Nucleon electromagnetic form factors in QCD

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(Received 17 March 2008; published 13 June 2008)

The nucleon electromagnetic form factors are calculated in a light cone QCD sum rules framework using the most general form of the nucleon interpolating current. Using two models for the distribution amplitudes, we predict the form factors. The predictions are also compared with existing experimental data. It is shown that our results describe remarkably well the existing experimental data.

DOI: 10.1103/PhysRevD.77.114014 PACS numbers: 14.20.Dh, 11.55.Hx, 13.40.Gp

I. INTRODUCTION

The nucleon electromagnetic (EM) form factors are the fundamental objects for understanding nucleons' internal structure. The internal structure of the nucleon is usually described in terms of the electromagnetic Dirac and Pauli form factors $F_1(q^2)$ and $F_2(q^2)$ or equivalently the electric and magnetic dipole Sachs form factors $G_E(q^2)$ and $G_M(q^2)$, respectively (for a recent status of experiments and phenomenology of the form factors see [1]).

Until a few years ago, the nucleon electromagnetic form factors were experimentally studied in unpolarized elastic electron-nucleon scattering through a virtual photon exchange. It is shown in the pioneering work [2] that the polarization effects, i.e., scattering of polarized electrons from a polarized target, can play an essential role for a more accurate determination of the nucleon electromagnetic form factors. The main result of [2] is that, unlike the unpolarized elastic cross section, which is proportional to the sum of squares of the form factors, the polarized cross section also contains interference terms of the form factors $G_E(q^2)$ and $G_M(q^2)$. Studying various polarization observables allows more accurate determination of these form factors.

Recent developments in experimental instruments allow production of polarized electron beams and polarized protons, which gives the opportunity for a more precise separation of the $G_E(q^2)$ and $G_M(q^2)$ form factors. The electron-proton scattering experiments, which are performed at Jefferson Laboratory using the polarized electrons and polarized proton, show strong deviation from the theoretical predictions [3–6], i.e., the ratio $F_2(q^2)/F_1(q^2)$ does not behave as is expected from previous experiments and as is predicted by the perturbative QCD (for a review see [7] and references therein). For understanding this unexpected result, some model-independent nonperturbative method is needed. Among all existing nonperturbative approaches the QCD sum rules approach is one of the more attractive and powerful methods, because it is based on the fundamental QCD Lagrangian.

The goal of our work is the calculation of the electromagnetic form factors of the nucleon using the light cone QCD sum rule (LCQSR) and general form of the interpolating current for the nucleon. In this approach the form factors of the nucleons are expressed in terms of distribution amplitude of the nucleon. Note that this problem is investigated for the Ioffe current in the framework of the LCQSR in [8] and the traditional sum rules in [9]. In [10], an improved version of the Chernyak-Zhitnitsky current is used. The paper is organized in following way: In Sec. II, we present the result for the nucleon electromagnetic form factors in the LCQSR method. Section III is devoted to the numerical analysis, discussion, and conclusion.

II. ELECTROMAGNETIC FORM FACTORS OF THE NUCLEON IN LCQSR

In this section EM form factors of the nucleon are calculated within the light cone QCD sum rules method. The electromagnetic form factors of the nucleon are defined by the matrix element of the electromagnetic current $J_\lambda^{\rm el}$ between the initial and final nucleon states $\langle N(p') \mid J_\lambda^{\rm el} \mid N(p)\rangle$. The most general form of this matrix element satisfying the Lorentz invariance and electromagnetic current conservation is

$$\langle N(p') \mid J_\lambda^{\rm el}(0) \mid N(p)\rangle = \bar{N}(p')\left[\gamma_\lambda F_1(Q^2) - \frac{i}{2m_N}\sigma_{\lambda\nu}q^\nu F_2(Q^2)\right]N(p), \quad (1)$$

where $Q^2 = -q^2$ is the negative of the square of the virtual photon momentum, $q = p - p'$, $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ and F_1 and F_2 are the Dirac and Pauli form factors, respectively.

Another set of nucleon form factors is the so-called Sachs form factors, which are defined in terms of the $F_1(Q^2)$ and $F_2(Q^2)$ as follows:

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2),$$
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2}F_2(Q^2), \quad (2)$$

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at the static limit, i.e., at $Q^2 = 0$, $G_E^P(0) = 1$, $G_E^n(0) = 0$, $G_M^P(0) = \mu_P = 2.792\,847\,337(29)$, and $G_M^n(0) = \mu_n = -1.913\,042\,72(45)$, where μ_P and μ_n are the anomalous magnetic moments of the proton and neutron in units of the Bohr magneton.

After these preliminary remarks, we proceed to calculate the electromagnetic form factors of the nucleon in LCQSR. The basic object of the LCQSR is a suitably chosen correlation function. In this study, it is chosen as

$$\Pi_\lambda(p, q) = i \int d^4x e^{iqx} \langle 0 \mid T\{J^N(0) J_\lambda^{\mathrm{el}}(x)\} \mid N(p)\rangle, \quad (3)$$

where J_λ^{el} is the electromagnetic current and J^N is the nucleon interpolating current. The correlation function given in Eq. (3) describes the transition of the nucleon $N(p)$ to the nucleon $N(p-q)$ via the electromagnetic current. The interpolating current for the nucleon is not uniquely defined. In principle, any operator which has the quantum numbers of the nucleon can be used. In this work, the interpolating current is chosen as

$$J^N(x) = 2\varepsilon^{abc} \sum_{l=1}^{2} (u^{Ta}(x) C A_1^l d^b(x)) A_2^l u^c(x), \quad (4)$$

where $A_1^1 = I$, $A_1^2 = A_2^1 = \gamma_5$, $A_2^2 = \beta$, and C is the charge conjugation operator, and a, b, c are the color indices. The parameter β is arbitrary, and the choice $\beta = -1$ corresponds to the Ioffe current which is also commonly used in the literature. The electromagnetic current is

$$J_\lambda^{\mathrm{el}}(x) = e_u \bar{u} \gamma_\lambda u + e_d \bar{d} \gamma_\lambda d. \quad (5)$$

The main idea of the LCQSR method is to calculate the correlation function, on the one hand, in terms of the form factors at the hadron level and, on the other hand, in terms of the quark and gluon degrees of freedom. Equating two representations of the correlation function and performing a Borel transformation in order to suppress the contributions of the higher states and continuum, we get sum rules for the EM form factors of the nucleon.

Let us first calculate the physical part of the correlator (3). The contribution of the nucleon to the correlation function (3) is given by

$$\Pi_\lambda(p, q) = \sum_s \frac{\langle 0 \mid J^N(0) \mid N(p', s)\rangle \langle N(p', s) \mid J_\lambda^{\mathrm{el}}(0) \mid N(p)\rangle}{m_N^2 - p'^2}. \quad (6)$$

The matrix element $\langle 0 \mid J^N(0) \mid N(p', s)\rangle$ in (6) is determined in the following way:

$$\langle 0 \mid J^N(0) \mid N(p', s)\rangle = \lambda_N N(p', s), \quad (7)$$

where λ_N is the residue of the nucleon. The matrix element $\langle N(p', s) \mid J_\lambda^{\mathrm{el}}(0) \mid N(p)\rangle$ is parameterized in terms of the form factors F_1 and F_2 via Eq. (1). Summing over spins of the nucleons

$$\sum_s N(p', s) \bar{N}(p', s) = \not{p}' + m_N, \quad (8)$$

and using Eqs. (1), (6), and (7), we obtain the following expression for the contribution of the nucleon to the correlation function

$$\Pi_\lambda(p, q) = \frac{\lambda_N}{m_N^2 - p'^2} (\not{p}' + m_N) \bigg[\gamma_\lambda F_1(Q^2) - \frac{i}{2m_N} \sigma_{\lambda\nu} q^\nu F_2(Q^2) \bigg] N(p) + \cdots, \quad (9)$$

where $\cdots$ stands for the contributions to the correlation functions from the higher states and continuum. It follows from expression (9) that the correlation function contains numerous structures and in principle all of them can be used in determination of the electromagnetic form factors of nucleons. In further analysis, we choose the independent structures containing p_λ and $p_\lambda \not{q}$ for obtaining F_1 and F_2, respectively.

The theoretical part of the correlation function can be calculated in LCQSR in terms of the nucleon distribution amplitudes (DA's) when the momentum squared, p'^2, is in the deep Euclidean region. These nucleon DA's for all three quarks have been studied in great detail in [8,10,11]. Using the explicit expression for the currents and carrying out all contractions, the correlation function takes the form

$$\begin{aligned}(\Pi_\lambda)_\rho = \frac{i}{2} \int d^4x e^{iqx} \sum_{l=1}^{2} \{ & e_u (CA_1^l)_{\alpha\gamma} [A_2^l S_u(-x) \gamma_\lambda]_{\rho\phi} 4\epsilon^{abc} \langle 0 | u_\alpha^a(0) u_\phi^b(x) d_\gamma^c(0) | N(p)\rangle \\ & + e_u (A_2^l)_{\rho\alpha} [(CA_1^l)^T S_u(-x) \gamma_\lambda]_{\gamma\phi} 4\epsilon^{abc} \langle 0 | u_\alpha^a(0) u_\phi^b(x) d_\gamma^c(0) | N(p)\rangle \\ & + e_d (A_2^l)_{\rho\phi} [CA_1^l S_d(-x) \gamma_\lambda]_{\alpha\gamma} 4\epsilon^{abc} \langle 0 | u_\alpha^a(0) u_\phi^b(0) d_\gamma^c(x) | N(p)\rangle \}, \end{aligned} \quad (10)$$

in x representation, where λ is a Lorentz index, and α, γ, ρ, and ϕ are spinor indices. $S(x)$ is the full light quark propagator expanded near the light cone [12]:

$$S(x) = \frac{i\not{x}}{2\pi^2 x^4} - \langle qq\rangle\left(1 + \frac{m_0^2 x^2}{16}\right) - ig_s\int_0^1 dv\left[\frac{\not{x}}{16\pi^2 x^2}G_{\mu\nu}\sigma^{\mu\nu} - vx^\mu G_{\mu\nu}\gamma^\nu\frac{i}{4\pi^2x^2}\right], \quad (11)$$

where $m_0^2 = (0.8 \pm 0.2)\ \mathrm{GeV}^2$ and $G_{\mu\nu}$ is the gluon field strength tensor. The terms proportional to the gluon strength tensor can contribute to four- and five-particle distribution functions, but they are expected to be very small [8,10,11], and for this reason we will neglect these amplitudes in further analysis. The terms proportional to $\langle\bar{q}q\rangle$ can also be omitted because Borel transformation eliminates these terms, and hence only the first term in Eq. (11) is relevant for our discussion. It follows from Eq. (10) that for the calculation of $\Pi_\lambda(p, q)$ we need the matrix element

$$\langle 0 | 4\epsilon^{abc}u_\alpha^a(a_1x)u_\phi^b(a_2x)d_\gamma^c(a_3x) | N(p)\rangle. \quad (12)$$

It is shown in [11] that the general Lorentz decomposition of this matrix element is symmetric with respect to the interchange of the momentum fractions of the u quarks:

$$\langle 0 | 4\epsilon^{abc}u_\alpha^a(a_1x)u_\phi^b(a_2x)d_\gamma^c(a_3x) | N(p)\rangle = \sum K\Gamma_1^{\alpha\phi}(\Gamma_2 N(p))^\gamma, \quad (13)$$

where $N(p)$ on the right is the nucleon spinor, $\Gamma_{1,2}$ are certain Dirac structures over which the sum is carried out, a_i are positive numbers which satisfy $a_1 + a_2 + a_3 = 1$, and K are the distribution amplitudes, depending on eight nonperturbative parameters. Explicit expressions of all DA'S and the values of eight nonperturbative parameters can be found in [8,10,11,13].

Omitting the details of calculations of the theoretical part, choosing the coefficients of the structures p_λ, and $p_\lambda\not{q}$, equating both representation of the correlation function, and applying the Borel transformation with respect to the variable $p'^2 = (p-q)^2$, we obtain the following sum rules for the form factors F_1 and F_2:

$$\begin{aligned}
F_1(Q^2) = \frac{-1}{2\lambda_N}e^{m_N^2/M_B^2}\bigg\{& e_u m_N\int_{t_0}^1 dx_2\int_0^{1-x_2}dx_1 e^{-s(x_2,Q^2)/M_B^2}[2\mathcal{H}_{5,-7}(x_i)(1-\beta) + 4(\mathcal{H}_{17}(x_i) - 2\mathcal{H}_{19}(x_i))(1+\beta)] \\
&+ e_u m_N\int_{t_0}^1 dx_2\int_0^{1-x_2}dx_1\int_{t_0}^{x_2}\frac{dt_1}{t_1}e^{-s(t_1,Q^2)/M_B^2}\bigg(-2[\mathcal{H}_{20,-18}(x_i)(1+\beta) - \mathcal{H}_6(x_i)(-1+\beta)] - \frac{1}{M_B^2} \\
&\times[\{2\mathcal{H}_{20,18}(x_i)(1+\beta)(Q^2 + s(t_1,Q^2) + m_N^2(-1+t_1))\} + m_N^2\{\mathcal{H}_{15,-14}(x_i)t_1(1-\beta) \\
&- 4\mathcal{H}_{21,24}(x_i)t_1(1+\beta) + 2\mathcal{H}_{10}(x_i)(-1+\beta)(t_1 - x_2) + 2(\mathcal{H}_{16}(x_i)(-1+\beta) + 2\mathcal{H}_{24}(x_i)(1+\beta))x_2\}]\bigg) \\
&- e_u m_N\int_{t_0}^1 dx_2\int_0^{1-x_2}dx_1 e^{-s_0/M_B^2}\frac{t_0}{Q^2 + m_N^2 t_0^2}(2\mathcal{H}_{20,18}(x_i)(1+\beta)(Q^2 + s_0 + m_N^2(-1+t_0)) \\
&+ m_N^2[\{\mathcal{H}_{-8,9}(x_i)(1-\beta) - (3\mathcal{H}_{21,24}(x_i) + 8\mathcal{H}_{23}(x_i))(1+\beta)\}t_0 + 2\mathcal{H}_{10}(x_i)(-1+\beta)(t_0 - x_2) \\
&+ 2(\mathcal{H}_{16}(x_i)(-1+\beta) + \mathcal{H}_{24}(x_i)(1+\beta))x_2]) + e_d\eta_1'(Q^2,\beta) + e_u\eta_1(Q^2,\beta)\bigg\},
\end{aligned} \quad (14)$$

$$\begin{aligned}
F_2(Q^2) = \frac{-m_N}{\lambda_N}e^{m_N^2/M_B^2}\bigg\{& e_u\int_{t_0}^1 dx_2\int_0^{1-x_2}dx_1 e^{-s(x_2,Q^2)/M_B^2}\left[\frac{2\mathcal{H}_5(x_i)(-1+\beta)}{x_2}\right](x_i) - e_u m_N^2\int_{t_0}^1 dx_2\int_0^{1-x_2}dx_1 \\
&\times\int_{t_0}^{x_2}\frac{dt_1}{t_1}e^{-s(t_1,Q^2)/M_B^2}\bigg(\frac{1}{M_B^2}[\mathcal{H}_{8,-9}(x_i)(1-\beta) + 2(\mathcal{H}_{18,20}(x_i) + 2\mathcal{H}_{21,22}(x_i) + 4\mathcal{H}_{23}(x_i))(1+\beta)] \\
&- \frac{4}{M_B^2 t_1}[\mathcal{H}_{22}(x_i)(1+\beta)x_2]\bigg) + e_u m_N^2\int_{t_0}^1 dx_2\int_0^{1-x_2}dx_1 e^{-s_0/M_B^2}\bigg(\frac{1}{Q^2 + m_N^2 t_0^2}[\mathcal{H}_{8,-9}(x_i)(-1+\beta)t_0 \\
&- 2(\mathcal{H}_{18,20}(x_i) + 2\mathcal{H}_{21,22}(x_i) + 4\mathcal{H}_{23}(x_i))(1+\beta)t_0 + 4\mathcal{H}_{22}(x_i)(1+\beta)x_2]\bigg) \\
&+ e_d\eta_2'(Q^2,\beta) + e_u\eta_2(Q^2,\beta)\bigg\},
\end{aligned} \quad (15)$$

where

$$\mathcal{F}(x_i) = \mathcal{F}(x_1, x_2, 1 - x_1 - x_2), \qquad \mathcal{F}(x_i') = \mathcal{F}(x_1, 1 - x_1 - x_3, x_3), \qquad s(y, Q^2) = (1-y)m_N^2 + \frac{(1-y)}{y}Q^2, \quad (16)$$

with $t_0(s_0, Q^2)$ being the solution of the equation $s(t_0, Q^2) = s_0$, and

$$\begin{aligned}
\eta_1(Q^2,\beta) = m_N\bigg\{&\int_{t_0}^1 dx_3\int_0^{1-x_3}dx_1 e^{-s(x_3,Q^2)/M_B^2}[(\mathcal{H}_{1,17,3}(x_i)-2\mathcal{H}_{19}(x_i))(1+\beta)+\mathcal{H}_{13,7}(x_i)(-1+\beta)]\\
&+\int_{t_0}^1 dx_3\int_0^{1-x_3}dx_1\int_{t_0}^{x_3}dt_1 e^{-s(t_1,Q^2)/M_B^2}\bigg(\frac{1}{M_B^4t_1}[-\mathcal{H}_{22}(x_i)m_N^2(-m_N^2+Q^2+s(t_1,Q^2))(1+\beta)x_3]+\frac{1}{M_B^4}\\
&\times[\mathcal{H}_{22}(x_i)m_N^2(m_N^2(-1+2t_1-2x_3)+Q^2+s(t_1,Q^2))(1+\beta)]+\frac{1}{2M_B^2t_1}[-(m_N^2-Q^2-s(t_1,Q^2))\\
&\times\{(\mathcal{H}_{18}(x_i)-3\mathcal{H}_{20}(x_i))(1+\beta)+2\mathcal{H}_{6,12}(x_i)(-1+\beta)\}+2(2\mathcal{H}_{22}(x_i)-\mathcal{H}_{24}(x_i))m_N^2(1+\beta)x_3]\\
&+\frac{1}{M_B^2}[m_N^2\{\mathcal{H}_{-12,15,-6,9}(x_i)(1-\beta)+(\mathcal{H}_{18,-2,24,4,21}(x_i)+2\mathcal{H}_{-20,-22,23}(x_i))(1+\beta)\}]+\frac{1}{t_1}\\
&\times[\mathcal{H}_{12,6}(x_i)(1-\beta)+\mathcal{H}_{-18,20}(x_i)(1+\beta)]\bigg)+\int_{t_0}^1 dx_3\int_0^{1-x_3}dx_1 e^{-s_0/M_B^2}\bigg(\frac{1}{M_B^2t_0(Q^2+m_N^2t_0^2)}\\
&\times[\mathcal{H}_{22}(x_i)m_N^2(1+\beta)t_0^2(Q^2+s_0+m_N^2(-1+2t_0))(t_0-x_3)]+\frac{1}{(Q^2+m_N^2t_0^2)^3}\\
&\times[2\mathcal{H}_{22}(x_i)m_N^4(1+\beta)t_0^4(Q^2+s_0+m_N^2(-1+2t_0))(t_0-x_3)]-\frac{1}{(Q^2+m_N^2t_0^2)^2}\\
&\times[\mathcal{H}_{22}(x_i)m_N^2(1+\beta)t_0^2((Q^2+s_0)(2t_0-x_3)+m_N^2(2t_0(-1+3t_0-2x_3)+x_3))]+\frac{1}{t_0(Q^2+m_N^2t_0^2)}\\
&\times[2\mathcal{H}_{22}(x_i)m_N^2(1+\beta)t_0^2x_3]+\frac{1}{2(Q^2+m_N^2t_0^2)}[-\mathcal{H}_{20}(x_i)(1+\beta)t_0\{3(Q^2+s_0)+m_N^2(-3+4t_0)\}\\
&+2\mathcal{H}_{6,12}(x_i)(-1+\beta)(Q^2+s_0+m_N^2(-1+t_0))t_0+2\mathcal{H}_{24}(x_i)m_N^2(1+\beta)t_0(t_0-x_3)+2\mathcal{H}_{18}(x_i)\\
&\times(1+\beta)t_0(Q^2+s_0+m_N^2(-1+2t_0))+2m_N^2\mathcal{H}_{9,-15}(x_i)(-1+\beta)t_0^2+2m_N^2(\mathcal{H}_{4,-2,21}(x_i)\\
&+2\mathcal{H}_{23,-22}(x_i))(1+\beta)t_0^2]\bigg)\bigg\},
\end{aligned}\tag{17}$$

$$\begin{aligned}
\eta_2(Q^2,\beta) = &\int_{t_0}^1 dx_3\int_0^{1-x_3}dx_1 e^{-s(x_3,Q^2)/M_B^2}\bigg[\frac{\mathcal{H}_{11,-5}(x_i)(-1+\beta)}{x_3}\bigg]+m_N\int_{t_0}^1 dx_3\int_0^{1-x_3}dx_1\int_{t_0}^{x_3}dt_1 e^{-s(t_1,Q^2)/M_B^2}\\
&\times\bigg(\frac{-2}{M_B^4}[\mathcal{H}_{22}(x_i)m_N^3(1+\beta)]+\frac{1}{M_B^4t_1}[\mathcal{H}_{22}(x_i)m_N(1+\beta)(-Q^2-s(t_1,Q^2)+m_N^2(1+2x_3))]+\frac{1}{M_B^4t_1^2}\\
&\times[\mathcal{H}_{22}(x_i)m_N(1+\beta)(Q^2+s(t_1,Q^2)-m_N^2)x_3]-\frac{3}{M_B^2t_1^2}[\mathcal{H}_{22}(x_i)m_N(1+\beta)x_3]+\frac{m_N}{M_B^2t_1}\\
&\times[\mathcal{H}_{12,15,6,-9}(x_i)(-1+\beta)+(\mathcal{H}_{2,-20,-21,-4}(x_i)+3\mathcal{H}_{22}(x_i)-2\mathcal{H}_{23}(x_i))(1+\beta)]\bigg)+m_N\int_{t_0}^1 dx_3\\
&\times\int_0^{1-x_3}dx_1 e^{-s_0/M_B^2}\bigg(-\frac{m_N}{M_B^2(Q^2+m_N^2t_0^2)}[\mathcal{H}_{22}(x_i)(1+\beta)(Q^2+s_0+m_N^2(-1+2t_0))(t_0-x_3)]\\
&+\frac{1}{(Q^2+m_N^2t_0^2)^3}[-2\mathcal{H}_{22}(x_i)m_N^3(1+\beta)t_0^3(Q^2+s_0+m_N^2(-1+2t_0))(t_0-x_3)]+\frac{1}{(Q^2+m_N^2t_0^2)^2}\\
&\times[\mathcal{H}_{22}(x_i)m_N(1+\beta)t_0^2(Q^2+s_0+m_N^2(-1+4t_0-2x_3))]+\frac{m_N}{(Q^2+m_N^2t_0^2)}[\mathcal{H}_{-12,-15,-6,9}(x_i)(1-\beta)\\
&+(\mathcal{H}_{2,-20,-21,22,-4}(x_i)-2\mathcal{H}_{23}(x_i))(1+\beta)t_0-\mathcal{H}_{22}(x_i)(1+\beta)(3x_3-2t_0)]\bigg),
\end{aligned}\tag{18}$$

and $\eta_i'(Q^2,\beta)$, $(i=1,2)$ are obtained from $\eta_i(Q^2,\beta)$ by replacing x_3 with x_2 and replacing $\mathcal{F}(x_i)$ with $\mathcal{F}(x_i')$ in the integrals. In the above equations, we have used the shorthand notations for the functions $\mathcal{H}_{\pm i,\pm j,\ldots}=\pm\mathcal{H}_i\pm\mathcal{H}_j\ldots$, and $\mathcal{H}_i$ are defined in terms of the distribution amplitudes as follows:

$$\mathcal{H}_1 = S_1, \quad \mathcal{H}_2 = S_{1,-2}, \quad \mathcal{H}_3 = P_1, \quad \mathcal{H}_4 = P_{1,-2}, \quad \mathcal{H}_5 = V_1, \quad \mathcal{H}_6 = V_{1,-2,-3}, \quad \mathcal{H}_7 = V_3,$$
$$\mathcal{H}_8 = -2V_{1,-5} + V_{3,4}, \quad \mathcal{H}_9 = V_{4,-3}, \quad \mathcal{H}_{10} = -V_{1,-2,-3,-4,-5,6}, \quad \mathcal{H}_{11} = A_1, \quad \mathcal{H}_{12} = -A_{1,-2,3},$$
$$\mathcal{H}_{13} = A_3, \quad \mathcal{H}_{14} = -2A_{1,-5} - A_{3,4}, \quad \mathcal{H}_{15} = A_{3,-4}, \quad \mathcal{H}_{16} = A_{1,-2,3,4,-5,6}, \quad \mathcal{H}_{17} = T_1,$$
$$\mathcal{H}_{18} = T_{1,2} - 2T_3, \quad \mathcal{H}_{19} = T_7, \quad \mathcal{H}_{20} = T_{1,-2} - 2T_7, \quad \mathcal{H}_{21} = -T_{1,-5} + 2T_8, \quad \mathcal{H}_{22} = T_{2,-3,-4,5,7,8},$$
$$\mathcal{H}_{23} = T_{7,-8}, \quad \mathcal{H}_{24} = -T_{1,-2,-5,6} + 2T_{7,8}, \tag{19}$$

where for any distribution amplitudes, $X_{\pm i,\pm j,\ldots} = \pm X_i \pm X_j \ldots$. The overlap amplitude of the nucleon interpolating current with the nucleon is determined from the mass sum rules, and its expression is [14]

$$\lambda_N^2 = e^{m_N^2/M_B^2}\Big\{\frac{M_B^6}{256\pi^4}E_2(x)(5+2\beta+\beta^2) + \frac{\langle\bar{u}u\rangle}{6}[-6(1-\beta^2)\langle\bar{d}d\rangle + (-1+\beta)^2\langle\bar{u}u\rangle] - \frac{m_0^2}{24M_B^2}\langle\bar{u}u\rangle[-12(1-\beta^2)\langle\bar{d}d\rangle + (-1+\beta)^2\langle\bar{u}u\rangle]\Big\}, \tag{20}$$

where $x = s_0/M_B^2$ and the functions

$$E_n(x) = 1 - e^{-x}\sum_{k=1}^{n}\frac{x^k}{k!} \tag{21}$$

correspond to the continuum subtraction.

III. NUMERICAL RESULTS

It follows from explicit expressions of the sum rules for the nucleon electromagnetic form factors that the nucleon DA's are the principal input parameters, whose explicit expressions can be found in [8]. These DA's contain nonperturbative parameters which should be determined in some framework. In the present work, we consider two different determinations of these input parameters: (a) all eight nonperturbative parameters f_N, λ_1, λ_2, V_1^d, A_1^u, f_1^d, f_1^u, and f_2^d are estimated within the QCD sum rules method [8,10,11] (set1) and (b) the condition that the next to leading conformal spin contributions vanish fixes five of the eight parameters. This is the so-called asymptotic set. The values of all nonperturbative parameters are (see [8])

$$\begin{aligned} f_N &= (5.0 \pm 0.5)\times 10^{-3}\ \mathrm{GeV}^2,\\ \lambda_1 &= -(2.7 \pm 0.9)\times 10^{-2}\ \mathrm{GeV}^2,\\ \lambda_2 &= (5.4 \pm 1.9)\times 10^{-2}\ \mathrm{GeV}^2, \end{aligned} \tag{22}$$

$$\begin{array}{ll} \text{set1} & \text{asymptotic}\\ A_1^u = 0.38 \pm 0.15, & A_1^u = 0,\\ V_1^d = 0.23 \pm 0.03, & V_1^d = \frac{1}{3},\\ f_1^d = 0.40 \pm 0.05, & f_1^d = \frac{3}{10},\\ f_2^d = 0.22 \pm 0.05, & f_2^d = \frac{4}{15},\\ f_1^u = 0.07 \pm 0.05, & f_1^u = \frac{1}{10}. \end{array} \tag{23}$$

The continuum threshold that appears in the continuum subtraction is determined from the mass sum rules as $s_0 = 2.25\ \mathrm{GeV}^2$. There are two auxiliary parameters of the sum rules: the Borel parameter M_B^2 and the parameter β. The Borel mass squared M_B^2 is an unphysical parameter of the sum rules, and therefore we need to find a region of M_B^2, where physically measurable quantities, in our case electromagnetic form factors, are independent of M_B^2. The lower bound of M_B^2 is determined from the condition that contributions from higher states and continuum in the correlator should be small enough; the upper bound of M_B^2 is determined by requiring that the series of the light cone expansion with increasing twist should be convergent. Our numerical analysis shows that both conditions are satisfied in the region $1\ \mathrm{GeV}^2 \le M_B^2 \le 2\ \mathrm{GeV}^2$, which we will use in numerical analysis.

The other auxiliary parameter β is chosen in a region such that the predictions are independent of the precise value of β in that region. In our analysis, it is shown that in the region $-0.5 \le \cos\theta \le 0.5$ the form factors are practically insensitive to the variation of β, where θ is defined as $\tan\theta = \beta$. Note that the analysis of mass sum rules and magnetic moments of octet baryons [14] leads to the very close region for $\cos\theta$, i.e., $-0.6 \le \cos\theta \le 0.3$. Also, it is observed in [15] that the optimal value of β is $\beta = -1.2(\cos\theta = -0.64)$, which follows from the Monte Carlo analysis.

In Fig. 1, we present the dependence of the proton magnetic form factor $G_M^p/\mu_p G_D$ on Q^2 at $s_0 = 2.25\ \mathrm{GeV}^2$, $M_B^2 = 1.2\ \mathrm{GeV}^2$ for two sets of DA's, at fixed values of parameter β. In this figure, we also present the experimental results [16–18]. From this figure, we see that the Q^2 dependencies as well as the magnitude of the proton magnetic form factor are rather in good agreement with the experimental data, especially for the set 1 of DA's and Ioffe current ($\beta = -1$). The dependence of the ratio of the proton electric form factor to the magnetic form factor $\mu_p G_E^p/G_M^p$ on Q^2 at $s_0 = 2.25\ \mathrm{GeV}^2$, $M_B^2 = 1.2\ \mathrm{GeV}^2$ for two sets of DA's, at fixed values of parameter β, is depicted in Fig. 2. From this figure it follows that, practically, both sets of DA's well describe the existing experimental results, except for $\beta = 5$ and $\beta = -1$ of set 1. For large values of Q^2, $Q^2 > 4\ \mathrm{GeV}^2$, the experimental results obtained in [4] and in [17,18] are not in agreement. Whereas $\beta = -1$ describes better the data in [4], larger values of $|\beta|$ describe better the data in [18].

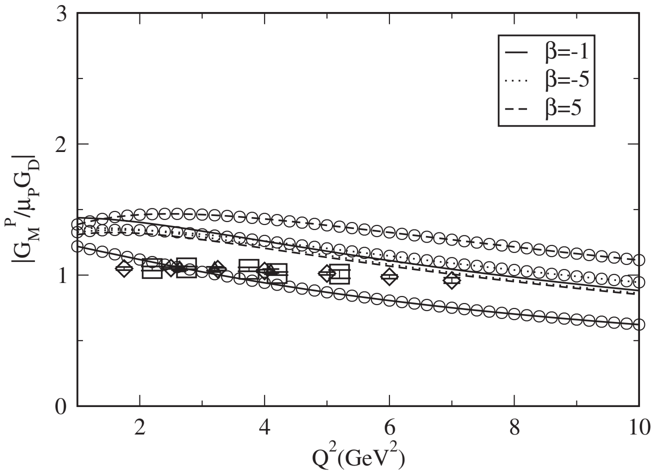

FIG. 1. The dependence of $G_M^P/\mu_P G_D$ on Q^2 at $s_0 = 2.25\ \mathrm{GeV}^2$, $M_B^2 = 1.2\ \mathrm{GeV}^2$ for $\beta = -1$, -5, and 5. The boxes correspond to experimental data in [16], the diamonds to [17], and the up triangles to [18]. The lines with circles correspond to set 1, and the lines without any circles correspond to the asymptotic DA's.

The LCQSR results for the neutron magnetic (normalized to the dipole form factor) and electric form factors are given in Figs. 3 and 4, respectively. From Fig. 3, we see that the magnetic form factor of the neutron reproduces experimental data very well at $\beta = -1$ for both sets of DA's. The neutron electric form factor is in good agreement with the experimental result for all cases.

Analysis of the experimental results (for a review see [7] and references therein) reveals that the magnetic form factors of the nucleon are very well described by the dipole formula

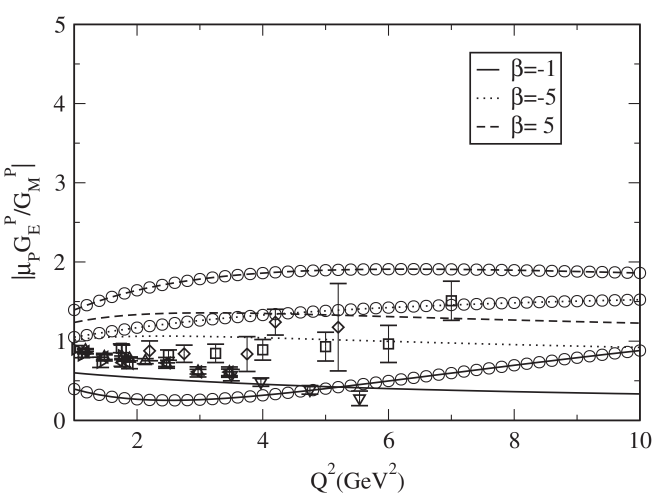

FIG. 2. The same as Fig. 1, but for $\mu_P G_E^P/G_M^P$. The boxes, diamonds, up triangles, down triangles, right triangles, left triangles correspond to experimental data given in [16], [17], [6], [4], [5], [3], respectively.

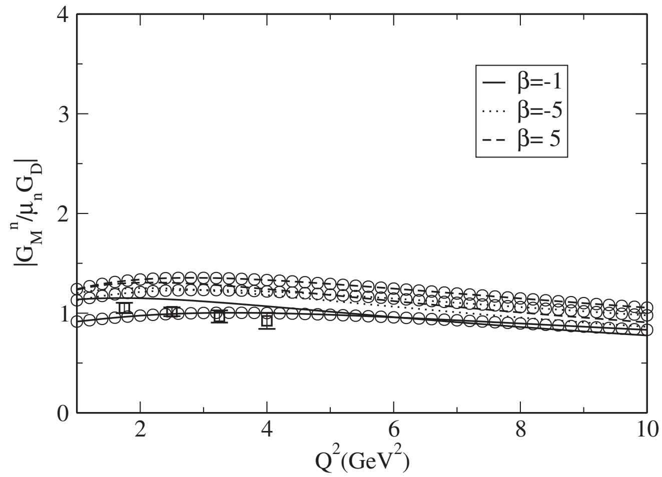

FIG. 3. The same as Fig. 1 but for $G_M^n/\mu_n G_D$. The boxes correspond to experimental data [24].

$$G_M^{n,p}(Q^2) = \frac{\mu_{n,p}}{(1 + \frac{Q^2}{(0.71\ \mathrm{GeV})^2})^2} = \mu_{n,p} G_D. \tag{24}$$

The measured values of the electric form factors of the neutron are given in [19,20].

In [21,22], the following large Q^2 behavior of the electromagnetic form factors is obtained:

$$\frac{F_2(Q^2)}{F_1(Q^2)} \sim \frac{\ln^2(Q^2/\Lambda^2)}{Q^2}, \tag{25}$$

where $\Lambda = 300$ MeV. In Fig. 5 (6), we present the logarithmic scale prediction, i.e., $(1/15)\ln^{-2}(Q^2/\Lambda^2) \times Q^2 F_2(Q^2)/F_1(Q^2)$ for the proton (neutron), with available experimental data [23] at fixed values of β for two sets of DA's. From these figures, we see that our prediction for the proton for $\ln^{-2}(Q^2/\Lambda^2)Q^2F_2(Q^2)/F_1(Q^2)$ is in good agreement with experimental data except for the $\beta = -1$ case for both DA's, and $\beta = -5$ case for set 1. For the

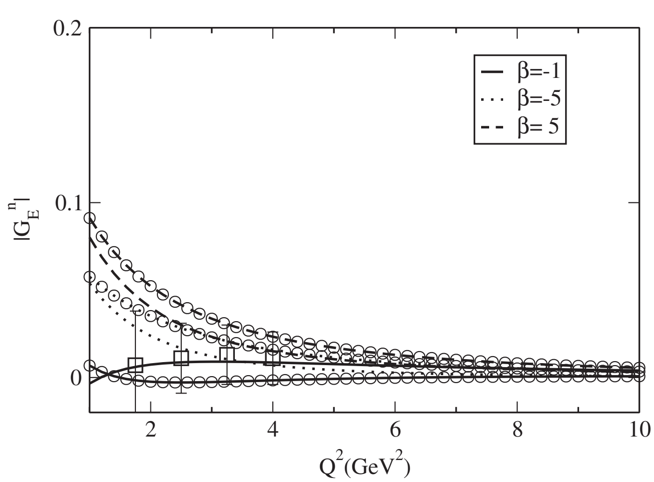

FIG. 4. The same as Fig. 1 but for G_E^n. The boxes correspond to experimental data [24].

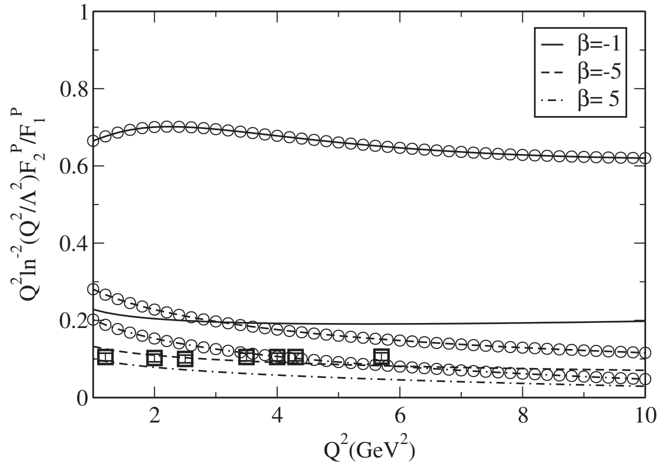

FIG. 5. The same as Fig. 1 but for $Q^2\ln^{-2}(\frac{Q^2}{\Lambda^2})F_2^p/F_1^p$, where $\Lambda = 300$ MeV. The boxes correspond to experimental data [23].

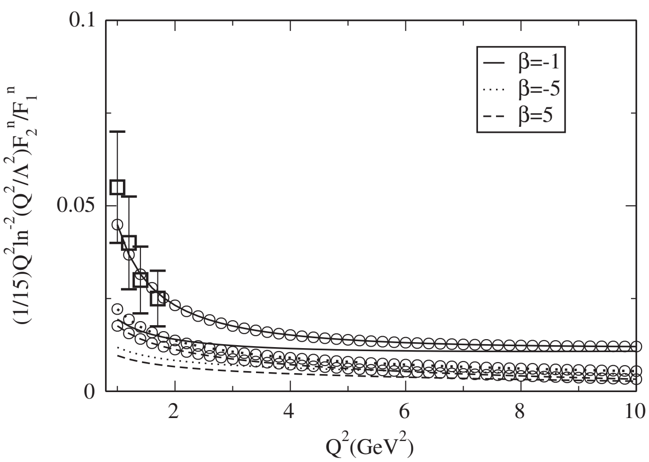

FIG. 6. The dependence of $\frac{1}{15}Q^2\ln^{-2}(\frac{Q^2}{\Lambda^2})F_2^n/F_1^n$ on Q^2 at $s_0 = 2.25$ GeV^2, $M_B^2 = 1.2$ GeV^2, $\Lambda = 300$ MeV. The boxes correspond to experimental data [23].

neutron case only set 1 for $\beta = -1$ describes quite successfully the existing experimental data.

Finally, in Fig. 7, as an example of the dependence of the predictions on β, we present the dependence of the proton magnetic form factor normalized to the dipole form factor $G_M^p/\mu_p G_D$ on $\cos\theta$, for both sets of DA's at two fixed values of Q^2. It follows from this graph that, in the chosen region of β, i.e., in the region $-0.5 \le \cos\theta \le 0.5$, the form factor G_M^p is practically insensitive to the variation of β.

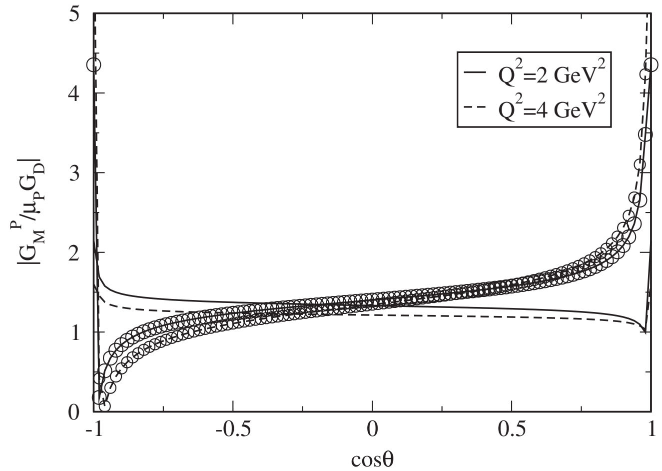

FIG. 7. The dependence of $G_M^P/\mu_P G_D$ on $\cos\theta$ at $s_0 = 2.25$ GeV^2, $M_B^2 = 1.2$ GeV^2 for two different values of Q^2, i.e., $Q^2 = 2$ GeV^2 and $Q^2 = 4$ GeV^2. The lines with circles correspond to set 1, and the lines without any circles correspond to the asymptotic wave functions.

In conclusion, in the present work, we calculate the nucleon electromagnetic form factors using the most general form of the nucleon interpolating current in the light cone QCD sum rules. The sum rules for these form factors are obtained. Using two forms of the DA's, we calculate sum rules predictions for these form factors and compare them with existing experimental data. We obtain that our results are in good agreement with the existing experimental data. More precisely, at different values of β, our results for the form factors reproduce the experimental data. Finally, we obtained the "working region for β."

Our final remark is that in order to answer to the question which β is more preferable, both theoretical and experimental studies have to be refined. From the theoretical viewpoint $\mathcal{O}(\alpha_s)$ corrections to the distribution amplitudes and more accurate determination of the DA's are needed. From experimental data, the discrepancies between various data have to be eliminated.

ACKNOWLEDGMENTS

Two of the authors (K. A. and A. O.), would like to thank TUBITAK, Turkish Scientific and Research Council, for its partial financial support both through the scholarship program and through Project No. 106T333. One of the authors (A. O.) would like to thank TUBA for partial support provided through the GEBIP program. We thank A. Lenz for stimulating discussion and providing us with experimental data.

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