Measuring $B^{\pm} \to \tau^{\pm}\nu$ and $B_c^{\pm} \to \tau^{\pm}\nu$ at the Z peak

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The measurement of $B^{\pm} \to \tau^{\pm}\nu_{\tau}$ at the B factories provides important constraints on the parameter $\tan\beta/m_{H^{\pm}}$ in the context of models with two Higgs doublets. Limits on this decay from e^+e^- collisions at the Z peak were sensitive to the sum of $B^{\pm} \to \tau^{\pm}\nu_{\tau}$ and $B_c^{\pm} \to \tau^{\pm}\nu_{\tau}$. Because of the possibly sizeable contribution from $B_c^{\pm} \to \tau^{\pm}\nu_{\tau}$ we suggest that a signal for this combination might be observed if the CERN LEP L3 Collaboration used its total data of $\sim 3.6 \times 10^6$ hadronic decays of the Z boson. Moreover, we point out that a future linear collider operating at the Z peak (Giga Z option) could constrain $\tan\beta/m_{H^{\pm}}$ from the sum of these processes with a precision comparable to that anticipated at proposed high luminosity B factories from $B^{\pm} \to \tau^{\pm}\nu_{\tau}$ alone.

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I. INTRODUCTION

In April 2006 the BELLE Collaboration announced the first observation of the purely leptonic decay $B^{\pm} \to \tau^{\pm}\nu_{\tau}$ [1] utilizing an integrated luminosity of 414 fb^{-1}. The measured branching ratio (BR) is in agreement with the standard model (SM) rate within theoretical and experimental errors:

$$\mathrm{BR}\,(B^{\pm} \to \tau^{\pm}\nu_{\tau}) = (1.79^{+0.56}_{-0.49}(\mathrm{stat})^{+0.46}_{-0.51}(\mathrm{syst})) \times 10^{-4}. \quad (1)$$

Subsequently, the *BABAR* collaboration reported a measurement with an integrated luminosity of 346 fb^{-1} which is an average of separate analyses with semileptonic [2] and hadronic [3] tags:

$$\mathrm{BR}\,(B^{\pm} \to \tau^{\pm}\nu_{\tau}) = (1.2 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4}. \quad (2)$$

The average of the BELLE and *BABAR* measurements is [4]

$$\mathrm{BR}\,(B^{\pm} \to \tau^{\pm}\nu_{\tau}) = (1.41^{+0.43}_{-0.42}) \times 10^{-4}. \quad (3)$$

Significantly improved precision for $\mathrm{BR}(B^{\pm} \to \tau^{\pm}\nu_{\tau})$ would require a high luminosity $\mathcal{L} \geq 10^{35}$ cm^{-2} s^{-1} B factory [5–11]. In the context of the SM the decay $B^{\pm} \to \tau^{\pm}\nu_{\tau}$ provides a direct measurement of the combination $f_B V_{ub}$, where f_B is the decay constant which can only be calculated by nonperturbative techniques such as lattice QCD. Charged Higgs bosons ($H^{\pm}$) present in the two Higgs doublet model (2HDM) and the minimal supersymmetric SM (MSSM) would also mediate $B^{\pm} \to \tau^{\pm}\nu_{\tau}$ [12] with the new physics contribution being sizeably enhanced if $\tan\beta$ (the ratio of vacuum expectation values of the two Higgs doublets) is large [13]. The above measurements of $B^{\pm} \to \tau^{\pm}\nu_{\tau}$ now provide a very important constraint on the parameter $\tan\beta/m_{H^{\pm}}$ in the context of the 2HDM and the MSSM. Hence this decay is of much interest in both the SM and models beyond the SM and improved precision in the above measurements is certainly desirable.

Prior to the era of the B factories three CERN LEP collaborations searched for $B^{\pm} \to \tau^{\pm}\nu_{\tau}$ and obtained upper bounds within an order of magnitude of the SM prediction [14–16]. Such limits were actually sensitive to the sum of $B^{\pm} \to \tau^{\pm}\nu_{\tau}$ and $B_c^{\pm} \to \tau^{\pm}\nu_{\tau}$ [17] since the center-of-mass energy ($\sqrt{s} = 91$ GeV) was above the $B_c^{\pm}$ production threshold (unlike the B factories). The strongest limits were set by the L3 Collaboration which obtained $\mathrm{BR}(B^{\pm} \to \tau^{\pm}\nu_{\tau}) < 5.7 \times 10^{-4}$ [14]. Since $\mathrm{BR}(B^{\pm} \to \tau^{\pm}\nu_{\tau})$ has now been measured at the B factories, the L3 limit can now be used to provide a limit on the product of the transition probability $f(b \to B_c)$ and $\mathrm{BR}(B_c^{\pm} \to \tau^{\pm}\nu_{\tau})$. A quantitative study of the magnitude of the contribution of $B_c^{\pm} \to \tau^{\pm}\nu_{\tau}$ to the LEP limits was performed in [17]. We update this analysis using the significant improvements in the measurements of the Cabibbo-Kobayashi-Maskawa matrix and calculations of f_B. Moreover, the measurements of the $B_c^{\pm}$ production cross section at the Fermilab Tevatron [18–21] provide the first measurements of the transition probability for $b \to B_c$ and suggest much larger values than the theoretical estimations used in the numerical analysis of [17]. The L3 limit on $\mathrm{BR}(B_c^{\pm} \to \tau^{\pm}\nu_{\tau})$ was obtained with $\sim 1.5 \times 10^6$ hadronic Z boson decays, which is slightly less than half the full L3 data taken at the Z peak. We suggest that a search for $B^{\pm}/B_c^{\pm} \to \tau^{\pm}\nu_{\tau}$ using the full L3 data sample ($\sim 3.6 \times 10^6$ hadronic Z decays [22]) would not only strengthen the limit on the product of $f(b \to B_c)$ and $\mathrm{BR}(B_c^{\pm} \to \tau^{\pm}\nu_{\tau})$ but also offer the possibility of a signal, which would be an additional observation of $B^{\pm} \to \tau^{\pm}\nu_{\tau}$ and the first observation of $B_c^{\pm} \to \tau^{\pm}\nu_{\tau}$. It is also pointed out that a future e^+e^- linear collider operating at the Z peak (the Giga Z option [23–26]) could offer similar sensitivity to the parameter $\tan\beta/m_{H^{\pm}}$ from these leptonic decays as the proposed high luminosity B

factories. This article is structured as follows: in Sec. II we present basic formulas for the decay rates for $B^\pm/B_c^\pm \to \tau^\pm \nu_\tau$ and discuss the $H^\pm$ contribution, we study the admixture of $B_c^\pm \to \tau^\pm \nu$ and $B^\pm \to \tau^\pm \nu$ at the Z peak in Sec. III and we give our conclusions in Sec. IV.

II. THE DECAYS $B^\pm \to \tau^\pm \nu$ AND $B_c^\pm \to \tau^\pm \nu$

In the SM, the purely leptonic decays ($\ell^\pm \nu_\ell$) of $B^\pm$ and $B_c^\pm$ proceed via annihilation to a W boson in the s channel. The decay rate is given by (where $q = u$ or c)

$$\Gamma(B_q^+ \to \ell^+ \nu_\ell) = \frac{G_F^2 m_{B_q} m_\ell^2 f_{B_q}^2}{8\pi} |V_{qb}|^2 \left(1 - \frac{m_\ell^2}{m_{B_q}^2}\right)^2. \quad (4)$$

Because of helicity suppression, the rate is proportional to m_ℓ^2 and one expects

$$\mathrm{BR}(B_q^+ \to \tau^+ \nu_\tau):\mathrm{BR}(B_q^+ \to \mu^+ \nu_\mu):\mathrm{BR}(B_q^+ \to e^+ \nu_e) = m_\tau^2 : m_\mu^2 : m_e^2. \quad (5)$$

These decays are relatively much more important for $B_c^\pm$ than $B_u^\pm$ due to the enhancement factor $|V_{cb}/V_{ub}|^2 (f_{B_c}/f_{B_u})^2$. Using the input parameters given in Table I, we obtain the SM predictions listed in Table II.

The effect of $H^\pm$ in the 2HDM (model II) on the decays $B_u^\pm \to \ell^+ \nu_\ell$ was considered in [13] and the analogous analysis for $B_c^\pm \to \ell^+ \nu_\ell$ was presented in [27]. In both cases the $H^\pm$ contribution modifies the SM prediction by a global factor r_H^q where

$$r_H^q = [1 - \tan^2\beta (M_{B_q}/m_{H^\pm})^2]^2 \equiv [1 - R^2 M_{B_q}^2]^2. \quad (6)$$

The $H^\pm$ contribution interferes destructively with that of $W^\pm$. There are two solutions for $r_H^q = 1$ which occur at $R = 0$ and $R \sim 0.27\ \mathrm{GeV}^{-1}$ for $B_u^\pm \to \ell^+ \nu_\ell$ ($R = 0$ and $R \sim 0.26\ \mathrm{GeV}^{-1}$ for $B_c^\pm \to \ell^+ \nu_\ell$). This is shown in Fig. 1 where $\mathrm{BR}(B_u \to \tau^+ \nu_\tau)$ is plotted as a function of $\tan\beta$ and $m_{H^\pm}$. For $\tan\beta/m_{H^\pm} \sim 0$, the BR remains at its SM value

TABLE I. Input parameters used in this paper, unless indicated otherwise in the text.

$G_F = 1.16639 \times 10^{-5}\ \mathrm{GeV}^{-2}$	$m_e = 0.511$ MeV
$m_\mu = 0.10566$ GeV	$m_\tau = 1.777$ GeV
$\lvert V_{ub}\rvert = 0.00386(28)$	$\lvert V_{cb}\rvert = 0.0416(9)$
$m_{B_u} = 5.279$ GeV	$\tau_{B_u} = 1.638 \times 10^{-12}$ s
$m_{B_c} = 6.271$ GeV	$\tau_{B_c} = 0.463(177) \times 10^{-12}$ s
$f_{B_u} = 0.216(22)$ GeV	$f_{B_c} = 0.450$ GeV

TABLE II. Standard model predictions for the branching ratios (central values).

	$\mathrm{BR}(B_q^+ \to \tau^+ \nu_\tau)$	$\mathrm{BR}(B_q^+ \to \mu^+ \nu_\mu)$	$\mathrm{BR}(B_q^+ \to e^+ \nu_e)$
B_u	1.2×10^{-4}	5.5×10^{-7}	1.3×10^{-11}
B_c	0.022	9.3×10^{-5}	2.2×10^{-9}

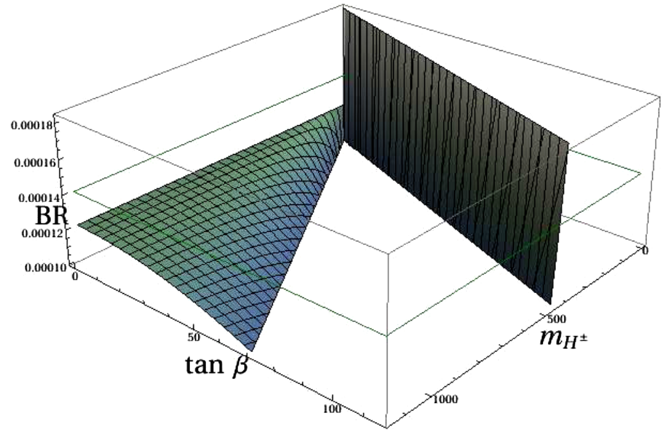

FIG. 1 (color online). $\mathrm{BR}(B_u \to \tau^+ \nu_\tau)$ as a function of $\tan\beta$ and $m_{H^\pm}$. The plotted range of the BR corresponds to the 1-σ range of the world average measurement $(1.42 \pm 0.43) \times 10^{-4}$, and the line indicates the central value.

(slightly higher than the thin line indicating the central value of the experimental measurement), but this SM value can also be achieved along a line through the steep part of the surface where $r_H = (1 - 2)^2 = 1$.

If the b quark couples to both Higgs doublets at tree level (which is referred to as the type III 2HDM), Eq. (6) is modified to [28]

$$r_H = \left(1 - \frac{\tan^2\beta}{1 + \tilde{\epsilon}_0 \tan\beta} \frac{m_B^2}{m_{H^\pm}^2}\right)^2. \quad (7)$$

In the MSSM, the parameter $\tilde{\epsilon}_0$ does not appear at tree level but is generated at the 1-loop level [29,30] (with the main contribution originating from gluino diagrams) and may reach values of 0.01. The redefinition of both the b quark Yukawa coupling and the Cabibbo-Kobayashi-Maskawa matrix element V_{ub} is encoded in $\tilde{\epsilon}_0$ [31,32]. The impact of $\tilde{\epsilon}_0 \neq 0$ on r_H has been developed in [33–36]. In particular, the value of R where $r_H = 1$ shifts depending on the magnitude and sign of $\tilde{\epsilon}_0$.

In Fig. 2 we show the impact of the measurement of $\mathrm{BR}(B_u \to \tau^+ \nu_\tau)$ on the plane of $[\tan\beta, m_{H^\pm}]$ in the 2HDM (type III) which updates the study of [28] (for a recent analogous plot with a somewhat lower value of f_B see [37]). The white regions are excluded and the shaded areas correspond to $\mathrm{BR}(B_u \to \tau^+ \nu_\tau)$ within the 1-σ experimental range. We plot overlapping bands for the 1-σ ranges of the input parameters and consider $\tilde{\epsilon}_0 = 0, 0.01, -0.01$. In the MSSM, positive values for $\tilde{\epsilon}_0$, corresponding to positive values of the μ parameter, are preferred in order to explain the $(g-2)_\mu$ anomaly [38], but in general both signs are possible.

The different values for $\tilde{\epsilon}_0$ result in significantly different allowed regions in the plane of $[\tan\beta, m_{H^\pm}]$. Importantly, these constraints from $\mathrm{BR}(B_u \to \tau^+ \nu_\tau)$ are from a tree-level process and when applied to the MSSM are only sensitive to the assumptions for the soft supersymmetry

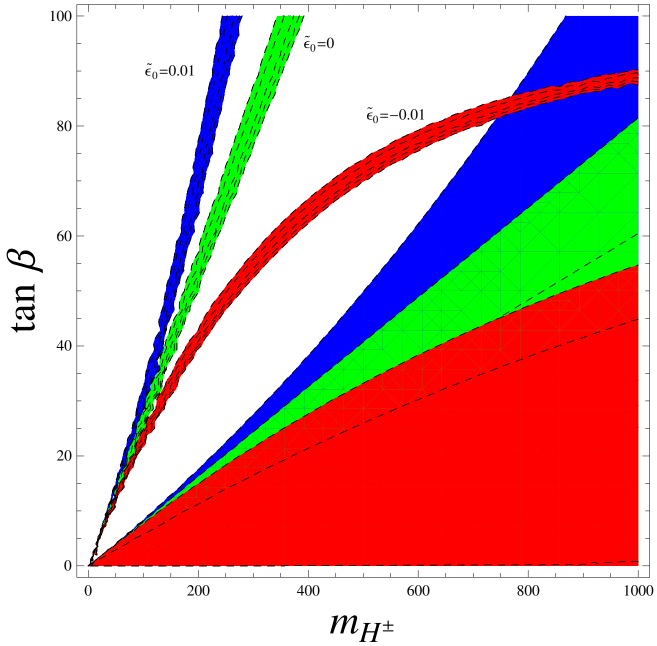

FIG. 2 (color online). The constraint on the $\tan\beta$-$m_{H^\pm}$ plane in the 2HDM (type III) from the measurement of $\mathrm{BR}(B^\pm \to \tau^\pm \nu_\tau)$. The shaded regions correspond to allowed ranges of $\tan\beta$ and $m_{H^\pm}$ for various values of $\tilde{\epsilon}_0 = 0$, 0.01, -0.01, $\mathrm{BR}(B_u \to \tau^+\nu_\tau)$ (1-σ range, overlapping) and f_B (overlapping).

breaking sector via $\tilde{\epsilon}_0$ (recently emphasized in [39]), i.e., a higher order effect. In contrast, other important B physics observables such as $b \to s\gamma$, $B_s - \bar{B}_s$ mixing and $B_{d,s} \to \mu\mu$ are all loop induced processes. Consequently, constraints on the plane $[\tan\beta, m_{H^\pm}]$ from such processes are very sensitive to the assumptions made for the sparticle masses, and in certain cases the constraints can be removed completely.[1] In global studies of B physics observables in specific MSSM scenarios [34,42] the measured $\mathrm{BR}(B_u \to \tau^+\nu_\tau)$ also plays an important role. Certainly, improved precision for $\mathrm{BR}(B_u \to \tau^+\nu_\tau)$ is desirable and very relevant in the era of the CERN LHC in which the plane $[\tan\beta, m_{H^\pm}]$ will be probed via direct production of Higgs bosons. Currently only high luminosity B factories operating at the $\Upsilon(4S)$ are discussed when considering future facilities which could offer improved precision for $\mathrm{BR}(B_u \to \tau^+\nu_\tau)$.

Another promising approach to probe the plane $[\tan\beta, m_{H^\pm}]$ is via the tree-level $H^\pm$ contribution to the semileptonic decays $B \to D\tau\nu$ [33,35,36,43]. We note here that $H^\pm$ can mediate the analogous leptonic decays $K^\pm \to \mu^\pm\nu$ [13,44] and $D_s^\pm \to \mu^\pm\nu$, $\tau^\pm\nu$ [13,45] but constraints on the plane $[\tan\beta, m_{H^\pm}]$ from these processes are not yet competitive. However, such processes might play a role in the future with increased experimental precision and reduced theoretical uncertainties.

[1]In the nonsupersymmetric 2HDM (model II) $b \to s\gamma$ constrains $m_{H^\pm}$ independently of $\tan\beta$. A recent study [40,41] obtains $m_{H^\pm} > 295$ GeV.

III. AT THE Z PEAK

In this section we discuss the searches for $B^\pm \to \tau^\pm\nu$ using data from e^+e^- collisions at the Z peak ($\sqrt{s} \sim 91$ GeV). It was pointed out in [17] that such searches would also be sensitive to the decay $B_c^\pm \to \tau^\pm\nu$. Assuming that the detection efficiencies are the same[2] the ratio of $\tau^\pm\nu$ events originating from $B^\pm \to \tau^\pm\nu$ and $B_c^\pm \to \tau^\pm\nu$ is given by

$$\frac{N_c}{N_u} = \left|\frac{V_{cb}}{V_{ub}}\right|^2 \frac{f(b \to B_c^\pm)}{f(b \to B^\pm)} \left(\frac{f_{B_c}}{f_B}\right)^2 \frac{M_{B_c}}{M_B} \frac{\tau_{B_c}}{\tau_B} \frac{(1 - \frac{m_\tau^2}{M_{B_c}^2})^2}{(1 - \frac{m_\tau^2}{M_B^2})^2}. \tag{8}$$

The largest uncertainty in the determination of N_c is from the transition probability $f(b \to B_c^\pm)$ and the decay constant f_{B_c}. The magnitude of N_c is suppressed by the small $f(b \to B_c^\pm)$ but this can be compensated by the large ratio $(V_{cb}f_{B_c})^2/(V_{ub}f_B)^2$. Consequently N_c can be similar in magnitude to N_u. In the analysis of [17] three scenarios were defined in order to account for the error in the determination of N_c/N_u: "central" and "max/min" ($\pm 1\sigma$ above/below the central values of the input parameters). Since the analysis of [17] there have been significant improvements in the measurements of V_{ub} and V_{cb}. In addition, the decay constant f_B has now been calculated in unquenched lattice QCD with smaller errors and a central value considerably larger [46] than the values used in both [17] and the L3 analysis [14]. We are unaware of an unquenched lattice QCD calculation of f_{B_c} and the error in this parameter has not been reduced significantly since [17]. The main uncertainty in the ratio N_c/N_u is from $f(b \to B_c^\pm)$, which in [17] was varied in the range suggested by theoretical estimations [47]: $2 \times 10^{-4} < f(b \to B_c^\pm) < 1 \times 10^{-3}$. At that time $B_c^\pm$ was still undiscovered and hence there was no measurement of $f(b \to B_c^\pm)$.

However, $f(b \to B_c^\pm)$ can now be extracted (although with a large uncertainty) from the measurement of the ratio of $B_c^\pm \to J/\Psi \ell^+\nu_\ell$ to $B^\pm \to J/\Psi K^\pm$ which is defined by

$$\mathcal{R}_\ell = \frac{\sigma(B_c^+) \cdot \mathrm{BR}(B_c \to J/\psi \ell^\pm \nu_\ell)}{\sigma(B^+) \cdot \mathrm{BR}(B \to J/\psi K^+)}. \tag{9}$$

Tevatron Run II data give $\mathcal{R}_e = 0.28 \pm 0.07$ [20], and the denominator in Eq. (9) has been measured precisely by various experiments. The transition probability $f(b \to B_c)$ determines $\sigma(B_c^+)$ and several theoretical calculations are available for $\mathrm{BR}(B_c \to J/\psi \ell^\pm \nu_\ell)$). In Fig. 3 we display contours of $\mathcal{R}_e$ as a function of $\mathrm{BR}(B_c \to J/\Psi e^+\nu_e)$ and $f(b \to B_c)$, and the band denotes the prediction of the

[2]In practice, the shorter lifetime of $B_c^\pm$ would result in a slightly inferior detection efficiency [17].

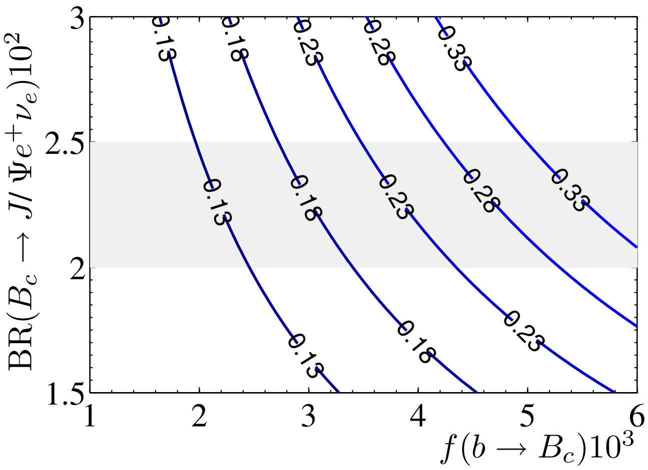

FIG. 3 (color online). Contours of $\mathcal{R}_e$ in the plane of $\mathrm{BR}(B_c \to J/\Psi e^+ \nu_e)$ and transition probability $f(b \to B_c)$. The shaded region denotes the theoretical prediction for $\mathrm{BR}(B_c \to J/\Psi e^+ \nu_e)$.

various theoretical calculations for $\mathrm{BR}(B_c \to J/\Psi e^+ \nu_e)$ whose values lie in the range $(2.0 \sim 2.5)\%$ [48]. From Fig. 3 one can see that the Tevatron Run I measurement of $\mathcal{R}_e = 0.13 \pm 0.05$ [18] is accommodated by $f(b \to B_c) = 1.3 \times 10^{-3}$. However, in order to satisfy the central value of the Run II measurement, the transition probability $f(b \to B_c)$ needs to be 4.5×10^{-3}. An even larger value for $f(b \to B_c)$ was suggested in Ref. [49]. Such unexpectedly large values of $f(b \to B_c)$, which are indicated by Tevatron Run II data, would significantly enhance the contribution of $B_c^\pm \to \tau^\pm \nu$ to the LEP searches for $B^\pm \to \tau^\pm \nu$. Of course, $f(b \to B_c)$ is dependent on the available center-of-mass energy (at higher energies there is more phase space to produce a charm quark instead of a light quark), but the value of $f(b \to B_c)$ is expected to be of comparable size at LEP and at the LHC [47]. In our numerical analysis in Sec. III A, we will consider values of $f(b \to B_c^\pm)$ up to 5×10^{-3}.

A. The LEP search for $B^\pm \to \tau^\pm \nu$ and the contribution of $B_c^\pm \to \tau^\pm \nu$

Three LEP collaborations searched for the decay $B^\pm \to \tau^\pm \nu$ using data taken at the Z peak ($\sqrt{s} = 91$ GeV). L3 [14] used around 1.5×10^6 hadronic decays of the Z boson which corresponds to about half its total data [22]. DELPHI [15] and ALEPH [16] used their full data samples of around 3.6×10^6 hadronic decays of the Z boson. The best sensitivity was from the L3 experiment which set the upper limit $\mathrm{BR}(B^\pm \to \tau^\pm \nu) < 5.7 \times 10^{-4}$. The L3 limit is of particular interest since it could be improved if the full data sample of $\sim 3.6 \times 10^6$ hadronic Z boson decays were used.

The LEP searches were sensitive to $\tau^\pm \nu$ events originating from both $B^\pm \to \tau^\pm \nu$ and $B_c^\pm \to \tau^\pm \nu$. Hence the published limits constrain the "effective branching ratio" defined by

$$\mathrm{BR}_{\mathrm{eff}} = \mathrm{BR}(B^\pm \to \tau^\pm \nu)\left(1 + \frac{N_c}{N_u}\right). \qquad (10)$$

This expression applies to searches for $B^\pm \to \tau^\pm \nu$ at the Z peak. For searches at the $\Upsilon(4S)$ clearly $N_c = 0$ and $\mathrm{BR}_{\mathrm{eff}} = \mathrm{BR}(B^\pm \to \tau^\pm \nu)$. In our numerical analysis in this section we will use Eq. (10) with the experimental value of $\mathrm{BR}(B^\pm \to \tau^\pm \nu)$ as input. The calculation of N_c/N_u in Eq. (10) uses Eq. (8) (i.e., the expression for the SM) with input parameters taken from Table I. Our analysis can be applied to any model for which $N_c/N_u \sim |N_c/N_u|_{\mathrm{SM}}$, which includes the 2HDM because the scale factors in Eq. (6) are almost equal.

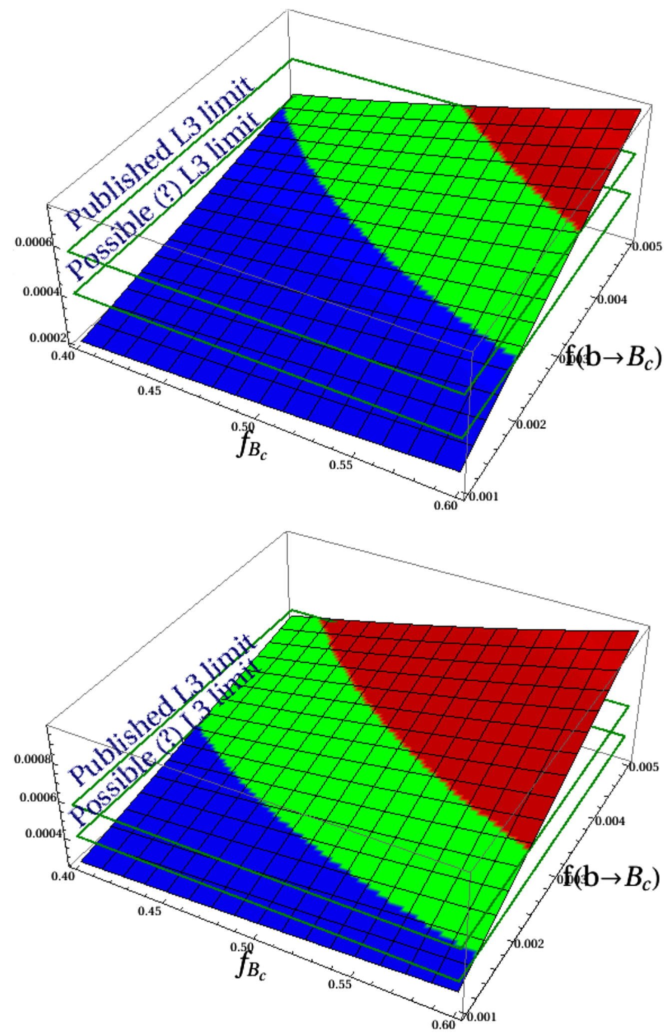

FIG. 4 (color online). The *effective* $\mathrm{BR}(B^\pm \to \tau^\pm \nu)$ at the Z peak in the plane $[f(b \to B_c^\pm), f_{B_c}]$. The published L3 limit and a possible stricter limit are indicated. In the upper (lower) panel $\mathrm{BR}(B^\pm \to \tau^\pm \nu)$ is taken to be the central value of the world average measurement (the 1-σ upper value).

TABLE III. Required number of B mesons (Z bosons) for a precision of 20% and 4% in the measurement of $\mathrm{BR}(B^{\pm}/B_c^{\pm} \to \tau^{\pm}\nu)$, assuming a signal of $\mathrm{BR}_{\mathrm{eff}} = 4 \pm 2 \times 10^{-4}$ at L3.

Error $\mathrm{BR}(B^{\pm}/B_c^{\pm} \to \tau^{\pm}\nu)$	High luminosity B factory (B mesons)	Giga Z (Z bosons)
20%	2.2×10^9	3.2×10^7
4%	8.1×10^{10}	8×10^8

The max scenario of [17] showed that the current limit of $\mathrm{BR}_{\mathrm{eff}} < 5.7 \times 10^{-4}$ would be sensitive to the SM rate for $\mathrm{BR}(B^{\pm} \to \tau^{\pm}\nu)$. The measurements of $\mathrm{BR}(B^{\pm} \to \tau^{\pm}\nu)$ at the B factories are consistent with the SM prediction which suggests that the L3 search was not so far from observing a signal.

In Fig. 4 we plot $\mathrm{BR}_{\mathrm{eff}}$ in the plane $[f(b \to B_c^{\pm}), f_{B_c}]$, for two values of $\mathrm{BR}(B^{\pm} \to \tau^{\pm}\nu)$ corresponding to the central value and 1σ above the world average. All other parameters are held at their central values from Table I. The region above the contour $\mathrm{BR}_{\mathrm{eff}} = 5.7 \times 10^{-4}$ (red/dark gray on the right) is excluded by the L3 limit [14], while the contour $\mathrm{BR}_{\mathrm{eff}} = 4 \times 10^{-4}$ represents the hypothetical sensitivity if the full data of 3.6×10^6 hadronic decays of the Z boson were used. The green/light gray area between the two contours is the area where a signal would be seen if the full data set were studied. Depending on the other input parameters and the $B^{\pm} \to \tau^{\pm}\nu$ branching ratio, this area can cover a very significant part of the $[f(b \to B_c^{\pm}), f_{B_c}]$ parameter space. We therefore consider a reanalysis using the full L3 data set very worthwhile.

A different way of studying the number of B_c events was followed in [17]. The number of B_c events per B_u event can be calculated as a function of $f(b \to B_c^{\pm})$, and the authors obtained $N_c/N_u = 1.2 f(b \to B_c^{\pm})/10^{-3}$ for central values of the input parameters (max scenario: 2.3). With updated values for the input parameters, we now find

$$\frac{N_c}{N_u} = \begin{array}{ll} 0.48 \cdot f_{b\to B_c}/10^{-3} & \text{(central values)} \\ 1.50 \cdot f_{b\to B_c}/10^{-3} & \text{(``optimistic'' values),} \end{array} \quad (11)$$

where for the "optimistic" values of the parameters from Table I we have chosen that end of the 1-σ range that results in a higher value for N_c/N_u, and the optimistic f_{B_c} was chosen to be 550 MeV.

These numbers are lower than those of [17] mainly because the central value of V_{ub}/V_{cb} has increased in the last ten years. The inverse of this ratio enters N_c/N_u quadratically and therefore reduces this quantity. On the other hand, experimental data do not preclude values of $f(b \to B_c^{\pm})$ which are much higher (a few $\times 10^{-3}$) than the theoretical estimates, and so the admixture of $B_c^{\pm} \to \tau^{\pm}\nu$ can still easily reach 100%.

B. Giga Z option at a future e^+e^- linear collider

A future e^+e^- linear collider operating at the Z peak with a luminosity of $5 \times 10^{33}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ could produce 10^9 Z bosons in 50–100 days of operation [23–26]. This corresponds to roughly 1000 times the number of Z bosons recorded at each LEP detector. Historically, limits on $B^{\pm} \to \tau^{\pm}\nu$ from Z decays have been comparable to (if not stronger than) those at $\Upsilon(4S)$ for the same number of Z bosons and B mesons. For example, the CLEO collaboration obtained $\mathrm{BR}(B^{\pm} \to \tau^{\pm}\nu) < 8.4 \times 10^{-4}$ with 9.7×10^6 B mesons [50], while L3 obtained $\mathrm{BR}(B^{\pm} \to \tau^{\pm}\nu) < 5.7 \times 10^{-4}$ with 1.5×10^6 hadronic decays of the Z boson.

High luminosity B factories [5–11] anticipate data samples of 10^{10} B mesons. By the time of operation of a Giga Z the two main sources of uncertainty in N_c (and hence $\mathrm{BR}_{\mathrm{eff}}$) will have been substantially reduced. The error in $f(b \to B_c)$ will be reduced from LHC-b measurements [51] of the cross section in Eq. (9), and improved lattice calculations of f_{B_c} and/or (f_{B_c}/f_B) would also reduce the error in N_c. In Table III we present the required number of B mesons and Z bosons for a precision of 20% and 4% in the measurement of $B^{\pm} \to \tau^{\pm}\nu$ at a high luminosity B factory and $\mathrm{BR}_{\mathrm{eff}}$ at Giga Z. The numbers for a high luminosity B factory are taken from [10]. For the Giga Z precision we assume a signal of $\mathrm{BR}_{\mathrm{eff}} = 4 \pm 2 \times 10^{-4}$ (50% error) at L3 with 3.6×10^6 hadronic Z decays, and scale the error by $1/\sqrt{N}$, where N is the total number of Z bosons at Giga Z divided by the full L3 data sample of $\sim 5.1 \times 10^6$ Z bosons.

It is clear from Table III that a Giga Z facility might be capable of measuring $\mathrm{BR}_{\mathrm{eff}}$ in Eq. (10) with similar precision to that anticipated for $B^{\pm} \to \tau^{\pm}\nu$ at high luminosity B factories. We believe that this competitiveness of the Giga Z facility has not been pointed out for the leptonic B decays although it has been emphasized for the decay $B \to X_s \nu\bar{\nu}$ in [24]. If both facilities were realized this would enable competitive and complementary constraints on $\tan\beta/m_{H^{\pm}}$ in the context of models with $H^{\pm}$.

IV. CONCLUSIONS

The decay $B^{\pm} \to \tau^{\pm}\nu$ has been observed at the e^+e^- B factories and is recognized as an important constraint on the parameter $\tan\beta/m_{H^{\pm}}$ in the context of models with two Higgs doublets. We studied the contribution of $B_c^{\pm} \to \tau^{\pm}\nu$ to the LEP searches for $B^{\pm} \to \tau^{\pm}\nu$ (first pointed out in [17]), whose main uncertainty is from the value for the transition probability $b \to B_c^{\pm}$ which is now being measured at the Tevatron Run II. Using values of this transition probability which are consistent with the current Tevatron measurements (which accommodate values significantly larger than the theoretical estimations), we found that the contribution of $B_c^\pm$ mesons to the search for $B^\pm/B_c^\pm \to \tau^\pm\nu$ can be as large as that of $B^\pm$. We suggested that a reanalysis of the L3 search for $B^\pm \to \tau^\pm\nu$ [14] using all the data taken at the Z peak could provide a signal for the admixture of $B^\pm/B_c^\pm \to \tau^\pm\nu$. Finally, it was pointed out that the Giga Z option of a future e^+e^- collider could offer measurements of these leptonic $B^\pm/B_c^\pm$ decays which are comparable in precision and complementary with those anticipated at the proposed high luminosity B factories.

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