Bang Observer will provide us with invaluable information on the cosmic thermal history after inflation, and they will be able to determine the reheat temperature T_R provided that it lies in the range preferred by the cosmological gravitino problem, $T_R \sim 10^{5-9}$ GeV. Therefore it is strongly desired that they will be put into practice as soon as possible.

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Although we can probe physics during inflation in the early Universe [1] rather precisely now using observations of the anisotropy in the cosmic microwave background (CMB) radiation [2,3], cosmic evolution from the end of inflation to the beginning of the primordial big bang nucleosynthesis (BBN) is still in the dark age, when many important phenomena such as (p)reheating, baryogenesis, generation, and freeze-out of cold dark matter particles, etc. have taken place. It is desirable to clarify the cosmic thermal history in this regime observationally.

Here we argue that future space-based gravitational-wave detectors such as DECIGO [4] or the Big Bang Observer (BBO), are very useful for this purpose and may be able to determine the reheat temperature after inflation by observing stochastic gravitational radiation background generated during inflation [5] in the frequency range around 0.1–10 Hz, where foregrounds from astrophysical objects are separable [6].[1]

We introduce tensor perturbations, h_{ij}, around a spatially flat Robertson-Walker metric as

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + 2h_{ij})dx^i dx^j, \qquad (1)$$

with $a(t)$ being the scale factor. Decomposing the tensor metric perturbation to Fourier modes as

$$h_{ij} = \sqrt{8\pi G} \sum_{A=+,\times} \int \frac{d^3k}{(2\pi)^{3/2}} \varphi_k^A(t) e^{i\mathbf{k}\cdot\mathbf{x}} e_{ij}^A, \qquad (2)$$

we find that the two independent degrees of freedom φ^A behave as two massless minimally coupled scalar fields, where e_{ij}^A represents polarization tensor with $e_{ij}^A e^{ijA'} = \delta^{AA'}$ for $A, A' = +, \times$. Applying quantum field theory of a massless minimally coupled field in de Sitter spacetime, we find that the Fourier modes are characterized by the following vacuum correlation,

$$\langle \varphi_k^A(t) \varphi_{k'}^{A'}(t) \rangle = \frac{H^2}{2k^3} \delta^3(k - k') \delta^{AA'}, \qquad (3)$$

so that the amplitude per logarithmic frequency interval is given by

$$h_F^2(f) \equiv 2\langle h_{ij} h^{ij}(f) \rangle = 4 \times 8\pi G \left(\frac{H(\phi)}{2\pi}\right)^2 = \frac{V[\phi(f)]}{3\pi^2 M_G^4} \equiv \frac{1}{2}\Delta_h^2(f), \qquad (4)$$

in the long wavelength regime. Here $M_G = (8\pi G)^{-1/2}$ is the reduced Planck mass and $V[\phi(f)]$ is the value of potential energy density of the inflaton when the scale corresponding to the frequency f today left the Hubble radius during inflation. Δ_h is an expression for the amplitude of tensor perturbation used frequently in the analysis of cosmological observations.

The above expression gives the initial condition to the solution of each Fourier mode in the post inflationary universe which behaves as

$$h(f, a) \propto a(t)^{(1-3p)/2p} J_{(3p-1)/(2(1-p))}\left(\frac{p}{1-p} \frac{k}{a(t)H(t)}\right), \quad k = 2\pi f a_0, \qquad (5)$$

in a power-law background $a(t) \propto t^p$ with $p < 1$. Here $J_n(x)$ is a Bessel function and a_0 denotes the current scale factor.

Thus the amplitude of the gravitational wave takes a constant value, $h(f, a) = h_F(f)$, until its wavelength falls shorter than the Hubble radius H^{-1} at $a = 2\pi f a_0 / H \equiv a_{\rm in}(f)$. From the asymptotic expansion of (5), one can see that the energy density stored in the tensor perturbation

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[1] Recently, Population III stars are proposed as a dominant component around the deci-Heltz band as a result of the gravitational radiation associated with neutrino emissions [7]. However, the amplitude is very uncertain due to the lack of understating on early star formation history. This signal would be separable if we adopt a reliable abundance of Population III stars and take the duty cycle and their angular distribution into account.

starts to decrease just as radiation once the wavelength falls to a subhorizon scale. Therefore in this regime, its relative energy density to the background density remains constant during radiation domination, while it decreases when the Universe is dominated by another form of energy such as nonrelativistic matter or coherent field oscillation. If such a stage lasts long, energy density in subhorizon tensor perturbations tends to be suppressed and the resultant spectrum is modified from a nearly scale-invariant one. The present density parameter of the gravitational radiation per logarithmic frequency interval is described as

$$\Omega_{\rm GW}(f, t_0) = \frac{(2\pi f)^2}{12H_0^2}\Delta_h^2(f)\left(\frac{a_{\rm in}(f)}{a_0}\right)^2. \qquad (6)$$

It behaves as $\Omega_{\rm GW}(f, t_0) \propto f^{-2}(f^0)$ for the mode which enters the horizon in the matter (radiation) dominated regime. Thus, the stochastic gravitational-wave background not only carries information on the inflationary regime, during which they are generated, but also serves as a probe of the equation of state in the early universe as studied in [8] and extended in [9].

This double role played by primordial gravitational-wave background in particle cosmology is similar to that played by high-redshift quasars in observational cosmology, which not only reflects the properties of a high-redshift universe at their location, but also carries line-of-sight information in their absorption spectra. In this sense, just as high-redshift quasars are regarded as lighthouses of the distant universe, we may regard the inflation-produced gravitational-wave background as a lighthouse that can shed light on the early Universe which serves as a laboratory for high energy physics. (see also [10] for other applications of stochastic gravitational-wave background).

Besides direct observation by space laser interferometers, cosmological stochastic gravitational-wave background can be detected indirectly through CMB. By measuring its B-mode polarization, we can probe the amplitude of tensor perturbation from inflation.

In a simple single-field slow-roll inflation model with a potential $V[\phi]$, observable quantities such as the amplitude of curvature perturbation, $\Delta_{\mathcal{R}}$, its spectral index, n_s, and running, $dn_s/d\ln k$, are described by the slow-roll parameters,

$$\epsilon = \frac{M_G^2}{2}\left(\frac{V'[\phi]}{V[\phi]}\right)^2, \qquad \eta = M_G^2\frac{V''[\phi]}{V[\phi]},$$
$$\xi = M_G^4\frac{V'[\phi]V'''[\phi]}{V^2[\phi]},$$

as

$$\Delta_{\mathcal{R}}^2 = \frac{V[\phi]}{24\pi^2 M_G^4\epsilon}, \qquad n_s = 1 - 6\epsilon + 2\eta,$$
$$\frac{dn_s}{d\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi,$$

where each quantity should be evaluated at the time of horizon crossing during inflation.

WMAP has measured both $\Delta_{\mathcal{R}}$ and n_s with an unprecedented accuracy. However, since $\Delta_{\mathcal{R}}$ depends not only on $V[\phi]$ but also on the slow-roll parameter ϵ [11], we have not been able to fix the energy scale of inflation yet. Detection of tensor perturbation is essentially important to determine the energy scale of inflation.

Theoretically, different inflation models predict different energy scales of accelerated expansion. Among them, the chaotic inflation model [12] with a massive scalar potential, $V[\phi] = m^2\phi^2/2$, which is attractive both from naturalness of initial condition and phenomenological point of view [13], we find a relatively large amplitude of tensor perturbation: $r \equiv \Delta_h^2/\Delta_{\mathcal{R}}^2(k_0) \cong 0.16$, where pivot scale, k_0, corresponds to the CMB scale. Other models such as small-field models may predict much a smaller value of r including a vanishingly small one, which would make us desperate with regards to the detection of B-mode polarization.

Recently, however, it has been claimed that, if we use the observed value of the scalar spectral index $n_s = 0.961 \pm 0.017$ by WMAP as a constraint, even small-field models would predict $r > 10^{-3}$ [14]. That is, it would be unusual to have $\eta \gg \epsilon$ because it would mean the inflaton is near an inflection point when the CMB scale left the Hubble radius and it would be very difficult to achieve the right number of e folds thereafter without severe fine-tuning. Then the observed deviation of the spectral index from unity, $n_s \simeq 0.95$–0.99, implies $\epsilon = \mathcal{O}(0.01)$ or $r \sim 0.1$. Indeed the authors of [14] have considered a number of potentials with different parameters and calculated n_s and r under the condition that the duration of inflation takes a proper value. As a result, they find that the observed value of n_s implies $r > 0.003$.

The same issue has been studied by Boyle, Steinhardt, and Turok [15] in a somewhat more model-independent manner. They count the number of zeros of the slow-roll parameters ϵ and η in the last 60 e folds of inflation as a conservative measure of how many derivatives of them must be fine-tuned to achieve a given set of (n_s, r). They find inflation models with no fine-tuning give $n_s < 0.98$ and $r > 10^{-2}$ and that models predicting $r < 10^{-3}$ require nine or more extra degrees of fine-tuning.

If r is indeed larger than, say, 0.003, we will be able to detect B-mode polarization by ongoing and planned projects and its implication is profound. Because we already know that $\Delta_{\mathcal{R}}^2 \simeq 2.0 \times 10^{-9}$ on the CMB scale [2], by measuring r we can fix the energy scale of inflation as

$$V[\phi] = (3.2 \times 10^{16}\ {\rm GeV})^4 r$$
$$= (7.5 \times 10^{15}\ {\rm GeV})^4\left(\frac{r}{0.003}\right).$$

Once we succeed in measuring r by B-mode polarization, we will be even more confident in simple slow-roll single-field inflation, on which the above arguments [14,15] are based, and we can predict the amplitude of gravitational waves on frequencies accessible by the other means of observation, namely, a space laser interferometer such as DECIGO [4] and BBO. Extrapolating[2] the amplitude of tensor perturbation on the CMB scale with the wave number $k_0 = 0.002\ \mathrm{Mpc}^{-1}$ corresponding to the frequency $f_h = 3 \times 10^{-18}$ Hz today, to higher frequency using the slow-roll parameters measured at k_0 we find

$$\Delta_h^2(f) = \Delta_h^2(f_h)\bigg[1 - 2\epsilon \ln\frac{f}{f_h} + 2\epsilon(\eta - \epsilon)\bigg(\ln\frac{f}{f_h}\bigg)^2 + \frac{1}{3}\epsilon(-12\epsilon^2 + 16\epsilon\eta - 4\eta^2 - 2\xi)\bigg(\ln\frac{f}{f_h}\bigg)^3\bigg]. \tag{7}$$

Among the planned space laser interferometers, LISA [18] is the most sensitive to frequency around 10^{-3} Hz where stochastic gravitational-wave background is dominated by astronomical sources such as white dwarf binaries, and it seems difficult to detect inflationary gravitational-wave background even if r is maximal. On the other hand, DECIGO or BBO targets the frequency window around $f = 0.1$–1 Hz where contamination of astrophysical foregrounds is separable, so they are ideal to probe inflation. In the standard cosmology, the gravitational-wave with its current frequency 0.1 Hz reentered the Hubble radius in the radiation dominated regime when the cosmic temperature was $T = 4 \times 10^6$ GeV assuming that there was no significant entropy production after that.

Interestingly, this temperature is in the expected range of the upper bound on the reheat temperature, T_R, after inflation imposed by the decay of unstable gravitinos [19] which are produced by thermal scattering in the reheating stage. Specifically, in order to ensure successful BBN in the presence of the hadronic decay of gravitinos, the reheat temperature should satisfy $T_R \lesssim 10^{6-8}$ GeV depending on the gravitino mass and the hadronic branching ratio [20]. In case the gravitino is the lightest supersymmetric particle and hence stable due to R-parity conservation, we also find a constraint on the reheat temperature so that it does not overclose the universe,

$$T_R < 7 \times 10^6 \bigg(\frac{m_{\tilde{g}}}{1\ \mathrm{TeV}}\bigg)^{-2}\bigg(\frac{m_{3/2}}{1\ \mathrm{GeV}}\bigg)\ \mathrm{GeV}, \tag{8}$$

for the gravitino mass $m_{3/2} = 10^{-4} - 10$ GeV [21], where $m_{\tilde{g}}$ denotes the gluino mass.

[2]Note that this extrapolation may not be valid if higher-order slow-roll parameters, in particular, the running spectral index ξ, take a large value [16]. In such a case, fluctuations on CMB scales and those on DECIGO/BBO band may have a different origin because inflation responsible for the former may not last long enough [17]. Fortunately, however, five-year WMAP data disfavors such a possibility [3].

If the reheat temperature indeed satisfies the above constraints, it is likely that the Universe is still dominated by coherent inflaton field oscillation when the frequency range probed by DECIGO/BBO reentered the Hubble radius. Since the cosmic expansion law is the same as in the matter-dominated era as field oscillation is driven by a mass term, the resultant spectrum of $\Omega_{\mathrm{GW}}(f, t_0)$ acquires a modulation proportional to f^{-2} for those frequency bands entering the horizon in this regime.

So far, we have implicitly assumed that there is no significant entropy production after the completion of reheating after inflation. If a nonnegligible amount of entropy is produced in the lower temperature regime, from the decay of a long-lived scalar field other than the inflaton, for example, not only the spectrum of $\Omega_{\mathrm{GW}}(f, t_0)$ but also constraint on the reheat temperature imposed by the thermal gravitino problem is modified. Defining the entropy increase factor F by the ratio of entropy in a fixed comoving volume before and after the entropy production [8], we find that the overall amplitude of the high-frequency part of $\Omega_{\mathrm{GW}}(f, t_0)$, the frequency of gravitational radiation entering the horizon at the end of reheating, and the constraint on the reheat temperature by the gravitino problem are multiplied by $F^{-4/3}$, $F^{-1/3}$, and F, respectively.

Taking all these factors into account, we can numerically calculate the spectrum of stochastic gravitational-wave background in the band probed by DECIGO/BBO using future-observed values of the tensor-to-scalar ratio and the slow-roll parameters. Figure 1 summarizes the result of numerical calculation for the case $r = 0.16$, $\epsilon = \eta = 0.01$, and $\xi = 0$, which are realized in chaotic inflation driven by a quadratic potential.

We find a nearly-flat spectrum

$$\Omega_{\mathrm{GW}}(f, t_0) = 2.8 \times 10^{-16} F^{-4/3} g_{*106.75}^{-1/3}\bigg(\frac{r}{0.1}\bigg)\bigg[\frac{\Delta_h(f)}{\Delta_h(f_h)}\bigg]^2 \equiv \Omega_{\mathrm{GW}}^{(L)}(f, t_0), \tag{9}$$

in the low frequency band reentering the horizon in the radiation dominated regime after reheating (and before possible entropy production),

$$f \lesssim 0.042\bigg(\frac{T_R F^{-1/3}}{10^7\ \mathrm{GeV}}\bigg)\ \mathrm{Hz} \equiv f_-. \tag{10}$$

Here $g_{*106.75}$ denotes an effective number of relativistic degrees of freedom normalized by 106.75 when the relevant mode reentered the Hubble radius. For higher frequency modes with

$$f \gtrsim 1.4\bigg(\frac{T_R F^{-1/3}}{10^7\ \mathrm{GeV}}\bigg)\ \mathrm{Hz} \equiv f_+, \tag{11}$$

we obtain a nearly power-law spectrum

$$\Omega_{\mathrm{GW}}(f, t_0) = \Omega_{\mathrm{GW}}^{(L)}(f, t_0)\bigg(\frac{f_T}{f}\bigg)^2, \tag{12}$$

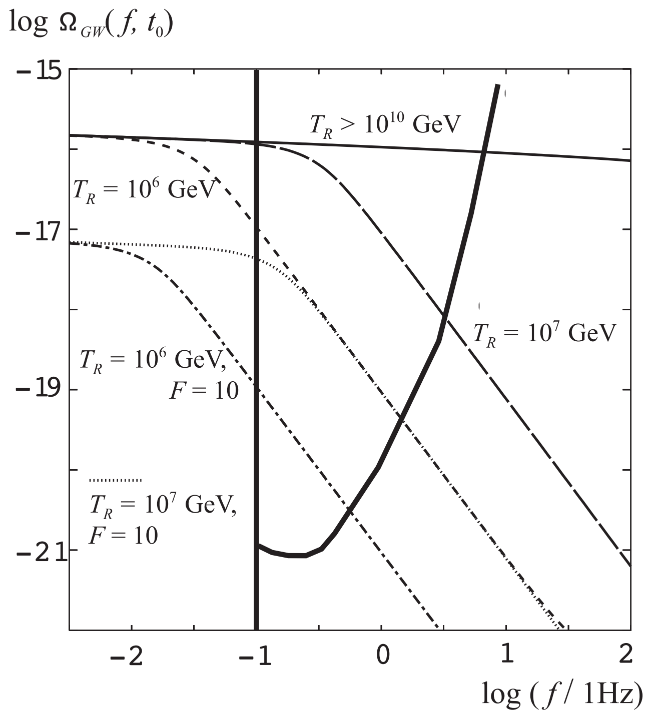

FIG. 1. Density parameter of gravitational radiation for different reheat temperature T_R and entropy increase factor F in chaotic inflation with a massive scalar field. Curves without F imply $F = 1$. The region above the thick curve with $f \gtrsim 0.1$ Hz can be observable by the ultimate DECIGO after its ten year operation.

$$f_T \equiv 0.31\left(\frac{T_R F^{-1/3}}{10^7\ \mathrm{GeV}}\right)\ \mathrm{Hz}. \tag{13}$$

The cosmological information we can obtain by observing these gravitational-wave backgrounds depends on the shape of the spectrum observable at the DECIGO/BBO band, namely, $0.1\ \mathrm{Hz} \lesssim f \lesssim 10\ \mathrm{Hz}$, which can be classified to the following three cases, **(a)**–**(c)**.

Case (a) $f_- \gtrsim 10$ Hz corresponding to $T_R \gtrsim 2.4 \times 10^9 F^{1/3}$ GeV: We can observe the nearly-flat region of the spectrum and determine F from the overall amplitude of Ω_{GW}. Since WMAP constrains $r < 0.55$ [2], F should satisfy $F \lesssim 3 \times 10^4$. Thus, for stable gravitino, the above lower bound on T_R indicates

$$m_{3/2} > 0.4\left(\frac{F}{3\times 10^4}\right)^{-2/3}\left(\frac{m_{\tilde{g}}}{1\ \mathrm{TeV}}\right)^2\ \mathrm{GeV}, \tag{14}$$

and for the unstable one, larger mass ($m_{3/2} \gtrsim 10$ TeV) is preferred.

Case (b) $f_- \lesssim 10$ Hz and $f_+ \gtrsim 0.1$ Hz corresponding to $7.1 \times 10^5 F^{1/3}\ \mathrm{GeV} < T_R < 2.4 \times 10^9 F^{1/3}$ GeV: We can fix F from the overall amplitude and T_R from the shape of the spectrum independently. This is the ideal case that space laser interferometers can practically determine the entire thermal history of the Universe between inflation and BBN.

Case (c) $f_+ \lesssim 0.1$ Hz corresponding to $T_R < 7.1 \times 10^5 F^{1/3}$ GeV: We can observe only the power-law region of the spectrum and measure only the ratio T_R/F. This ratio, however, fixes the gravitino-to-entropy ratio uniquely apart from a logarithmic correction.

In conclusion, space laser interferometers such as DECIGO and BBO will bring about invaluable information on the delayed reheating stage required by the gravitino problem after inflation and will be able to determine the reheat temperature and/or the entropy increase factor. Hence, it is desired that they are put into practice as soon as possible.

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