Collisional energy loss of a fast heavy quark in a quark-gluon plasma

Stéphane Peigné and André Peshier

SUBATECH, UMR 6457, Université de Nantes, Ecole des Mines de Nantes, IN2P3/CNRS, 4 rue Alfred Kastler,
44307 Nantes cedex 3, France

We discuss the average collisional energy loss dE/dx of a heavy quark crossing a quark-gluon plasma, in the limit of high quark energy $E \gg M^2/T$, where M is the quark mass and $T \ll M$ is the plasma temperature. In the fixed coupling approximation, at leading order $dE/dx \propto \alpha_s^2$, with a coefficient which is logarithmically enhanced. The soft logarithm arising from t-channel scattering off thermal partons is well known, but a collinear logarithm from u-channel exchange had previously been overlooked. We also determine the constant beyond those leading logarithms. We then generalize our calculation of dE/dx to running coupling. We estimate the remaining theoretical uncertainty of dE/dx, which turns out to be quite large under Relativistic Heavy Ion Collider conditions. Finally, we point out an approximate relation between dE/dx and the QCD Debye mass, from which we derive an upper bound to dE/dx for all quark energies.

I. INTRODUCTION

Jet quenching, as first anticipated by Bjorken [1], is a crucial probe of the state of matter created at the Relativistic Heavy Ion Collider (RHIC). The quenching of light hadron spectra at large $p_\perp$ [2,3] is usually attributed to the radiative energy loss of the (light) parent parton when crossing the hot or dense medium. On the other hand, recent data on heavy flavor quenching [4,5] suggest that for *heavy* quarks, purely radiative energy loss *might* be insufficient to explain the observed attenuation. This has renewed the interest in the *collisional* part ΔE_{coll} of the parton energy loss, in particular, in the case of a heavy quark [6]. It is not clear yet whether such a collisional contribution can help in understanding the data on heavy flavor quenching. For instance, although some studies support the possibility of a quite large collisional loss [7–9], a recent comparison between collisional and radiative losses [10] indicates that the collisional contribution to parton energy loss might be small ($\sim 20\%$) compared to the radiative one, even for heavy quarks. It should however be noted that a relatively small *average* collisional loss might be compatible with an important effect of collisions on quenching [11,12], due to differently behaved collisional and radiative energy loss *probability distributions*.

In any case, in the present (unclear) situation where the importance of collisional energy loss is reconsidered, we believe it is not useless to state the correct result for the (average) collisional loss of an energetic heavy quark.

A basic quantity required to study collisional quenching is the rate of energy loss per unit distance, dE/dx, of a parton produced in the remote past and traveling in a large size medium, as studied in Refs. [1,13,14]. For heavy-ion collisions, where a parton initially produced in a hard subprocess crosses a medium of finite size L, we expect deviations from the linear law $\Delta E_{\mathrm{coll}}(L) = (dE/dx) \cdot L$

[15]. Nonetheless, knowing the "asymptotic" rate dE/dx is a prerequisite before attempting a more refined evaluation of ΔE_{coll}, including, in particular, finite size effects.

So far, the most detailed calculation of dE/dx for a heavy quark in a quark-gluon plasma (QGP) is due to Braaten and Thoma (BT) [14], and is based on their previous QED calculation of dE/dx for a muon [16]. As we recently analyzed [17], the BT QED calculation relies on an assumption of the momentum exchange q in elastic scattering, namely $q \ll E$, which is incorrect in the domain $E \gg M^2/T$. Therefore, the BT results for dE/dx, both in QED [16] and in QCD [14], need to be corrected in this limit. This was done in [17] in the QED case. Here we consider the QCD case of a fast heavy quark.

We assume the heavy quark to be produced in the hard partonic subprocess of the heavy-ion collision, i.e. to be present in the "initial" stage of the QGP evolution. It then crosses the QGP (supposed thermally equilibrated) by losing some energy, before hadronizing (within a jet) into a heavy-flavored hadron. The situation we consider should be appropriate when dealing with heavy-quark tagged jets, where the energy loss is defined as the difference between the initial and final energies of the *flagged* heavy quark. This is different from light parton (or untagged heavy-quark) energy loss, which should be defined (at the partonic level) as the energy difference between the *leading* partons in the initial and final states. Indeed, without tagging, the flavor or even the type (quark or gluon) of the initial energetic parton does not have to be conserved in the "energy loss" process.

In Sec. II we study the fixed coupling approximation which allows us to closely follow the lines of our QED calculation [17]. In Sec. III we show how the result with a running coupling can be simply inferred and discuss the theoretical uncertainty. The latter is illustrated by a numerical estimate in Sec. IV.

II. FIXED COUPLING APPROXIMATION

In this section we assume a fixed coupling, which is justified when the running coupling does not change much in the range of probed momenta (we will specify this condition below). We calculate, in the limit $E \gg M^2/T$, the heavy-quark collisional loss dE/dx beyond logarithmic accuracy. For the sake of clarity we first focus on the leading logarithms before calculating the constant next to those logarithms.

A. Leading logarithmic terms

The leading logarithms in dE/dx for a heavy quark in the quark-gluon plasma have the same origin as in the QED analog of a muon crossing an $e^{\pm}\gamma$ plasma. We thus make a detour to QED and outline the derivation of [17], which is straightforward at leading logarithmic accuracy. The QCD result for dE/dx of a heavy quark will then be obtained from simple considerations.

1. Muon energy loss in a QED plasma

At leading order the energy loss of a muon (of mass M and energy E) arises from elastic scattering off thermal electrons or positrons [Fig. 1(a)] and photons [Figs. 1(b) and 1(c)]. Scattering off electrons corresponds to t-channel exchange, whereas scattering off photons (Compton scattering) receives contributions from s and u-channels. The muon energy loss is given by [16]

$$
\frac{dE^{(\mu)}}{dx} = \sum_i \frac{1}{2Ev} \int_k \frac{n_i(k)}{2k} \int_{k'} \frac{\bar{n}_i(k')}{2k'} \int_{p'} \frac{1}{2E'} (2\pi)^4
$$
$$
\times \delta^{(4)}(P + K - P' - K') \frac{1}{d} \sum_{\text{spins}} |\mathcal{M}_i|^2 \omega, \quad (2.1)
$$

where $\omega = E - E'$. The tree-level amplitude $\mathcal{M}_i$ corresponds to scattering off a thermal particle of type $i = e^+$, e^-, and γ. Each $|\mathcal{M}_i|^2$ is summed over initial and final spin states, and we divide by the degeneracy factor $d = 2$ of the incoming muon. Furthermore, $n_i(k) = (\exp(k/T) \mp 1)^{-1}$ is the thermal distribution of the target particles, and $\bar{n}_i = 1 \pm n_i$ accounts for the Bose enhancement or Pauli blocking for the scattered state. We also use the shorthand notation $\int_k \equiv \int d^3\mathbf{k}/(2\pi)^3$.
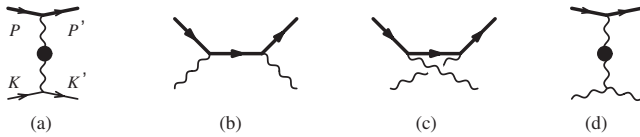

FIG. 1. Amplitudes for heavy muon (quark) elastic scattering in a QED (QCD) plasma. A curly line denotes a photon (QED) or a gluon (QCD). The amplitude (d) is specific to the QCD case. The blob in (a) and (d) denotes the resummed hard thermal loop boson propagator, which is necessary to screen the t-channel contribution in the infrared.

In the $E \gg T$ limit, (2.1) can be simplified to [17]

$$
\frac{dE^{(\mu)}}{dx} = \sum_i d_i \int_k \frac{n_i(k)}{2k} \int_{t_{\min}}^{t_{\max}} dt(-t) \frac{d\sigma_i}{dt}, \quad (2.2)
$$

where d_i is the degeneracy factor of the target particles, and

$$
\frac{d\sigma_i}{dt} = \frac{1}{16\pi \tilde{s}^2} \frac{1}{dd_i} \sum_{\text{spins}} |\mathcal{M}_i|^2 \quad (2.3)
$$

is the corresponding differential cross section. We also define $\tilde{s} \equiv s - M^2$, as well as for later reference $\tilde{u} \equiv u - M^2$. In Eq. (2.2) the bounds on t are set by kinematics, $t_{\min} = -\tilde{s}^2/s$, and $t_{\max} = 0$. We will focus on the limit $E \gg M^2/T$, which implies $s = (P + K)^2 = M^2 + 2PK \sim \mathcal{O}(ET) \gg M^2$, so that $t_{\min} \simeq -s$.

To obtain the leading logarithm from the t-channel contribution we can assume $|t| \ll s$ and approximate

$$
\sum_{i=e^{\pm}} \sum_{\text{spins}} |\mathcal{M}_i|^2 \simeq 32e^4 \frac{\tilde{s}^2}{t^2}. \quad (2.4)
$$

This contributes to (2.2) as

$$
\frac{e^4}{\pi} \int_k \frac{n_F(k)}{2k} \int_{t_{\min}}^{t_{\max}} \frac{dt}{-t} \simeq \frac{e^4 T^2}{48\pi} \left[\ln \frac{ET}{m_D^2} + \mathcal{O}(1) \right]. \quad (2.5)
$$

To obtain the right-hand side (rhs), we replaced $t_{\min} \simeq -s \rightarrow -ET$, which is justified to logarithmic accuracy. Strictly speaking, with $t_{\max} = 0$ the integral in (2.5) would be infrared divergent. As is well known, this divergence is screened by medium effects. To logarithmic accuracy it is sufficient to take $t_{\max} = -m_D^2$ as an effective infrared cutoff, where $m_D = eT/\sqrt{3}$ is the Debye screening mass in the QED plasma [18]. The logarithm in (2.5) thus arises from the domain $m_D^2 \ll |t| \ll s \sim ET$. A more accurate description of screening requires using the resummed hard thermal loop (HTL) [18,19] photon propagator in the t-channel (instead of the bare one with effective cutoff), as pictured in Fig. 1(a). This is needed to control the constant $\sim \mathcal{O}(1)$ in (2.5), as will be recalled in Sec. II B. Finally, in order to obtain (2.5) we used

$$
\int_k \frac{n_F(k)}{2k} = \frac{T^2}{48}. \quad (2.6)
$$

The contribution from Compton scattering to the muon energy loss brings another logarithm, arising from the square of the u-channel amplitude, more specifically from the domain $\tilde{u}_{\min} \ll \tilde{u} \ll \tilde{u}_{\max}$ [17]. In this domain the Compton scattering amplitude squared can be approximated as

$$
\sum_{\text{spins}} |\mathcal{M}_\gamma|^2 \simeq 8e^4 \frac{\tilde{s}}{-\tilde{u}}. \quad (2.7)
$$

Changing variables from t to $\tilde{u}$ in (2.2), and using the

bounds $\tilde{u}_{\min} = -\tilde{s} \simeq -s$ and $\tilde{u}_{\max} = -M^2 \tilde{s}/s \simeq -M^2$, (2.7) contributes to dE/dx as

$$\frac{e^4}{4\pi} \int_k \frac{n_B(k)}{2k} \int_{\tilde{u}_{\min}}^{\tilde{u}_{\max}} \frac{d\tilde{u}}{-\tilde{u}} \simeq \frac{e^4 T^2}{96\pi} \left[\ln \frac{ET}{M^2} + \mathcal{O}(1) \right], \quad (2.8)$$

where we used

$$\int_k \frac{n_B(k)}{2k} = \frac{T^2}{24}. \quad (2.9)$$

Adding (2.5) and (2.8) we obtain the muon energy loss to logarithmic accuracy,

$$\frac{dE^{(\mu)}}{dx} = \frac{e^4 T^2}{48\pi} \left[\ln \frac{ET}{m_D^2} + \frac{1}{2} \ln \frac{ET}{M^2} + \mathcal{O}(1) \right]. \quad (2.10)$$

2. Heavy-quark energy loss in a QGP

The heavy-quark collisional energy loss arises from elastic scattering off thermal quarks [Fig. 1(a)] and gluons [Figs. 1(b)–1(d)]. Compared to the QED case, there is one additional amplitude [Fig. 1(d)], corresponding to t-channel scattering off thermal gluons. Using the cross sections [20] corresponding to the QCD amplitudes of Fig. 1 (with tree-level gluon propagators in the t-channel amplitudes), we can easily identify the origin of a logarithmic enhancement in the QCD analog of (2.2). There is a soft logarithm $\sim \int dt/t$ arising from $|\mathcal{M}_q|^2$ and $|\mathcal{M}_g^t|^2$, and a collinear logarithm $\sim \int d\tilde{u}/\tilde{u}$ from $|\mathcal{M}_g^u|^2$. The interference terms are not enhanced by any logarithm—they however contribute to the constant to be evaluated in Sec. II B.

The heavy-quark energy loss is readily derived from the QED result as follows. The contribution from $|\mathcal{M}_q|^2$ is obtained by multiplying (2.5) by the number of quark flavors n_f and by the color factor $(N_c^2 - 1)/(4N_c) = 2/3$. To get the t-channel contribution from $|\mathcal{M}_g^t|^2$ for scattering off thermal gluons, we also start from (2.5) and multiply by $1/2$ since contrary to the electron, a gluon is its own antiparticle. This factor is compensated by a factor 2 arising from the difference between bosons and fermions when performing the integral over k [compare (2.6) and (2.9)]. The color factor for this contribution is $(N_c^2 - 1)/2 = 4$. Finally, the u-channel contribution from $|\mathcal{M}_g^u|^2$ is obtained from (2.8) by multiplying by the color factor $C_F^2 = 16/9$. Introducing the QCD coupling by $e^2 \to g^2 = 4\pi\alpha_s$, we obtain the heavy-quark energy loss in a QGP by summing all contributions,

$$\frac{dE}{dx} = \frac{4\pi\alpha_s^2 T^2}{3} \left[\left(1 + \frac{n_f}{6} \right) \ln \frac{ET}{m_D^2} + \frac{2}{9} \ln \frac{ET}{M^2} + c(n_f) \right]. \quad (2.11)$$

Here $m_D^2 = 4\pi\alpha_s T^2 (1 + n_f/6)$ [18] is the Debye mass squared in the QGP. The constant $c(n_f) \sim \mathcal{O}(1)$ is evaluated in the next section, see (2.17).

B. The constant beyond leading logarithms

In the QED case, the constant $\sim \mathcal{O}(1)$ in (2.10) was determined in Ref. [17]. Here we infer from the QED calculation the constant $c(n_f)$ appearing in the QCD expression (2.11).

The logarithms $\ln(ET/m_D^2)$ and $\ln(ET/M^2)$ arise from the ranges $m_D^2 \ll |t| \ll s$ and $M^2 \ll |\tilde{u}| \ll s$, with $s \sim ET$. This is why they could be easily obtained in Sec. II A, using approximate expressions for the squared amplitudes [see (2.4) and (2.7)] in those kinematical domains. Controlling the constant next to the leading logarithms, however, requires considering the complete phase space $0 \le |t| \le |t_{\min}| \simeq s$ and $|\tilde{u}_{\max}| \simeq M^2 \le |\tilde{u}| \le |\tilde{u}_{\min}| \simeq s$.

In order to treat correctly the domain $|t| \sim m_D^2$, it is convenient to introduce an intermediate scale $t^\star$ chosen as $m_D^2 \ll |t^\star| \ll T^2$ [17]. The contribution to dE/dx from $|t| < |t^\star|$ is determined by the HTL self-energy of the exchanged gluon in Figs. 1(a) and 1(d), similar to the QED case where it depends on the photon HTL self-energy [and where only Fig. 1(a) contributes]. In both cases, the HTL self-energies have the same form, up to the replacement of the QED Debye mass by its QCD counterpart. Introducing the overall color factor $C_F = 4/3$, the QCD result is thus inferred from [17] to be

$$\frac{dE}{dx} \bigg|_{|t| < |t^\star|} = \frac{\alpha_s m_D^2}{3} \left[\ln \frac{|t^\star|}{m_D^2} + \ln 2 \right]. \quad (2.12)$$

We stress that the latter equation is valid *beyond* logarithmic accuracy, i.e. the constant next to the leading logarithm (written as ln2 here) is meaningful.

The contribution from $|t| > |t^\star|$ can be evaluated by using tree-level internal propagators in the amplitudes of Fig. 1. [Since $|\tilde{u}| \ge M^2 \gg T^2$, HTL corrections to the internal quark propagator in Figs. 1(b) and 1(c) are irrelevant.] However, in order to control the constant beyond the leading logarithms, accurate expressions for the tree-level cross sections have to be used.

For the contribution from scattering off light quarks [Fig. 1(a)], the calculation is similar to the QED case, which is done by using

$$\sum |\mathcal{M}_q|^2 \propto \left[\frac{\tilde{s}^2}{t^2} + \frac{s}{t} + \frac{1}{2} \right] \quad (2.13)$$

instead of (2.4), and by performing the integrals in (2.2) with $|t^\star| \le |t| \le s$. The QCD result reads

$$\frac{dE_q}{dx} \bigg|_{|t| > |t^\star|} = \frac{4\pi\alpha_s^2 T^2}{3} \frac{n_f}{6} \left[\ln \frac{8ET}{|t^\star|} - \frac{3}{4} + c \right], \quad (2.14)$$

where $c = \zeta'(2)/\zeta(2) - \gamma \simeq -1.147$, with $\gamma \simeq 0.577$ being Euler's constant.

Similarly, scattering off gluons [Figs. 1(b)–1(d)] should in principle be evaluated with the exact tree-level cross section [20]. However, in the limit $s \sim ET \gg M^2$ we are considering, this cross section can be approximated as

$$\sum |\mathcal{M}_g|^2 \propto \left[\frac{\tilde{s}^2}{t^2} + \frac{s}{t} + \frac{1}{2}\right] + \frac{2}{9}\left[\frac{-\tilde{u}}{\tilde{s}} + \frac{\tilde{s}}{-\tilde{u}}\right]. \quad (2.15)$$

The contribution of the first term of (2.15) to dE/dx is evaluated as for scattering off quarks [see (2.13)], except for the factor $n_B(k)$ instead of $n_F(k)$ in (2.2). Up to the color factor 2/9, the second term of (2.15) has the same form as QED Compton scattering, and its contribution to dE/dx can be directly obtained from [17]. Summing the two contributions we get

$$\frac{dE_g}{dx}\bigg|_{|t|>|t^\star|} = \frac{4\pi\alpha_s^2 T^2}{3}\left[\left(\ln\frac{4ET}{|t^\star|} - \frac{3}{4} + c\right) + \frac{2}{9}\left(\ln\frac{4ET}{M^2} - \frac{5}{6} + c\right)\right]. \quad (2.16)$$

Adding now the contributions from (2.12), (2.14), and (2.16) we obtain for the constant $c(n_f)$ in (2.11)

$$c(n_f) = an_f + b \simeq 0.146n_f + 0.050, \quad (2.17)$$

where the exact values of a and b are $a = (2/3)\ln 2 - 1/8 + c/6$ and $b = (31/9)\ln 2 - 101/108 + 11c/9$.

III. IMPLEMENTING RUNNING COUPLING

Implementing the running of α_s in the calculation of dE/dx has already been done in Ref. [21] for the logarithmically enhanced contribution from t-channel exchange. Here we will generalize the argument to the collinear logarithm stemming from the u-channel. We also discuss the influence of running on the term $\propto \alpha_s^2 c(n_f)$ in (2.11), which allows us to define precisely the level of accuracy of our final result, see (3.7).

We first note that the QCD result (2.11) [together with (2.17)] obtained with a fixed coupling suffers from a lack of predictability. Indeed, the fixed coupling calculation does not specify at which scale α_s should be evaluated. As in PQCD calculations at zero temperature, "fixing the scale" in α_s demands to calculate the next order in the perturbative series for the observable of interest. This next-to-leading order (NLO) generally brings large logarithms, which can be "absorbed," via renormalization, by setting the scale of α_s in the leading order result. In the case of t-channel scattering [Figs. 1(a) and 1(d)], the *vacuum* contributions to the self-energy and vertex corrections conspire to yield a logarithmic dependence on the invariant transfer t. In order to avoid a large NLO contribution, one must set the scale of α_s to $\sim\mathcal{O}(|t|)$ [22].

In order to obtain a predictive result for the t-channel contribution to dE/dx, we consider the term derived in Sec. II A, which is logarithmically enhanced with a fixed coupling, and recalculate it with a running coupling $\alpha_s(|t|)$. The procedure is rather trivial,

$$\alpha_s^2 \int_{m_D^2}^{ET} \frac{d|t|}{|t|} \longrightarrow \int_{m_D^2}^{ET} \frac{d|t|}{|t|} \alpha_s^2(|t|). \quad (3.1)$$

Using

$$\alpha_s(|t|) = [4\pi\beta_0 \ln(|t|/\Lambda^2)]^{-1}, \quad (3.2)$$

where $\beta_0 = (11 - \frac{2}{3}n_f)/(4\pi)^2$ is the leading coefficient of the QCD β-function, we see that using a running coupling amounts to perform the replacement

$$\alpha_s^2 \ln\frac{ET}{m_D^2} \longrightarrow \alpha_s(m_D^2)\alpha_s(ET)\ln\frac{ET}{m_D^2}. \quad (3.3)$$

As noted in [21], the latter result becomes E independent in the $E \to \infty$ limit, where the logarithmic enhancement of the fixed coupling result (2.11) is invalid.

The accuracy of the above procedure is inferred by noting that the scale of α_s in the rhs of (3.1) can be in principle chosen as $C|t|$ rather than $|t|$, with C a constant of order unity. This implies an ambiguity of order $\alpha_s^3 \ln ET/m_D^2$ to the rhs of (3.3) [24]. Thus specifying the term $\propto \alpha_s^2 c(n_f)$ in (2.11) is meaningful provided we assume

$$\alpha_s(m_D^2)\ln\frac{ET}{m_D^2} \ll 1 \Longleftrightarrow \ln\frac{ET}{m_D^2} \ll \ln\frac{m_D^2}{\Lambda^2}. \quad (3.4)$$

For the u-channel contribution [Fig. 1(c)] to the cross section, α_s should be evaluated at a scale $\sim\mathcal{O}(|\tilde{u}|)$. Similar to the above discussion, the logarithmic integral appearing in (2.8) is modified to $\sim \int d|\tilde{u}|\alpha_s^2(|\tilde{u}|)/|\tilde{u}|$, and the collinear logarithm in (2.11) becomes

$$\alpha_s^2 \ln\frac{ET}{M^2} \longrightarrow \alpha_s(M^2)\alpha_s(ET)\ln\frac{ET}{M^2}. \quad (3.5)$$

Since $M \gg m_D$ and we already assumed (3.4), the uncertainty when choosing $|\tilde{u}|$ as the scale in α_s is of relative order $\alpha_s(M^2)\ln ET/M^2 \ll 1$ compared to the contribution $\propto \alpha_s^2 c(n_f)$ in (2.11).

After we have shown how the leading logarithmic terms in the fixed coupling result (2.11) are modified when the running is taken into account [see (3.3) and (3.5)], we now discuss the modification of the constant term $\propto \alpha_s^2 c(n_f)$. Since this term is the difference between the full result (2.11) and the leading logarithmic terms, it should be clear from Sec. II B that it is determined by integrals over t dominated by either $|t| \sim m_D^2$ or $|t| \sim ET$, and by integrals over $\tilde{u}$ dominated by $|\tilde{u}| \sim ET$ [25]. There is no logarithmic spread there. Thus with running coupling, the term $\alpha_s^2 c(n_f)$ should be evaluated at a scale chosen *arbitrarily* [26] between m_D^2 and ET.

We can now give the result for dE/dx of a fast heavy quark, at leading order in the running coupling,

$$\frac{dE}{dx} = \frac{4\pi T^2}{3}\left[\left(1 + \frac{n_f}{6}\right)\alpha_s(m_D^2)\alpha_s(ET)\ln\frac{ET}{m_D^2} + \frac{2}{9}\alpha_s(M^2)\alpha_s(ET)\ln\frac{ET}{M^2} + \alpha_s^2 c(n_f)\right]. \quad (3.6)$$

This result has been derived for $E \gg M^2/T$ and assuming (3.4). The constant $c(n_f)$ is given in (2.17). As discussed above, we are free to set the scale in $\alpha_s^2 c(n_f)$ as $\alpha_s^2 \rightarrow \alpha_s(m_D^2)\alpha_s(ET)$, in order to rewrite (3.6) as

$$\frac{dE}{dx} = \frac{4\pi T^2}{3} \alpha_s(m_D^2)\alpha_s(ET)\left[\left(1 + \frac{n_f}{6}\right)\ln\frac{ET}{m_D^2} + \frac{2}{9}\frac{\alpha_s(M^2)}{\alpha_s(m_D^2)}\right.$$
$$\left. \times \ln\frac{ET}{M^2} + c(n_f) + \mathcal{O}\left(\alpha_s(m_D^2)\ln\frac{ET}{m_D^2}\right)\right], \quad (3.7)$$

where we displayed the order of neglected terms.

As a final remark, we note that when the logarithmic term $\propto \ln ET/m_D^2$ from t-channel exchange is dominant in (3.7), we obtain an interesting relation between the fast heavy-quark collisional loss and the gluon Debye mass,

$$\frac{dE}{dx} \simeq \frac{m_D^2}{3}\alpha_s(ET)\ln\frac{ET}{m_D^2}. \quad (3.8)$$

We used the self-consistent equation for the QCD Debye mass derived with running coupling [27],

$$m_D^2 = 4\pi\left(1 + \frac{n_f}{6}\right)\alpha_s(m_D^2)T^2. \quad (3.9)$$

Using (3.2), the $E \rightarrow \infty$ limit of (3.8) yields an upper bound for dE/dx,

$$\frac{dE}{dx} \leq \frac{m_D^2}{12\pi\beta_0} = \frac{4\pi}{33 - 2n_f}m_D^2. \quad (3.10)$$

For $n_f = 3$ and $m_D \simeq 0.66$ GeV (at $T = 0.2$ GeV, see below), this gives the bound $dE/dx \leq 1.0$ GeV/fm.

IV. A NUMERICAL ESTIMATE

Our final result (3.7) for the fast heavy-quark collisional energy loss is predictive: the scales at which to evaluate the different factors of α_s are determined, and the order of its theoretical uncertainty is known. Consider the condition (3.4), in the limit of very high energy E and very high temperature T, such that the logarithm $\ln ET/m_D^2 \sim \ln E/T$ is kept fixed. Then the neglected contributions in (3.7) are indeed small, due to $\alpha_s(m_D^2) \ll 1$ in the (very) high temperature limit. The explicit terms in (3.7) are then all meaningful, including that $\propto c(n_f)$.

In practice, say under RHIC conditions, the values of m_D^2 and ET are not very different (on a logarithmic scale) from Λ^2. Therefore, the condition (3.4) is not satisfied in the strict sense, and the theoretical uncertainty affecting (3.7) is rather large. We take $n_f = 3$ and the generic values (in GeV) $\Lambda = 0.2$, $T = 0.2$, $M = 1.3$ (charm quark), and $E = 20$. We use (3.2) and (3.9) and find $m_D \simeq 0.66$ GeV (in good agreement with lattice QCD calculations, cf. [27]), $\alpha_s(m_D^2) \simeq 0.58$ and $dE/dx \simeq 0.6$ GeV/fm. The separate contributions in the bracket of Eq. (3.7) read $[3.31 + 0.12 + 0.49]$, to be compared to the magnitude of neglected terms $\alpha_s(m_D^2)\ln ET/m_D^2 \simeq 1.29$.

Thus the theoretical uncertainty is larger than the collinear logarithm from u-channel exchange and than the constant $c(n_f)$ in (3.7)—making the approximation (3.8) reasonable under RHIC conditions. In the absence of an explicit calculation of the neglected terms, our result $dE/dx \simeq 0.6$ GeV/fm in the above conditions might be accurate only up to a factor ~ 2 or so, and should be considered at best as a sound estimate.

ACKNOWLEDGMENTS

[1] J. D. Bjorken, Fermilab Report No. PUB-82/59-THY, 1982.

[2] K. Adcox *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **88**, 022301 (2001); S. S. Adler *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **91**, 072301 (2003).

[3] C. Adler *et al.* (STAR Collaboration), Phys. Rev. Lett. **89**, 202301 (2002).

[4] S. S. Adler *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **96**, 032301 (2006).

[5] J. Bielcik *et al.* (STAR Collaboration), Nucl. Phys. **A774**, 697 (2006).

[6] S. Wicks, W. Horowitz, M. Djordjevic, and M. Gyulassy, Nucl. Phys. **A784**, 426 (2007).

[7] M. G. Mustafa and M. H. Thoma, Acta Phys. Hung. A **22**, 93 (2005).

[8] M. G. Mustafa, Phys. Rev. C **72**, 014905 (2005).

[9] A. K. Dutt-Mazumder, Jan-e Alam, P. Roy, and B. Sinha, Phys. Rev. D **71**, 094016 (2005).

[10] B. G. Zakharov, JETP Lett. **86**, 444 (2007).

[11] G. Y. Qin, J. Ruppert, C. Gale, S. Jeon, G. D. Moore, and M. G. Mustafa, Phys. Rev. Lett. **100**, 072301 (2008).

[12] A. Peshier, arXiv:0801.0595.

[13] M. H. Thoma and M. Gyulassy, Nucl. Phys. **B351**, 491 (1991).

[14] E. Braaten and M. H. Thoma, Phys. Rev. D **44**, R2625 (1991).

[15] P. B. Gossiaux, J. Aichelin, C. Brandt, T. Gousset, and S. Peigné, J. Phys. G **34**, S817 (2007).

[16] E. Braaten and M. H. Thoma, Phys. Rev. D **44**, 1298 (1991).

[17] S. Peigné and A. Peshier, Phys. Rev. D **77**, 014015 (2008).

[18] J. P. Blaizot and E. Iancu, Phys. Rep. **359**, 355 (2002).

[19] R. D. Pisarski, Physica (Amsterdam) **158A**, 146 (1989); E. Braaten and R. D. Pisarski, Phys. Rev. Lett. **64**, 1338 (1990); Nucl. Phys. **B337**, 569 (1990); **B339**, 310 (1990).

[20] B. L. Combridge, Nucl. Phys. **B151**, 429 (1979).

[21] A. Peshier, Phys. Rev. Lett. **97**, 212301 (2006).

[22] Setting this scale in the factor α_s^2 of the $2 \to 2$ cross section amounts to perform a specific all-order resummation, corresponding physically to the final heavy quark being accompanied by collinear gluon radiation. The true *exclusive* $2 \to 2$ cross section involving charged (colored) particle vanishes [23].

[23] Y. L. Dokshitzer, V. A. Khoze, A. H. Mueller, and S. I. Troian, *Basics Of Perturbative QCD* (Frontieres, Gif-sur-Yvette, France, 1991), p. 274 (basics of).

[24] Using the two-loop running coupling instead of the one-loop expression (3.2) yields similar corrections, provided we assume $\ln\ln(|t|/\Lambda^2) \sim \mathcal{O}(1)$.

[25] The absence of $\tilde{u}$-integrals dominated by $|\tilde{u}| \sim M^2$ (after subtraction of the logarithmic term $\propto \ln ET/M^2$) is due to the fact that the u-channel amplitude squared is *exactly* $\propto 1/|\tilde{u}|$ when $|\tilde{u}| \sim M^2$; see the last term of (2.15). This is in contrast with the t-channel amplitude squared in the region $|t| \sim m_D^2$, which is not exactly $\propto 1/t$ (when weighted by t), because of HTL complications. The latter lead to a remnant contribution dominated by $|t| \sim m_D^2$ after subtraction of the t-channel logarithmic term $\sim \ln ET/m_D^2$.

[26] Splitting the $\alpha_s^2 c(n_f)$ term as $c_1 \alpha_s^2(m_D^2) + c_2 \alpha_s^2(ET)$ is an easy task in our calculation. However, the distinction between $\alpha_s^2(m_D^2)$ and $\alpha_s^2(ET)$ would be meaningless since we neglect contributions of order $\alpha_s^3 \ln ET/m_D^2$.

[27] A. Peshier, arXiv:hep-ph/0601119.