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Generalizing tensor-vector-scalar cosmology

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I consider an extended version of Bekenstein's Tensor-Vector-Scalar theory where the action of the vector field is of a general Einstein-Ether form. This work presents the cosmological equations of this theory, both at the background and perturbed level, for scalar, vector and tensor perturbation modes. By solving the background equations in the radiation era analytically, to an excellent approximation, I construct the primordial adiabatic perturbation for a general family of scalar field kinetic functions.

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I. INTRODUCTION

In the last two decades, cosmology has undergone a "precision" revolution, with a large influx of data such as observations of the cosmic microwave background [1], large scale structure [2], and supernovae observations [3]. There is now a consensus cosmological standard model, named ΛCDM, based on general relativity as the theory of gravity, which requires only about 4% of the energy budget of the universe to be in known baryonic form, while the rest is divided into two apparently distinct, dark components: *cold dark matter* and *dark energy*. It is unfortunate however, that apart from their phenomenology as fluids, we know nothing of their actual nature at the present time.

Cold dark matter, typically composed of very massive slowly moving and weakly interacting particles, is required on cosmological scales mainly to source large scale structure. To dramatize its importance, assuming general relativity, if dark matter was absent, structure as we know it would not have even formed yet. A plethora of such particles generally arises in particle physics models beyond the standard model quite naturally with the right cross sections to create the right abundance (see [4,5] for reviews). Yet, while its phenomenology as a dust fluid has been shown to agree with observations to a very good degree, the actual nature of cold dark matter is left to speculation as no cold dark matter particle has been observed so far. Moreover there are still some mishaps within the ΛCDM paradigm, for example, the problem of voids [6] and the recent observations of the Abel 520 cluster [7].

Given that the law of gravity plays a key role, to all observations from which dark matter and dark energy are inferred, it is conceivable that general relativity breaks down at small enough gradients and curvatures, and an alternative theory of gravity might also provide an explanation to the dark sector. One such theory was proposed some time ago by Bekenstein [8], building on key work by Sanders [9]. This theory was dubbed Tensor-Vector-Scalar (TeVeS) because it relies on a bimetric transformation involving a scalar and a vector field. It was designed to reduce to the Aquadratic Lagrangian nonrelativistic theory of Bekenstein and Milgrom [10]. Thus, it provides essentially the same phenomenology as Milgrom's Modified Newtonian Dynamics (MOND) [11] for galactic rotation curves, for which MOND has had tremendous success [12].

TeVeS theory has since been shown [13] to be able to source structure in a similar way as dark matter. The vector field in the theory plays a key role [14], as for a wide range of parameters it has a power-law growing mode which sources potential wells. In contrast with dark matter, the vector field has shear which creates a mismatch between the two scalar gravitational potentials. This has been identified as prospective discriminator between dark matter and theories like TeVeS [15–18]. Other noncosmological tests of TeVeS have also been studied, for example, gravitational lensing [19,20]. Probing the difference between the arrival times of neutrinos and gravitational waves from distant supernovae is also a possibility [21].

Having shown that one can cast TeVeS in a single metric form, with the scalar field absorbed into the vector field [22], Zlosnik, Ferreira and Starkman, have explored a sister theory, based solely on a unit-timelike vector field with a noncanonical kinetic term [23] which is a noncanonical Einstein-ether theory [24]. This theory has also been shown to source structure in a similar way as TeVeS [25] (cosmology within the context of canonical Einstein-ether theory has been extensively studied in [26–28]) while its predictions for corrections to Newtonian gravity in the solar system have also been studied [29]. Further exploration of this theory into different directions has been considered by Halle and Zhao [30,31].

In this work I initiate a study of a version of TeVeS which involves a generalization of the action of the vector field into the Einstein-ether form. This is motivated in part from the instability present in spherically symmetric solutions to the original TeVeS theory [32], that possibly stems from the fact that the kinetic term for the unit-timelike vector field was of Maxwellian form which can violate the dominant energy condition [33]. Seifert has shown that more general Einstein-ether actions can be stable depending on the parameters. It is therefore of importance to check whether forms of TeVeS exist which have stable

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spherically symmetric solution and still can form large scale structure. Possible directions for deriving TeVeS-type theories from more fundamental theories are discussed in [34–36].

In Sec.-II, I lay down the action and derive the field equations. The cosmological equations for a (possibly curved) Friedman-Lemaître-Robertson-Walker (FLRW) metric are studied in Sec.-III, where it is shown that they are identical with the TeVeS equations up to a rescaling of Hubble's constant. The cosmological solutions are therefore the same as those in TeVeS. I conclude that section by deriving an approximate (to an excellent degree) solution in the radiation era for a general family of scalar field functions.

To study large scale structure we need the cosmological perturbation equations about an FLRW universe. The gauge form–invariant perturbed equations are derived using the techniques in [37] and are given in Sec.-IV for all types of perturbations, namely, scalar vector and tensor perturbations. Using the approximate background solution in the radiation era from Sec.-III, I proceed to construct the primordial adiabatic perturbation. The construction of the most general, regular primordial perturbation is quite involved and is given elsewhere [38]. Throughout the paper I use the conventions of Wald [39].

II. FUNDAMENTALS: ACTION AND FIELD EQUATIONS

A. Preliminaries

TeVeS theory and the generalization herein is a bimetric theory where gravity is mediated by a tensor field $\tilde{g}_{ab}$ with associated metric-compatible connection $\tilde{\nabla}_a$ and well-defined inverse $\tilde{g}^{ab}$ such that $\tilde{g}^{ac}\tilde{g}_{cb} = \delta^a{}_b$, a timelike (dual) vector field A_a such that $\tilde{g}^{ab}A_aA_b = -1$, and a scalar field ϕ. Matter is required to obey the weak equivalence principle, which means that there is a metric g_{ab} with associated metric-compatible connection ∇_a, universal to all matter fields, such that test particles follow its geodesics. The tensor field $\tilde{g}_{ab}$ will be called the *Einstein-Hilbert frame metric* (see below) while g_{ab} the *matter frame metric*.

The relation between the four above tensor fields (when the field equations are satisfied) is

$$g_{ab} = e^{-2\phi}\tilde{g}_{ab} - 2\sinh(2\phi)A_aA_b \tag{1}$$

with inverse

$$g^{ab} = e^{2\phi}\tilde{g}^{ab} + 2\sinh(2\phi)A^aA^b \tag{2}$$

where $A^a = \tilde{g}^{ab}A_b$.

B. The action principle

The theory is based on an action S, which splits as $S = S_g + S_A + S_\phi + S_m$, where S_g, S_A, S_ϕ and S_m are the actions for $\tilde{g}_{ab}$, vector field A_a, scalar field ϕ and matter, respectively.

The action for $\tilde{g}_{ab}$, A_a and ϕ is most easily written in the Einstein-Hilbert frame, and is such that S_g is of Einstein-Hilbert form

$$S_g = \frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}\tilde{R}, \tag{3}$$

where $\tilde{g}$ and $\tilde{R}$ are the determinant and scalar curvature of $\tilde{g}_{\mu\nu}$ respectively and G is the bare gravitational constant. Because of the complicated nature of the equations, the numerical value of G will not be the measured value of Newton's constant as measured on Earth. The precise relation between them depends on the spherically symmetric solution which apart from depending on the arbitrary function V (see below) is not expected to be unique, just like the case of standard TeVeS [8,40].

The action for the vector field A_a is given by

$$S_A = -\frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}[K^{abcd}\tilde{\nabla}_aA_b\tilde{\nabla}_cA_d - \lambda(A_aA^a + 1)], \tag{4}$$

where

$$K^{abcd} = K_B(\tilde{g}^{ac}\tilde{g}^{bd} - \tilde{g}^{ad}\tilde{g}^{bc}) + K_+(\tilde{g}^{ac}\tilde{g}^{bd} + \tilde{g}^{ad}\tilde{g}^{bc}) + K_0\tilde{g}^{ab}\tilde{g}^{cd} + K_A\tilde{g}^{bd}A^aA^c; \tag{5}$$

λ is a Lagrange multiplier ensuring the timelike constraint on A_a, and K_B, K_+, K_0 and K_A are dimensionless constants.

The action for the scalar field ϕ is given by

$$S_\phi = -\frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}[\mu(\tilde{g}^{ab} - A^aA^b)\tilde{\nabla}_a\phi\tilde{\nabla}_b\phi + V(\mu)] \tag{6}$$

where μ is a nondynamical dimensionless scalar field, and $V(\mu)$ is an arbitrary function which must be such that $\frac{dV}{d\mu} \rightarrow \mu^2$ as $\mu \rightarrow 0$ in order to have exact MOND limit, while it must diverge as $\mu \rightarrow \mu_0$ where μ_0 is a constant in order to have exact Newtonian limit [41]. One example is the form considered in [41] which is

$$\frac{dV}{d\mu} = \frac{\mu_0^2}{16\pi\ell_B^2}\frac{\hat{\mu}^2}{\hat{\mu} - 1}(\hat{\mu} - \mu_a)^n \tag{7}$$

where ℓ_B is a scale, μ_a is a constant,n is an integer power and $\hat{\mu} = \frac{\mu}{\mu_0}$. This general class of functions will also be used in this work.

The matter is coupled only to the matter frame metric g_{ab} and thus its action is of the form $S_m[g, \chi^A] = \int d^4x\sqrt{-g}L[g, \chi^A]$ for some generic collection of matter fields χ^A.

One can further generalize the vector field action by making the constants K_B, K_+, K_0 and K_A functions of the scalar field ϕ, as well as generalizing the scalar field action by inserting ϕ-dependent functions as coefficients of the terms $\tilde{g}^{ab}$ and A^aA^b in the kinetic term and a potential for ϕ. I leave this for future investigations (if warranted).

C. The field equations

Variation with respect to the Lagrange multiplier λ gives back the timelike constraint on the vector field, $\tilde{g}^{ab}A_aA_b = -1$. The matter stress-energy tensor T_{ab} is defined by varying of the matter action with respect to the matter frame metric as $\delta S_m = -\frac{1}{2}\int d^4x\sqrt{-g}T_{ab}\delta g^{ab}$.

Now consider the tensors $S^{efcd}{}_{ab}$ and J^{abcde} defined as

$$\begin{aligned} S^{efcd}{}_{ab} = \frac{\delta K^{efcd}}{\delta \tilde{g}^{ab}} &\equiv K_B[\delta^e{}_{(a}\delta^c{}_{b)}\tilde{g}^{fd} + \tilde{g}^{ec}\delta^f{}_{(a}\delta^d{}_{b)} - \delta^e{}_{(a}\delta^d{}_{b)}\tilde{g}^{fc} \\ &- \delta^f{}_{(a}\delta^c{}_{b)}\tilde{g}^{ed}] + K_+[\delta^e{}_{(a}\delta^c{}_{b)}\tilde{g}^{fd} + \tilde{g}^{ec}\delta^f{}_{(a}\delta^d{}_{b)} \\ &+ \delta^e{}_{(a}\delta^d{}_{b)}\tilde{g}^{fc} + \delta^f{}_{(a}\delta^c{}_{b)}\tilde{g}^{ed}] + K_0[\delta^e{}_{(a}\delta^f{}_{b)}\tilde{g}^{cd} \\ &+ \delta^c{}_{(a}\delta^d{}_{b)}\tilde{g}^{ef}] + K_A[\delta^f{}_{(a}\delta^d{}_{b)}A^eA^c \\ &+ \tilde{g}^{fd}\delta^e{}_{(a}A_{b)}A^c + \tilde{g}^{fd}\delta^c{}_{(a}A_{b)}A^e] \end{aligned} \tag{8}$$

and

$$J^{abcde} \equiv \frac{\delta K^{abcd}}{\delta A_e} = K_A\tilde{g}^{bd}(\tilde{g}^{ae}A^c + \tilde{g}^{ce}A^a) \tag{9}$$

respectively.

Then the field equations for $\tilde{g}_{ab}$ are given by

$$\begin{aligned} \tilde{G}_{ab} &= 8\pi G[T_{ab} + 2(1 - e^{-4\phi})A^cT_{c(a}A_{b)}] - \lambda A_aA_b \\ &+ \mu[\tilde{\nabla}_a\phi\tilde{\nabla}_b\phi - 2A^c\tilde{\nabla}_c\phi A_{(a}\tilde{\nabla}_{b)}\phi] + \frac{1}{2}(\mu V' \\ &- V)\tilde{g}_{ab} + \left[S^{efcd}{}_{ab} - \frac{1}{2}K^{efcd}g_{ab}\right]\tilde{\nabla}_eA_f\tilde{\nabla}_cA_d \\ &- \tilde{\nabla}_e[(A_{(a}K^{ecd}{}_{b)} + A_{(a}K_{b)}{}^{ecd} - A^eK_{(ab)}{}^{cd})\tilde{\nabla}_cA_d] \end{aligned} \tag{10}$$

where $\tilde{G}_{ab}$ is the Einstein tensor of $\tilde{g}_{ab}$.

The field equations for the vector field A_a are

$$\begin{aligned} K^{abc}{}_d\tilde{\nabla}_c\tilde{\nabla}_aA_b &= \left[\frac{1}{2}J^{abce}{}_d - J^a{}_d{}^{ceb}\right]\tilde{\nabla}_aA_b\tilde{\nabla}_cA_e \\ &- \lambda A_d - \mu A^b\tilde{\nabla}_b\phi\tilde{\nabla}_d\phi \\ &+ 8\pi G(1 - e^{-4\phi})A^bT_{ba}. \end{aligned} \tag{11}$$

The field equation for the scalar field ϕ is

$$\tilde{\nabla}_a[\mu(\tilde{g}^{ab} - A^aA^b)\tilde{\nabla}_b\phi] = 8\pi Ge^{-2\phi}[g^{ab} + 2e^{-2\phi}A^aA^b]T_{ab} \tag{12}$$

where the nondynamical field μ is found by inverting

$$(\tilde{g}^{ab} - A^aA^b)\tilde{\nabla}_a\phi\tilde{\nabla}_b\phi = -V', \tag{13}$$

and therefore the arbitrary function V and its derivatives are nothing but functions of kinetic terms for ϕ, contracted with $\tilde{g}^{ab}$ and A^a.

III. FLRW COSMOLOGY

A. Equations

A most convenient coordinate system that is commonly used in cosmological perturbation theory is the conformal synchronous coordinate system with t denoting conformal time and $x^{\hat{a}}$ the spatial coordinates. This gives the matter frame metric with scale factor a as

$$ds^2 = a^2[-dt^2 + q_{ij}dx^idx^j] \tag{14}$$

where q_{ab} is the metric of a space of constant curvature $\frac{K}{r_c^2}$, with radius of curvature r_c and where $K = 0$ for a flat, $K = 1$ for positively curved and $K = -1$ for negatively curved space. The scale factor of the Einstein-frame metric is $b = ae^{\phi}$. The vanishing of the Lie derivative with respect to all the Killing vectors of the background spacetime gives $\phi = \bar{\phi}(t)$ only, while the vector field is pure gauge for this background.

The scalar field is governed by the TeVeS constraint which in this coordinate system reads

$$\dot{\bar{\phi}}^2 = \frac{1}{2}a^2e^{-2\phi}\frac{dV}{d\mu} \tag{15}$$

which must be inverted to get $\bar{\mu}(a, \bar{\phi}, \dot{\bar{\phi}})$, and the second-order equation

$$\ddot{\bar{\phi}} = \dot{\bar{\phi}}\left(\frac{\dot{a}}{a} - \dot{\bar{\phi}}\right) - \frac{1}{U}\left[3\bar{\mu}\frac{\dot{b}}{b}\dot{\bar{\phi}} + 4\pi Ga^2e^{-4\bar{\phi}}(\bar{\rho} + 3\bar{P})\right], \tag{16}$$

where $U = \mu + 2\frac{dV}{d\mu}/\frac{d^2V}{d\mu^2}$. Both of the above are unchanged from TeVeS, because they are not affected by the vector field action.

Defining the constant $K_F = 1 + K_0 + \frac{3}{2}K_+$, the Friedmann equation gives

$$3K_F\frac{\dot{b}^2}{b^2} = a^2e^{-4\phi}\left[\frac{1}{2}e^{2\phi}\left(\mu\frac{dV}{d\mu} + V\right) + 8\pi G\bar{\rho} - \frac{3K}{r_c^2a^2}\right] \tag{17}$$

while the Raychaudhuri equation is

$$K_F\left[-2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} - 4\frac{\dot{b}}{b}\dot{\bar{\phi}}\right] = a^2e^{-4\phi}\left[\frac{1}{2}e^{2\bar{\phi}}\left(\mu\frac{dV}{d\mu} - V\right) + 8\pi G\bar{P} + \frac{K}{r_c^2a^2}\right] \tag{18}$$

where $\bar{\rho}$ and $\bar{P}$ are the energy density and pressure of a matter fluid, and evolve as

$$\dot{\bar{\rho}} + 3\frac{\dot{a}}{a}(1+w)\bar{\rho} = 0 \tag{19}$$

with $w = \bar{P}/\bar{\rho}$.

One quickly notices that the only change from the original TeVeS theory, at the background level, is a constant rescaling of the Friedman and Raychaudhuri equations. Thus FLRW solutions to this theory are identical to those in TeVeS up to a rescaling of the Hubble's constant.

B. Solution in the radiation era

For the form of the function above (7), it has been shown [13,14,41] that the scalar field tracks the dominant fluid. This simply means that the energy density of the fluid relative to the energy density in the scalar field is constant. This tracker behavior is found in many scalar field dark energy (quintessence) models [42]. On the other hand it has also been shown [14] for the special case of the Bekenstein toy model ($n = 2$ and $\mu_a = 2$ in the function above), that in the radiation era for realistic baryon and radiation densities (where by radiation, I mean the total contribution from all relativistic species, like photons, neutrinos, etc.), the radiation tracker is almost never reached until just before the transition to the matter era. Instead the solution is such that $\dot{\phi}$ evolves as a power law of the scale factor. The purpose of this subsection is to show that this is also true for the generalized function above.

In the deep radiation era, for the function (7), μ is very large and we get that

$$C_\phi \equiv \frac{\mu}{U} \rightarrow \frac{1+n}{3+n}. \tag{20}$$

During this time $\dot{\bar{\phi}}$ is very subdominant and we may assume that $\frac{\dot{b}}{b} \approx \frac{\dot{a}}{a}$. The Friedmann equation then assumes the standard form

$$3K_F\frac{\dot{a}^2}{a^2} = 8\pi G a^2 e^{-4\bar{\phi}_i}\bar{\rho}_r \tag{21}$$

up to a rescaling of the radiation energy density by $e^{-4\bar{\phi}_i}/K_F$, where $\bar{\phi}_i$ is the initial condition of ϕ and $\bar{\rho}_r$ is the radiation energy density. Let us define

$$\Omega_{0r} \equiv \frac{8\pi G\rho_{0r}e^{-4\bar{\phi}_i}}{3K_F H_0^2} \tag{22}$$

where ρ_{0r} is the proper radiation density today (as given by the radiation temperature) and H_0 is the Hubble constant today. The solution to the Friedman equation during the radiation era gives the well-known form

$$a = \sqrt{\Omega_{0r}}H_0 t \tag{23}$$

for the scale factor a as a function of conformal time t.

Although $\dot{\bar{\phi}}$ is subdominant, we expect it to grow in order to approach the tracker solution. We therefore assume the ansatz

$$\phi = \phi_i - \phi_1 a^m \tag{24}$$

where ϕ_1 and m are constants to be determined below. The second initial condition for $\bar{\phi}$ is assumed to be $\dot{\bar{\phi}} = 0$. Consider now the variable $q = -2\mu\frac{e^{\bar{\phi}}}{a}\dot{\bar{\phi}}$. Using the constraint (15) and the evolution equation for the scalar field (16) we get that in the radiation era under the above assumptions, q evolves as

$$\frac{dq}{d\ln a} + 3q = 6K_F\frac{\dot{a}}{a^2}. \tag{25}$$

Therefore μ (again under the above assumptions) evolves as

$$\frac{d\mu}{d\ln a} + (m+1)\mu = \frac{3}{m\phi_1}K_F e^{-m\ln a}. \tag{26}$$

The solution is

$$\mu = \frac{3}{m\phi_1}K_F a^{-m}, \tag{27}$$

and from the positivity of μ we get that $\phi_1 > 0$. Combining (24) with (27) we get the relation $\mu = \frac{3K_F}{m(\phi_i - \phi)}$ which holds irrespective of the constants ℓ_B and μ_0.

We can now determine the constants m and ϕ_1 which appear in the approximated solutions above. To do so we use the TeVeS constraint (15) in the large μ limit to get

$$\dot{\bar{\phi}}^2 = \frac{\mu_0^2}{32\pi\ell_B^2}a^2 e^{-2\bar{\phi}_i}\hat{\mu}^{1+n}. \tag{28}$$

Using the solution for the scale factor $a(t)$ (23) to find $\phi(t)$ and $\mu(t)$ in terms of conformal time t, and then using the above equation we get that the constant m is given by

$$m = \frac{4}{3+n} \tag{29}$$

while the constant ϕ_1 is given as

$$\phi_1 = \frac{1}{m}\left[\frac{e^{-2\bar{\phi}_i}(3K_F)^{1+n}}{32\pi\ell_B^2\mu_0^{n-1}\Omega_{0r}H_0^2}\right]^{1/(3+n)}. \tag{30}$$

For the TeVeS case, i.e. $n = 2$, and $\bar{\phi}_i = 0$, we get the Dodelson-Liguori [14] result $m = 4/5$ and $\phi_1 = \frac{5}{4}\times\left[\frac{27}{32\pi\ell_B^2\Omega_{0r}H_0^2\mu_0}\right]^{1/5}$.

It is important to note that the approximate solutions for $a(t)$, $\phi(t)$ and $\mu(t)$ found in this subsection are excellent up to the radiation-matter equality. Figure 1 shows the exact numerical solution (solid curve) for $\bar{\phi}$ (upper panel) and μ (lower panel) as well as the approximated solution of this subsection (dashed curve), both plotted against $H_0 t$. Similarly, Fig. 2 shows the exact numerical solution (solid curve) for $\dot{\bar{\phi}}$ (upper panel) and $\mu\dot{\bar{\phi}}t$ (lower panel) and the

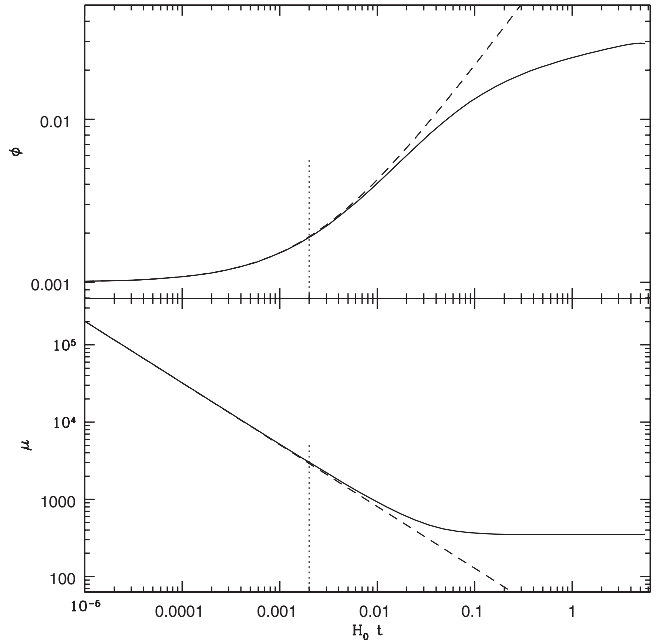

FIG. 1. Exact numerical evolution (solid) and the analytical approximation (dashed) for $\bar{\phi}$ (upper panel) and μ (lower panel). The initial condition for $\bar{\phi}$ is 10^{-3}. Both approximations are excellent in the radiation era and depart from the true solution once the Universe starts to enter the matter era. The radiation-matter equality is indicated by the vertical dotted lines.

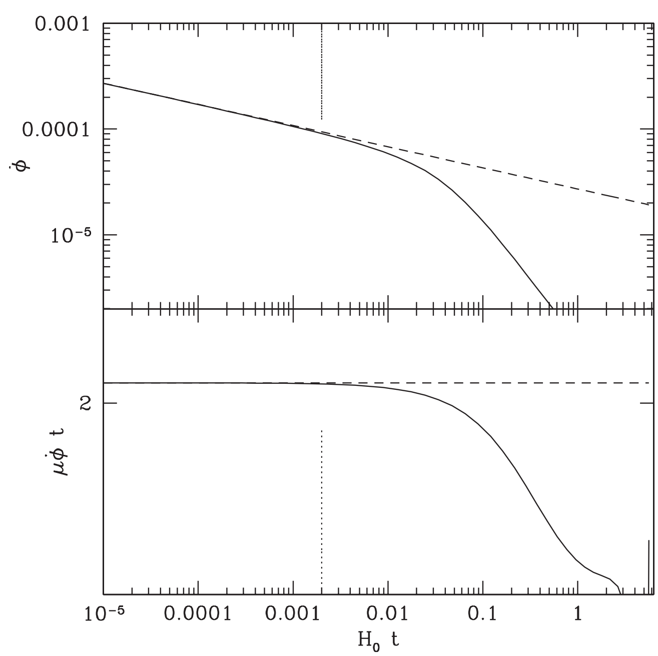

FIG. 2. Exact numerical evolution (solid) and the analytical approximation (dashed) for $\dot{\bar{\phi}}$ (upper panel) and $\mu\dot{\bar{\phi}}t$ (lower panel) for the same model in Fig. 1. Both approximations are excellent in the radiation era and depart from the true solution once the Universe starts to enter the matter era. The radiation-matter equality is indicated by the vertical dotted lines.

approximated solution of this subsection (dashed curve) again plotted against H_0t. The vertical lines in both figures show the position of the radiation-matter equality.

IV. COSMOLOGICAL PERTURBATIONS

The perturbed equations are written directly in Fourier space, and are thus dependent on the wave number k.

We will find it easier to define the following combinations of constants :

$$K_t = K_B + K_+ - K_A \tag{31}$$

$$\kappa_d = K_+ + \frac{1}{2}K_0 \tag{32}$$

$$K_F = 1 + K_0 + \kappa_d \tag{33}$$

$$R_K = 1 - \frac{3\kappa_d}{K_F} \tag{34}$$

as it is these combinations which appear in the perturbed equations. Moreover in the limit that the scalar field is switched off, these constants must obey certain relations by requiring that the energy density and square speed of sound on the linear modes are both positive.

As it turns out, for scalar and tensor modes K_A never appears by itself to linear order and is always absorbed in to the combination K_t defined above. Moreover the constant κ_d functions as a damping constant in the scalar mode vector field equation when expressed in gauge invariant variables. When $\kappa_d = 0$, the vector field equation is independent of k. Thus κ_d must be positive and very close to zero, for if it were negative, the perturbations would have negative square speed of sound on flat space and thus be greatly unstable on small scales, while if it were large and positive, it would damp the vector field on cosmological scales, which would render the theory irrelevant for structure formation. The constants K_t and R_K must be obey $0 < K_t < 2$ and $0 < R_K \leq 1$ respectively for the energy densities and the square speed of sound of all modes to be positive (the last conditions are sufficient to ensure that $\kappa_d \geq 0$). Scalar modes depend on K_t, K_F and R_K and tensor modes on K_F and R_K only.

For vector modes the situation is slightly different and all constants K_i are needed. In that case one can use the parameters K_t, K_F and R_K defined above but one more is needed which can be either K_A or K_B which must obey $K_B \geq \frac{K_F R_K - 1}{2K_F R_K}$.

A. Scalar modes

Scalar modes are defined as in [37]. The scalar field is perturbed as $\phi = \bar{\phi} + \varphi$. The vector field has only one scalar mode, α, because the other one is fixed by the timelike constraint and is defined as $A_i = ae^{-\bar{\phi}}\vec{\nabla}_i\alpha$. The matter frame metric has four scalar modes, Ξ, χ, ζ

and ν, such that $g_{00} = -a^2(1-2\Xi)$, $g_{0i} = -a^2\vec{\nabla}_i\zeta$, $g_{ij} = a^2(1+\frac{1}{3}\chi)q_{ij} + a^2(q_i{}^k q_j{}^l - \frac{1}{3}q_{ij}q^{kl})\vec{\nabla}_k\vec{\nabla}_l\nu$. The Einstein-frame metric has also four scalar modes, $\tilde{\Xi}$, $\tilde{\chi}$, $\tilde{\zeta}$ and $\tilde{\nu}$, defined in a similar way, related to the matter frame metric as $\tilde{\Xi} = \Xi + \varphi$, $\tilde{\chi} = \chi + 6\varphi$, $\tilde{\zeta} = \zeta - (1 - e^{-4\bar{\phi}})\alpha$ and $\tilde{\nu} = \nu$. The fluid variables are the density contrast $\delta = \frac{\delta\rho}{\bar{\rho}}$ and momentum divergence θ such that the fluid velocity perturbation is defined as $u_i = a\vec{\nabla}_i\theta$, where u_a is the unit-timelike with respect to g_{ab} fluid velocity.

1. Fluid equations

The density contrast evolves as

$$\dot{\delta} = 3\frac{\dot{a}}{a}(w - C_s^2)\delta + (1+w)\left(-k^2\theta - \frac{1}{2}\dot{\chi} + k^2\zeta\right) \tag{35}$$

where $C_s^2 = \frac{\delta P}{\delta\rho}$ is the speed of sound.

The momentum divergence evolves as

$$\dot{\theta} = -\Xi + \frac{\dot{a}}{a}(3w-1)\theta + \frac{C_s^2}{1+w}\delta - \frac{\dot{w}}{1+w}\theta - \frac{2}{3}\left(k^2 - \frac{3K}{r_c^2}\right)\Sigma \tag{36}$$

where Σ is the fluid's scalar anisotropic stress (see [37]).

2. Scalar field equations

The scalar field perturbation evolves as

$$\dot{\varphi} = -\frac{C_\phi}{2\mu\dot{\bar{\phi}}}\gamma - \dot{\bar{\phi}}\tilde{\Xi} \tag{37}$$

where the auxiliary scalar field perturbation γ [43] evolves as

$$\dot{\gamma} = -\left[(1+3C_\phi)\frac{\dot{b}}{b} + 4\dot{\bar{\phi}} + 8\pi G a^2 e^{-4\bar{\phi}}\frac{C_\phi}{2\bar{\mu}\dot{\bar{\phi}}}\times(\bar{\rho}+3\bar{P})\right]\gamma + \bar{\mu}\dot{\bar{\phi}}k^2 e^{-4\bar{\phi}}[\varphi + \dot{\bar{\phi}}\alpha] + \bar{\mu}\dot{\bar{\phi}}^2[\dot{\tilde{\chi}} - 2k^2\tilde{\zeta}] + 8\pi G a^2 e^{-4\bar{\phi}}\bar{\rho}\dot{\bar{\phi}}[(1+3C_s^2)\delta - (1+3w)(\tilde{\Xi}+2\varphi)]. \tag{38}$$

3. Vector equation

The vector field equation is

$$\dot{\alpha} = E - \tilde{\Xi} + \left(\dot{\bar{\phi}} - \frac{\dot{a}}{a}\right)\alpha \tag{39}$$

where the auxiliary gauge invariant vector mode E evolves as

$$K_t\left[\dot{E} + \frac{\dot{b}}{b}E\right] + \frac{K}{r_c^2}(1 - K_F e^{4\bar{\phi}})(\dot{\nu} + 2(\zeta-\alpha)) - e^{4\bar{\phi}}\kappa_d\left(k^2 - \frac{3K}{r_c^2}\right)[\dot{\nu} + 2(\zeta-\alpha)] + \frac{1}{3}(K_F e^{4\bar{\phi}} - 1)\times\left[\dot{\tilde{\chi}} + k^2\dot{\nu} + 6\frac{\dot{b}}{b}\tilde{\Xi} + 6\left(-\frac{\ddot{b}}{b} + 2\frac{\dot{b}^2}{b^2} - 2\frac{\dot{b}}{b}\dot{\bar{\phi}}\right)\alpha\right] + (2e^{4\bar{\phi}} - 1)\bar{\mu}\dot{\bar{\phi}}\left(\varphi - \dot{\bar{\phi}}\alpha\right) = 0. \tag{40}$$

The coupling to the matter velocity in the equation above has been eliminated with the use of (42) below.

4. Einstein equations

The two Einstein constraint equations are

$$\frac{\dot{b}}{b}K_F\left[\dot{\tilde{\chi}} + 2k^2(\alpha-\zeta) + 6\frac{\dot{b}}{b}\tilde{\Xi}\right] + \frac{1}{3}\left(k^2 - \frac{3K}{r_c^2}\right)\times e^{-4\phi}[\tilde{\chi} + k^2\nu] = 8\pi G a^2 e^{-4\phi}\bar{\rho}[\delta - 2\varphi] + e^{-4\phi}k^2\left(K_t E + 2\frac{\dot{b}}{b}\alpha\right) - \gamma \tag{41}$$

and

$$-K_F\left[\frac{1}{3}(\dot{\tilde{\chi}} + k^2\dot{\nu}) + 2\frac{\dot{b}}{b}\tilde{\Xi}\right] = 8\pi G a^2 e^{-4\phi}(\bar{\rho}+\bar{P})\theta + 2\bar{\mu}\dot{\bar{\phi}}\varphi - \frac{2K}{r_c^2}e^{-4\bar{\phi}}\alpha - \left[K_F\frac{K}{r_c^2} + \kappa_d\left(k^2 - \frac{3K}{r_c^2}\right)\right][\dot{\nu} + 2(\zeta-\alpha)] \tag{42}$$

while the propagation equations are

$$-K_F\left\{\ddot{\tilde{\chi}} + 2k^2(\dot{\alpha}-\dot{\zeta}) + 6\frac{\dot{b}}{b}\dot{\tilde{\Xi}} + 6\left[2\frac{\ddot{b}}{b} - \frac{\dot{b}^2}{b^2} + 4\dot{\bar{\phi}}\frac{\dot{b}}{b}\right]\tilde{\Xi} + 2\left(\frac{\dot{b}}{b} + \dot{\bar{\phi}}\right)[\dot{\tilde{\chi}} + 2k^2(\alpha-\zeta)]\right\} - \frac{1}{3}e^{-4\bar{\phi}}\left(k^2 - \frac{3K}{r_c^2}\right)(\tilde{\chi} + k^2\nu) + 3C_\phi\gamma = 24\pi G a^2 e^{-4\bar{\phi}}\bar{\rho}(C_s^2\delta - 2w\varphi) - 2k^2 e^{-4\bar{\phi}}E - 2k^2 e^{-4\bar{\phi}}\frac{\dot{b}}{b}\alpha \tag{43}$$

for the coupling to the perturbed pressure and

$$K_F R_K\left[\ddot{\nu} + 2(\dot{\zeta}-\dot{\alpha}) + 2\left(\dot{\bar{\phi}} + \frac{\dot{b}}{b}\right)(\dot{\nu} + 2\zeta - 2\alpha)\right] + 2e^{-4\bar{\phi}}E + e^{-4\bar{\phi}}\left[2\frac{\dot{b}}{b}\alpha - \frac{1}{3}(\tilde{\chi} + k^2\nu)\right] = 16\pi G a^2 e^{-4\bar{\phi}}(\bar{\rho}+\bar{P})\Sigma \tag{44}$$

for the coupling to the shear.

B. Vector modes

Vector modes are defined as in [37]. All vector modes have two polarizations and are purely spacial and divergenceless. The vector field has a vector mode β_i and is defined as $A_i = ae^{-\bar\phi}\beta_i$. The matter frame metric has two vector modes r_i and f_i such that $g_{0i} = -a^2 r_i$ and $g_{ij} = 2a^2\vec{\nabla}_{(i}f_{j)}$. The Einstein-frame metric has also two vector modes $\tilde r_i$ and $\tilde f_i$ defined in a similar way, related to the matter frame metric as $\tilde r_i = r_i - (1-e^{-4\bar\phi})\beta_i$ and $\tilde f_i = f_i$. The fluid variable is the vector mode v_i in the fluid momentum such that the fluid velocity perturbation is defined as $u_i = av_i$.

1. Fluid equations

The fluid vector mode v evolves as

$$\dot v = -\left[(1-3w)\frac{\dot a}{a} + \frac{\dot w}{1+w}\right]v - \left(k^2 - \frac{2K}{r_c^2}\right)\sigma^{(v)} \qquad (45)$$

where $\sigma^{(v)}$ is the fluid's vector anisotropic stress (see [37]).

2. Vector field equations

The vector mode equation is

$$\dot\beta = \epsilon + \left(\dot{\bar\phi} - \frac{\dot a}{a}\right)\beta \qquad (46)$$

while the auxiliary vector mode ϵ evolves as

$$-K_t\left(\dot\epsilon + \frac{\dot b}{b}\epsilon\right) - \frac{1}{2}[1+(2K_B-1)e^{-4\phi}]\left(k^2+\frac{2K}{r_c^2}\right)\beta + \frac{1}{2}(1-K_FR_Ke^{4\bar\phi})\left(k^2-\frac{2K}{r_c^2}\right)(\dot f + r - \beta) + (K_Fe^{4\phi}-1)\left(2\frac{\ddot b}{b} - 4\frac{\dot b^2}{b^2} + 4\frac{\dot b}{b}\dot{\bar\phi}\right)\beta + (2e^{4\phi}-1)\mu\dot{\bar\phi}^2\beta = 0.$$

Notice that unlike scalar modes, the parameter K_A is no longer redundant. In this case we can parametrize the vector field with K_t, K_B, K_F and R_K.

3. Einstein field equations

We have the constraint equations

$$\left(k^2 - \frac{2K}{r_c^2}\right)[K_FR_K(\dot f + r - \beta) + e^{-4\bar\phi}\beta] = -16\pi Ga^2e^{-4\bar\phi}(\bar\rho + \bar P)v$$

and the propagation equation

$$K_FR_K\left[\ddot f + \dot r - \dot\beta + 2\left(\frac{\dot b}{b} + \dot{\bar\phi}\right)(\dot f + r - \beta)\right] + e^{-4\bar\phi}\left(\dot\beta + 2\frac{\dot a}{a}\beta\right) = 16\pi Ga^2e^{-4\bar\phi}(\bar\rho + \bar P)\sigma^{(v)}.$$

C. Tensor modes

Tensor modes are defined as in [37]. All tensor modes have two polarizations and are purely spacial and divergenceless. The matter frame metric has a tensor mode H_{ij}, such that $g_{ij} = a^2H_{ij}$. The Einstein-frame metric has also a tensor mode $\tilde H_{ij}$ defined in a similar way, related to the matter frame metric as $\tilde H_{ij} = H_{ij}$. The equation of motion for the tensor mode H is

$$K_FR_K\left[\ddot H + 2\left(\frac{\dot b}{b} + \dot{\bar\phi}\right)\dot H\right] + e^{-4\bar\phi}\left(k^2 + \frac{2K}{r_c^2}\right)H = 16\pi Ga^2e^{-4\bar\phi}(\bar\rho + \bar P)\sigma^{(T)}$$

where $\sigma^{(T)}$ is the fluid's tensor anisotropic stress (see [37]).

D. Adiabatic initial conditions for scalar modes

1. Conformal synchronous gauge in radiation era

We start by adopting the scalar mode perturbation equations, to the synchronous gauge (defined as $\Xi = \zeta = 0$, $\chi = h$ and $-k^2\nu = h + 6\eta$), in the radiation era, using the background solution discussed above. Let us also define the following dimensionless variables : $x = kt$, $v = k\theta$, $u = k\alpha$, $\sigma = \frac{2}{3}k^2\Sigma$ and $y = \frac{\gamma}{k^2}$. All equations are then written in dimensionless form, where derivatives with respect to x are denoted by a prime. For simplicity let us also define $\phi_r = m\phi_1\Omega_{0r}^{m/2}H_k^m$ where $H_k = H_0/k$, such that $\bar\phi' = -\phi_r x^{m-1}$ (see Sec.-III B). Furthermore let $S_\nu = \frac{\Omega_{0\nu}}{\Omega_{0\nu}+\Omega_{0\gamma}}$ and $S_\gamma = \frac{\Omega_{0\gamma}}{\Omega_{0\nu}+\Omega_{0\gamma}}$.

We have the fluid equations for photons given by

$$\delta'_\gamma = -\frac{4}{3}v_\gamma - \frac{2}{3}h' \qquad (47)$$

and

$$v'_\gamma = \frac{1}{4}\delta_\gamma \qquad (48)$$

where the photon shear as well as higher moments of the Boltzmann hierarchy are vanishingly small due to the tight-coupling of photons to baryons and are ignored. Likewise the fluid equations for neutrinos are

$$\delta'_\nu = -\frac{4}{3}v_\nu - \frac{2}{3}h' \qquad (49)$$

$$v'_\nu = \frac{1}{4}\delta_\nu - \sigma_\nu \qquad (50)$$

and

$$\sigma'_\nu = \frac{4}{15}v_\nu + \frac{2}{15}(h' + 6\eta') \qquad (51)$$

where higher moments of the Boltzmann hierarchy are small because they are of higher powers in expansions about x and are ignored.

The scalar field evolves according to

$$\varphi' = \frac{C_\phi}{6K_F} xy + \phi_r x^{m-1}\varphi \tag{52}$$

and

$$y' = -\frac{(1+2C_\phi)}{x} y - \frac{3K_F}{x} e^{-4\bar{\phi}_i}\varphi + \frac{3K_F}{x}\phi_r x^{m-1} \times [h' + 6\varphi'] + \frac{3K_F}{x}\phi_r x^{m-1}[2 - e^{-4\bar{\phi}_i}]u - 6K_F\phi_r x^{m-3}[S_\gamma\delta_\gamma + S_\nu\delta_\nu - 3\varphi]. \tag{53}$$

The vector field obeys

$$u' = E - \varphi - \frac{1}{x}u \tag{54}$$

and

$$K_t\left[E' + \frac{1}{x}E\right] + e^{4\bar{\phi}_i}\kappa_d[h' + 2u] + 2(1 - K_F R_K e^{4\bar{\phi}_i})\eta' + 2(K_F e^{4\bar{\phi}_i} - 1)\left[\varphi' + \frac{1}{x}\varphi + \frac{2}{x^2}u\right] - \frac{3K_F}{x}(2e^{4\bar{\phi}_i} - 1)(\varphi + \phi_r x^{m-1}u) = 0.$$

Finally we need the two Einstein constraint equations

$$\frac{1}{x}K_F\left[h' + 6\varphi' + 2u + \frac{12}{x}\varphi\right] = \frac{3K_F}{x^2}[S_\gamma\delta_\gamma + S_\nu\delta_\nu] + 2e^{-4\bar{\phi}_i}[\eta - \varphi] + e^{-4\bar{\phi}_i}\left(K_t E + \frac{2}{x}u\right) - \gamma$$

and

$$R_K\eta' = \frac{2}{x^2}(S_\gamma v_\gamma + S_\nu v_\nu) + \varphi' - \frac{2}{x}\varphi + \frac{\kappa_d}{2K_F}[h' + 2u].$$

2. Adiabatic ansatz

The adiabatic mode is such that $\eta \to 1$ for $x \to 0$ while all other perturbations vanish in this limit (regularity assumption). The adiabatic mode ansatz $\eta = 1 + \eta_2 x^2$, $h = h_2 x^2$ solves the matter equations to give $\delta_\nu = \delta_\gamma = -\frac{2}{3}h_2 x^2$, $v_\gamma = -\frac{1}{18}h_2 x^3$, $v_\nu = -\frac{1}{18}h_2 x^3 - \frac{2}{45}\times(h_2 + 6\eta_2)x^3$ and $\sigma_\nu = \frac{2}{15}(h_2 + 6\eta_2)x^2$. We seek solutions to the scalar and vector field variables which are regular as $x \to 0$. All the scalar and vector field terms in the Einstein constraint equations are then subdominant to lowest order in x, and we can solve them to get $h_2 = \frac{e^{-4\bar{\phi}_i}}{2K_F}$ and $\eta_2 = \frac{10 - 15R_K - 4S_\nu}{6(15R_K + 4S_\nu)}h_2$.

Now consider the scalar field Eq. (52) where the first term clearly dominates at early times because of the regularity condition on φ and y. Let $y = y_0 x^p$ for some power $p \geq 0$ to leading order. We can then solve (52) to get

$$\varphi = \varphi_0 x^{2+p} = \frac{C_\phi}{6(2+p)K_F} y_0 x^{2+p} \tag{55}$$

and using the above solution along with the one already found for h and δ into (53) we get

$$(1 + p + 2C_\phi)y_0 x^p = 10K_F\phi_r h_2 x^m - 3K_F e^{-4\bar{\phi}_i}\varphi_0 x^{2+p} + 18(3+p)K_F\phi_r\varphi_0 x^{m+p} + 3K_F\phi_r[2 - e^{-4\bar{\phi}_i}]u x^{m-1}. \tag{56}$$

Since all terms above save the last one are regular as $x \to 0$, then the u-term must also be regular which means that $u = u_0 x^l$ with $l + m > 1$. Since $0 < m < 1$ and $p > 0$, we have that $2 + p > m$ and $m + p > m$ which means that as $x \to 0$, the first term would always dominate over the second and third term and the above equation is reduced to

$$(1 + p + 2C_\phi)y_0 x^p = 10K_F\phi_r h_2 x^m + 3K_F\phi_r[2 - e^{-4\bar{\phi}_i}]u_0 x^{l+m-1}. \tag{57}$$

Now, the vector field equations become

$$(l+1)u_0 x^{l-1} = E_0 x^q - \varphi_0 x^{2+p} \tag{58}$$

and

$$K_t(1+q)E_0 x^q + 2e^{4\bar{\phi}_i}\kappa_d h_2 x^2 + 4(1 - K_F R_K e^{4\bar{\phi}_i})\eta_2 x^2 + 4(K_F e^{4\bar{\phi}_i} - 1)u_0 x^{l-1} = 0 \tag{59}$$

where I have ignored the φ-terms because they are all $\sim x^{2+p}$, and so the h- and η-terms always dominate them as $x \to 0$. I have also kept only the leading u-term. Using (58) into (59) we get

$$\left[K_t(1+q) + \frac{4}{1+l}(K_F e_i^{4\bar{\phi}} - 1)\right]E_0 x^q + 2e^{4\bar{\phi}_i}\kappa_d h_2 x^2 + 4(1 - K_F R_K e^{4\bar{\phi}_i})\eta_2 x^2 = 0 \tag{60}$$

where once again the φ-term has been ignored as it is of higher order than the h- and η-terms. Therefore consistency requires that $q = 2$ from which we get that the φ-term is of higher order than the E-term in (58) which gives $l = 3$. Hence, the u-term in (57) is of higher order and we get $p = m$.

Reconstructing the full solution by using the above powers and matching coefficients then gives

$$h = \frac{e^{-4\bar{\phi}_i}}{2K_F}x^2 \qquad \eta = 1 + \frac{10 - 15R_K - 4S_\nu}{6(15R_K + 4S_\nu)}h$$

$$\frac{4}{3}\delta_b = \delta_\gamma = \delta_\nu = -\frac{2}{3}h \qquad \theta_\gamma = -\frac{1}{18}ht$$

$$\theta_\nu = -\frac{15R_K + 8 + 4S_\nu}{18(15R_K + 4S_\nu)}ht \qquad \sigma_\nu = \frac{4}{3(15R_K + 4S_\nu)}h$$

$$E = \frac{20(K_F R_K e^{4\bar{\phi}_i} - 1) + 2(15R_K + 4S_\nu)(1 - e^{4\bar{\phi}_i}K_F)}{3(15R_K + 4S_\nu)(3K_t + K_F e_i^{4\bar{\phi}} - 1)}h$$

$$\alpha = \frac{1}{4}Et \qquad \gamma = \frac{5m\phi_1 \Omega_{0r}^{m/2} H_k^m e^{-4\bar{\phi}_i}}{1 + m + 2C_\phi}k^2 x^m$$

$$\varphi = \frac{C_\phi}{6(2+m)K_F}\gamma t^2.$$

Note that for standard TeVeS with $\bar{\phi}_i = 0$ we get that $E = 0$ and $\alpha = 0$ to this order. In this very special case $E = O(3)$ and $\alpha = O(4)$ and depend on higher powers of h and η.

Figure 3 shows the exact numerical evolution (solid) compared with the approximate solution above (dashed) for the variables h (upper left panel), δ_γ (lower left panel), v_γ (upper right panel) and σ_ν (lower right panel) for $k = 10^{-10}$ Mpc^{-1}. Similarly Fig. 4 shows the exact numerical evolution (solid) compared with the approximate solution above (dashed) for the variables φ (upper left panel), y (lower left panel), u (upper right panel) and E (lower right panel). Notice the excellent agreement in the deep radiation era. The figures indicate that initial conditions should be set around $kt \sim 10^{-10}$. Thus for typical k values used in numerical simulations, initial conditions are conservatively set at $t \sim 10^{-5}$, i.e. much earlier than initial times in ΛCDM.

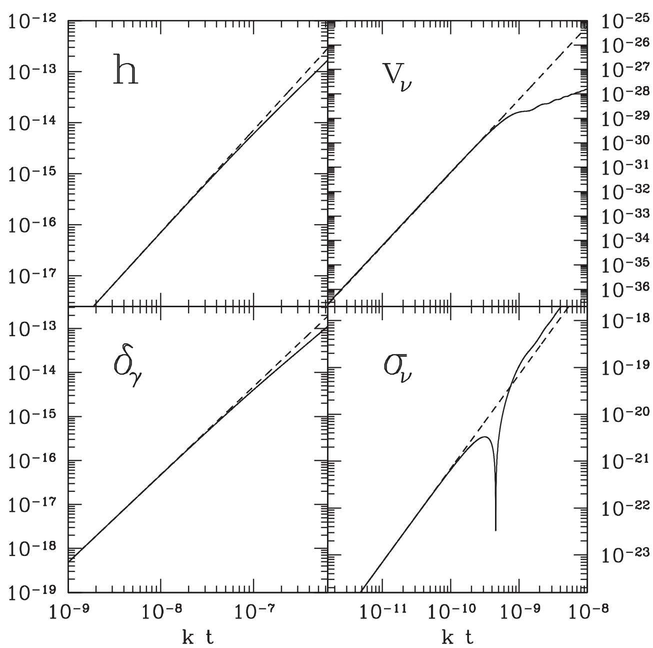

FIG. 3. Exact numerical evolution (solid) and the analytical approximation (dashed) for h (upper left panel), v_ν (upper right panel), δ_γ (lower left panel), and σ_ν (lower right panel), for the same model in Fig. 1. All approximations are excellent in the radiation era during tight-coupling and depart from the exact numerical solution once the Universe starts to enter the matter era and/or departs from tight-coupling.

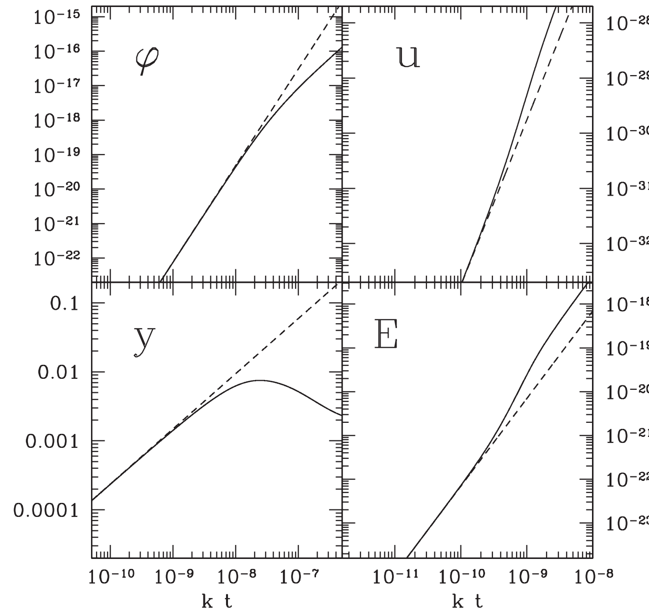

FIG. 4. Exact numerical evolution (solid) and the analytical approximation (dashed) for φ (upper left panel), y (lower left panel), u (upper right panel) and E (lower right panel), for the same model as in Fig. 1. All approximations are excellent in the radiation era during tight-coupling and depart from the exact numerical solution once the Universe starts to enter the matter era and/or departs from tight-coupling.

V. CONCLUSION

I have formulated the cosmological equations both at the background and linear perturbation level for a version of TeVeS theory with a generalized vector field action. Using an analytical solution to the background equations for a general family of scalar field functions, I constructed the primordial adiabatic perturbation. The most general type of regular primordial perturbation is studied elsewhere [38]. These equations can be used to study large scale structure for these theories, to check whether there are stable versions of TeVeS which can agree with observations.

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