

# Rescattering effects in $\bar{B}_{u,d,s} \rightarrow DP, \bar{D}P$ decays

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We study quasielastic rescattering effects in  $\bar{B}_{u,d,s} \rightarrow DP, \bar{D}P$  decays, where  $P$  is a light pseudoscalar. The updated measurements of  $\bar{B}_{u,d} \rightarrow DP$  decays are used to extract the effective Wilson coefficients  $a_1^{\text{eff}} \simeq 0.90$  and  $a_2^{\text{eff}} \simeq 0.23$ , three strong phases  $\delta \simeq 56^\circ$ ,  $\theta \simeq 18^\circ$ , and  $\sigma \simeq -104^\circ$ , and the mixing angle  $\tau \simeq 7^\circ$ . This information is used to predict rates of 19  $\bar{B}_s \rightarrow DP$  and  $\bar{B}_{u,d,s} \rightarrow \bar{D}P$  decay modes, including modes of interests in the  $\gamma/\phi_3$  program. Many decay rates are found to be enhanced. In particular, the  $\bar{B}_s \rightarrow D^0 K^0$  rate is predicted to be  $7 \times 10^{-4}$ , which could be measured soon. The rescattering effects on the corresponding  $\bar{B}_{u,d,s} \rightarrow \bar{D}P, DP$  amplitude ratios  $r_B$  and  $r_{B_s}$ , and the relative strong phases  $\delta_B$  and  $\delta_{B_s}$  are studied. Although the decay rates are enhanced in most cases,  $r_{B,B_s}$  values are similar to factorization expectation.

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## I. INTRODUCTION

Color-suppressed  $b \rightarrow c$  decays  $\bar{B}^0 \rightarrow D^{(*)0} \pi^0$  [1,2],  $D^0 \eta$ ,  $D^0 \omega$  [1],  $D^0 \eta'$  [3],  $D_s^+ K^-$  and  $D^0 \bar{K}^0$  [4,5] started to emerge in 2001 (for updated measurements, see [6,7]), with branching ratios that are significantly larger than earlier theoretical expectations based on naive factorization. When combined with color-allowed  $\bar{B} \rightarrow D^{(*)} \pi$  modes in an SU(2) framework, the enhancement in the  $D^{(*)0} \pi^0$  rate indicates the presence of nonvanishing strong phases, which has attracted much attention [8–18]. We proposed [11] a quasielastic final state rescattering (FSI) picture, where the enhancement of color-suppressed  $D^0 h^0$  modes is due to rescattering from the color-allowed  $D^+ \pi^-$  final state. This approach was also applied to study final state interaction in charmless  $B$  decays [19].

The quasielastic approach was recently extended to  $\bar{B} \rightarrow D\bar{K}, \bar{D}\bar{K}$  decays [20,21]. The color-allowed  $B^- \rightarrow D^0 K^-$  and color-suppressed  $B^- \rightarrow \bar{D}^0 K^-$  decays are of interest for the determination of the unitary phase angle  $\phi_3$  (or  $\gamma$ )  $\equiv \arg V_{ub}^*$ , where  $V$  is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. The Gronau-London-Wyler (GLW) [22], Atwood-Dunietz-Soni (ADS) [23] and “ $DK$  Dalitz plot” [24,25] methods probe, in varying ways, the interference of the two types of amplitudes in a common final state. The enhancement of color-suppressed  $DP$  modes (where  $P$  stands for a light pseudoscalar) could imply a larger  $\bar{D}K$  rate [15]. Since the strong interaction respects SU(3) and charge conjugation symmetries, the FSI in  $DP$  and  $\bar{D}P$  modes should be related. It is thus of interest to study  $DP$  and  $\bar{D}P$  modes together.

Besides making an update with recently available data, we note that data for  $\bar{B}_s$  is starting to emerge from the Tevtron [7] and from  $B$  factories [26], and we anticipate more to come in the near future, from CERN LHCb and other LHC experiments. Some  $B_s$  modes will be useful in

the extraction of  $\gamma/\phi_3$  [27–29]. It is thus timely to study  $\bar{B}_s$  decays. In this work, we extend the scope of the quasielastic rescattering approach to  $\bar{B}_s \rightarrow DP, \bar{D}P$  decays, as well as update our previous results using the latest  $\bar{B}_{u,d} \rightarrow DP$  data [6,7].

In Sec. II we briefly summarize and extend the quasielastic rescattering formula for  $\bar{B}_{u,d,s} \rightarrow DP, \bar{D}P$  decays. Numerical results are reported in Sec. III. The effective Wilson coefficients and rescattering parameters are obtained by using current  $\bar{B} \rightarrow DP$  data. By SU(3) symmetry and charge conjugation invariance of the strong interactions, we make predictions on  $\bar{B}_s \rightarrow DP$  and  $\bar{B}_{u,d,s} \rightarrow \bar{D}P$  rates. The conclusion is then offered in Sec. IV. An Appendix specifies the source amplitudes used to fit data with rescattering formalism.

## II. FINAL STATE RESCATTERING FRAMEWORK

We only briefly summarize, as well as extend, the decay amplitudes obtained in the quasielastic approach for  $\bar{B} \rightarrow DP, \bar{D}P$  decays, and refer the reader to Refs. [11,12] for more detail.

The quasielastic strong rescattering amplitudes can be put in four different classes, as given below. For  $\bar{B}$  decaying to  $DP$  with  $C = +1, S = 0, -1$  final states, we have

$$\begin{aligned}
 A_{B^- \rightarrow D^0 \pi^- (D^0 K^-)} &= (1 + ir'_0 + ir'_e) A_{B^- \rightarrow D^0 \pi^- (D^0 K^-)}^0, \\
 \begin{pmatrix} A_{\bar{B}^0 \rightarrow D^+ K^-} \\ A_{\bar{B}^0 \rightarrow D^0 \bar{K}^0} \end{pmatrix} &= \mathcal{S}_1^{1/2} \begin{pmatrix} A_{\bar{B}^0 \rightarrow D^+ K^-}^0 \\ A_{\bar{B}^0 \rightarrow D^0 \bar{K}^0}^0 \end{pmatrix}, \\
 \begin{pmatrix} A_{\bar{B}^0 \rightarrow D^+ \pi^-} \\ A_{\bar{B}^0 \rightarrow D^0 \pi^0} \\ A_{\bar{B}^0 \rightarrow D_s^+ K^-} \\ A_{\bar{B}^0 \rightarrow D^0 \eta_8} \\ A_{\bar{B}^0 \rightarrow D^0 \eta_1} \end{pmatrix} &= \mathcal{S}_2^{1/2} \begin{pmatrix} A_{\bar{B}^0 \rightarrow D^+ \pi^-}^0 \\ A_{\bar{B}^0 \rightarrow D^0 \pi^0}^0 \\ A_{\bar{B}^0 \rightarrow D_s^+ K^-}^0 \\ A_{\bar{B}^0 \rightarrow D^0 \eta_8}^0 \\ A_{\bar{B}^0 \rightarrow D^0 \eta_1}^0 \end{pmatrix}. \tag{1}
 \end{aligned}$$

Extending to  $\bar{B}_s$  to  $DP$  decays with  $C = +1$ ,  $S = 0, +1$  final states, one has

$$\begin{aligned} \begin{pmatrix} A_{\bar{B}_s^0 \rightarrow D_s^+ \pi^-} \\ A_{\bar{B}_s^0 \rightarrow D^0 K^0} \end{pmatrix} &= \mathcal{S}_1^{1/2} \begin{pmatrix} A_{\bar{B}_s^0 \rightarrow D_s^+ \pi^-}^0 \\ A_{\bar{B}_s^0 \rightarrow D^0 K^0}^0 \end{pmatrix}, \\ \begin{pmatrix} A_{\bar{B}_s^0 \rightarrow D^+ \pi^-} \\ A_{\bar{B}_s^0 \rightarrow D^0 \pi^0} \\ A_{\bar{B}_s^0 \rightarrow D_s^+ K^-} \\ A_{\bar{B}_s^0 \rightarrow D^0 \eta_8} \\ A_{\bar{B}_s^0 \rightarrow D^0 \eta_1} \end{pmatrix} &= \mathcal{S}_2^{1/2} \begin{pmatrix} A_{\bar{B}_s^0 \rightarrow D^+ \pi^-}^0 \\ A_{\bar{B}_s^0 \rightarrow D^0 \pi^0}^0 \\ A_{\bar{B}_s^0 \rightarrow D_s^+ K^-}^0 \\ A_{\bar{B}_s^0 \rightarrow D^0 \eta_8}^0 \\ A_{\bar{B}_s^0 \rightarrow D^0 \eta_1}^0 \end{pmatrix}. \end{aligned} \quad (2)$$

For  $\bar{B}_{u,d} \rightarrow \bar{D}P$  decays with  $C = -1$ ,  $S = \pm 1$  final states,

$$\begin{aligned} \begin{pmatrix} A_{\bar{B}^0 \rightarrow D^- K^+} \\ A_{\bar{B}^0 \rightarrow \bar{D}^0 \bar{K}^0} \end{pmatrix} &= \mathcal{S}_1^{1/2} \begin{pmatrix} A_{\bar{B}^0 \rightarrow D^- K^+}^0 \\ A_{\bar{B}^0 \rightarrow \bar{D}^0 \bar{K}^0}^0 \end{pmatrix}, \\ \begin{pmatrix} A_{B^- \rightarrow \bar{D}^0 K^-} \\ A_{B^- \rightarrow D^- \bar{K}^0} \\ A_{B^- \rightarrow D_s^- \pi^0} \\ A_{B^- \rightarrow D_s^- \eta_8} \\ A_{B^- \rightarrow D_s^- \eta_1} \end{pmatrix} &= \mathcal{S}_3^{1/2} \begin{pmatrix} A_{B^- \rightarrow \bar{D}^0 K^-}^0 \\ A_{B^- \rightarrow D^- \bar{K}^0}^0 \\ A_{B^- \rightarrow D_s^- \pi^0}^0 \\ A_{B^- \rightarrow D_s^- \eta_8}^0 \\ A_{B^- \rightarrow D_s^- \eta_1}^0 \end{pmatrix}. \end{aligned} \quad (3)$$

And for  $\bar{B}_s^0 \rightarrow \bar{D}P$  decays with  $C = -1$ ,  $S = 0$  final states,

$$\begin{pmatrix} A_{\bar{B}_s^0 \rightarrow D^- \pi^+} \\ A_{\bar{B}_s^0 \rightarrow \bar{D}^0 \pi^0} \\ A_{\bar{B}_s^0 \rightarrow \bar{D}_s^- K^+} \\ A_{\bar{B}_s^0 \rightarrow \bar{D}^0 \eta_8} \\ A_{\bar{B}_s^0 \rightarrow \bar{D}^0 \eta_1} \end{pmatrix} = \mathcal{S}_2^{1/2} \begin{pmatrix} A_{\bar{B}_s^0 \rightarrow D^- \pi^+}^0 \\ A_{\bar{B}_s^0 \rightarrow \bar{D}^0 \pi^0}^0 \\ A_{\bar{B}_s^0 \rightarrow \bar{D}_s^- K^+}^0 \\ A_{\bar{B}_s^0 \rightarrow \bar{D}^0 \eta_8}^0 \\ A_{\bar{B}_s^0 \rightarrow \bar{D}^0 \eta_1}^0 \end{pmatrix}. \quad (4)$$

In these expressions, the square root of the rescattering  $S$  matrix is denoted as  $\mathcal{S}_i^{1/2} = (1 + i\mathcal{T}'_i)^{1/2} = 1 + i\mathcal{T}'_i$ , with

$$\begin{aligned} \mathcal{T}_1 &= \begin{pmatrix} r_0 & r_e \\ r_e & r_0 \end{pmatrix}, \\ \mathcal{T}_2 &= \begin{pmatrix} r_0 + r_a & \frac{r_a - r_e}{\sqrt{2}} & r_a & \frac{r_a + r_e}{\sqrt{6}} & \frac{\bar{r}_a + \bar{r}_e}{\sqrt{3}} \\ \frac{r_a - r_e}{\sqrt{2}} & r_0 + \frac{r_a + r_e}{2} & \frac{r_a}{\sqrt{2}} & \frac{r_a + r_e}{2\sqrt{3}} & \frac{\bar{r}_a + \bar{r}_e}{\sqrt{6}} \\ r_a & \frac{r_a}{\sqrt{2}} & r_0 + r_a & \frac{r_a - 2r_e}{\sqrt{6}} & \frac{\bar{r}_a + \bar{r}_e}{\sqrt{3}} \\ \frac{r_a + r_e}{\sqrt{6}} & \frac{r_a + r_e}{2\sqrt{3}} & \frac{r_a - 2r_e}{\sqrt{6}} & r_0 + \frac{r_a + r_e}{6} & \frac{\bar{r}_a + \bar{r}_e}{3\sqrt{2}} \\ \frac{\bar{r}_a + \bar{r}_e}{\sqrt{3}} & \frac{\bar{r}_a + \bar{r}_e}{\sqrt{6}} & \frac{\bar{r}_a + \bar{r}_e}{\sqrt{3}} & \frac{\bar{r}_a + \bar{r}_e}{3\sqrt{2}} & \bar{r}_0 + \frac{\bar{r}_a + \bar{r}_e}{3} \end{pmatrix}, \\ \mathcal{T}_3 &= \begin{pmatrix} r_0 + r_a & r_a & \frac{r_e}{\sqrt{2}} & \frac{r_e - 2r_a}{\sqrt{6}} & \frac{\bar{r}_e + \bar{r}_a}{\sqrt{3}} \\ r_a & r_0 + r_a & -\frac{r_e}{\sqrt{2}} & \frac{r_e - 2r_a}{\sqrt{6}} & \frac{\bar{r}_a + \bar{r}_e}{\sqrt{3}} \\ \frac{r_e}{\sqrt{2}} & -\frac{r_e}{\sqrt{2}} & r_0 & 0 & 0 \\ \frac{r_e - 2r_a}{\sqrt{6}} & \frac{r_e - 2r_a}{\sqrt{6}} & 0 & r_0 + \frac{2}{3}(r_a + r_e) & -\frac{\sqrt{2}}{3}(\bar{r}_a + \bar{r}_e) \\ \frac{\bar{r}_e + \bar{r}_a}{\sqrt{3}} & \frac{\bar{r}_a + \bar{r}_e}{\sqrt{3}} & 0 & -\frac{\sqrt{2}}{3}(\bar{r}_a + \bar{r}_e) & \bar{r}_0 + \frac{\bar{r}_a + \bar{r}_e}{3} \end{pmatrix}, \end{aligned} \quad (5)$$

where  $r_e$ ,  $r_a$  and  $r_0$  are charge exchange, annihilation, and singlet exchange rescattering parameters [11], respectively, while  $\bar{r}_i$  and  $\tilde{r}_i$  are those for  $D\Pi(\mathbf{8}) \leftrightarrow D^0\eta_1$  and

$D^0\eta_1 \leftrightarrow D^0\eta_1$  scattering, respectively [20]. SU(3) symmetry requires that  $\mathcal{T}'_i$  has the same structure as  $\mathcal{T}_i$ . Hence the  $\mathcal{T}'_i$  is basically  $\mathcal{T}_i$ , but with  $r_j$ ,  $\bar{r}_j$  and  $\tilde{r}_j$  replaced by  $r'_j$ ,  $\bar{r}'_j$  and  $\tilde{r}'_j$ , respectively. We note that some of the above formulas were already reported in [11,20], while all formulas for the second and fourth cases, and some for the third case, are new. We have used charge conjugation invariance and SU(3) symmetry of the strong interactions; hence the  $r_i^{(l)}$ ,  $\bar{r}_i^{(l)}$  and  $\tilde{r}_i^{(l)}$  coefficients in  $\mathcal{T}_i^{(l)}$  of  $\bar{B}_{u,d,s} \rightarrow \bar{D}P$  rescattering amplitudes are identical to those in  $\mathcal{T}_i^{(l)}$  of  $\bar{B}_{u,d,s} \rightarrow DP$  rescattering amplitudes.

Using SU(3) symmetry and  $\mathcal{S}^\dagger \mathcal{S} = 1$ , the rescattering parameters are given by [20]

$$\begin{aligned} (1 + ir_0) &= \frac{1}{2}(1 + e^{2i\delta}), & ir_e &= \frac{1}{2}(1 - e^{2i\delta}), \\ ir_a &= \frac{1}{8}(3\mathcal{U}_{11} - 2e^{2i\delta} - 1), & i(\bar{r}_a + \bar{r}_e) &= \frac{3}{2\sqrt{2}}\mathcal{U}_{12}, \\ i\left(\tilde{r}_0 + \frac{\tilde{r}_a + \tilde{r}_e}{3}\right) &= \mathcal{U}_{22} - 1, \end{aligned} \quad (6)$$

where

$$\begin{aligned} \mathcal{U} &= \mathcal{U}^T \\ &= \begin{pmatrix} \cos\tau & \sin\tau \\ -\sin\tau & \cos\tau \end{pmatrix} \begin{pmatrix} e^{2i\theta} & 0 \\ 0 & e^{2i\sigma} \end{pmatrix} \begin{pmatrix} \cos\tau & -\sin\tau \\ \sin\tau & \cos\tau \end{pmatrix}, \end{aligned} \quad (7)$$

and we have set the overall phase factor  $(1 + ir_0 + ir_e)$  in  $\mathcal{S}$  to unity. This phase convention is equivalent to choosing the  $A_{\bar{B}^0 \rightarrow D^0 \pi^-}$  amplitude to be real. The  $r'_i$ ,  $\bar{r}'_i$  and  $\tilde{r}'_i$  in  $\mathcal{S}^{1/2}$  can be obtained by using the above formulas with phases  $(\delta, \theta, \sigma)$  reduced by half. We need three phases and one mixing angle to specify FSI effects in  $DP$  and  $\bar{D}P$  rescattering. The interpretation of these phases and mixing angle in term of SU(3) decomposition can be found in [20].

The FSI rescattering matrices  $\mathcal{S}_{1,2,3}$  may be subjected to SU(3) breaking effects. The FSI phases and mixing angle in  $\mathcal{S}_{1,3}$  may be different from those in  $\mathcal{S}_2$ . In order to incorporate such effects, the FSI phase  $\delta$  in  $\mathcal{S}_1$  is multiplied by a SU(3) violating factor  $\kappa$  and the three FSI phases and one mixing angle in  $\mathcal{S}_3$  are multiplied by four SU(3) violating factors  $\kappa_{1,2,3,4}$ , respectively. In the SU(3) limit, the values of  $\kappa$  and  $\kappa_{1,2,3,4}$  should reduce to unity.

To use the FSI formulas, we need to specify  $A^0$ . We use naive factorization amplitudes  $A^f$  for  $A^0$  to avoid double counting of FSI effects [11,20]. And the explicit forms of  $A^0$  are given in the Appendix. A certain amount of SU(3) breaking effects which have to do with meson formation are included in the factorization amplitudes via decay constants and form factors.

Two real and universal parameters  $a_{1,2}^{\text{eff}}$  are used in the factorization amplitudes. The color-allowed tree amplitude has been demonstrated for the  $D\pi$  system by Ref. [13]. The use of a real and universal  $a_{1,2}^{\text{eff}}$  in factorization amplitudes is therefore reasonable (see also [17]). On the other hand,

since color-suppressed modes are more sensitive to rescattering effects, it is possible that rescattering from excited intermediate states, such as  $\bar{B} \rightarrow D^*M \rightarrow DP$ , which are not included in our FSI formalism, may lead to a complex  $a_2^{\text{eff}}$ . For example, rescattering from  $\bar{B} \rightarrow D^*P, DV, D^*V$  modes may contribute to  $DP$  if the scope of the quasielastic FSI is extended from SU(3) to ‘‘SU(6)’’ (flavor  $\times$  spin). In fact, the final states  $D^*P$  and  $DV$  are in  $p$ -wave configuration, their parities are different from that of  $DP$  and, consequently, these rescattering channels are not allowed. On the other hand, the  $D^*V \rightarrow DP$  channel may still have remnant effects. However, it has been shown from a statistical approach [30], where inelastic FSI amplitudes with random phases tend to cancel each other and lead to small FSI phases. Hence, we do not expect a large  $a_2^{\text{eff}}$  phase as a result of FSI contributions from all inelastic states.

### III. RESULTS

In our numerical study, masses and lifetimes are taken from the Particle Data Group [7], and  $B$  to charm meson decay branching ratios are taken from [6,7]. We fix  $V_{ud} = 0.97419$ ,  $V_{us} = 0.22568$ ,  $V_{cb} = 0.04166$ ,  $V_{cs} =$

$0.997334$ ,  $|V_{ub}| = 3.624 \times 10^{-3}$  [31], and use the decay constants  $f_\pi = 131$  MeV,  $f_K = 156$  MeV [7] and  $f_{D(s)} = 200$  (230) MeV.

We have seven parameters to describe the processes with rescattering from factorization amplitudes: the two effective Wilson coefficients  $a_1^{\text{eff}}$  and  $a_2^{\text{eff}}$ , the three rescattering phases  $\delta$ ,  $\theta$  and  $\sigma$ , one mixing angle  $\tau$  in  $S^{1/2}$  and one SU(3) violating factor  $\kappa$ . These parameters are fitted with rates of nine  $\bar{B}$  decay to  $C = 1, S = 0, -1$  modes, namely,  $\bar{B} \rightarrow D^+\pi^-, D^0\pi^-, D^0\pi^0, D^0\eta, D^0\eta', D_s^+K^-, D^0K^-, D^+K^-$  and  $D^0\bar{K}^0$  decays, given in Table I. The fitted FSI parameters are listed in Table II. We then use the extracted parameters to predict 19  $\bar{B}_s \rightarrow DP$  (Table III) and  $\bar{B}_{u,d,s} \rightarrow \bar{D}P$  (Table IV) decays. Predictions on the ratios of  $\bar{B} \rightarrow \bar{D}P$  and  $\bar{B} \rightarrow DP$  amplitudes are also given. Note that the SU(3) violating parameters  $\kappa_{1,2,3,4}$  in  $S_3$  are not being fitted since the modes involving  $S_3$  are not being measured yet. We use  $\kappa_{1,2,3,4} = 1 \pm 0.25$  to estimate the SU(3) breaking effects in the numerical study.

To obtain the errors for FSI results given in Tables I, II, III, IV, and VI, we scan the parameter space by requiring  $\chi^2 \leq \chi_{\text{min}}^2 + 1$ . The sources of errors are from: the factorization amplitudes, where 10% uncertainties on form

TABLE I. Branching ratios of various  $\bar{B} \rightarrow DP$  and  $D\bar{K}$  modes in  $10^{-4}$  units. The second column is the experimental data [6,7], which is taken as input. The naive factorization model results are given in third column. Fitting the experimental data shown in the first column with quasielastic FSI (fit parameters as given in Table II), we obtain the FSI fit results given in the last column. The factorization results are recovered by setting FSI phases in Table II to zero.

Mode	$\mathcal{B}^{\text{exp}}$ ( $10^{-4}$ )	$\mathcal{B}^{\text{fac}}$ ( $10^{-4}$ )	$\mathcal{B}^{\text{FSI}}$ ( $10^{-4}$ )
$B^- \rightarrow D^0\pi^-$	$48.4 \pm 1.5$	$48.4^{+15.5}_{-10.7}$	$48.4 \pm 1.0$
$\bar{B}^0 \rightarrow D^+\pi^-$	$26.8 \pm 1.3$	$31.9^{+8.7}_{-6.2}$	$26.8 \pm 0.9$
$\bar{B}^0 \rightarrow D^0\pi^0$	$2.61 \pm 0.24$	$0.57^{+0.34}_{-0.20}$	$2.61^{+0.20}_{-0.17}$
$\bar{B}^0 \rightarrow D_s^+K^-$	$0.28 \pm 0.05$	0	$0.28^{+0.05}_{-0.04}$
$\bar{B}^0 \rightarrow D^0\eta$	$2.02 \pm 0.35$	$0.33^{+0.20}_{-0.12}$	$2.02^{+0.24}_{-0.27}$
$\bar{B}^0 \rightarrow D^0\eta'$	$1.25 \pm 0.23$	$0.20^{+0.12}_{-0.07}$	$1.25^{+0.20}_{-0.18}$
$B^- \rightarrow D^0K^-$	$4.02 \pm 0.21$	$4.02^{+1.32}_{-0.92}$	$4.02^{+0.15}_{-0.16}$
$\bar{B}^0 \rightarrow D^+K^-$	$2.04 \pm 0.57$	$2.43^{+0.66}_{-0.47}$	$2.05^{+0.08}_{-0.07}$
$\bar{B}^0 \rightarrow D^0\bar{K}^0$	$0.52 \pm 0.07$	$0.14^{+0.08}_{-0.05}$	$0.52 \pm 0.05$

TABLE II. Fit parameters in the SU(3) FSI picture, where results are from using  $\bar{B} \rightarrow D^0\pi^-, D^+\pi^-, D^0\pi^0, D^0\eta, D^0\eta', D_s^+K^-, D^0K^-, D^+K^-$  and  $D^0\bar{K}^0$  decay rates (Table I) as fit input. There is a twofold ambiguity (the overall sign of phases) in the solutions. The SU(3) phases and mixing angle are reexpressed in terms of the rescattering parameters  $r'_i, \bar{r}'_i$ , and  $\bar{r}'_i$  of  $S_2$ . Note that  $\kappa_{1,2,3,4}$  are not being fitted.

Parameter	Result	Parameter	Result
$\chi_{\text{min}}^2/\text{d.o.f.}$	0.00/2		
$a_1^{\text{eff}}$	$0.90^{+0.11}_{-0.09}$	$a_2^{\text{eff}}$	$0.23^{+0.06}_{-0.05}$
$\delta$	$\pm(55.8^{+1.9}_{-2.0})^\circ$	$\theta$	$\pm(17.9^{+1.4}_{-1.3})^\circ$
$\sigma$	$\mp(104.0^{+22.0}_{-32.4})^\circ$	$\tau$	$(7.0^{+3.7}_{-2.5})^\circ$
$\kappa$	$0.85^{+0.08}_{-0.09}$	$(\kappa_{1,2,3,4})$	$(1 \pm 0.25)$
$1 + ir'_0$	$(0.78 \pm 0.01) \pm (0.41 \pm 0.01)i$	$ir'_e$	$(0.22 \pm 0.01) \mp (0.41 \pm 0.01)i$
$ir'_a$	$(0.08 \pm 0.01) \mp (0.10 \pm 0.01)i$	$i(\bar{r}'_a + \bar{r}'_e)$	$(-0.15^{+0.03}_{-0.02}) \mp (0.16^{+0.08}_{-0.09})i$
$1 + i\bar{r}'_0 + i\frac{\bar{r}'_a + \bar{r}'_e}{3}$	$(-0.22^{+0.39}_{-0.49}) \mp (0.95^{+0.27}_{-0.03})i$		

TABLE III. The predictions on branching ratios of various  $\bar{B}_s \rightarrow DP$  modes in  $10^{-4}$  and  $10^{-5}$  units, respectively. The errors for the FSI results are from  $\bar{B} \rightarrow DP$  data only.

Mode	$\mathcal{B}^{\text{exp}}$	$\mathcal{B}^{\text{fac}}$ ( $10^{-4}$ )	$\mathcal{B}^{\text{FSI}}$ ( $10^{-4}$ )
$\bar{B}_s^0 \rightarrow D_s^+ \pi^-$	$30 \pm 7$	$30.5^{+8.3}_{-5.9}$	$25.8^{+1.0}_{-0.9}$
$\bar{B}_s^0 \rightarrow D^0 K^0$	...	$2.2^{+1.3}_{-0.8}$	$6.9^{+0.7}_{-0.9}$
Mode	$\mathcal{B}^{\text{exp}}$ ( $10^{-5}$ )	$\mathcal{B}^{\text{fac}}$ ( $10^{-5}$ )	$\mathcal{B}^{\text{FSI}}$ ( $10^{-5}$ )
$\bar{B}_s^0 \rightarrow D^+ \pi^-$	...	0	$0.16 \pm 0.03$
$\bar{B}_s^0 \rightarrow D^0 \pi^0$	...	0	$0.08 \pm 0.02$
$\bar{B}_s^0 \rightarrow D_s^+ K^-$	...	$23.2^{+6.3}_{-4.5}$	$19.4^{+0.7}_{-0.7}$
$\bar{B}_s^0 \rightarrow D^0 \eta$	...	$0.6^{+0.4}_{-0.2}$	$3.3 \pm 0.6$
$\bar{B}_s^0 \rightarrow D^0 \eta'$	...	$0.9^{+0.5}_{-0.3}$	$1.7^{+0.5}_{-0.7}$

factor values are used, SU(3) violating effects ( $\kappa$ ,  $\kappa_{1,2,3,4}$ ) and the experimental uncertainties of data.

The values of fitted parameters given in Table II are similar to those in our previous analysis [20].<sup>1</sup> There is a twofold ambiguity (the overall sign of the phases) in the solutions. We obtain  $\chi_{\text{min}}^2/\text{d.o.f.} = 0.00$  indicating a perfect fit to these modes. The effective Wilson coefficients  $a_{1,2}^{\text{eff}}$  are close to expectation [35,36]. From  $|r'_e| > |r'_a|$  we infer that exchange rescattering is dominant over annihilation rescattering.

We show in the fourth column of Table I the fit output for the nine fitted  $\bar{B} \rightarrow DP$  and  $D\bar{K}$  modes. These fitted branching ratios (in units of  $10^{-4}$ ) should be compared with data and naive factorization results given in the second and third columns. The FSI results reproduce the data quite well, as they should. The errors for the FSI results are from data only. The factorization results can be recovered by using the parameters of Table II but with FSI phases set to zero. Note that unitarity is implied automatically; i.e. the sum of rates within coupled modes is unchanged by FSI.

Our main interest here is the color-suppressed  $B_s$  decays. The predicted branching ratios of various  $\bar{B}_s \rightarrow DP$  modes with  $C = +1$ ,  $S = 0, +1$  final states are shown in Table III, where the second column gives naive factorization results and the third column gives the FSI results. Again, the factorization results are recovered by using the same parameters of Table II but with FSI phases set to zero, and the errors for the FSI results are from  $\bar{B} \rightarrow DP$  data only. Analogous to  $\bar{B}^0 \rightarrow D^0 \pi^0$  enhancement being fed from  $\bar{B}^0 \rightarrow D^+ \pi^-$  rescattering, it is interesting to note that  $\bar{B}_s^0 \rightarrow D_s^+ \pi^-$  with FSI rescattering to  $D^0 K^0$  brings  $\bar{B}_s^0 \rightarrow D^0 K^0$  rate to the  $10^{-3}$  level, which can be measured soon. This is helped by the absence of annihilation rescattering. The  $\bar{B}_s \rightarrow D^0 \eta, D^0 \eta'$  modes are the direct analogs of  $\bar{B}^0 \rightarrow D^0 \pi^0$ . One can see that their rates are brought up to levels similar to  $\bar{B}^0 \rightarrow D^0 \pi^0$ . Rescattering slightly reduces the  $\bar{B}_s \rightarrow D_s^+ K^-$  and  $B_s \rightarrow D_s^+ \pi^-$  rates. The  $D_s^+ K$  mode will be used to extract  $\gamma/\phi_3$  at LHCb [28], while the  $D^0 \eta$  and  $D^0 \eta'$  modes could also be useful [29].

<sup>1</sup>We found and corrected a numerical error in our previous analysis, resulting in the value of  $\sigma$  taking opposite sign.

$\bar{B}_{u,d,s} \rightarrow \bar{D}P$  decays are  $V_{ub}$  suppressed, and mostly not measured yet, except  $D_s^- \pi^+$ . The quasielastic FSI formalism allows us to make predictions even for such modes that are color-suppressed. Predictions for  $\bar{B}_{u,d,s} \rightarrow \bar{D}P$  decays are shown in Table IV, where again, experimental results and limits [7,32] are shown in the second column, and the third and fourth columns are naive factorization and FSI results, respectively. The same comments on FSI parameters apply. Agreement of the only observed  $\bar{B}^0 \rightarrow D_s^- \pi^+$  mode with theoretical prediction is improved by including FSI effects.  $\bar{B}_s \rightarrow D_s^- K^+$  is slightly reduced, but overall, the redistribution of decay rates by rescattering is not very significant.

Combining  $\bar{B}_s \rightarrow D_s^+ K^-$  and  $\bar{B}_s \rightarrow D_s^- K^+$ , we can compare our predicted ratio

$$R \equiv \frac{\mathcal{B}(\bar{B}_s \rightarrow D_s^+ K^-)}{\mathcal{B}(\bar{B}_s \rightarrow D_s^- K^+)} = 0.086 \pm 0.003, \quad (8)$$

with the recent experimental result of  $R = 0.107 \pm 0.019 \pm 0.008$  [37] from the Collider Detector at Fermilab (CDF). The agreement is reasonable. In fact, the larger rescattering of  $\bar{B}_s \rightarrow D_s^+ \pi^-$  to  $\bar{B}_s \rightarrow D^0 K^0$  has helped enhance the ratio from the lower factorization value.

In Table V, we compare our predictions for various  $\bar{B}_s \rightarrow DP$  rates with results obtained in other approaches [38,39] that differ in the application of SU(3) symmetry. Most of our results agree with others. For modes with  $\eta^{(0)}$ , our results are closer to those in [38] obtained using earlier data. In both approaches, U(3) symmetry is not imposed and  $D\eta_1$  is treated as an independent component. Although predictions on  $\bar{B}_s \rightarrow DP$  rates are similar in all three works, it should be noted that there is a major difference between ours and the other two approaches. In this work, the information obtained in  $\bar{B}_{u,d} \rightarrow DP$  rescattering from data is used to predict not only  $\bar{B}_s \rightarrow DP$  decays [via SU(3) symmetry], but also  $\bar{B}_{u,d,s} \rightarrow \bar{D}P$  decays [through charge conjugation invariance of the  $S$  matrix]. SU(3) symmetry itself is not sufficient to relate  $\bar{B}_{u,d,s} \rightarrow DP$  and  $\bar{B}_{u,d,s} \rightarrow \bar{D}P$  amplitudes. Hence, in the two other works, which employed solely SU(3) symmetry to decay amplitudes,

TABLE IV. Predictions for  $\bar{B}_{u,d,s} \rightarrow \bar{D}P$  rates. Experimental results and limits [7,32] are shown in the second column, and naive factorization and FSI results are given in the third and fourth columns.

Mode	$\mathcal{B}^{\text{exp}}$ ( $10^{-5}$ )	$\mathcal{B}^{\text{fac}}$ ( $10^{-5}$ )	$\mathcal{B}^{\text{FSI}}$ ( $10^{-5}$ )
$B^- \rightarrow \bar{D}^0 K^-$	...	$0.2 \pm 0.1$	$0.3^{+0.1}_{-0.3}$
$B^- \rightarrow D^- \bar{K}^0$	$<0.5$	0	$0.04^{+0.07}_{-0.03}$
$B^- \rightarrow D_s^- \pi^0$	$<20$	$0.9 \pm 0.2$	$0.7 \pm 0.1$
$B^- \rightarrow D_s^- \eta$	$<50$	$0.5 \pm 0.1$	$0.3 \pm 0.2$
$B^- \rightarrow D_s^- \eta'$	...	$0.3 \pm 0.1$	$0.6 \pm 0.2$
$\bar{B}^0 \rightarrow D_s^- \pi^+$	$1.4 \pm 0.3$	$1.7 \pm 0.1$	$1.4^{+0.0}_{-0.1}$
$\bar{B}^0 \rightarrow \bar{D}^0 K^0$	...	$0.2 \pm 0.1$	$0.5 \pm 0.0$
$\bar{B}_s^0 \rightarrow D^- \pi^+$	...	0	$0.02 \pm 0.01$
$\bar{B}_s^0 \rightarrow \bar{D}^0 \pi^0$	...	0	$0.01 \pm 0.00$
$\bar{B}_s^0 \rightarrow D_s^- K^+$	...	$3.4^{+0.9}_{-0.7}$	$2.9 \pm 0.1$
$\bar{B}_s^0 \rightarrow \bar{D}^0 \eta$	...	$0.09^{+0.05}_{-0.03}$	$0.5 \pm 0.1$
$\bar{B}_s^0 \rightarrow \bar{D}^0 \eta'$	...	$0.12^{+0.07}_{-0.04}$	$0.3 \pm 0.1$

 TABLE V. Comparison of predictions for branching ratios of various  $\bar{B}_s \rightarrow DP$  modes to other approaches.

$\mathcal{B}$ ( $10^{-4}$ )	This work	Colangelo and Ferrandes (CF) [38]	Chiang and Senaha (CS) [39]
$\bar{B}_s^0 \rightarrow D_s^+ \pi^-$	$25.8 \pm 1.0$	$29 \pm 6$	$22 \pm 1$
$\bar{B}_s^0 \rightarrow D^0 K^0$	$6.9^{+0.7}_{-0.9}$	$8.1 \pm 1.8$	$5.3 \pm 0.3$
$\mathcal{B}$ ( $10^{-5}$ )	This work	CF [38]	CS [39]
$\bar{B}_s^0 \rightarrow D^+ \pi^-$	$0.16 \pm 0.03$	$0.20 \pm 0.06$	$0.14 \pm 0.03$
$\bar{B}_s^0 \rightarrow D^0 \pi^0$	$0.08 \pm 0.02$	$0.10 \pm 0.03$	$0.07 \pm 0.01$
$\bar{B}_s^0 \rightarrow D_s^+ K^-$	$19.4 \pm 0.7$	$18 \pm 3$	$20 \pm 1$
$\bar{B}_s^0 \rightarrow D^0 \eta$	$3.3 \pm 0.6$	$2.1 \pm 1.2$	$1.4 \pm 0.1$
$\bar{B}_s^0 \rightarrow D^0 \eta'$	$1.7^{+0.5}_{-0.7}$	$0.98 \pm 0.76$	$2.9 \pm 0.2$

no prediction on  $\bar{B}_{u,d,s} \rightarrow \bar{D}P$  decays were given by analyzing  $\bar{B}_{u,d} \rightarrow DP$  data.

In Table VI,  $r_{B(s)}$  and  $\delta_{B(s)}$  for various modes are predicted and compared with data, where the amplitude ratio  $r_{B(s)}$  and the strong phase difference  $\delta_{B(s)}$  are defined as

$$r_{B(s)}(DP) = \left| \frac{A(\bar{B}_{(s)} \rightarrow \bar{D}P)}{A(\bar{B}_{(s)} \rightarrow DP)} \right|, \quad (9)$$

$$\delta_{B(s)}(DP) = \arg \left[ \frac{e^{i\phi_3} A(\bar{B}_{(s)} \rightarrow \bar{D}P)}{A(\bar{B}_{(s)} \rightarrow DP)} \right].$$

 TABLE VI. Naive factorization and FSI results on  $r_{B(s)}$  and  $\delta_{B(s)}$  with  $|V_{ub}| = 3.67 \times 10^{-3}$ , and compared to the experimental results [33,34]. The errors for the FSI results are from  $DP$  data only.

	Expt	Fac	FSI
$r_B(D^0 K^-)$	$0.16 \pm 0.05 \pm 0.01 \pm 0.05$ (Belle) $<0.14$ ( $1\sigma$ ) (BABAR) $0.071 \pm 0.024$ (UTfit)	$0.07 \pm 0.02$	$0.09 \pm 0.02$
$\delta_B(D^0 K^-)$	$(146^{+19}_{-20} \pm 3 \pm 23)^\circ$ (Belle) $(118 \pm 63 \pm 19 \pm 36)^\circ$ (BABAR)	$180^\circ$	$180^\circ \mp (27.2^{+12.0}_{-10.0})^\circ$
$r_B(D^0 K^0)$	...	$0.38 \pm 0.00$	$0.29 \pm 0.01$
$\delta_B(D^0 K^0)$	...	$180^\circ$	$180^\circ \pm (9.1^{+0.6}_{-0.8})^\circ$
$r_{B_s}(D_s^+ K^-)$	...	$0.38 \pm 0.00$	$0.38 \pm 0.00$
$\delta_{B_s}(D_s^+ K^-)$	...	$180^\circ$	$180^\circ \pm (0.1 \pm 0.0)^\circ$
$r_{B_s}(D^0 \eta)$	...	$0.38 \pm 0.00$	$0.38 \pm 0.00$
$\delta_{B_s}(D^0 \eta)$	...	$180^\circ$	$180^\circ \mp (0.6 \pm 0.0)^\circ$
$r_{B_s}(D^0 \eta')$	...	$0.38 \pm 0.00$	$0.38 \pm 0.00$
$\delta_{B_s}(D^0 \eta')$	...	$180^\circ$	$180^\circ \mp (0.1^{+0.1}_{-0.2})^\circ$



The weak phase  $\phi_3$  is removed from  $A(\bar{B}_{(s)} \rightarrow \bar{D} \bar{P})$  in defining  $\delta_{B(B_s)}$ . Except  $r_B(D^0 K^0)$  the effects from final state interaction are mild. We see that our  $r_B(D^0 K^-)$  and  $\delta_B(D^0 K^-)$  agree with the Dalitz analysis results of Belle and BABAR. Our  $r_B(D^0 K^-)$  is also in agreement with the fit from UTfit group obtained by using all three methods of GLW, ADS and  $DK$  Dalitz analysis [34].

#### IV. CONCLUSION

We study quasielastic rescattering effects in  $\bar{B}_{u,d,s} \rightarrow DP, \bar{D}P$  modes. The updated data for nine  $\bar{B}_{u,d} \rightarrow DP$  modes are used to extract  $a_{1,2}^{\text{eff}}$  and four rescattering parameters. We find the effective Wilson coefficients  $a_1^{\text{eff}} \simeq 0.90$  and  $a_2^{\text{eff}} \simeq 0.23$ , the strong phases  $\delta \simeq 56^\circ$ ,  $\theta \simeq 18^\circ$ , and  $\sigma \simeq -104^\circ$  and mixing angle  $\tau \simeq 7^\circ$ . Since strong interaction respects SU(3) symmetry and charge conjugation symmetry, the formalism can be used to predict  $\bar{B}_s \rightarrow DP$  and  $\bar{B}_{u,d,s} \rightarrow \bar{D}P$  rates and  $r_{B(B_s)}$ , without referring to any experimental information on  $\bar{B}_{u,d,s} \rightarrow \bar{D}P$  decays, which is limited by the smallness of the corresponding decay rates.

Our results are summarized as following: (a) The  $\bar{B}_s^0 \rightarrow D^0 K^0, D^0 \eta, D^0 \eta'$  rates are enhanced in the presence of FSI. In particular, the  $\bar{B}_s^0 \rightarrow D^0 K^0$  rate is close to  $10^{-3}$  level and can be measured soon. (b) The predicted  $\bar{B}_s \rightarrow D_s^+ \pi^-$  rate and the ratio of  $\mathcal{B}(\bar{B}_s \rightarrow D_s^+ K^-)/\mathcal{B}(\bar{B}_s \rightarrow D_s^+ \pi^-)$  is in better agreement with experimental results. (c) Except the  $\bar{B}^0 \rightarrow D^0 K^0$  mode, the FSI effects on  $r_{B(B_s)}$  are mild. (d) The predicted  $r_B(D^0 K^-)$  agree with data and the fit from the UTfit Collaboration, while  $\delta_B(D^0 K^-)$  agree with data.

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#### APPENDIX: EXPLICIT EXPRESSION OF $A^0$

As mentioned in the text, we use naive factorization amplitudes  $A^f$  for  $A^0$  to avoid double counting of FSI effects. For each final state, we have

$$\begin{aligned}
A_{B^- \rightarrow D^0 \pi^-}^f &= V_{cb} V_{ud}^* (T_f + C_f), & A_{\bar{B}^0 \rightarrow D^+ \pi^-}^f &= V_{cb} V_{ud}^* (T_f + E_f), & A_{\bar{B}^0 \rightarrow D^0 \pi^0}^f &= \frac{V_{cb} V_{ud}^*}{\sqrt{2}} (-C_f + E_f), \\
A_{\bar{B}^0 \rightarrow D_s^+ K^-}^f &= V_{cb} V_{ud}^* E_f, & A_{\bar{B}^0 \rightarrow D^0 \eta_8}^f &= \frac{V_{cb} V_{ud}^*}{\sqrt{6}} (C_f + E_f), & A_{\bar{B}^0 \rightarrow D^0 \eta_1}^f &= \frac{V_{cb} V_{ud}^*}{\sqrt{3}} (C_f + E_f), \\
A_{B^- \rightarrow D^0 K^-} &= V_{cb} V_{us}^* (T_f + C_f), & A_{\bar{B}^0 \rightarrow D^+ K^-} &= V_{cb} V_{us}^* T_f, & A_{\bar{B}^0 \rightarrow D^0 \bar{K}^0} &= V_{cb} V_{us}^* C_f, & A_{\bar{B}_s^0 \rightarrow D_s^+ \pi^-}^f &= V_{cb} V_{ud}^* T_f, \\
A_{\bar{B}_s^0 \rightarrow D^0 K^0}^f &= V_{cb} V_{ud}^* C_f, & A_{\bar{B}_s^0 \rightarrow D^+ \pi^-}^f &= V_{cb} V_{us}^* E_f, & A_{\bar{B}_s^0 \rightarrow D^0 \pi^0}^f &= \frac{V_{cb} V_{us}^*}{\sqrt{2}} E_f, \\
A_{\bar{B}_s^0 \rightarrow D^0 \eta_8}^f &= \frac{V_{cb} V_{us}^*}{\sqrt{6}} (-2C_f + E_f), & A_{\bar{B}_s^0 \rightarrow D^0 \eta_1}^f &= \frac{V_{cb} V_{us}^*}{\sqrt{3}} (C_f + E_f), & A_{\bar{B}_s^0 \rightarrow D_s^+ K^-}^f &= V_{cb} V_{us}^* (T_f + E_f), \quad (A1)
\end{aligned}$$

where the super- and subscripts  $f$  indicate naive factorization amplitude, and

$$\begin{aligned}
T_f &= \frac{G_F}{\sqrt{2}} a_1^{\text{eff}} (m_B^2 - m_D^2) f_P F_0^{BD}(m_P^2), & C_f &= \frac{G_F}{\sqrt{2}} a_2^{\text{eff}} (m_B^2 - m_P^2) f_D F_0^{BP}(m_D^2), \\
E_f &= \frac{G_F}{\sqrt{2}} a_2^{\text{eff}} (m_D^2 - m_P^2) f_B F_0^{0 \rightarrow DP}(m_B^2). \quad (A2)
\end{aligned}$$

$F_0^{BD(BP)}$  is the  $\bar{B}_{u,d,s} \rightarrow D_{u,d,s}(P)$  transition form factor and  $F_0^{0 \rightarrow DP}$  is the vacuum to  $DP$  (timelike) form factor.

For  $\bar{B}_{u,d,s} \rightarrow \bar{D}_{u,d,s} P$  decays, we have

$$\begin{aligned}
A_{B^- \rightarrow \bar{D}^0 K^-}^f &= V_{ub} V_{cs}^* (c_f + a_f), & A_{B^- \rightarrow D^- \bar{K}^0}^f &= V_{ub} V_{cd}^* a_f, & A_{B^- \rightarrow \bar{D}^0 \eta_8}^f &= \frac{V_{ub} V_{cs}^*}{\sqrt{6}} (t_f - 2a_f), \\
A_{B^- \rightarrow \bar{D}^0 \eta_1}^f &= \frac{V_{ub} V_{cd}^*}{\sqrt{3}} (t_f + a_f), & A_{B^- \rightarrow D_s^- \pi^0}^f &= \frac{V_{ub} V_{cs}^*}{\sqrt{2}} t_f, & A_{\bar{B}^0 \rightarrow D_s^- \pi^+} &= V_{ub} V_{cs}^* t_f, & A_{\bar{B}^0 \rightarrow \bar{D}^0 \bar{K}^0} &= V_{ub} V_{cs}^* c_f, \\
A_{\bar{B}_s^0 \rightarrow D^- \pi^+}^f &= V_{ub} V_{cs}^* e_f, & A_{\bar{B}_s^0 \rightarrow \bar{D}^0 \pi^0}^f &= \frac{V_{ub} V_{cs}^*}{\sqrt{2}} e_f, & A_{\bar{B}_s^0 \rightarrow \bar{D}^0 \eta_8}^f &= \frac{V_{ub} V_{cs}^*}{\sqrt{6}} (-2c_f + e_f), \\
A_{\bar{B}_s^0 \rightarrow \bar{D}^0 \eta_1}^f &= \frac{V_{ub} V_{cs}^*}{\sqrt{3}} (c_f + e_f), & A_{\bar{B}_s^0 \rightarrow D_s^- K^+}^f &= V_{ub} V_{cs}^* (t_f + e_f), \quad (A3)
\end{aligned}$$

TABLE VII. Central values of form factors taken from [41,42]. For  $B_{(s)} \rightarrow \eta'$  form factors the mixing angle and Clebsch-Gordan coefficients are included [see Eq. (A6)]. Note that 10% uncertainties on these form factors are estimated.

Form factor	Value	Form factor	Value
$F_0^{B\pi}(m_{D,D_s}^2)$	0.28	$F_0^{B_{(s)}D_{(s)}}(m_{\pi,K}^2)$	0.67
$F_0^{B\eta}(m_{D,D_s}^2)$	0.15	$F_0^{B\eta'}(m_{D,D_s}^2)$	0.13
$F_0^{BK}(m_{D,D_s}^2)$	0.43	$F_0^{B_s K}(m_D^2)$	0.40
$F_0^{B_s \eta}(m_D^2)$	-0.29	$F_0^{B_s \eta'}(m_{D,D_s}^2)$	0.35

where, as before, the super- and subscripts  $f$  indicate naive factorization amplitude, and

$$\begin{aligned} t_f &= \frac{G_F}{\sqrt{2}} a_1^{\text{eff}}(m_B^2 - m_P^2) f_D F_0^{BP}(m_D^2), \\ c_f &= \frac{G_F}{\sqrt{2}} a_2^{\text{eff}}(m_B^2 - m_P^2) f_D F_0^{BP}(m_D^2), \\ e_f &= \frac{G_F}{\sqrt{2}} a_2^{\text{eff}}(m_P^2 - m_D^2) f_B F_0^{0 \rightarrow PD}(m_B^2). \end{aligned} \quad (\text{A4})$$

Note that we have  $(t_f, e_f) = (T_f, E_f)$  with  $D$  and  $P$  interchanged and  $c_f = C_f$  (without the interchange of  $D$  and  $P$ ).

The  $D^0 \eta_8$  and  $D^0 \eta_1$  are not physical final states. The physical  $\eta$  and  $\eta'$  mesons are defined through

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \vartheta & -\sin \vartheta \\ \sin \vartheta & \cos \vartheta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}, \quad (\text{A5})$$

with the mixing angle  $\vartheta = -15.4^\circ$  [40]. Form factors are

taken from [41,42], where we list the relevant values in Table VII. For  $B_{(s)} \rightarrow \eta'$  form factors the mixing angle and Clebsch-Gordan coefficients are included:

$$\begin{aligned} F^{B\eta}(m_{D,D_s}^2) &= \left( \frac{\cos \vartheta}{\sqrt{6}} - \frac{\sin \vartheta}{\sqrt{3}} \right) F_0^{B\pi}(m_{D,D_s}^2), \\ F^{B\eta'}(m_{D,D_s}^2) &= \left( \frac{\sin \vartheta}{\sqrt{6}} + \frac{\cos \vartheta}{\sqrt{3}} \right) F_0^{B\pi}(m_{D,D_s}^2), \\ F^{B_s \eta}(m_{D,D_s}^2) &= \left( -2 \frac{\cos \vartheta}{\sqrt{6}} - \frac{\sin \vartheta}{\sqrt{3}} \right) F_0^{B_s \eta_s}(m_{D,D_s}^2), \\ F^{B_s \eta'}(m_{D,D_s}^2) &= \left( -2 \frac{\sin \vartheta}{\sqrt{6}} + \frac{\cos \vartheta}{\sqrt{3}} \right) F_0^{B_s \eta_s}(m_{D,D_s}^2), \end{aligned} \quad (\text{A6})$$

where  $\eta_s$  is the  $s\bar{s}$  component of  $\eta$  and  $\eta'$  and the form factor  $F_0^{B\pi}(m_{D,D_s}^2)$  and  $F_0^{B_s \eta_s}(m_{D,D_s}^2)$  are taken from [41,42], respectively.

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