Chiral corrections to the vector and axial couplings of quarks and baryons

Amand Faessler,[1] Thomas Gutsche,[1] Barry R. Holstein,[2] and Valery E. Lyubovitskij[1,*]
[1]*Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D–72076 Tübingen, Germany*
[2]*Department of Physics–LGRT, University of Massachusetts, Amherst, Massachusetts 01003, USA*
(Received 21 December 2007; published 9 June 2008)

We calculate chiral corrections to the semileptonic vector and axial quark coupling constants using a manifestly Lorentz covariant chiral quark approach up to order $\mathcal{O}(p^4)$ in the two- and three-flavor pictures. These couplings are then used in the evaluation of the corresponding couplings which govern the semileptonic transitions between octet baryon states. In the calculation of baryon matrix elements we use a general ansatz for the spatial form of the quark wave function, without referring to a specific realization of hadronization and confinement of quarks in baryons. Matching the physical amplitudes calculated within our approach to the model-independent predictions of baryon chiral perturbation theory allows us to deduce a connection between our parameters and those of baryon chiral perturbation theory.

DOI: 10.1103/PhysRevD.77.114007 PACS numbers: 12.39.Fe, 12.39.Ki, 13.30.Ce, 14.20.Dh

I. INTRODUCTION

The study of the semileptonic decays of the baryon octet $B_i \to B_j e \bar{\nu}_e$ presents an opportunity to shed light on the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{us}. At zero momentum transfer, the weak baryon matrix elements for the $B_i \to B_j e \bar{\nu}_e$ transitions are determined by just two constants—the vector coupling $g_V^{B_iB_j}$ and its axial counterpart $g_A^{B_iB_j}$. In the limit of exact SU(3) symmetry, $g_V^{B_iB_j}$ and $g_A^{B_iB_j}$ are expressed in terms of basic parameters—the vector couplings are given in terms of well-known Clebsch-Gordan coefficients which are fixed due to the conservation of the vector current (CVC), while the axial couplings are given in terms of the simple SU(3) octet axial-vector couplings F and D. The Ademollo-Gatto theorem (AGT) [1] protects the vector form factors from leading SU(3)-breaking corrections generated by the mass difference of strange and nonstrange quarks. The first nonvanishing breaking effects start at second order in symmetry breaking. As stressed in Ref. [2] this vanishing of the first-order correction to the vector hyperon form factors $g_V^{B_iB_j}$ presents an opportunity to determine V_{us} from the direct measurement of $V_{us} g_V^{B_iB_j}$. The axial form factors, on the other hand, contain symmetry-breaking corrections already at first order. Note that the experimental data on baryon semileptonic decays [3] are well described by Cabibbo theory [4], which assumes SU(3) invariance of strong interactions. However, for a precise determination of V_{us} one needs to include leading and perhaps subleading SU(3)-breaking corrections.

The theoretical analysis of SU(3)-breaking corrections to hyperon semileptonic decay form factors has been performed in various approaches [5–22], including quark and soliton models, $1/N_c$ expansion of QCD, chiral perturbation theory (ChPT), lattice QCD, etc. Quark models, in particular, have had a major impact on the understanding of the phenomenology of hyperon semileptonic decays. The original predictions of the naive SU(6) model [23] have been substantially improved by inclusion of relativistic [24] and SU(3) symmetry-breaking effects [8,12,13], and gluon [25] and meson-cloud corrections [12,26]. However, a fully consistent presentation of chiral corrections [both SU(3) symmetric and SU(3) breaking] to semileptonic form factors of baryons is still awaited, although a model-independent inclusion of some chiral corrections to semileptonic form factors of baryons has been performed using different versions of the chiral effective theory of baryons (including baryon ChPT and heavy baryon ChPT) [16–22]. Recently a complete calculation of the SU(3)-breaking corrections to the hyperon vector form factors up to $\mathcal{O}(p^4)$ in covariant baryon ChPT has been presented in [22]. Note that a detailed ChPT analysis of the nucleon axial coupling/form factor has also been performed in Refs. [27–33].

In the present paper we evaluate chiral corrections to the semileptonic vector and axial quark coupling constants, using a manifestly Lorentz covariant chiral quark approach up to order $\mathcal{O}(p^4)$ in the two- and three-flavor pictures. Here SU(3)-breaking corrections are generated by the mass difference of strange (s) and nonstrange (u, d) current quarks. We proceed as follows. First, we calculate the vector and axial *quark* couplings including chiral effects. Then we use the weak quark transition operators containing these couplings to evaluate the octet baryon matrix elements involved in the semileptonic transitions. Performing the matching of the baryon matrix elements to those derived in baryon ChPT, we deduce relations between the parameters of the two approaches. This matching guarantees inclusion of chiral corrections to baryon observables, which is consistent with QCD. In the calculation of the baryon matrix elements, we employ a general ansatz for the spatial form of the quark wave function,

*On leave of absence from Department of Physics, Tomsk State University, 634050 Tomsk, Russia.

without referring to any specific realization of hadronization and confinement of quarks in baryons. In a forthcoming paper [34] we will consider the evaluation of the baryon matrix elements within a specific Lorentz- and gauge-invariant quark model [35] explicitly including the internal quark dynamics. Note that in Refs. [36,37] we performed an analogous study of the electromagnetic properties of the baryon octet and the $\Delta(1230)$ resonance. In particular, we developed an approach based on a nonlinear chirally symmetric Lagrangian, which involves constituent quarks *and* chiral fields. In a first step, this Lagrangian was used to dress the constituent quarks by a cloud of light pseudoscalar mesons and other (virtual) heavy states using the calculational technique of infrared dimensional regularization [38]. Then within a formal chiral expansion, we calculated the dressed transition operators relevant for the interaction of the quarks with external fields in the presence of a virtual meson cloud. In a following step, these dressed operators were used to calculate baryon matrix elements including internal dynamics of valence quarks. (Note that a simpler and more phenomenological quark model based on similar ideas regarding the dressing of constituent quarks by the meson cloud has been developed in Refs. [39].) We treat the constituent quarks as the intermediate degrees of freedom between the current quarks (building blocks of the QCD Lagrangian) and the hadrons (building blocks of ChPT). This concept dates back to the pioneering works of Refs. [40,41]. Furthermore, our strategy in dressing the constituent quarks by a cloud of pseudoscalar mesons is motivated by the procedure pursued in Ref. [41]. Recent analyses of experiments at Jefferson Lab (TJLAB), Fermilab, BNL, and IHEP (Protvino) renewed the interest in the concept of constituent quarks. The obtained data can be interpreted in a picture, where the hadronic quasiparticle substructure is assumed to consist of constituent quarks with nontrivial form factors. These experiments also initiated new progress in the manifestation of constituent degrees of freedom in hadron phenomenology (see, e.g., Refs. [42]).

The present approach has the intrinsic advantage that it is *a priori* not restricted to small energy or momentum transfers. In a full evaluation, when taking into account the effects of the internal dynamics of valence quarks, one can describe hadron form factors at much higher Euclidean momentum squared when compared to ChPT. When restricting to the inclusion of valence quark effects, our approach was successfully applied to different problems of light baryons and also heavy baryons containing one, two, and three heavy quarks (see Refs. [35,37]). We achieved good agreement with existing data and gave certain predictions for future experiments. For example, our predictions for the semileptonic, nonleptonic, and strong decays of heavy-light baryons were later confirmed experimentally. On the other hand, in Refs. [36,37] we developed the formalism in order to include chiral effects in a way consistent with low-energy theorems and the infrared structure of QCD. Consistency in the present formalism with ChPT is limited since we cannot consider baryonic matrix elements in Minkowski space. Also, our approach does not provide any constraints for the expansion parameter of standard ChPT.

In the present manuscript we proceed as follows. First, in Sec. II, we discuss the basic notions of our approach, which is in direct line with our previous work of Refs. [36,37]. That is, we derive a chiral Lagrangian motivated by baryon ChPT [38,43], and formulate it in terms of quark and mesonic degrees of freedom. Using constituent quarks dressed with a cloud of light pseudoscalar mesons and other heavy states, we derive dressed transition operators within the chiral expansion, which are in turn used in the quark model to produce baryon matrix elements. In Sec. III we derive specific expressions for the vector and axial baryon semileptonic decay constants, while in Sec. IV we give the numerical analysis of the axial nucleon charge and the vector and axial hyperon semileptonic couplings. Finally, in Sec. V we present a short summary of our results.

II. APPROACH

A. Chiral Lagrangian

The SU(3) chiral quark Lagrangian $\mathcal{L}_{qU}$ [up to $\mathcal{O}(p^3)$], which dynamically generates dressing of the constituent quarks by mesonic degrees of freedom, consists of three primary pieces, $\mathcal{L}_q$, $\mathcal{L}_{qq}$, and $\mathcal{L}_U$:

$$\begin{aligned} \mathcal{L}_{qU} &= \mathcal{L}_q + \mathcal{L}_{qq} + \mathcal{L}_U, \\ \mathcal{L}_q &= \mathcal{L}_q^{(1)} + \mathcal{L}_q^{(2)} + \mathcal{L}_q^{(3)} + \cdots, \\ \mathcal{L}_{qq} &= \mathcal{L}_{qq}^{(3)} + \cdots, \\ \mathcal{L}_U &= \mathcal{L}_U^{(2)} + \cdots. \end{aligned} \tag{1}$$

The superscript (i) attached to $\mathcal{L}_U^{(i)}$ and $\mathcal{L}_{q(qq)}^{(i)}$ denotes the low-energy dimension of the Lagrangian:

$$\mathcal{L}_U^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle, \qquad \mathcal{L}_q^{(1)} = \bar{q}\Big[i\not{D} - m + \frac{1}{2} g \not{u}\gamma^5 \Big] q, \tag{2a}$$

$$\mathcal{L}_q^{(2)} = \frac{C_3^q}{2}\langle u_\mu u^\mu\rangle \bar{q}q + \frac{C_4^q}{4}\bar{q} i\sigma^{\mu\nu}[u_\mu, u_\nu] q + \cdots, \tag{2b}$$

$$\mathcal{L}_q^{(3)} = \frac{D_{16}^q}{2}\bar{q}\not{u}\gamma^5 q\langle\chi_+\rangle + \frac{D_{17}^q}{8}\bar{q}\{\not{u}\gamma^5, \hat{\chi}_+\} q + \cdots, \tag{2c}$$

$$\begin{aligned} \mathcal{L}_{qq}^{(3)} = {} & \frac{D_1^{qq}}{2}\bar{q}\not{u}\gamma^5 q\bar{q}q\langle\chi_+\rangle + \frac{D_2^{qq}}{8}\bar{q}\{\not{u}\gamma^5, \hat{\chi}_+\} q\bar{q}q \\ & + \frac{D_3^{qq}}{2}\bar{q}\not{u}\gamma^5 q\bar{q}\hat{\chi}_+ q + \cdots, \end{aligned} \tag{2d}$$

where the symbols $\langle\,\rangle$, $[\,]$, and $\{\,\}$ occurring in Eq. (2) denote the trace over flavor matrices, commutator, and anticommutator, respectively. In Eq. (2) we display only the terms involved in the calculation of vector and axial quark/baryon coupling constants. In addition to the one-body quark Lagrangian we included also the two-body part $\mathcal{L}_{qq}$. The detailed form of the chiral Lagrangian used in the calculations of electromagnetic properties of baryons can be found in Refs. [36,37].

The Lagrangians (2) contain the basic building blocks of our approach. The couplings m and g denote the quark mass and axial charge in the chiral limit [i.e., they are counted as quantities of order $\mathcal{O}(1)$ in the chiral expansion], q is the triplet of u-, d-, s-quark fields, C_i^q and D_i^q are the second- and third-order, one-body quark, low-energy coupling constants (LEC's), respectively, while D_i^{qq} are the third-order, two-body quark LEC's. The LEC's encode the (virtual) contributions due to heavy states. We denote the SU(3) quark LEC's by capital letters in order to distinguish them from the SU(2) LEC's c_i^q and d_i^q. Also, for the one-body quark LEC's we use the additional superscript " q" to differentiate them from the analogous ChPT LEC's: C_i, D_i in SU(3) and c_i, d_i in SU(2). For the two-body quark LEC's we attach the superscript " qq." The octet of pseudoscalar fields

$$\phi = \sum_{i=1}^{8} \phi_i \lambda_i = \sqrt{2}\begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix} \tag{3}$$

is contained in the SU(3) matrix $U = u^2 = \exp(i\phi/F)$ where F is the octet decay constant. We use the standard notations [38,43]

$$\begin{aligned} D_\mu &= \partial_\mu + \Gamma_\mu, \\ \Gamma_\mu &= \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2}u^\dagger r_\mu u - \frac{i}{2}u l_\mu u^\dagger, \\ u_\mu &= i\{u^\dagger, \partial_\mu u\} + u^\dagger r_\mu u - u l_\mu u^\dagger, \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u\chi^\dagger u, \\ \chi &= 2B\mathcal{M} + \cdots, \\ \hat{\chi}_+ &= \chi_+ - \frac{1}{3}\langle\chi_+\rangle. \end{aligned} \tag{4}$$

The fields r_μ and l_μ include external vector (v_μ) and axial (a_μ) fields: $r_\mu = v_\mu + a_\mu$, $l_\mu = v_\mu - a_\mu$. $\mathcal{M} = \mathrm{diag}\{\hat{m}, \hat{m}, \hat{m}_s\}$ is the mass matrix of current quarks (we work in the isospin symmetry limit with $\hat{m}_u = \hat{m}_d = \hat{m} = 7$ MeV, and the mass of the strange quark $\hat{m}_s$ is related to the nonstrange one via $\hat{m}_s \simeq 25\hat{m}$). The quark vacuum condensate parameter is denoted by $B = -\langle 0|\bar{u}u|0\rangle/F^2 = -\langle 0|\bar{d}d|0\rangle/F^2$. To distinguish between constituent and current quark masses we attach the symbol ˆ ("hat") when referring to the current quark masses. We rely on the standard picture of chiral symmetry breaking ($B \gg F$). To leading order in the chiral expansion the masses of pseudoscalar mesons are given by $M_\pi^2 = 2\hat{m}B$, $M_K^2 = (\hat{m} + \hat{m}_s)B$, $M_\eta^2 = \frac{2}{3}(\hat{m} + 2\hat{m}_s)B$. For the numerical analysis we will use $M_\pi = 139.57$ MeV, $M_K = 493.677$ MeV (the charged pion and kaon masses), $M_\eta = 547.51$ MeV, $F = F_\pi = 92.4$ MeV in SU(2), and $F = (F_\pi + F_K)/2$ in SU(3) with $F_K/F_\pi = 1.22$ [44].

Reduction of the SU(3) Lagrangian to its SU(2) counterpart is straightforward. The quark triplet (u, d, s) and meson octet are replaced by the quark doublet (u, d) and the pion triplet, respectively. Likewise, the LEC's C_i^q, D_i^q, and D_i^{qq} should be replaced by their SU(2) analogues c_i^q, d_i^q, and d_i^{qq}. Note that the SU(2) Lagrangian does *not* contain the LEC's d_{17}^q and $d_{2(3)}^{qq}$. Also, we should use $\mathcal{M} = \mathrm{diag}\{\hat{m}, \hat{m}\}$ and $\hat{\chi}_+ = \chi_+ - \frac{1}{2}\langle\chi_+\rangle$ instead of the corresponding quantities defined in Eq. (4).

B. Dressing of quark operators

Any bare quark operator can be dressed by a cloud of pseudoscalar mesons and heavy states in a straightforward manner by use of the effective chirally invariant Lagrangian $\mathcal{L}_{qU}$. In Refs. [36,37] we illustrated the technique of dressing in the case of the electromagnetic quark operator and performed a detailed analysis of the electromagnetic properties of the baryon octet and of the $\Delta \to N\gamma$ transition. Here we extend our method to the case of vector and axial quark operators. First, we define the bare vector and axial quark transition operators constructed from quark fields of flavor i and j as

$$\begin{aligned} J_{\mu,V}(q) &= \int d^4x e^{-iqx} j_{\mu,V}(x), \\ j_{\mu,V}(x) &= \bar{q}_j(x)\gamma_\mu q_i(x), \\ J_{\mu,A}(q) &= \int d^4x e^{-iqx} j_{\mu,A}(x), \\ j_{\mu,A}(x) &= \bar{q}_j(x)\gamma_\mu\gamma_5 q_i(x). \end{aligned} \tag{5}$$

Next, using the chiral Lagrangian derived in Sec. II A, we construct the vector/axial currents with quantum numbers of the bare quark currents which include mesonic degrees of freedom. Then these currents are projected on corresponding quark (initial and final) states in order to evaluate the dressed vector and axial quark form factors encoding the chiral corrections. Finally, using the dressed quark form factors in momentum space, we can determine their Fourier transform in coordinate space.

The dressed quark operators $j_{\mu,V(A)}^{\mathrm{dress}}(x)$ include one-body, $j_{\mu,V(A)}^{\mathrm{dress}(1)}(x)$, and two-body, $j_{\mu,V(A)}^{\mathrm{dress}(2)}(x)$, operators:

$$j_{\mu,V(A)}^{\mathrm{dress}}(x) = j_{\mu,V(A)}^{\mathrm{dress}(1)}(x) + j_{\mu,V(A)}^{\mathrm{dress}(2)}(x). \tag{6}$$

In Refs. [36,37] we restricted to the consideration of one-body quark operators only. Here we discussed also the two-body operators. In Figs. 1–4 we display the tree and loop diagrams which contribute to the dressed one- and two-body vector and axial operators, respectively, up to and including third order in the chiral expansion.

The dressed one-body quark operators $j^{\rm dress(1)}_{\mu,V(A)}(x)$ and their Fourier transforms $J^{\rm dress(1)}_{\mu,V(A)}(q)$ have the forms

$$\begin{aligned} j^{\rm dress(1)}_{\mu,V}(x) &= f_1^{ij}(-\partial^2)[\bar{q}_j(x)\gamma_\mu q_i(x)] \\ &\quad + \frac{f_2^{ij}(-\partial^2)}{m_i+m_j}\partial^\nu[\bar{q}_j(x)\sigma_{\mu\nu}q_i(x)] \\ &\quad - \frac{f_3^{ij}(-\partial^2)}{m_i+m_j} i\partial_\mu[\bar{q}_j(x)q_i(x)], \\ J^{\rm dress(1)}_{\mu,V}(q) &= \int d^4x e^{-iqx}\bar{q}_j(x)\bigg[\gamma_\mu f_1^{ij}(q^2) \\ &\quad + \frac{i\sigma_{\mu\nu}q^\nu}{m_i+m_j}f_2^{ij}(q^2) + \frac{q_\mu}{m_i+m_j}f_3^{ij}(q^2)\bigg]q_i(x), \end{aligned} \tag{7}$$

and

$$\begin{aligned} j^{\rm dress(1)}_{\mu,A}(x) &= g_1^{ij}(-\partial^2)[\bar{q}_i(x)\gamma_\mu\gamma_5 q_j(x)] \\ &\quad + \frac{g_2^{ij}(-\partial^2)}{m_i+m_j}\partial^\nu[\bar{q}_j(x)\sigma_{\mu\nu}\gamma_5 q_i(x)] \\ &\quad - \frac{g_3^{ij}(-\partial^2)}{m_i+m_j} i\partial_\mu[\bar{q}_j(x)\gamma_5 q_i(x)], \\ J^{\rm dress(1)}_{\mu,A}(q) &= \int d^4x e^{-iqx}\bar{q}_j(x)\bigg[\gamma_\mu\gamma_5 g_1^{ij}(q^2) \\ &\quad + \frac{i\sigma_{\mu\nu}q^\nu}{m_i+m_j}\gamma_5 g_2^{ij}(q^2) \\ &\quad + \frac{q_\mu}{m_i+m_j}\gamma_5 g_3^{ij}(q^2)\bigg]q_i(x), \end{aligned} \tag{8}$$

where $m_{i(j)}$ is the dressed constituent quark mass of $i(j)$th flavor generated by the chiral Lagrangian (2) (see details in Ref. [36]); $f^{ij}_{1,2,3}(q^2)$ and $g^{ij}_{1,2,3}(q^2)$ are the one-body quark vector and axial $i \to j$ flavor changing form factors encoding the chiral corrections. In Figs. 1 and 2 we only show the one-body diagrams which are relevant for the calculation of the vector $f_1^{ij} = f_1^{ij}(0)$ and axial $g_1^{ij} = g_1^{ij}(0)$ couplings at the order of accuracy to which we are working in. The ellipses $\cdots$ in Figs. 1 and 2 denote higher-order diagrams, i.e., diagrams which contribute only to the q^2 dependence of $f_1^{ij}(q^2)$ and $g_1^{ij}(q^2)$ and/or to the remaining four form factors $f^{ij}_{2(3)}(q^2)$ and $g^{ij}_{2(3)}(q^2)$. The full analysis of all six form factors goes beyond the scope of the present paper. The contributions of the various graphs in Figs. 1 and 2 to the vector and axial couplings are discussed in Appendix A and are listed in Tables I and II. Evaluation of the one-body diagrams in Figs. 1 and 2 was performed using the method of *infrared dimensional regularization* suggested in Ref. [38] in order to guarantee a straightforward connection between loop and chiral expansions in terms of quark masses and small external momenta. We relegate a detailed discussion of the calculational technique to Ref. [36].

In Figs. 3 and 4 we display the two-body diagrams (contributions to the vector and axial operators, respectively) which are generated by the chiral Lagrangian (2) at the order of accuracy we are working in. These diagrams include the terms generated by meson exchange and by the contact terms representing contributions due to heavy states and generated by the two-body $O(p^3)$ chiral

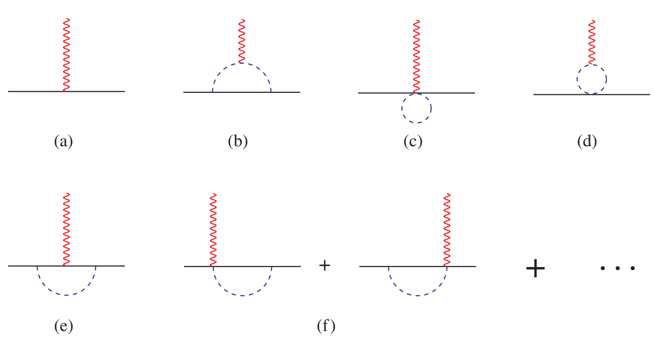

FIG. 1 (color online). Diagrams contributing to the one-body vector quark transition operator up to fourth order. Solid, dashed, and wiggly lines refer to quarks, pseudoscalar mesons, and the external vector field, respectively.

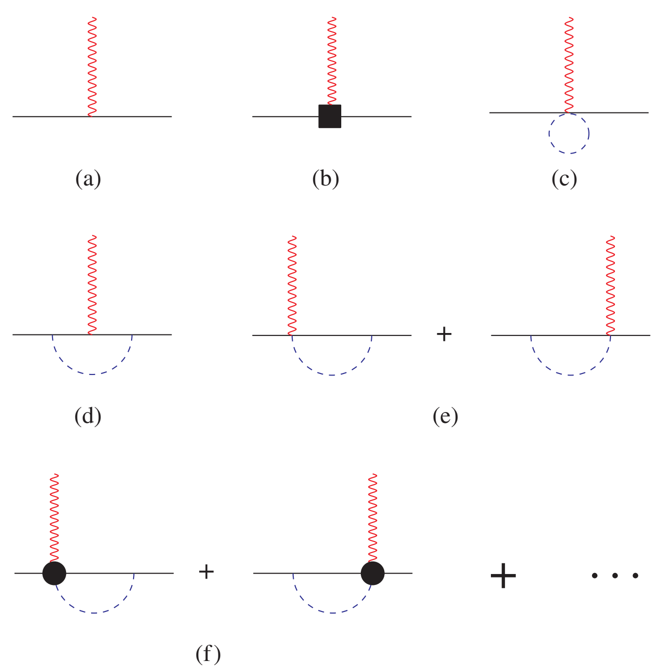

FIG. 2 (color online). Diagrams contributing to the one-body axial quark transition operator up to fourth order. Solid, dashed, and wiggly lines refer to quarks, pseudoscalar mesons, and the external axial field, respectively. Vertices denoted by a black filled circle and box correspond to insertions from the second and third order chiral Lagrangians.

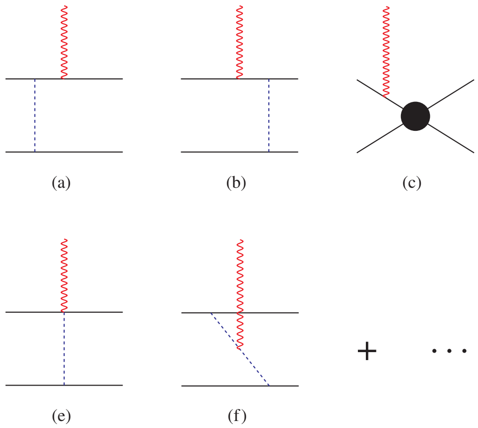

FIG. 3 (color online). Diagrams contributing to the two-body vector quark transition operator up to fourth order. Solid, dashed, and wiggly lines refer to quarks, pseudoscalar mesons, and the external vector field, respectively. The vertex denoted by a black filled circle corresponds to insertion of the two-body mass counterterm due to one-meson exchange.

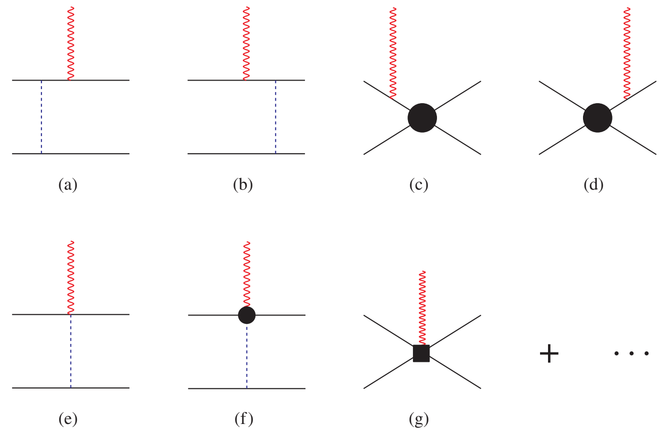

FIG. 4 (color online). Diagrams contributing to the two-body axial quark transition operator up to fourth order. Solid, dashed, and wiggly lines refer to quarks, pseudoscalar mesons, and the external axial field, respectively. Vertices denoted by a big (small) black filled circle and box correspond to insertions of the two-body mass counterterm due to one-meson exchange, from the second and third order chiral Lagrangians.

Lagrangian $\mathcal{L}_{qq}^{(3)}$ (2d). Inclusion of the two-body quark operators in the evaluation of the vector and axial couplings of baryons goes beyond the scope of the present paper and will be done in the future (therefore, in the numerical calculations we will restrict to the one-body approximation). Here we just give the general expressions for the Fourier transforms of the two-body operators $J^{\rm dress(2)}_{\mu,V(A)}(q)$:

$$j^{\rm dress(2)}_{\mu,V}(q) = \int d^4x e^{-iqx} \sum_m (\bar q_j(x) \Gamma^V_{ij,m} q_i(x) \bar q_l(x) \times \Gamma^V_{kl,m} q_k(x))_\mu f^{ijkl}_m(q^2), \tag{9a}$$

$$j^{\rm dress(2)}_{\mu,A}(q) = \int d^4x e^{-iqx} \sum_m (\bar q_j(x) \Gamma^A_{ij,m} q_i(x) \bar q_l(x) \times \Gamma^A_{kl,m} q_k(x))_\mu g^{ijkl}_m(q^2), \tag{9b}$$

where $\Gamma^{V(A)}_{ij,m}$ reflects the corresponding spin structures, $f(g)^{ijkl}_m(q^2)$ are the two-body quark form factors encoding chiral effects, and m is the summation index over possible contributions to the two-body operators. We discuss the two-body operators in Appendix B.

In order to calculate the vector and axial current transitions between baryons, we project the dressed quark operators between the corresponding baryon states. The master formulas are

$$\langle B_j(p')|J^{\rm dress}_{\mu,V(A)}(q)|B_i(p)\rangle = (2\pi)^4 \delta^4(p' - p - q) \times M^{B_iB_j}_{\mu,V(A)}(p,p'), \tag{10}$$

$$\begin{aligned} M^{B_iB_j}_{\mu,V}(p,p') &= \sum_{k=1}^{3} f^{ij}_k(q^2) \langle B_j(p')|V^{ij}_{\mu,k}(0)|B_i(p)\rangle + \sum_m f^{ijkl}_m(q^2)\langle B_j(p')|V^{ijkl}_{\mu,m}(0)|B_i(p)\rangle \\ &= \bar u_{B_j}(p')\bigg\{\gamma_\mu F_1^{B_iB_j}(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{m_{B_i}+m_{B_j}} F_2^{B_iB_j}(q^2) + \frac{q_\mu}{m_{B_i}+m_{B_j}} F_3^{B_iB_j}(q^2)\bigg\} u_{B_i}(p), \end{aligned} \tag{11}$$

$$\begin{aligned} M^{B_iB_j}_{\mu,A}(p,p') &= \sum_{k=1}^{3} g^{ij}_k(q^2) \langle B_j(p')|A^{ij}_{\mu,k}(0)|B_i(p)\rangle + \sum_m g^{ijkl}_m(q^2)\langle B_j(p')|A^{ijkl}_{\mu,m}(0)|B_i(p)\rangle \\ &= \bar u_{B_j}(p')\bigg\{\gamma_\mu\gamma_5 G_1^{B_iB_j}(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{m_{B_i}+m_{B_j}} \times \gamma_5 G_2^{B_iB_j}(q^2) + \frac{q_\mu}{m_{B_i}+m_{B_j}}\gamma_5 G_3^{B_iB_j}(q^2)\bigg\} \times u_{B_i}(p), \end{aligned} \tag{12}$$

where $B_i(p)$ and $u_{B_i}(p)$ are the baryon state and spinor, respectively, normalized via

$$\langle B_i(p')|B_i(p)\rangle = 2E_{B_i}(2\pi)^3\delta^3(\vec p - \vec p'), \quad \bar u_{B_i}(p) u_{B_i}(p) = 2m_{B_i} \tag{13}$$

with $E_{B_i} = \sqrt{m^2_{B_i} + \vec p^2}$ being the baryon energy and m_{B_i} the baryon mass. The index $i(j)$ attached to the baryon state/field indicates the flavor of the quark involved in the semileptonic transition. Here, $F^{B_iB_j}_k(q^2)$ and $G^{B_iB_j}_k(q^2)$ with

$k = 1, 2, 3$ are the vector and axial semileptonic form factors of baryons.

The main idea of the above relations is to express the matrix elements of the dressed quark operators in terms of the matrix elements of the one- and two-body valence quark operators $V^{ij}_{\mu,k}(0)$, $A^{ij}_{\mu,k}(0)$, $V^{ijkl}_{\mu,m}(0)$, and $A^{ijkl}_{\mu,m}(0)$, and encode the chiral effects in the form factors $f^{ij}_k(q^2)$, $g^{ij}_k(q^2)$, $f^{ijkl}_m(q^2)$, and $g^{ijkl}_m(q^2)$. The set of valence quark operators is defined as

$$V^{ij}_{\mu,k}(0) = \bar{q}_j(0)\Gamma^V_{\mu,k}q_i(0), \qquad A^{ij}_{\mu,k}(0) = \bar{q}_j(0)\Gamma^A_{\mu,k}q_i(0), \tag{14a}$$

$$\begin{aligned} V^{ijkl}_{\mu,m}(0) &= (\bar{q}_j(0)\Gamma^V_{ij,m}q_i(0)\bar{q}_l(0)\Gamma^V_{kl}q_k(0))_\mu, \\ A^{ijkl}_{\mu,m}(0) &= (\bar{q}_j(0)\Gamma^A_{ij,m}q_i(0)\bar{q}_l(0)\Gamma^A_{kl,m}q_k(0))_\mu \end{aligned} \tag{14b}$$

where

$$\Gamma^V_{\mu,1} = \gamma_\mu, \qquad \Gamma^V_{\mu,2} = \frac{i\sigma_{\mu\nu}q^\nu}{m_i + m_j}, \qquad \Gamma^V_{\mu,3} = \frac{q_\mu}{m_i + m_j},$$
$$\Gamma^A_{\mu,1} = \gamma_\mu\gamma_5, \qquad \Gamma^A_{\mu,2} = \frac{i\sigma_{\mu\nu}q^\nu}{m_i + m_j}\gamma_5,$$
$$\Gamma^A_{\mu,3} = \frac{q_\mu}{m_i + m_j}\gamma_5. \tag{15}$$

The set of Eqs. (10)–(15) contains our main result: we perform a separation of the effects of internal dynamics of valence quarks contained in the matrix elements of the bare quark operators $V(A)^{ij}_{\mu,k}(0)$, $V(A)^{ijkl}_{\mu,m}(0)$ and the effects dictated by chiral dynamics which are encoded in the relativistic form factors $f^{ij}_k(q^2)$, $g^{ij}_k(q^2)$, $f^{ijkl}_m(q^2)$, and $g^{ijkl}_m(q^2)$. In particular, the results for the baryon properties (static characteristics and form factors in the Euclidean region) derived using the above formulas satisfy the low-energy theorems and identities dictated by the infrared singularities of QCD (see, e.g., the detailed discussion in Refs. [36,37]). Let us stress that consistency in the present formalism with ChPT is limited since we cannot consider baryonic matrix elements in Minkowski space. Because of the factorization of the chiral effects and effects of internal dynamics of valence quarks, the calculation of the form factors $f(g)^{ij}_k(q^2)$ and $f(g)^{ijkl}_m(q^2)$ encoding chiral dynamics, on one side, and the matrix elements of $V(A)^{ij}_{\mu,k}(0)$ and $V(A)^{ijkl}_{\mu,m}(0)$ encoding effects of valence quarks, on the other side, can be done independently. The evaluation of the matrix elements $V(A)^{ij}_{\mu,k}(0)$ and $V(A)^{ijkl}_{\mu,m}(0)$ is not restricted to the small squared momenta and, therefore, can shed light on baryon form factors at higher Euclidean momentum squared in comparison with ChPT. In particular, as a first step we employ a formalism motivated by the ChPT Lagrangian, which is formulated in terms of constituent quark degrees of freedom, for the calculation of $f(g)^{ij}_k(q^2)$ and $f(g)^{ijkl}_m(q^2)$. The calculation of the matrix elements of the bare quark operators can then be relegated to quark models based on specific assumptions about internal quark dynamics, hadronization, and confinement. Note that Eqs. (10)–(15) are valid for the calculations of dressed vector and axial quark operators of *any* flavor content.

C. Chiral expansion of vector and axial quark couplings

In this section we present the results for the chiral expansion of the vector $f^{ij}_1 = f^{ij}_1(0)$ and axial $g^{ij}_1 = g^{ij}_1(0)$ quark couplings for various $i \to j$ flavor transitions in the SU(2) (isospin) limit. We begin by defining the quark wave function renormalization constant Z. In particular, the tree graphs $T_{\rm tree}$ in Figs. 1(a) and 2(a) should be renormalized in terms of Z via $1/2\{T_{\rm tree}, Z\}$. Details of the calculation of Z can be found in [36]. In SU(2) we find

$$Z = 1 - \tfrac{9}{4}R_\pi \tag{16}$$

while in SU(3) we have

$$\mathrm{diag}\{Z, Z, Z_s\} = I - \sum_{P=\pi,K,\eta}\alpha_P R_P, \tag{17}$$

where in both cases $Z \equiv Z_u \equiv Z_d$ and

$$\begin{aligned} R_P &= \frac{g^2}{F^2}\Delta_P + \frac{g^2M_P^2}{24\pi^2F^2}\left\{1 - \frac{3\pi}{2}\mu_P\right\}, \qquad \mu_P = \frac{M_P}{m}, \\ P &= \pi, K, \eta, \qquad \alpha_\pi = \frac{9}{2}Q + \frac{3}{2}I - \frac{9}{4}\lambda_3, \\ \alpha_K &= -3Q + 2I + \frac{3}{2}\lambda_3, \qquad \alpha_\eta = -\frac{3}{2}Q + \frac{1}{2}I + \frac{3}{4}\lambda_3. \end{aligned} \tag{18}$$

Here $Q = \mathrm{diag}\{2/3, -1/3, -1/3\}$ and the quantity Δ_P is defined as

$$\begin{aligned} \Delta_P &= 2M_P^2\lambda_P, \\ \lambda_P &= \frac{M_P^{d-4}}{16\pi^2}\left\{\frac{1}{d-4} - \frac{1}{2}(\ln 4\pi + \Gamma'(1) + 1)\right\}. \end{aligned} \tag{19}$$

In Appendix A we explicitly list the contributions of the various graphs to the vector (Fig. 1) and axial (Fig. 2) couplings. First, we discuss the vector couplings. For completeness, we consider vector currents conserving the quark flavor, i.e., corresponding to electric and isospin charge, as well as those involving $d \to u$ and $s \to u$ transitions. Because of charge conservation and isospin invariance, the total contribution of the diagrams of Fig. 1 properly reproduces the quark electric and isospin charges. In addition, the vector coupling governing the $d \to u$ transition is equal to unity—$f^{du}_1 = 1$ [a detailed discussion for both SU(2) and SU(3) is presented in Appendix A]. As stressed in Ref. [20] the total contribution of diagrams shown in Fig. 1 to these quantities is finite, and no unknown LEC's appear at the order of accuracy to which we are working in. In the case of the $s \to u$ transition, the total contribution of the diagrams given in Fig. 1 to the corresponding vector coupling f_1^{su} is finite but contains symmetry-breaking corrections of second order in SU (3)— $\mathcal{O}((M_K - M_\pi)^2)$ and $\mathcal{O}((M_K - M_\eta)^2)$. Note, that the Ademollo-Gatto theorem (AGT) protects the coupling f_1^{su} from *first-order* symmetry-breaking corrections. As shown explicitly (cf. Appendix A) the AGT holds for the two sets of diagrams, set I and set II, independently. For set I, including the diagrams of Figs. 1(a), 1(b), 1(e), and 1(f), and for II, including the diagrams of Figs. 1(c) and 1(d), we have

$$f_1^{su;I} = \sum_{i=a,b,e,f} f_1^{su;(i)} = 1 - \frac{9g^2}{16}(H_{\pi K} + H_{\eta K} + G_{\pi K} + G_{\eta K}) \tag{20}$$

and

$$f_1^{su;II} = \sum_{i=c,d} f_1^{su;(i)} = -\frac{3}{16}(H_{\pi K} + H_{\eta K}). \tag{21}$$

The $\mathcal{O}(p^2)$ functions H_{ab} and G_{ab}, which show up in the context of ChPT (see, e.g., Refs. [9,16,17,19,20,22,45]) are defined as

$$\begin{aligned} H_{ab} &= \frac{1}{(4\pi F)^2}\left(M_a^2 + M_b^2 - \frac{2M_a^2 M_b^2}{M_a^2 - M_b^2}\ln\frac{M_a^2}{M_b^2}\right) \\ &= \mathcal{O}((M_a^2 - M_b^2)^2), \\ G_{ab} &= -\frac{1}{(4\pi F)^2}\frac{2\pi}{3m}\frac{(M_a - M_b)^2}{M_a + M_b}(M_a^2 + 3M_a M_b + M_b^2) \\ &= \mathcal{O}((M_a^2 - M_b^2)^2). \end{aligned} \tag{22}$$

Therefore, the final result for the $s \to u$ quark transition vector coupling is

$$\begin{aligned} f_1^{su} &= f_1^{su;I} + f_1^{su;II} \\ &= 1 + \delta f_1^{su} = 1 - \frac{3}{16}((1 + 3g^2)(H_{\pi K} + H_{\eta K}) \\ &\quad + 3g^2(G_{\pi K} + G_{\eta K})) \end{aligned} \tag{23}$$

where δf_1^{su} is the total SU(3)-breaking correction.

Next we turn to the discussion of the axial couplings governing the $d \to u$ and $s \to u$ quark flavor transitions. The expressions for the axial (isovector) charge g_1 and the axial couplings responsible for the $d \to u$ and $s \to u$ transitions g_1^{du} and g_1^{su} are given in the following (cf. Appendix A for the expressions of the separate diagrams in Fig. 2). In SU(2) we have

$$\begin{aligned} g_1 = g_1^{du} = g\bigg\{1 &- \frac{g^2 M_\pi^2}{16\pi^2 F^2} + \frac{M_\pi^3}{24\pi m F^2} \\ &\times (3 + 3g^2 - 4c_3^q m + 8c_4^q m)\bigg\} \\ &+ 4M_\pi^2\left\{d_{16}^q - \frac{g}{F^2}\left(\frac{1}{2} + g^2\right)\lambda_\pi\right\}. \end{aligned} \tag{24}$$

Absorbing the infinity in the LEC d_{16}^q,

$$d_{16}^q = \bar{d}_{16}^q + \frac{g}{F^2}\left(\frac{1}{2} + g^2\right)\lambda_\pi, \tag{25}$$

we arrive at the ultraviolet-finite expression for g_1:

$$\begin{aligned} g_1 &= g_1^{du} = g + \delta g_1^{\chi_2} \\ &= g\bigg\{1 - \frac{g^2 M_\pi^2}{16\pi^2 F^2} + \frac{M_\pi^3}{24\pi m F^2}(3 + 3g^2 - 4c_3^q m \\ &\quad + 8c_4^q m)\bigg\} + 4M_\pi^2 \bar{d}_{16}^q, \end{aligned} \tag{26}$$

where $\delta g_1^{\chi_2}$ is the SU(2) chiral correction.

In SU(3) the corresponding expression for the isovector axial coupling g_1 is

$$\begin{aligned} g_1 = g_1^{du} = g\bigg\{1 &- \frac{g^2}{16\pi^2 F^2}\left(M_\pi^2 + M_K^2 + \frac{M_\eta^2}{3}\right) \\ &+ \frac{M_\pi^3}{24\pi m F^2}(3 + 3g^2 - 4C_3^q m + 8C_4^q m) + \frac{M_K^3}{48\pi m F^2}\left(3 + \frac{9}{2}g^2 + 8C_4^q m\right) + \frac{g^2 M_\eta^3}{48\pi m F^2}\bigg\} \\ &+ 2M_\pi^2\left(D_{16}^q + \frac{1}{3}D_{17}^q - \frac{g}{F^2}\left(1 + \frac{17}{9}g^2\right)\bar{\lambda} - \frac{g(1 + 2g^2)}{32\pi^2 F^2}\ln\frac{M_\pi^2}{m^2} + \frac{g^3}{288\pi^2 F^2}\ln\frac{M_\eta^2}{m^2}\right) \\ &+ 2M_K^2\left(2D_{16}^q - \frac{1}{3}D_{17}^q - \frac{g}{2F^2}\left(1 + \frac{35}{9}g^2\right)\bar{\lambda} - \frac{g(1 + 3g^2)}{64\pi^2 F^2}\ln\frac{M_K^2}{m^2} - \frac{g^3}{72\pi^2 F^2}\ln\frac{M_\eta^2}{m^2}\right), \end{aligned} \tag{27}$$

where the divergent quantity,

$$\bar{\lambda} = \frac{m^{d-4}}{16\pi^2}\left\{\frac{1}{d-4} - \frac{1}{2}(\ln 4\pi + \Gamma'(1) + 1)\right\}, \tag{28}$$

coincides with λ_P when $m = M_P$. The last two lines in Eq. (27), containing divergences, can be written in more compact form in terms of one of the divergent quantities λ_P ($P = \pi, K, \eta$)—e.g., in terms of λ_η, these lines have the succinct form

$$2M_\pi^2\left(D_{16}^q+\frac{1}{3}D_{17}^q-\frac{g}{F^2}\left(1+\frac{17}{9}g^2\right)\lambda_\eta-(1+2g^2)L_{\pi\eta}\right)+2M_K^2\left(2D_{16}^q-\frac{1}{3}D_{17}^q-\frac{g}{2F^2}\left(1+\frac{35}{9}g^2\right)\lambda_\eta-\frac{1}{2}(1+3g^2)L_{K\eta}\right), \tag{29}$$

where

$$L_{ab}=\frac{g}{32\pi^2F^2}\ln\frac{M_a^2}{M_b^2}. \tag{30}$$

Again we remove the divergences in g_1 using the renormalized LEC's $\bar{D}_{16}^q$ and $\bar{D}_{17}^q$:

$$D_{16}^q=\bar{D}_{16}^q+\frac{g}{2F^2}\left(1+\frac{23}{9}g^2\right)\lambda_\eta,\qquad D_{17}^q=\bar{D}_{17}^q+\frac{3g}{2F^2}\left(1+\frac{11}{9}g^2\right)\lambda_\eta. \tag{31}$$

Therefore, the result for the renormalized coupling g_1^{du} in SU(3) is expressed in terms of the axial charge and quark mass in the chiral limit, the meson masses, and two unknown LEC's, $\bar{D}_{16}$ and $\bar{D}_{17}$:

$$\begin{aligned}g_1=g_1^{du}=g+\delta g_1^{\chi_3}=g\bigg\{&1-\frac{g^2}{16\pi^2F^2}\left(M_\pi^2+M_K^2+\frac{M_\eta^2}{3}\right)+\frac{M_\pi^3}{24\pi mF^2}(3+3g^2-4C_3^qm+8C_4^qm)\\&+\frac{M_K^3}{48\pi mF^2}\left(3+\frac{9}{2}g^2+8C_4^qm\right)+\frac{g^2M_\eta^3}{48\pi mF^2}\bigg\}+2M_\pi^2\left(\bar{D}_{16}^q+\frac{1}{3}\bar{D}_{17}^q-(1+2g^2)L_{\pi\eta}\right)\\&+2M_K^2\left(2\bar{D}_{16}^q-\frac{1}{3}\bar{D}_{17}^q-\frac{1}{2}(1+3g^2)L_{K\eta}\right)\end{aligned} \tag{32}$$

where $\delta g_1^{\chi_3}$ is the SU(3) chiral correction.

Also we perform an expansion of $g_1=g_1^{du}$ in powers of the SU(3)-breaking parameter $m_s-\hat{m}$:

$$g_1=g_1^{du}=g_1^{\mathrm{SU}_3}+\delta g_1 \tag{33}$$

where

$$g_1^{\mathrm{SU}_3}=g\left\{1-\frac{7g^2\bar{M}^2}{48\pi^2F^2}+\frac{\bar{M}^3}{48\pi mF^2}\left(9+\frac{23}{2}g^2-8C_3^qm+24C_4^qm\right)\right\}+6\bar{M}^2\bar{D}_{16}^q \tag{34}$$

is the SU(3) symmetric term, and

$$\begin{aligned}\delta g_1&=(M_K^2-M_\pi^2)\left\{\frac{g}{96\pi^2F^2}\left(9+\frac{59}{3}g^2\right)-\frac{g\bar{M}}{96\pi mF^2}\left(9+\frac{11}{2}g^2-16C_3^qm+24C_4^qm\right)-\frac{2}{3}\bar{D}_{17}^q\right\}+\mathcal{O}((M_K^2-M_\pi^2)^2)\\&\equiv h_1(M_K^2-M_\pi^2)+\mathcal{O}((M_K^2-M_\pi^2)^2)\end{aligned} \tag{35}$$

is the SU(3)-breaking term, where we display the first-order term. Here for convenience we define the so-called SU(3) symmetric octet mass $\bar{M}$ of pseudoscalar mesons as $\bar{M}=2\bar{m}B$ with $\bar{m}=(m_u+m_d+m_s)/3=(2\hat{m}+m_s)/3$.

Likewise, we have the result for the $s\to u$ flavor transition axial coupling g_1^{su} in terms of $\bar{D}_{16}^q$ and $\bar{D}_{17}^q$:

$$\begin{aligned}g_1^{su}=g+\delta g_1^{\chi_{su}}=g\bigg\{&1-\frac{3g^2}{64\pi^2F^2}\left(M_\pi^2+2M_K^2+\frac{M_\eta^2}{9}\right)+\frac{M_\pi^3}{64\pi mF^2}\left(3+\frac{9}{2}g^2+16C_4^qm\right)\\&+\frac{M_K^3}{32\pi mF^2}\left(3+\frac{9}{2}g^2-\frac{16}{3}C_3^qm+\frac{16}{3}C_4^qm\right)+\frac{M_\eta^3}{64\pi mF^2}\left(3+\frac{11}{6}g^2+\frac{16}{3}C_4^qm\right)\bigg\}\\&+2M_\pi^2\left(\bar{D}_{16}^q-\frac{1}{6}\bar{D}_{17}^q-\frac{3}{8}(1+3g^2)L_{\pi\eta}\right)+2M_K^2\left(2\bar{D}_{16}^q+\frac{1}{6}\bar{D}_{17}^q-\frac{3}{4}(1+3g^2)L_{K\eta}\right),\end{aligned} \tag{36}$$

where $\delta g_1^{\chi_{su}}$ is the strangeness changing chiral correction. The $m_s-\hat{m}$ expansion for the g_1^{su} coupling reads

$$g_1^{su}=g_1^{\mathrm{SU}_3}+\delta g_1^{su} \tag{37}$$

where

$$\delta g_1^{su} = (M_K^2 - M_\pi^2)\Big\{\frac{g}{192\pi^2 F^2}\Big(9 + \frac{79}{3}g^2\Big) + \frac{g\bar{M}}{192\pi m F^2}\Big(9 + \frac{11}{2}g^2 - 16C_3^q m - 16C_4^q m\Big) + \frac{1}{3}\bar{D}_{17}^q\Big\} + \mathcal{O}((M_K^2 - M_\pi^2)^2) \equiv h_2(M_K^2 - M_\pi^2) + \mathcal{O}((M_K^2 - M_\pi^2)^2). \tag{38}$$

D. Bare quark matrix elements

Now we are in the position to discuss the calculation of the matrix elements of the bare quark operators derived in Eqs. (11) and (12) and restrict in the following to the one-body approximation:

$$V_{\mu,1}^{ij}(0) = \bar{q}_j(0)\gamma_\mu q_i(0), \qquad A_{\mu,1}^{ij}(0) = \bar{q}_j(0)\gamma_\mu\gamma_5 q_i(0). \tag{39}$$

As stressed earlier, the evaluation of these matrix elements can be done independently of the calculation of chiral effects. Therefore, it can be relegated to a quark model based on a specific scenario about hadronization and confinement of quarks within baryons, including internal quark dynamics.

As mentioned in the Introduction, in the calculation of the baryon matrix elements $\langle V_{\mu,1}\rangle = \langle B_j(p')|V_{\mu,1}^{ij}(0)|B_i(p)\rangle$ and $\langle A_{\mu,1}\rangle = \langle B_j(p')|A_{\mu,1}^{ij}(0)|B_i(p)\rangle$, we employ a general ansatz for the spatial form of the quark wave functions, without referring to any specific realization. In a forthcoming paper [34] we will evaluate the baryon matrix elements using a Lorentz- and gauge-invariant quark model based on a specific hadronization ansatz—i.e. modeling internal quark dynamics, which goes beyond the additive quark model. Note that in Ref. [37] we did an analogous study of the electromagnetic properties of the baryon octet and the $\Delta(1230)$ resonance. One should stress that the approach [37] is restricted to the evaluation of baryon matrix elements in the Euclidean space to avoid unphysical cuts. Therefore, it pretends to the evaluation of the baryon matrix elements only for the Euclidean transferred momentum squared.

In the evaluation of the bare matrix elements $\langle V_{\mu,1}\rangle$ and $\langle A_{\mu,1}\rangle$, we follow, e.g., Refs. [5,39]. We begin by introducing the ground-state wave function of the quark with flavor f moving in a spin-independent central potential:

$$q_f(x) = q_f(\vec{x})e^{-iEt}, \qquad q_f(\vec{x}) = \begin{pmatrix} u_f(r) \\ il_f(r)\frac{\vec{\sigma}\cdot\vec{x}}{r}\end{pmatrix}\chi_s\chi_f\chi_c, \tag{40}$$

where $u_f(r)$ and $l_f(r)$ signify the upper and lower components of the quark wave function (in the nonrelativistic limit l_f vanishes); χ_s, χ_f, χ_c are the spin, flavor, and color quark wave functions, respectively. Note that this form of the quark wave function also appears in relativistic harmonic oscillator models utilizing a central potential (see references in [39]). In the following we use the notation ($u = u_u = u_d$, $l = l_u = l_d$) for nonstrange and (u_s, l_s) for strange quark wave functions. In practice it is also convenient to introduce ratios between the two sets of wave functions via

$$\xi_u = \frac{u_s}{u}, \qquad \xi_l = \frac{l_s}{l}. \tag{41}$$

The normalization condition for the spatial wave function is

$$\int d^3x q_f^\dagger(\vec{x})q_f(\vec{x}) = \int d^3x(u_f^2(r) + l_f^2(r)) = 1. \tag{42}$$

Now we are in the position to pin down the matrix elements $\langle V_{\mu,1}\rangle$ and $\langle A_{\mu,1}\rangle$ by considering quark operators with different flavor structure. We begin by calculating the vector matrix elements for the respective initial and final baryon states:

$$V_1^{B_iB_j} = \Big\langle B_j\uparrow\Big|\int d^3x\bar{q}_j(x)\gamma^0 q_i(x)\Big|B_i\uparrow\Big\rangle = \int d^3x[u_i(r)u_j(r) + l_i(r)l_j(r)]\Big\langle B_j\uparrow\Big|\sum_{k=1}^3\lambda_{ji}^k\Big|B_i\uparrow\Big\rangle, \tag{43}$$

where the spin-flavor matrix elements $\langle B_j\uparrow|\sum_{k=1}^3\lambda_{ji}^k|B_i\uparrow\rangle$ are evaluated using the simple SU(6) quark model:

$$\Big\langle B_j\uparrow\Big|\sum_{k=1}^3\lambda_{ji}^k\Big|B_i\uparrow\Big\rangle = \begin{cases} 1 & \text{for } n\to p, \\ -\sqrt{3/2} & \text{for } \Lambda\to p, \\ -1 & \text{for } \Sigma^-\to n, \\ 0 & \text{for } \Sigma^-\to\Lambda, \\ \sqrt{3/2} & \text{for } \Xi^-\to\Lambda, \\ \sqrt{1/2} & \text{for } \Xi^-\to\Sigma^0, \\ 1 & \text{for } \Xi^0\to\Sigma^+, \end{cases} \tag{44}$$

λ_{ji}^k are the linear combinations of the Gell-Mann flavor matrices, and relativistic effects are included in the overlap of the spatial quark wave functions. It is clear that $F_1^{np}(0) = 1$ as required by conservation of the vector current (CVC)—the CVC prediction of unity emerges as the normalization condition (42) for the spatial quark wave functions. Note that in the SU(3) limit, wherein quarks, independent of their flavor, have identical wave functions, the "spatial" integral in Eq. (43) is identical to the wave function normalization condition (42). As a result, in $\Delta S = 1$ transitions the corresponding vector current is also conserved to the extent that the s and u quarks are degenerate in mass.

In the case of the corresponding baryon axial matrix elements, we have

$$A_1^{B_iB_j} = \left\langle B_j\uparrow \left| \int d^3x \bar{q}_j(x)\gamma^3\gamma^5 q_i(x) \right| B_i\uparrow \right\rangle = \int d^3x \left[u_i(r)u_j(r) - \frac{1}{3} l_i(r) l_j(r) \right] \times \left\langle B_j\uparrow \left| \sum_{k=1}^3 \sigma_3^k \lambda_{ji}^k \right| B_i\uparrow \right\rangle, \quad (45)$$

where the spin-flavor matrix elements $\langle B_j\uparrow|\sum_{k=1}^3 \sigma_3^k\lambda_{ji}^k|B_i\uparrow\rangle$ are evaluated using SU(6):

$$\left\langle B_j\uparrow \left| \sum_{k=1}^3 \sigma_3^k \lambda_{ji}^k \right| B_i\uparrow \right\rangle = \begin{cases} 5/3 & \text{for } n\to p, \\ -\sqrt{3/2} & \text{for } \Lambda\to p, \\ 1/3 & \text{for } \Sigma^-\to n, \\ \sqrt{2/3} & \text{for } \Sigma^-\to\Lambda, \\ \sqrt{1/6} & \text{for } \Xi^-\to\Lambda, \\ 5/(3\sqrt{2}) & \text{for } \Xi^-\to\Sigma^0, \\ 5/3 & \text{for } \Xi^0\to\Sigma^+, \end{cases} \quad (46)$$

and, again, relativistic effects are included in the overlap of the spatial quark wave functions.

Using the normalization condition the integrals over the spatial quark wave functions can be simplified. There are three possible situations: (1) overlap of nonstrange quark wave functions; (2) overlap of strange quark wave functions; (3) overlap of nonstrange with strange quark wave functions. In the first case we have

$$I_V = \int d^3x[u^2(r) + l^2(r)] = 1, \quad (47)$$

$$I_A = \int d^3x\left[u^2(r) - \frac{1}{3}l^2(r)\right] = 1 - \frac{4}{3}\int d^3x l^2(r). \quad (48)$$

In the second case,

$$I_V^{ss} = \int d^3x[u_s^2(r) + l_s^2(r)] = \int d^3x[\xi_u^2 u^2(r) + \xi_l^2 l^2(r)] = 1, \quad (49)$$

$$I_A^{ss} = \int d^3x\left[u_s^2(r) - \frac{1}{3}l_s^2(r)\right] = 1 - \frac{4}{3}\xi_l^2\int d^3x l^2(r) = 1 + \xi_l^2(I_A - 1), \quad (50)$$

where ξ_u and ξ_l can be written in terms of the axial structure integral I_A via

$$\xi_u = \sqrt{1 + 3(1-\xi_l^2)\frac{1 - I_A}{1 + 3I_A}}. \quad (51)$$

It is clear that in the SU(3) limit— $\xi_u = \xi_l \to 1$—the expressions for the overlap integrals I_V^{ss} and I_A^{ss} reduce to I_V and I_A, respectively.

In the third case we have

$$I_V^s = \int d^3x[u(r)u_s(r) + l(r)l_s(r)] = \int d^3x[\xi_u u^2(r) + \xi_l l^2(r)] = \xi_u + \frac{3}{4}(\xi_u - \xi_l)(I_A - 1), \quad (52)$$

$$I_A^s = \int d^3x\left[u(r)u_s(r) - \frac{1}{3}l(r)l_s(r)\right] = \xi_u - \left(\xi_u + \frac{\xi_l}{3}\right)\int d^3x l^2(r) = \xi_u + \left(\frac{3\xi_u}{4} + \frac{\xi_l}{4}\right)(I_A - 1). \quad (53)$$

Here the parameter ξ_u can be rewritten by using identity (51). Therefore, all structure integrals (I_V, I_A, I_V^s, I_A^s, I_V^{ss}, I_A^{ss}) involving spatial quark wave functions are either fixed precisely (like $I_V = I_V^{ss} = 1$) or are expressed in terms of I_A and the parameter ξ_l. In the case of exact SU(3) symmetry the vector and axial integrals are degenerate— $I_V = I_V^s = I_V^{ss} = 1$ and $I_A = I_A^s = I_A^{ss}$. In the nonrelativistic limit $I_A = 1$, $\xi_u = 1$, and all these structure integrals are unity—

$$I_V = I_V^s = I_V^{ss} = I_A = I_A^s = I_A^{ss} = 1. \quad (54)$$

It should be stressed that all the vector integrals satisfy the AGT—either they are exactly equal to unity (like I_V and I_V^{ss}), or deviate from 1 by the corrections of second order in SU(3) breaking. Specifically,

$$I_V^s = 1 + \frac{3}{2}\frac{I_A - 1}{1 + 3I_A}\delta^2 + \mathcal{O}(\delta^3), \quad (55)$$

where $\delta = \xi_l - 1$ is an SU(3)-breaking parameter. In the case of the axial overlap integrals I_A^s and I_A^{ss}, the SU(3)-breaking corrections begin at order $\mathcal{O}(\delta)$.

Finally we note that the bare matrix elements (which contain the effects of valence quarks) can be expressed in terms of the axial structure integral I_A and the parameter δ, which encode the effects of SU(3) breaking, i.e., distinguish the lower components of the strange and nonstrange quarks—

$$I_V = I_V^{ss} = 1, \qquad I_V^s = 1 + \delta I_V^s, \qquad I_A^s = I_A + \delta I_A^s, \qquad I_A^{ss} = I_A + \delta I_A^{ss}, \quad (56)$$

where

$$\begin{aligned}
\delta I_V^s &= \xi_u - 1 + \frac{3}{4}(\xi_u - \xi_l)(I_A - 1)\\
&= \frac{3}{2}\frac{I_A - 1}{1 + 3I_A}\delta^2 + \mathcal{O}(\delta^3) = \mathcal{O}(\delta^2),\\
\delta I_A^s &= \xi_u - 1 + (1 - I_A)\left(1 - \frac{3\xi_u}{4} - \frac{\xi_l}{4}\right)\\
&= (I_A - 1)\delta + \mathcal{O}(\delta^2) = \mathcal{O}(\delta),\\
\delta I_A^{ss} &= (1 - I_A)(1 - \xi_l^2) = 2(I_A - 1)\delta + \mathcal{O}(\delta^2)\\
&= \mathcal{O}(\delta).
\end{aligned} \tag{57}$$

In Sec. III [see Eqs. (62), (65), and (88)] doing the matching of our results to the model-independent expressions derived in ChPT and by Ademollo and Gatto in Ref. [1], we express the quantities I_A and $\delta = \xi_l - 1$ in terms of parameters of the chiral Lagrangian (1).

III. SEMILEPTONIC VECTOR AND AXIAL COUPLINGS OF BARYONS

In this section we combine chiral and valence quark effects in order to derive the expressions for vector $g_V^{B_iB_j}$ and axial $g_A^{B_iB_j}$ couplings which govern the semileptonic transitions between octet baryons. Note, we use the phase convention [9] which gives e.g. the positive sign for the axial coupling g_A^{np} of the neutron β decay. In particular, neglecting contributions of order $q = p' - p$ the matrix elements of semileptonic decays of the baryon octet are determined by two constants, $g_V^{B_iB_j}$ and $g_A^{B_iB_j}$, as

$$\begin{aligned}
M^{B_iB_j}_{\mu,V-A}(p,p) &= M^{B_iB_j}_{\mu,V}(p,p) - M^{B_iB_j}_{\mu,A}(p,p)\\
&= \bar u_{B_j}(p)\gamma_\mu(g_V^{B_iB_j} - \gamma_5 g_A^{B_iB_j})u_{B_i}(p).
\end{aligned} \tag{58}$$

Using Eqs. (11) and (12), and the expressions for the couplings encoding chiral effects and valence quark contributions, the quantities $g_V^{B_iB_j}$ and $g_A^{B_iB_j}$ are defined as

$$\begin{aligned}
g_V^{B_iB_j} &= F_1^{B_iB_j}(0) = f_1^{ij}V_1^{B_iB_j},\\
g_A^{B_iB_j} &= G_1^{B_iB_j}(0) = g_1^{ij}A_1^{B_iB_j}.
\end{aligned} \tag{59}$$

A. Nucleon axial charge

First we examine the nucleon axial charge and perform the matching to ChPT—we relate the parameters of our Lagrangian to those of ChPT. In SU(2) the expression for the nucleon axial charge is

$$\begin{aligned}
g_A = \overset{\circ}{g}_A - \frac{\overset{\circ}{g}{}_A^3 M_\pi^2}{16\pi^2F^2} + \frac{\overset{\circ}{g}_A M_\pi^3}{24\pi \overset{\circ}{m}_N F^2}\\
\times(3 + 3\overset{\circ}{g}{}_A^2 - 4c_3\overset{\circ}{m}_N + 8c_4\overset{\circ}{m}_N) + 4M_\pi^2\bar d_{16}
\end{aligned} \tag{60}$$

in ChPT [27–29], and

$$\begin{aligned}
g_A &= g_1A_1^{np}\\
&= \frac{5}{3}I_A\bigg\{g - \frac{g^3M_\pi^2}{16\pi^2F^2} + \frac{gM_\pi^3}{24\pi mF^2}\\
&\quad\times(3 + 3g^2 - 4c_3^q m + 8c_4^q m) + 4M_\pi^2\bar d_{16}^q\bigg\}
\end{aligned} \tag{61}$$

in our approach, where $\overset{\circ}{g}_A$ and $\overset{\circ}{m}_N$ are the values of the nucleon axial charge and nucleon mass in the chiral limit. Matching these expressions for the axial charge up to order $\mathcal{O}(p^4)$, we derive the following relations between the parameters of the two approaches:

$$\overset{\circ}{g}_A = gR = \frac{5}{3}gI_A = \frac{5}{3}g\left(1 - \frac{4}{3}\int d^3x l^2(r)\right), \tag{62}$$

$$\bar d_{16} - \frac{\overset{\circ}{g}{}_A^3}{64\pi^2F^2} = \frac{5}{3}I_A\left(\bar d_{16}^q - \frac{g^3}{64\pi^2F^2}\right), \tag{63}$$

$$\frac{1 + \overset{\circ}{g}{}_A^2}{8\overset{\circ}{m}_N} + \frac{c_4}{3} - \frac{c_3}{6} = \frac{1 + g^2}{8m} + \frac{c_4^q}{3} - \frac{c_3^q}{6}. \tag{64}$$

Here, for convenience, we introduce the definition $R = \overset{\circ}{g}_A/g$. Then from the first matching condition we can derive constraints on the "axial" integral I_A (48) and on the integral over the square of the lower/upper components of the spatial quark wave functions—

$$I_A = \tfrac{3}{5}R \tag{65}$$

and

$$\int d^3xl^2(r) = 1 - \int d^3xu^2(r) = \frac{3}{4}\left(1 - \frac{3}{5}R\right). \tag{66}$$

Therefore, the second matching condition (63) reduces to

$$\bar d_{16} - \frac{\overset{\circ}{g}{}_A^3}{64\pi^2F^2} = R\left(\bar d_{16}^q - \frac{g^3}{64\pi^2F^2}\right). \tag{67}$$

Taking into account the relation between the mass and axial charge of both the nucleon and the quark in the chiral limit,

$$\frac{\overset{\circ}{m}_N}{m} = \left(\frac{\overset{\circ}{g}_A}{g}\right)^2 = R^2 \tag{68}$$

as derived in Ref. [36] from the matching of the nucleon mass in the two approaches, the third condition (64) can be simplified as

$$c_3 - 2c_4 = c_3^q - 2c_4^q + \frac{3}{4\overset{\circ}{m}_N}(1 - R^2). \tag{69}$$

We have two essential remarks: (1) the matching condition (68) is very important in our approach, because it allows us to remove the unknown scale parameter—constituent quark mass—from the explicit expressions of the matrix elements; (2) for the evaluation of the nucleon axial charge we do not require an explicit form for the spatial quark wave functions [see Eq. (66)].

Having dealt with SU(2), we note the corresponding expression for the nucleon axial charge in SU(3):

$$g_A = gR\biggl\{1 - \frac{g^2}{16\pi^2F^2}\biggl(M_\pi^2 + M_K^2 + \frac{M_\eta^2}{3}\biggr) + \frac{M_\pi^3}{24\pi mF^2}(3 + 3g^2 - 4C_3^q m + 8C_4^q m) + \frac{M_K^3}{48\pi mF^2}\biggl(3 + \frac{9}{2}g^2 + 8C_4^q m\biggr) + \frac{g^2M_\eta^3}{48\pi mF^2}\biggr\} + 2M_\pi^2 R\biggl(\bar{D}_{16}^q + \frac{1}{3}\bar{D}_{17}^q - (1+2g^2)L_{\pi\eta}\biggr) + 2M_K^2 R\biggl(2\bar{D}_{16}^q - \frac{1}{3}\bar{D}_{17}^q - \frac{1}{2}(1+3g^2)L_{K\eta}\biggr). \tag{70}$$

Substituting $g = \overset{\circ}{g}_A/R$ and $m = \overset{\circ}{m}_N/R^2$ we finally get

$$g_A = \overset{\circ}{g}_A\biggl\{1 - \frac{\overset{\circ}{g}{}_A^2}{16\pi^2F^2R^2}\biggl(M_\pi^2 + M_K^2 + \frac{M_\eta^2}{3}\biggr) + \frac{M_\pi^3}{8\pi\overset{\circ}{m}_N F^2}\biggl(R^2 + \overset{\circ}{g}{}_A^2 - \frac{4}{3}C_3^q\overset{\circ}{m}_N + \frac{8}{3}C_4^q\overset{\circ}{m}_N\biggr) + \frac{M_K^3}{16\pi\overset{\circ}{m}_N F^2}\biggl(R^2 + \frac{3}{2}\overset{\circ}{g}{}_A^2 + \frac{8}{3}C_4^q\overset{\circ}{m}_N\biggr) + \frac{\overset{\circ}{g}{}_A^2 M_\eta^3}{48\pi\overset{\circ}{m}_N F^2}\biggr\} + 2M_\pi^2 R\biggl(\bar{D}_{16}^q + \frac{1}{3}\bar{D}_{17}^q - \frac{R^2 + 2\overset{\circ}{g}{}_A^2}{R^3}\overset{\circ}{L}_{\pi\eta}\biggr) + 2M_K^2 R\biggl(2\bar{D}_{16}^q - \frac{1}{3}\bar{D}_{17}^q - \frac{R^2 + 3\overset{\circ}{g}{}_A^2}{2R^3}\overset{\circ}{L}_{K\eta}\biggr), \tag{71}$$

where

$$\overset{\circ}{L}_{ab} = \frac{\overset{\circ}{g}_A}{32\pi^2F^2}\ln\frac{M_a^2}{M_b^2}. \tag{72}$$

B. Baryon octet semileptonic couplings

Now we turn to the discussion of the vector and axial couplings $g_V^{B_iB_j}$ and $g_A^{B_iB_j}$ (59) governing the semileptonic decays of the octet baryons. Our results are summarized in Table III, and have a relatively simple structure.

In the case of the vector coupling $g_V^{B_iB_j}$, our results are unchanged from those of the SU(3) limit in the case of the two $\Delta S = 0$ transitions, while in the case of the five $\Delta S = 1$ transitions, our predictions are found by multiplying the simple SU(3) limit forms by the common factor $1 + \delta_V$. Here

$$\delta_V = \delta f_1^{su} + \delta I_V^s + \delta f_1^{su}\delta I_V^s \tag{73}$$

where the factors δf_1^{su} and δI_V^s have been defined in Eqs. (23) and (57) and are both second order in SU(3) breaking, in accord with the Ademollo-Gatto theorem [1].

In the case of the axial coupling $g_A^{B_iB_j}$ the SU(3) symmetry breaking is first order and, as derived in Ref. [1] and discussed e.g. in Refs. [7,11], can be described in terms of an effective Lagrangian containing two SU(3) symmetric terms proportional to the conventional couplings D plus F and four first order SU(3)-breaking terms proportional to the couplings H_i ($i = 1\cdots 4$):

$$\begin{aligned}\mathcal{L} &= D\langle\bar{B}\gamma^\mu\gamma^5\{a_\mu B\}\rangle + F\langle\bar{B}\gamma^\mu\gamma^5[a_\mu B]\rangle \\ &+ \frac{H_1}{\sqrt{3}}\langle\bar{B}\gamma^\mu\gamma^5 B\{a_\mu\lambda_8\}\rangle + \frac{H_2}{\sqrt{3}}\langle\bar{B}\gamma^\mu\gamma^5\{a_\mu\lambda_8\}B\rangle \\ &+ \frac{H_3}{\sqrt{3}}\langle\bar{B}\gamma^\mu\gamma^5 a_\mu B\lambda_8 - \bar{B}\gamma^\mu\gamma^5\lambda_8 B a_\mu\rangle \\ &+ \frac{H_4}{\sqrt{3}}(\langle\bar{B}a_\mu\rangle\gamma^\mu\gamma^5\langle B\lambda_8\rangle + \langle\bar{B}\lambda_8\rangle\gamma^\mu\gamma^5\langle Ba_\mu\rangle)\end{aligned}$$

where

$$B = \begin{pmatrix} \Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & \Sigma^+ & p \\ \Sigma^- & -\Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & n \\ \Xi^- & \Xi^0 & -2\Lambda/\sqrt{6} \end{pmatrix} \tag{74}$$

is the octet of baryon fields, while a_μ denotes the external axial field

$$a_\mu = \begin{pmatrix} 0 & a_\mu^{du}V_{ud} & a_\mu^{su}V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{75}$$

with V_{ud} and V_{us} being the usual CKM matrix elements.

The axial semileptonic couplings of baryons are then expressed in terms of the constants D, F, and H_i as (see also Refs. [7,11])

$$\begin{aligned}
g_A^{np} &= D + F + \frac{2}{3}(H_2 - H_3),\\
g_A^{\Lambda p} &= -\sqrt{\frac{3}{2}}\left(F + \frac{D}{3} + \frac{1}{9}(H_1 - 2H_2 - 3H_3 - 6H_4)\right),\\
g_A^{\Sigma^- n} &= D - F - \frac{1}{3}(H_1 + H_3),\\
g_A^{\Sigma^- \Lambda} &= \sqrt{\frac{2}{3}}\left(D + \frac{1}{3}(H_1 + H_2 + 3H_4)\right),\\
g_A^{\Xi^- \Lambda} &= \sqrt{\frac{3}{2}}\left(F - \frac{D}{3} + \frac{1}{9}(2H_1 - H_2 - 3H_3 + 6H_4)\right),\\
g_A^{\Xi^- \Sigma^0} &= \sqrt{\frac{1}{2}}\left(D + F - \frac{1}{3}(H_2 - H_3)\right),\\
g_A^{\Xi^0 \Sigma^+} &= D + F - \frac{1}{3}(H_2 - H_3).
\end{aligned} \tag{76}$$

In our approach the axial couplings are expressed in terms of the SU(3) symmetric contribution $g_A^{\mathrm{SU}_3} = \frac{5}{3} g_1^{\mathrm{SU}_3} I_A$ and two symmetry-breaking factors, δ_{A_1} and δ_{A_2}, using the definitions (see also Table III)

$$\tfrac{5}{3} g_1 I_A = g_A^{\mathrm{SU}_3}(1 + \delta_{A_1}), \tag{77}$$

$$\tfrac{5}{3} g_1^{su} I_A^s = g_A^{\mathrm{SU}_3}(1 + \delta_{A_2}). \tag{78}$$

The basic symmetry-breaking pattern is then similar to that in the case of the vector current—the two $\Delta S = 0$ transitions are altered from their SU(3) values by one factor $1 + \delta_{A_1}$, while the five $\Delta S = 1$ transitions are modified by a different factor $1 + \delta_{A_2}$, where

$$\delta_{A_1} = \frac{\delta g_1}{g_1^{\mathrm{SU}_3}}, \qquad \delta_{A_2} = \frac{\delta g_1^{su}}{g_1^{\mathrm{SU}_3}} + \frac{\delta I_A^s}{I_A} + \frac{\delta g_1^{su}}{g_1^{\mathrm{SU}_3}}\frac{\delta I_A^s}{I_A}. \tag{79}$$

Note that both factors δ_{A_i} include not only leading—$\mathcal{O}(m_s - \hat{m})$—but also higher-order SU(3)-breaking corrections. In order to identify the effective couplings D, F, and H_i in the Lagrangian (74), we reduce the expressions (79) to the pieces first order in SU(3) breaking. Then the leading-order factors are given by [see Eqs. (35), (38), and (57)]

$$\delta_{A_1}^{(1)} = \frac{h_1}{g_1^{\mathrm{SU}_3}}(M_K^2 - M_\pi^2), \tag{80}$$

$$\delta_{A_2}^{(1)} = \frac{h_2}{g_1^{\mathrm{SU}_3}}(M_K^2 - M_\pi^2) - \frac{1 - I_A}{I_A}\delta, \tag{81}$$

where the superscript (1) indicates that we have truncated the full expressions to include only the pieces first order in SU(3) breaking. Matching our results then for the axial couplings $g_A^{B_iB_j}$ to the model-independent predictions (76), we find

$$D = \tfrac{3}{2}F = \tfrac{3}{5}g_A^{\mathrm{SU}_3} \tag{82}$$

for the SU(3) symmetric contribution, and

$$\begin{aligned}
H_1 = \tfrac{1}{5}H_2 &= \tfrac{1}{2}I_A h_1(M_K^2 - M_\pi^2)\\
&= -I_A h_2(M_K^2 - M_\pi^2) + g_1^{\mathrm{SU}_3}(1 - I_A)\delta,
\end{aligned} \tag{83}$$

$$H_3 = H_4 = 0 \tag{84}$$

for the SU(3)-breaking terms. Therefore, to first order in SU(3) breaking the factors $\delta_{A_1}^{(1)}$ and $\delta_{A_2}^{(1)}$ are not independent and are related via $\delta_{A_1}^{(1)} = -2\delta_{A_2}^{(1)}$, which, in terms of parameters of chiral Lagrangian (1) or the model-independent Lagrangian (74), can be written as

$$\begin{aligned}
\delta_A^{(1)} = \frac{2H_1}{D} &= \frac{h_1}{g_1^{\mathrm{SU}_3}}(M_K^2 - M_\pi^2)\\
&= -2\left(\frac{h_2}{g_1^{\mathrm{SU}_3}}(M_K^2 - M_\pi^2) - \frac{1 - I_A}{I_A}\delta\right).
\end{aligned} \tag{85}$$

Then using the relation (83) or (85) we deduce the following constraint involving the parameters of the chiral Lagrangian and the quantities defining the matrix elements of valence quarks (bare quark matrix elements):

$$G\frac{M_K^2 - M_\pi^2}{(4\pi F)^2} = \frac{1 - I_A}{I_A}\delta, \tag{86}$$

where

$$G = \frac{g}{g_1^{\mathrm{SU}_3}}\left(\frac{3}{2} + \frac{23}{6}g^2 - \frac{10\pi}{3}\bar{M}C_4^q\right). \tag{87}$$

The latter equation can be used to express the unknown quantity δ in terms of the parameters of the chiral Lagrangian (1)—

$$\delta = \frac{GI_A}{1 - I_A}\frac{M_K^2 - M_\pi^2}{(4\pi F)^2}. \tag{88}$$

Substituting Eqs. (57) and (88) into Eqs. (73) we can then in turn express $\delta_V^{(2)}$ [the leading contribution to δ_V including second-order SU(3) breaking] in terms of the parameters of the chiral Lagrangian (1):

$$\delta_V^{(2)} = \delta f_1^{su} - \frac{3}{2}\frac{G^2 I_A^2}{(1 - I_A)(1 + 3I_A)}\frac{(M_K^2 - M_\pi^2)^2}{(4\pi F)^4}. \tag{89}$$

IV. NUMERICAL ANALYSIS

Now we perform the numerical analysis of the vector and axial couplings of quarks and baryons. First, we deduce constraints on the quark LEC's from the data on semileptonic decays of the baryon octet. Then we compare our results for the axial baryon couplings to the ones of baryon ChPT in the large-N_c limit [21], obtained from a fit to the measured decays and ratios $g_A^{B_iB_j}/g_V^{B_iB_j}$.

For the quark parameters in the chiral limit we use $g = 0.9$ and $m = 420$ MeV, values fixed previously in [36,37]. Note that these parameters are related to the corresponding nucleon quantities $\overset{\circ}{g}_A$ and $\overset{\circ}{m}_N$ via the matching condition (68). In particular, using the values $\overset{\circ}{g}_A = 1.2$, 1.2695 (data), 1.3, we find for the nucleon mass in the chiral limit the results $\overset{\circ}{m}_N = 746.7$, 835.7, 876.3 MeV, respectively, which are consistent with the values deduced in the context of the baryon ChPT (see discussion in [30,32,36,37,46–48]).

First, we analyze the axial charges of the quark and the nucleon in SU(2) and SU(3), respectively. In SU(2) the corresponding quantities in terms of the LEC's c_3^q, c_4^q, and d_{16}^q are given by

$$g_1 = 0.939 + 0.078\alpha, \qquad \alpha = 0.195(2c_4^q - c_3^q)\ \mathrm{GeV} + \bar{d}_{16}^q\ \mathrm{GeV}^2 \tag{90}$$

and

$$g_A = \begin{cases} 1.251 + 0.104\alpha & \text{for } \overset{\circ}{g}_A = 1.2, \\ 1.324 + 0.110\alpha & \text{for } \overset{\circ}{g}_A = 1.2695, \\ 1.356 + 0.113\alpha & \text{for } \overset{\circ}{g}_A = 1.3. \end{cases} \tag{91}$$

Matching the expression for g_A (91) to its experimental value we derive the following constraints on the SU(2) quark LEC's:

$$\alpha = \begin{cases} 0.178 & \text{for } \overset{\circ}{g}_A = 1.2, \\ -0.495 & \text{for } \overset{\circ}{g}_A = 1.2695, \\ -0.765 & \text{for } \overset{\circ}{g}_A = 1.3. \end{cases} \tag{92}$$

Using the matching condition (69), relating the combination $2c_4^q - c_3^q$ of quark LEC's to the corresponding ChPT LEC's c_3 and c_4, and using the averaged values of $c_3 = -4.7\ \mathrm{GeV}^{-1}$ and $c_4 = 3.5\ \mathrm{GeV}^{-1}$ from [49], we estimate the LEC $\bar{d}_{16}^q = -1.957, -2.605, -2.868\ \mathrm{GeV}^{-2}$ (the corresponding ChPT LEC $\bar{d}_{16}$ is equal to -2.469, -3.486, $-3.931\ \mathrm{GeV}^{-2}$). Note that for the axial charge of the quark at one loop we find the values $g_1 = 0.952$, 0.9, 0.879, respectively, which correspond to the values of $\overset{\circ}{g}_A = 1.2$, 1.2695, 1.3.

In SU(3) the corresponding results for the axial charges are

$$g_1 = 2.163 + 1.014\beta, \qquad \beta = (-0.012C_3^q + 0.563C_4^q)\ \mathrm{GeV} + (\bar{D}_{16}^q - 0.147\bar{D}_{17}^q)\ \mathrm{GeV}^2 \tag{93}$$

and

$$g_A = \begin{cases} 2.884 + 1.352\beta & \text{for } \overset{\circ}{g}_A = 1.2, \\ 3.051 + 1.430\beta & \text{for } \overset{\circ}{g}_A = 1.2695, \\ 3.124 + 1.464\beta & \text{for } \overset{\circ}{g}_A = 1.3. \end{cases} \tag{94}$$

Matching the expression for g_A (94) to its experimental value, we derive the following constraints on the SU(3) quark LEC's:

$$\beta = \begin{cases} -1.194 & \text{for } \overset{\circ}{g}_A = 1.2, \\ -1.246 & \text{for } \overset{\circ}{g}_A = 1.2695, \\ -1.267 & \text{for } \overset{\circ}{g}_A = 1.3. \end{cases} \tag{95}$$

Next we estimate the quark vector coupling f_1^{su}. We determine

$$f_1^{su} = 1 + \delta f_1^{su} = 1.070, \tag{96}$$

i.e., an SU(3)-breaking correction $\delta f_1^{su} = 7\%$.

Next we extract information about the SU(3)-breaking parameters $\delta_V^{(2)}$ and $\delta_A^{(1)}$ and find an additional constraint for the linear combination of LEC's C_3^q, C_4^q, and $\bar{D}_{17}^q$ using data for the ratios $r^{B_iB_j} = g_A^{B_iB_j}/g_V^{B_iB_j}$. Direct calculation of $\delta_V^{(2)}$ and $\delta_A^{(1)}$ gives

$$\delta_V^{(2)} = \begin{cases} 0.070 - 0.074r_V^2 & \text{for } \overset{\circ}{g}_A = 1.2, \\ 0.070 - 0.103r_V^2 & \text{for } \overset{\circ}{g}_A = 1.2695, \\ 0.070 - 0.123r_V^2 & \text{for } \overset{\circ}{g}_A = 1.3, \end{cases} \tag{97}$$

and

$$\delta_A^{(1)} = -0.136r_A \tag{98}$$

independent of the value for $\overset{\circ}{g}_A$, where r_V and r_A are given by

$$r_V = \frac{1 - 0.935C_4^q\ \mathrm{GeV}}{1 + 0.415\gamma_1}, \qquad r_A = \frac{1 + 0.449\gamma_2}{1 + 0.415\gamma_1}. \tag{99}$$

Here γ_1 and γ_2 are the combinations of the quark LEC's:

$$\gamma_1 = \bar{D}_{16}^q\ \mathrm{GeV}^2 - 0.311(C_3^q - 3C_4^q)\ \mathrm{GeV}, \qquad \gamma_2 = \bar{D}_{17}^q\ \mathrm{GeV}^2 - 1.400(2C_3^q - 3C_4^q)\ \mathrm{GeV}. \tag{100}$$

Matching our results for $r^{B_iB_j}$ to data [3] for the five semileptonic modes, we have the following conditions involving the parameters δ_V and δ_A:

$$\begin{aligned} r^{np} &= g_A = g_A^{\mathrm{SU}_3}(1 + \delta_A^{(1)}) = 1.2695 \pm 0.0029, \\ r^{\Lambda p} &= \frac{3g_A^{\mathrm{SU}_3}}{5}\frac{1 - \frac{1}{2}\delta_A^{(1)}}{1 + \delta_V^{(2)}} = 0.718 \pm 0.015, \\ r^{\Sigma n} &= -\frac{g_A^{\mathrm{SU}_3}}{5}\frac{1 - \frac{1}{2}\delta_A^{(1)}}{1 + \delta_V^{(2)}} = -0.34 \pm 0.017, \\ r^{\Xi^-\Lambda} &= \frac{g_A^{\mathrm{SU}_3}}{5}\frac{1 - \frac{1}{2}\delta_A^{(1)}}{1 + \delta_V^{(2)}} = 0.25 \pm 0.05, \\ r^{\Xi^0\Sigma^+} &= g_A^{\mathrm{SU}_3}\frac{1 - \frac{1}{2}\delta_A^{(1)}}{1 + \delta_V^{(2)}} = 1.20 \pm 0.04 \pm 0.03. \end{aligned} \tag{101}$$

Restricting to the central values of the data, we deduce the following constraints on $\delta_V^{(2)}$ and $\delta_A^{(1)}$:

$$g_A^{SU_3}(1+\delta_A^{(1)}) = 1.2695 \qquad \text{from the } n \to p \text{ transition,} \tag{102}$$

$$g_A^{SU_3}\frac{1-\frac{1}{2}\delta_A^{(1)}}{1+\delta_V^{(2)}} = \begin{cases} 1.197 & \text{from the } \Lambda \to p \text{ transition,} \\ 1.700 & \text{from the } \Sigma^- \to n \text{ transition,} \\ 1.250 & \text{from the } \Xi^- \to \Lambda \text{ transition,} \\ 1.200 & \text{from the } \Xi^0 \to \Sigma^+ \text{ transition.} \end{cases} \tag{103}$$

The three modes ($\Lambda \to p$, $\Xi^- \to \Lambda$, $\Xi^0 \to \Sigma^+$) are quite consistent with each other. Future, more precise data for the $\Sigma^- \to n$ mode will probably yield a smaller value for $r^{\Sigma n}$. As already stated above, in order to get a better quantitative agreement with experiment, we plan to go beyond the simple SU(6) quark model. Then we intend to evaluate the valence quark matrix elements (see discussion in Sec. II D) in a fully relativistic quark model based on a specific scenario about hadronization and confinement of quarks inside the baryon [34]. Roughly speaking, instead of the trivial identities (102) and (103) involving only two SU(3)-breaking parameters δ_V and δ_A, we will derive more general identities involving additional symmetry-breaking parameters.

Using Eq. (102) we derive the following constraint on the quark LEC's:

$$\gamma_1 - 0.147\gamma_2 = \begin{cases} -1.144 & \text{for } \overset{\circ}{g}_A = 1.2, \\ -1.195 & \text{for } \overset{\circ}{g}_A = 1.2695, \\ -1.216 & \text{for } \overset{\circ}{g}_A = 1.3. \end{cases} \tag{104}$$

Next, using the typical value $\simeq 1.2$ for the ratios in Eq. (103), we deduce the following constraints on the linear combinations of quark LEC's:

$$\begin{aligned} \gamma_1 + 0.078\gamma_2 - 0.130C_4^q \text{ GeV} &= -1.787 \quad \text{for } \overset{\circ}{g}_A = 1.2, \\ \gamma_1 + 0.079\gamma_2 - 0.174C_4^q \text{ GeV} &= -1.899 \quad \text{for } \overset{\circ}{g}_A = 1.2695, \\ \gamma_1 + 0.080\gamma_2 - 0.205C_4^q \text{ GeV} &= -1.962 \quad \text{for } \overset{\circ}{g}_A = 1.3. \end{aligned} \tag{105}$$

Finally, using two equations (104) and (105) on four LEC's, C_3^q, C_4^q, $\bar{D}_{16}^q$, and $\bar{D}_{17}^q$, we can express two of them (e.g. $\bar{D}_{16}^q$ and $\bar{D}_{17}^q$) through the other two (C_3^q and C_4^q) as follows:

For $\overset{\circ}{g}_A = 1.2$

$$\begin{aligned} \bar{D}_{16}^q &= -1.565 \text{ GeV}^{-2} + 0.311(C_3^q - 2.727C_4^q) \text{ GeV}^{-1}, \\ \bar{D}_{17}^q &= -2.862 \text{ GeV}^{-2} + 2.800(C_3^q - 1.294C_4^q) \text{ GeV}^{-1}. \end{aligned} \tag{106}$$

For $\overset{\circ}{g}_A = 1.2695$

$$\begin{aligned} \bar{D}_{16}^q &= -1.652 \text{ GeV}^{-2} + 0.311(C_3^q - 2.637C_4^q) \text{ GeV}^{-1}, \\ \bar{D}_{17}^q &= -3.111 \text{ GeV}^{-2} + 2.800(C_3^q - 1.225C_4^q) \text{ GeV}^{-1}. \end{aligned} \tag{107}$$

For $\overset{\circ}{g}_A = 1.3$

$$\begin{aligned} \bar{D}_{16}^q &= -1.699 \text{ GeV}^{-2} + 0.311(C_3^q - 2.573C_4^q) \text{ GeV}^{-1}, \\ \bar{D}_{17}^q &= -3.283 \text{ GeV}^{-2} + 2.800(C_3^q - 1.177C_4^q) \text{ GeV}^{-1}. \end{aligned} \tag{108}$$

Note that the constraint (95) on the SU(3) quark LEC's was obtained without dropping the higher-order terms in SU(3) breaking, while the constraints (106)–(108) were derived using the approximation for SU(3)-breaking terms $\delta_V \to \delta_V^{(2)}$ and $\delta_{A_i} \to \delta_{A_i}^{(1)}$ restricting to their leading terms.

Finally, for completeness we also present numerical results for the axial couplings at values of $\overset{\circ}{g}_A = 1.2$ and $C_4^q = 1.07 \text{ GeV}^{-1}$. The other three LEC's, C_3^q, $\bar{D}_{16}^q$, and $\bar{D}_{17}^q$, are then constrained as

$$\bar{D}_{16}^q - 0.311C_3^q = -1.668 \text{ GeV}^{-2}, \tag{109a}$$

$$\bar{D}_{17}^q - 2.800C_3^q = -6.739 \text{ GeV}^{-2}. \tag{109b}$$

Predictions for $g_A^{B_iB_j}$ of different semileptonic modes are given in Table IV. We also indicate the results of heavy baryon ChPT in the large-N_c limit [21]. In Table V we additionally present our results for the semileptonic decay widths of hyperons, which are calculated using the expression [50] at order $\mathcal{O}((m_{B_i} - m_{B_j})^6)$ and without inclusion of radiative corrections:

$$\begin{aligned} \Gamma(B_i \to B_j + l + \nu_l) = \frac{G_F^2}{60\pi^3}|V_{\rm CKM}|^2(\Delta m)^5(1-3\delta) \\ \times ((g_V^{B_iB_j})^2 + 3(g_A^{B_iB_j})^2)r(x). \end{aligned} \tag{110}$$

In the last expression we have $\Delta m = m_{B_i} - m_{B_j}$, $\delta = (m_{B_i} - m_{B_j})/(m_{B_i} + m_{B_j})$, and $G_F = 1.16637 \times 10^{-5}$ GeV^{-2} is the Fermi coupling constant. For the corresponding CKM matrix elements $V_{\rm CKM} = V_{ud}$ or V_{us}, we use the central values from [3]: $V_{ud} = 0.97377$ and $V_{us} = 0.225$. Here $r(x)$ is the function which takes into account the charged lepton mass m_l:

$$r(x) = \sqrt{1-x^2}\left(1 - \frac{9}{2}x^2 - 4x^4\right) + \frac{15}{4}x^4 \ln\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}, \qquad (111)$$

where $x = m_l/\Delta m$ and $r(0) = 1$.

V. SUMMARY

In this paper we have analyzed the semileptonic vector and axial quark coupling constants using a manifestly Lorentz covariant chiral quark approach up to order $\mathcal{O}(p^4)$ in the two- and three-flavor pictures. The resulting quark couplings were then used in the evaluation of the corresponding hadronic couplings which govern semileptonic transitions between baryon octet states. In the calculation of baryon matrix elements we utilized a general ansatz for the spatial form of the quark wave function, without referring to any specific realization of baryon hadronization and confinement. Matching physical amplitudes, calculated within our approach, to the model-independent predictions of baryon ChPT allowed us to deduce the relations between the chiral quark parameters and those of baryon ChPT.

Our main results can be summarized as follows:

(i) Evaluating the chiral and SU(3) symmetry-breaking corrections to the semileptonic vector and axial quark coupling constants, we determined that the SU(3) symmetry-breaking correction to the vector coupling f_1^{su}, governing the $s \to u$ quark flavor transition, is positive and equal to 7%.

(ii) Performing the matching to ChPT we reproduced the analytical result for the nucleon axial charge g_A in SU(2). We also determined the expression for g_A in SU(3).

(iii) We derived results for the vector and axial couplings governing the semileptonic decays of the baryon octet, revealing both chiral and SU(3) symmetry-breaking corrections.

(iv) We presented a numerical analysis of the calculated quantities and derived constraints on the parameters of the chiral quark Lagrangian (LEC's) using experimental data for g_A and the ratios $r^{B_iB_j} = g_A^{B_iB_j}/g_V^{B_iB_j}$. We also gave estimates for the semileptonic decay widths of hyperons.

In the future we plan to improve the quantitative determination of the valence quark effects by resorting to a relativistic quark model [35,37], describing the internal quark dynamics. This procedure will allow us to give predictions for all six form factors showing up in the matrix elements of the semileptonic decays of the baryon octet. With the explicit form factors and with additional radiative corrections included, we intend to give accurate predictions for the corresponding decay widths and asymmetries.

ACKNOWLEDGMENTS

This work was supported by the DFG under Contract No. FA67/31-1 and No. GRK683. B. R. H. is supported by the US National Science Foundation under Grant No. PHY 05-53304. This research is also part of the EU Integrated Infrastructure Initiative Hadronphysics project under Contract No. RII3-CT-2004-506078 and President Grant of Russia "Scientific Schools" No. 871.2008.2.

APPENDIX A: CONTRIBUTIONS OF DIFFERENT DIAGRAMS TO THE VECTOR AND AXIAL QUARK COUPLINGS

In this appendix we discuss the contributions of the various graphs in Figs. 1 and 2 to the vector and axial couplings with different flavor structures. The separate contributions of these graphs to the vector and axial couplings are listed in Tables I and II, respectively. We use the following notations: the quark charge matrix $Q = \mathrm{diag}\{2/3, -1/3\}$ in SU(2) and $Q = \mathrm{diag}\{2/3, -1/3, -1/3\}$ in SU(3); the unit 2×2 matrix $I = \mathrm{diag}\{1, 1\}$ and the 3×3 matrix $I = \mathrm{diag}\{1, 1, 1\}$. All further flavor matrices are expressed in terms of the charge, unit, Pauli (τ_i), and Gell-Mann (λ_i) matrices:

TABLE I. Contribution of different diagrams in Fig. 1 to the electric charges f_1^Q [in SU(2)] and f_1^Q [in SU(3)], isotopic (vector) charges $f_1\tau_3/2$ [in SU(2)] and $f_1\lambda_3/2$ [in SU(3)], vector couplings ($d \to u$ flavor transition) $f_1^{du}\tau_{ud}$ [in SU(2)] and $f_1^{du}\lambda_{ud}$ [in SU (3)], and vector coupling ($s \to u$ flavor transition) $f_1^{su}\lambda_{us}$. The contribution of the diagram in Fig. 1(a) is multiplied by the Z factor.

Coupling	Figure 1(a)	Figure 1(b)	Figure 1(c)	Figure 1(d)	Figure 1(e)	Figure 1(f)
f_1^Q, SU(2)	$Q(1-\frac{9}{4}R_\pi)$	$\tau_3 R_\pi^b$	$-\tau_3\frac{\Delta_\pi}{2F^2}$	$\tau_3\frac{\Delta_\pi}{2F^2}$	$\frac{1}{2}(I-\tau_3)R_\pi^e$	$\tau_3 R_\pi^f$
f_1^Q, SU(3)	$Q - \sum_P \beta_P^Q R_P$	$\sum_P \lambda_P^b R_P^b$	$-\sum_P \lambda_P^c \frac{\Delta_P}{2F^2}$	$\sum_P \lambda_P^d \frac{\Delta_P}{2F^2}$	$\sum_P \lambda_P^e R_P^e$	$\sum_P \lambda_P^f R_P^f$
$f_1 = f_1^{du}$, SU(2)	$1-\frac{9}{4}R_\pi$	$2R_\pi^b$	$-\frac{\Delta_\pi}{F^2}$	$\frac{\Delta_\pi}{F^2}$	$-R_\pi^e$	$2R_\pi^f$
$f_1 = f_1^{du}$, SU(3)	$1-\sum_P \beta_P R_P$	$2R_\pi^b + R_K^b$	$-\frac{1}{2F^2}(2\Delta_\pi + \Delta_K)$	$\frac{1}{2F^2}(2\Delta_\pi+\Delta_K)$	$-R_\pi^e + \frac{1}{3}R_\eta^e$	$2R_\pi^f + R_K^f$
f_1^{su}, SU(3)	$1-\sum_P \gamma_P R_P$	$I_{\pi K} + I_{\eta K}$	$-\frac{3}{8F^2}(\Delta_\pi + 2\Delta_K + \Delta_\eta)$	$J_{\pi K} + J_{\eta K}$	$-\frac{2}{3}R_\eta^e$	$\frac{3}{4}(R_\pi^f + 2R_K^f + R_\eta^f)$

TABLE II. Contribution of different diagrams in Fig. 2 to the isotopic (axial) charges $g_1\tau_3/2$ [in SU(2)] and $g_1\lambda_3/2$ [in SU(3)], axial couplings ($d\to u$ flavor transition) $g_1^{du}\tau_{ud}$ [in SU(2)] and $g_1^{du}\lambda_{ud}$ [in SU(3)], and vector coupling ($s\to u$ flavor transition) $g_1^{su}\lambda_{us}$. The contribution of the diagram in Fig. 2(a) is multiplied by the Z factor.

Coupling	Figure 2(a)	Figure 2(b)	Figure 2(c)	Figure 2(d)	Figure 2(e)	Figure 2(f)
$g_1 = g_1^{du}$, SU(2)	$g(1-\frac{9}{4}R_\pi)$	$4M_\pi^2 d_{16}^q$	$-\frac{g}{F^2}\Delta_\pi$	T_π^d	T_π^e	$(c_3^q - 2c_4^q)T_\pi^f$
$g_1 = g_1^{du}$, SU(3)	$g(1-\sum_P \beta_P R_P)$	$(2M_\pi^2 + 4M_K^2)D_{16}^q + \frac{2}{3}(M_\pi^2 - M_K^2)D_{17}^q$	$-\frac{g}{2F^2}(2\Delta_\pi + \Delta_K)$	$T_\pi^d - \frac{1}{3}T_\eta^d$	$T_\pi^e - \frac{1}{2}T_K^e$	$(C_3^q - 2C_4^q)T_\pi^f - C_4^q T_K^f$
g_1^{su}, SU(3)	$g(1-\sum_P \gamma_P R_P)$	$(2M_\pi^2 + 4M_K^2)D_{16}^q + \frac{1}{3}(M_K^2 - M_\pi^2)D_{17}^q$	$-\frac{3g}{8F^2}(\Delta_\pi + 2\Delta_K + \Delta_\eta)$	$\frac{2}{3}T_\eta^d$	$\frac{3}{8}(T_\pi^e + 2T_K^e + T_\eta^e)$	$(C_3^q - C_4^q)T_K^f - \frac{1}{2}C_4^q(3T_\pi^f + T_\eta^f)$

$$\tau_{ud} = \frac{1}{2}(\tau_1 + i\tau_2), \qquad \lambda_{ud} = \frac{1}{2}(\lambda_1 + i\lambda_2), \qquad \lambda_{us} = \frac{1}{2}(\lambda_4 + i\lambda_5),$$
$$\beta_\pi^Q = \frac{3}{4}Q + \frac{1}{4}I + \frac{3}{4}\lambda_3, \qquad \beta_K^Q = \frac{5}{2}Q - \frac{1}{6}I - \frac{1}{2}\lambda_3, \qquad \beta_\eta^Q = \frac{3}{4}Q - \frac{1}{12}I - \frac{1}{4}\lambda_3,$$
$$\beta_\pi = \frac{9}{4}, \qquad \beta_K = \frac{3}{2}, \qquad \beta_\eta = \frac{1}{4},$$
$$\gamma_\pi = \frac{9}{8}, \qquad \gamma_K = \frac{9}{4}, \qquad \gamma_\eta = \frac{5}{8},$$
$$\lambda_\pi^b = \lambda_\pi^c = \lambda_\pi^d = \lambda_\pi^f = \lambda_3, \qquad \lambda_K^b = \lambda_K^c = \lambda_K^d = \lambda_K^f = 3Q - \lambda_3, \qquad \lambda_\eta^b = \lambda_\eta^c = \lambda_\eta^d = \lambda_\eta^f = 0,$$
$$\lambda_\pi^e = Q + \frac{1}{3}I - \lambda_3, \qquad \lambda_K^e = -\frac{8}{3}Q - \frac{2}{9}I + \frac{4}{3}\lambda_3, \qquad \lambda_\eta^e = Q - \frac{1}{9}I - \frac{1}{3}\lambda_3.$$

We introduce the functions R_P, R_P^i, T_P^i, I_{ab}, and J_{ab}:

$$\begin{aligned} R_P &= \frac{g^2}{F^2}\Delta_P + \frac{g^2M_P^2}{24\pi^2F^2}\left\{1 - \frac{3\pi}{2}\mu_P\right\}, \\ R_P^b &= \frac{3g^2}{2F^2}\Delta_P + \frac{g^2M_P^2}{16\pi^2F^2}\left\{1 - \frac{5\pi}{2}\mu_P\right\}, \\ R_P^e &= \frac{3}{4}R_P, \\ R_P^f &= \frac{g^2M_P^2}{16\pi F^2}\mu_P, \\ T_P^d &= \frac{g^3}{4F^2}\Delta_P + \frac{g^3M_P^2}{32\pi^2F^2}\left\{1 - \frac{\pi}{2}\mu_P\right\}, \\ T_P^e &= \frac{g}{8\pi F^2}M_P^2\mu_P, \\ T_P^f &= -\frac{g}{6\pi F^2}M_P^3, \end{aligned} \tag{A1}$$

where $\mu_P = \frac{M_P}{m}$, and

$$I_{ab} = \frac{3g^2}{4F^2}\left[\frac{3}{2}\frac{\Delta_a M_a^2 - \Delta_b M_b^2}{M_a^2 - M_b^2} + \frac{M_a^2 + M_b^2}{64\pi^2} - \frac{M_a^3 + M_b^3}{8\pi m} - \frac{M_a^2M_b^2}{8\pi m(M_a + M_b)}\right] \tag{A2}$$

and

$$J_{ab} = \frac{3}{8F^2}\left[\frac{\Delta_a M_a^2 - \Delta_b M_b^2}{M_a^2 - M_b^2} - \frac{M_a^2 + M_b^2}{32\pi^2}\right]. \tag{A3}$$

Below we discuss the vector couplings in detail.

(1) Electric charges: Summing up the individual contributions of the graphs in Fig. 1 to the electric charges, we find

$$f_1^Q = Q(1 - \tfrac{9}{4}R_\pi) + \tau_3(R_\pi^b + R_\pi^f) + \tfrac{1}{2}(I - \tau_3)R_\pi^e \tag{A4}$$

in SU(2) and

$$f_1^Q = Q + \sum_P(-\beta_P^Q R_P + \lambda_P^b R_P^b + \lambda_P^e R_P^e + \lambda_P^f R_P^f) \tag{A5}$$

in SU(3).

Using the identities $R_P^b + R_P^f = \frac{3}{2}R_P$ and $\beta_P^Q = \frac{3}{2}\lambda_P^b + \frac{3}{4}\lambda_P^e$ we verify electric charge conservation—$f_1^Q = Q$ in SU(2) and $f_1^Q = Q$ in SU(3).

(2) The isotopic charge $f_1/2$ and the $d\to u$ transition vector coupling f_1^{du} are given by the expressions

$$f_1 = f_1^{du} = 1 - \tfrac{9}{4}R_\pi + 2(R_\pi^b + R_\pi^f) - R_\pi^e \tag{A6}$$

in SU(2) and

$$f_1 = f_1^{du} = 1 - \sum_P \beta_P R_P + 2(R_\pi^b + R_\pi^f) + R_K^b + R_K^f - R_\pi^e + \frac{1}{3}R_\eta^e \tag{A7}$$

in SU(3).

Again, using identities involving the functions R_P^i we arrive at

TABLE III. Semileptonic decay constants of baryons $g_V^{B_iB_j}$ and $g_A^{B_iB_j}$.

Decay mode	$g_V^{B_iB_j}$	$g_A^{B_iB_j}$
$n \to p$	1	$\frac{5}{3}g_1 I_A = g_A = g_A^{SU_3}(1+\delta_{A_1})$
$\Lambda \to p$	$-\sqrt{\frac{3}{2}}f_1^{su}I_V^s = -\sqrt{\frac{3}{2}}(1+\delta_V)$	$-\sqrt{\frac{3}{2}}g_1^{su}I_A^s = -\frac{3}{5}\sqrt{\frac{3}{2}}g_A^{SU_3}(1+\delta_{A_2})$
$\Sigma^- \to n$	$-f_1^{su}I_V^s = -(1+\delta_V)$	$\frac{1}{3}g_1^{su}I_A^s = \frac{1}{5}g_A^{SU_3}(1+\delta_{A_2})$
$\Sigma^- \to \Lambda$	0	$\sqrt{\frac{2}{3}}g_1 I_A = \frac{\sqrt{6}}{5}g_A = \frac{\sqrt{6}}{5}g_A^{SU_3}(1+\delta_{A_1})$
$\Xi^- \to \Lambda$	$\sqrt{\frac{3}{2}}f_1^{su}I_V^s = \sqrt{\frac{3}{2}}(1+\delta_V)$	$\sqrt{\frac{1}{6}}g_1^{su}I_A^s = \frac{1}{5}\sqrt{\frac{3}{2}}g_A^{SU_3}(1+\delta_{A_2})$
$\Xi^- \to \Sigma^0$	$\sqrt{\frac{1}{2}}f_1^{su}I_V^s = \sqrt{\frac{1}{2}}(1+\delta_V)$	$\frac{5}{3\sqrt{2}}g_1^{su}I_A^s = \frac{1}{\sqrt{2}}g_A^{SU_3}(1+\delta_{A_2})$
$\Xi^- \to \Sigma^+$	$f_1^{su}I_V^s = 1+\delta_V$	$\frac{5}{3}g_1^{su}I_A^s = g_A^{SU_3}(1+\delta_{A_2})$

$$f_1 = f_1^{du} = 1 \tag{A8}$$

in both the two- and three-flavor pictures.

(3) The $s \to u$ transition vector coupling f_1^{su}: The coupling f_1^{su} is finite but contains pieces $\mathcal{O}((M_K - M_\pi)^2)$ and $\mathcal{O}((M_K - M_\eta)^2)$ which are of second order in SU(3) symmetry breaking. The AGT protects the $f_1^{su}(0)$ from *first*-order symmetry-breaking corrections. Moreover, the AGT holds independently for two sets of diagrams—for set I, including the diagrams of Figs. 1(a), 1(b), 1(e), and 1(f), and for set II, including the diagrams of Figs. 1(c) and 1(d). In our derivation we use the identity

$$\frac{\Delta_a M_a^2 - \Delta_b M_b^2}{M_a^2 - M_b^2} = 2\lambda_a(M_a^2 + M_b^2) + \frac{1}{16\pi^2} \times \frac{M_b^4}{M_a^2 - M_b^2}\ln\frac{M_a^2}{M_b^2}. \tag{A9}$$

Then the results for set I and set II are

$$f_1^{su;I} = \sum_{i=a,b,e,f} f_1^{su;(i)} = 1 - \frac{9g^2}{16}(H_{\pi K} + H_{\eta K} + G_{\pi K} + G_{\eta K}) \tag{A10}$$

and

$$f_1^{su;II} = \sum_{i=c,d} f_1^{su;(i)} = -\frac{3}{16}(H_{\pi K} + H_{\eta K}), \tag{A11}$$

where the functions $H_{ab} = O((M_a^2 - M_b^2)^2)$ and $G_{ab} = O((M_a^2 - M_b^2)^2)$ are defined in Eq. (22) of Sec. II C.

The final result for the $s \to u$ quark transition vector coupling is

$$f_1^{su} = f_1^{su;I} + f_1^{su;II} = 1 - \frac{3}{16}((1+3g^2)(H_{\pi K} + H_{\pi K}) + 3g^2(G_{\pi K} + G_{\pi K})). \tag{A12}$$

APPENDIX B: TWO-BODY OPERATORS

The diagrams contributing to the two-body vector and axial quark transition operators up to fourth order are displayed in Figs. 3 and 4. First, let us discuss the diagrams in Figs. 3(a)–3(e) and 4(a)–4(e). Note, the diagrams in Figs. 3(c), 3(d), 4(c), and 4(d) are generated by an insertion of the two-body mass counterterm due to one-meson exchange, which is given by the four-quark operator

TABLE IV. Numerical results for $g_A^{B_iB_j}$.

Decay mode	Reference [21]	Our results
$n \to p$	1.272	1.2695
$\Lambda \to p$	−0.904	−0.944
$\Sigma^- \to n$	0.375	0.257
$\Sigma^+ \to \Lambda$	0.653	0.622
$\Sigma^- \to \Lambda$	0.624	0.622
$\Xi^- \to \Lambda$	0.139	0.315
$\Xi^- \to \Sigma^0$	0.869	0.908
$\Xi^0 \to \Sigma^+$	1.312	1.284

TABLE V. Numerical results for the semileptonic decay widths of hyperons (in units of $10^6\ s^{-1}$).

Decay mode	Our results	Data [3]
$\Lambda \to pe^-\bar{\nu}_e$	3.21	3.16 ± 0.06
$\Lambda \to p\mu^-\bar{\nu}_\mu$	0.52	0.60 ± 0.13
$\Sigma^- \to ne^-\bar{\nu}_e$	5.50	6.88 ± 0.24
$\Sigma^- \to n\mu^-\bar{\nu}_\mu$	2.45	3.0 ± 0.2
$\Sigma^+ \to \Lambda e^+\nu_e$	0.24	0.25 ± 0.06
$\Sigma^- \to \Lambda e^-\bar{\nu}_e$	0.40	0.39 ± 0.02
$\Xi^- \to \Lambda e^-\bar{\nu}e$	3.11	3.35 ± 0.37
$\Xi^- \to \Lambda\mu^-\bar{\nu}_\mu$	0.84	$2.1^{+2.1}_{-1.3}$
$\Xi^- \to \Sigma^0 e^-\bar{\nu}_e$	0.51	0.53 ± 0.10
$\Xi^- \to \Sigma^0\mu^-\bar{\nu}_\mu$	0.01	<0.05
$\Xi^- \to \Sigma^+ e^-\bar{\nu}_e$	0.90	0.88 ± 0.04
$\Xi^- \to \Sigma^+\mu^-\bar{\nu}_\mu$	0.01	0.02 ± 0.01

$$O_{\rm ct}(x, y) = \frac{g^2 m^2}{2F^2} \sum_{i=1}^{8} \bar{q}(x)\gamma_5\lambda_i q(x)\Delta_{ij}(x-y)\bar{q}(y)\gamma_5\lambda_j q(y) \tag{B1}$$

where

$$\Delta_{ij}(x-y) = \langle 0|T\phi_i(x)\phi_j(y)|0\rangle = \delta_{ij}\int \frac{d^4k}{(2\pi)^4 i}\frac{e^{-ik(x-y)}}{M_i^2 - k^2} \tag{B2}$$

is the meson propagator. Writing down the expressions for the diagrams in Figs. 3(a)–3(e) in the momentum space, it is easy to show that the contribution of the diagrams in Figs. 3(a), 3(b), and 3(e) is exactly equal to the contribution of the diagrams in Figs. 3(c) and 3(d) but with opposite sign. Therefore, their total contribution vanishes. Such cancellation guarantees the charge conservation and excludes a double-counting of the one-meson exchange corrections. The diagram in Fig. 3(f) does not contribute to the time component of the vector current (only to the spatial component); therefore we have no contribution to the baryon vector couplings from the two-body operators displayed in Fig. 3.

In the case of the two-body axial diagrams, the diagrams in Figs. 4(a)–4(e) do not cancel each other. Their total contribution in momentum space is given by

$$\frac{mg}{4F^2}(g^2-1)\sum_{i=1}^{8}\frac{1}{M_i^2-k^2}\bar{u}(p_1')[\lambda_A, \lambda_i]\gamma^\mu u(p_1)\bar{u}(p_2') \times \lambda_i\gamma^5 u(p_2) + (1 \leftrightarrow 2) \tag{B3}$$

where $u(p_i)$ and $\bar{u}(p_i')$ are the quark spinors, and λ_A is the flavor matrix corresponding to the axial quark flavor exchange. One can see, that the contribution (B3) vanishes for $g = 1$. The other two diagrams in Figs. 4(f) and 4(g) are generated by the one-body and two-body Lagrangians, and they contribute to the axial couplings of the baryon octet. Note that nonvanishing two-body operators corresponding to the meson exchange can be simplified. One can do the expansion of the meson propagators in powers of meson masses M as

$$\frac{1}{\Lambda^2 - M^2} = \frac{1}{\Lambda^2}\left(1 + \frac{M^2}{\Lambda^2} + \mathcal{O}(M^4)\right). \tag{B4}$$

Λ is a free parameter representing an averaged exchanged momenta between quarks, and we remove the infrared-regular parts proportional to the $1/\Lambda^N$; i.e. they do not contain powers of meson masses. Numerical analysis of the contributions of the two-body diagrams will be done in the future. Let us stress again that the vector couplings of the baryons do not receive contributions from the two-body quark operators (see diagrams in Fig. 3), while the axial couplings receive the corrections quadratic in meson masses. It will not damage the nonanalytical chiral corrections derived in the one-body approximation (see Sec. III) and will only redefine the expressions for the quadratic corrections. Note that such change of the quadratic chiral corrections will be consistent with ChPT due to the matching condition involving additional two-body quark LEC's.

[1] M. Ademollo and R. Gatto, Phys. Rev. Lett. **13**, 264 (1964).
[2] N. Cabibbo, E. C. Swallow, and R. Winston, Phys. Rev. Lett. **92**, 251803 (2004); Annu. Rev. Nucl. Part. Sci. **53**, 39 (2003).
[3] W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).
[4] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
[5] J. F. Donoghue, E. Golowich, and B. R. Holstein, Cambridge Monogr. Part. Phys., Nucl. Phys., Cosmol. **2**, 1 (1992); Phys. Rep. **131**, 319 (1986).
[6] A. Garcia, Phys. Rev. D **3**, 2638 (1971); A. Garcia and P. Kielanowski, Phys. Lett. **110B**, 498 (1982).
[7] A. Garcia, Phys. Rev. D **10**, 884 (1974).
[8] J. F. Donoghue and B. R. Holstein, Phys. Rev. D **25**, 206 (1982); **25**, 2015 (1982); J. F. Donoghue, B. R. Holstein, and S. W. Klimt, Phys. Rev. D **35**, 934 (1987).
[9] H. Leutwyler and M. Roos, Z. Phys. C **25**, 91 (1984).
[10] M. Roos, Phys. Lett. B **246**, 179 (1990); C. Avenarius, Phys. Lett. B **272**, 71 (1991); B. Ehrnsperger and A. Schafer, Phys. Lett. B **348**, 619 (1995); P. G. Ratcliffe, Phys. Lett. B **365**, 383 (1996); R. Flores-Mendieta, A. Garcia, and G. Sanchez-Colon, Phys. Rev. D **54**, 6855 (1996); P. G. Ratcliffe, Phys. Rev. D **59**, 014038 (1998); S. B. Gerasimov, Phys. Part. Nucl. **35**, S102 (2004).
[11] X. Song, P. K. Kabir, and J. S. McCarthy, Phys. Rev. D **54**, 2108 (1996); T. Yamanishi, Phys. Rev. D **76**, 014006 (2007).
[12] K. Kubodera, Y. Kohyama, K. Oikawa, and C. W. Kim, Nucl. Phys. **A439**, 695 (1985); T. P. Cheng and L. F. Li, Phys. Rev. D **57**, 344 (1998); X. Song, J. S. McCarthy, and H. J. Weber, Phys. Rev. D **55**, 2624 (1997); H. C. Kim, M. Praszalowicz, and K. Goeke, Phys. Rev. D **61**, 114006 (2000).
[13] F. Schlumpf, Phys. Rev. D **51**, 2262 (1995).
[14] J. Dai, R. F. Dashen, E. E. Jenkins, and A. V. Manohar, Phys. Rev. D **53**, 273 (1996); R. Flores-Mendieta, E. E. Jenkins, and A. V. Manohar, Phys. Rev. D **58**, 094028 (1998).
[15] R. Flores-Mendieta, Phys. Rev. D **70**, 114036 (2004); V. Mateu and A. Pich, J. High Energy Phys. 10 (2005) 041; V. Mateu, Nucl. Phys. B, Proc. Suppl. **164**, 75 (2007).

[16] A. Krause, Helv. Phys. Acta **63**, 3 (1990).
[17] J. Anderson and M. A. Luty, Phys. Rev. D **47**, 4975 (1993).
[18] M. J. Savage and J. Walden, Phys. Rev. D **55**, 5376 (1997).
[19] N. Kaiser, Phys. Rev. C **64**, 028201 (2001).
[20] G. Villadoro, Phys. Rev. D **74**, 014018 (2006).
[21] R. Flores-Mendieta and C. P. Hofmann, Phys. Rev. D **74**, 094001 (2006).
[22] A. Lacour, B. Kubis, and U. G. Meissner, J. High Energy Phys. 10 (2007) 083.
[23] F. E. Close, *An Introduction to Quarks and Partons* (Academic Press, London 1979), p. 481; J. Franklin, Phys. Rev. **172**, 1807 (1968).
[24] A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D **10**, 2599 (1974); B. H. Kellett, Phys. Rev. D **10**, 2269 (1974); A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Phys. Rev. D **15**, 844 (1977); E. Eich, D. Rein, and R. Rodenberg, Z. Phys. C **28**, 225 (1985); M. Beyer and S. K. Singh, Z. Phys. C **31**, 421 (1986); S. Migura, D. Merten, B. Metsch, and H. R. Petry, Eur. Phys. J. A **28**, 55 (2006).
[25] H. Hogaasen and F. Myhrer, Phys. Rev. D **37**, 1950 (1988); L. J. Carson, R. J. Oakes, and C. R. Willcox, Phys. Rev. D **37**, 3197 (1988).
[26] A. W. Thomas, Adv. Nucl. Phys. **13**, 1 (1984); M. A. Morgan, G. A. Miller, and A. W. Thomas, Phys. Rev. D **33**, 817 (1986); R. Tegen and W. Weise, Z. Phys. A **314**, 357 (1983); K. Dannbom, L. Y. Glozman, C. Helminen, and D. O. Riska, Nucl. Phys. **A616**, 555 (1997); K. Khosonthongkee, V. E. Lyubovitskij, T. Gutsche, A. Faessler, K. Pumsa-ard, S. Cheedket, and Y. Yan, J. Phys. G **30**, 793 (2004).
[27] J. Kambor and M. Mojzis, J. High Energy Phys. 04 (1999) 031.
[28] J. Schweizer, Diploma thesis, University of Bern, 2000.
[29] T. Becher and H. Leutwyler, J. High Energy Phys. 06 (2001) 017.
[30] T. R. Hemmert, M. Procura, and W. Weise, Phys. Rev. D **68**, 075009 (2003).
[31] V. Bernard and U. G. Meissner, Phys. Lett. B **639**, 278 (2006).
[32] M. Procura, B. U. Musch, T. R. Hemmert, and W. Weise, Phys. Rev. D **75**, 014503 (2007).
[33] M. R. Schindler, T. Fuchs, J. Gegelia, and S. Scherer, Phys. Rev. C **75**, 025202 (2007).
[34] A. Faessler, T. Gutsche, B. R. Holstein, M. A. Ivanov, J. G. Korner, and V. E. Lyubovitskij (to be published).
[35] M. A. Ivanov, M. P. Locher, and V. E. Lyubovitskij, Few-Body Syst. **21**, 131 (1996); M. A. Ivanov, V. E. Lyubovitskij, J. G. Körner, and P. Kroll, Phys. Rev. D **56**, 348 (1997); M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, and A. G. Rusetsky, Phys. Rev. D **57**, 5632 (1998); Phys. Lett. B **442**, 435 (1998); M. A. Ivanov, J. G. Körner, and V. E. Lyubovitskij, Phys. Lett. B **448**, 143 (1999); M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, and A. G. Rusetsky, Phys. Rev. D **60**, 094002 (1999); Phys. Lett. B **476**, 58 (2000); M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, M. A. Pisarev, and A. G. Rusetsky, Phys. Rev. D **61**, 114010 (2000); A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner, and V. E. Lyubovitskij, Phys. Lett. B **518**, 55 (2001); A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, D. Nicmorus, and K. Pumsa-ard, Phys. Rev. D **73**, 094013 (2006).
[36] A. Faessler, T. Gutsche, V. E. Lyubovitskij, and K. Pumsa-ard, Phys. Rev. D **73**, 114021 (2006); Prog. Part. Nucl. Phys. **55**, 12 (2005); AIP Conf. Proc. **884**, 43 (2007).
[37] A. Faessler, T. Gutsche, B. R. Holstein, V. E. Lyubovitskij, D. Nicmorus, and K. Pumsa-ard, Phys. Rev. D **74**, 074010 (2006); arXiv:hep-ph/0612246.
[38] T. Becher and H. Leutwyler, Eur. Phys. J. C **9**, 643 (1999).
[39] V. E. Lyubovitskij, T. Gutsche, and A. Faessler, Phys. Rev. C **64**, 065203 (2001); V. E. Lyubovitskij, T. Gutsche, A. Faessler, and E. G. Drukarev, Phys. Rev. D **63**, 054026 (2001); V. E. Lyubovitskij, T. Gutsche, A. Faessler, and R. Vinh Mau, Phys. Lett. B **520**, 204 (2001); V. E. Lyubovitskij, P. Wang, T. Gutsche, and A. Faessler, Phys. Rev. C **66**, 055204 (2002); T. Inoue, V. E. Lyubovitskij, T. Gutsche, and A. Faessler, Phys. Rev. C **69**, 035207 (2004).
[40] G. Altarelli, N. Cabibbo, L. Maiani, and R. Petronzio, Nucl. Phys. **B69**, 531 (1974); N. Cabibbo and R. Petronzio, Nucl. Phys. **B137**, 395 (1978).
[41] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
[42] R. Petronzio, S. Simula, and G. Ricco, Phys. Rev. D **67**, 094004 (2003); **68**, 099901 (2003).
[43] J. Gasser, M. E. Sainio, and A. Švarc, Nucl. Phys. **B307**, 779 (1988); E. Jenkins and A. V. Manohar, Phys. Lett. B **255**, 558 (1991); V. Bernard, N. Kaiser, J. Kambor, and U. G. Meissner, Nucl. Phys. **B388**, 315 (1992); P. J. Ellis and H. B. Tang, Phys. Rev. C **57**, 3356 (1998); N. Fettes, U. G. Meissner, and S. Steininger, Nucl. Phys. **A640**, 199 (1998); B. Kubis and U. G. Meissner, Nucl. Phys. **A679**, 698 (2001); Eur. Phys. J. C **18**, 747 (2001); M. R. Schindler, J. Gegelia, and S. Scherer, Phys. Lett. B **586**, 258 (2004); T. Fuchs, J. Gegelia, and S. Scherer, Eur. Phys. J. A **19**, 35 (2004); M. R. Schindler, J. Gegelia, and S. Scherer, Eur. Phys. J. A **26**, 1 (2005); V. Bernard, Prog. Part. Nucl. Phys. **60**, 82 (2008).
[44] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).
[45] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 517 (1985).
[46] B. Borasoy and U. G. Meissner, Ann. Phys. (N.Y.) **254**, 192 (1997).
[47] M. Frink, U. G. Meissner, and I. Scheller, Eur. Phys. J. A **24**, 395 (2005).
[48] A. Faessler, T. Gutsche, V. E. Lyubovitskij, and C. Oonariya, J. Phys. G **35**, 025005 (2008).
[49] N. Fettes, U. G. Meissner, and S. Steininger, Nucl. Phys. **640**, A199 (1998); P. Buettiker and U. G. Meissner, Nucl. Phys. **A668**, 97 (2000).
[50] H. Pietschmann, Acta Phys. Austriaca Suppl. **12**, 1 (1974); A. Kadeer, J. G. Korner, and U. Moosbrugger, arXiv:hep-ph/0511019.