

Study of $B_c \rightarrow B^{(*)}P, BV$ decays with QCD factorization

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The $B_c \rightarrow B_q^{(*)}P, B_qV$ decays are studied with the QCD factorization approach (where P and V denote pseudoscalar and vector mesons, respectively; $q = u, d$, and s). Considering the contributions of both current-current and penguin operators, the amplitudes of branching ratios are estimated at the leading approximation. We find that the contributions of the penguin operators are very small due to the serious suppression by the Cabibbo-Kobayashi-Maskawa elements. The most promising decay modes are $B_c \rightarrow B_s^{(*)}\pi, B_s\rho$, which might be easily detected at hadron colliders.

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I. INTRODUCTION

The B_c meson is one of the unique “double heavy-flavored” binding systems in the standard model (SM). The study of the B_c meson has received a great interest, due to its special properties: (1) The B_c meson carries open flavors. We can study the two heavy flavors of both b and c quarks simultaneously with the B_c meson. (2) The B_c meson can serve as a great laboratory for potential models, QCD sum rules, heavy quark effective theory, lattice QCD, etc. (3) The B_c meson has rich decay channels because of its sufficiently large mass and that the b and c quarks can decay individually. The B_c meson decays may provide windows for testing the predictions of the SM and can shed light on new physics beyond SM.

The B_c mesons are too massive to access at the B factories near $Y(4S)$. They can be produced in significant numbers at hadron colliders. The B_c meson has been firstly discovered by the CDF Collaboration [1]. Recently the CDF and D0 Collaborations announced some accurate measurements [2,3] with part of their available data. Much more B_c mesons and detailed information about their decay properties are expected at the Large Hadron Collider (LHC) which is scheduled to run in this year. It is estimated that one could expect around 5×10^{10} B_c events per year at LHC [4,5] due to the relatively large production cross section [6] plus the huge luminosity $\mathcal{L} = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ and high center-of-mass energy $\sqrt{s} = 14 \text{ TeV}$ [7]. There seems to exist a real possibility to study not only some B_c rare decays, but also CP violation and polarization asymmetries. The study of the B_c meson will highlight the advantages of B physics at hadron colliders.

The B_c meson is stable for strong interaction because it lies below the threshold of the $B - D$ mesons. The electromagnetic interaction cannot transform the B_c meson into other hadrons containing both b and c heavy quarks, because the B_c meson itself is the ground state. The B_c meson decays via weak interaction only, which can be divided into three classes: (1) the b quark decay ($b \rightarrow c$,

u) with c quark as a spectator, (2) the c quark decay ($c \rightarrow s, d$) with b quark as a spectator, and (3) the weak annihilation channels. In the B_c meson, both heavy quark can decay weakly, resulting in its much shorter lifetime than other b -flavored mesons, i.e., $\tau_{B_c} \lesssim \frac{1}{3}\tau_{B_q}$ (where $q = u, d$, and s) [8]. Rates of the class (1) and (2) are competitive in magnitude. The Cabibbo-Kobayashi-Maskawa (CKM) [9] matrix elements $|V_{cb}| \ll |V_{cs}|$ that are in favor of the c quark decay greatly, whereas the phase space factor $m_b^5 \gg m_c^5$ compensates the CKM matrix elements a lot for the two flavors [10]. In fact, the dominant contributions to the B_c lifetime comes from the c -quark decays [class (2)] ($\approx 70\%$), while the b -quark decay [class (1)] and weak annihilation [class (3)] are expected to add about 20% and 10%, respectively [4].

The B_c meson decays have been widely studied in the literature due to some of its outstanding features. (1) The pure leptonic B_c decays belong to the class (3), which are free from strong interaction in final states and can be used to measure the decay constant f_{B_c} and the CKM elements $|V_{cb}|$, but they are not fully reconstructed due to the missing neutrino. (2) The semileptonic B_c decays provide an excellent laboratory to measure the CKM elements $|V_{cb}|$, $|V_{ub}|$, $|V_{cs}|$, $|V_{cd}|$, and form factors for transitions of $B_c \rightarrow b$ - and c -flavored mesons. The first signal of B_c is observed via this mode [1]. The most difficult theoretical work at present is how to evaluate the hadronic matrix elements properly and accurately. (3) The nonleptonic B_c decays are the most complicated due to the participation of the strong interaction, which complicate the extraction of parameters in SM, but they also provide great opportunities to study perturbative and nonperturbative QCD, final state interactions, etc.

The earlier nonleptonic decays of the B_c meson has been studied in [4,5,11–37]. While c -quark decays take the lion's share of the B_c lifetime, the study on the class (2) has not received enough attention. This can be explained by the fact that on the one hand the available data on the B_c meson is very few, on the other hand it is assumed that the long-distance effects and final state interferences might be

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quite huge and that the class (2) decays were hard to detect experimentally. Accompanied by the LHC being about to run, the future copious data require more accurate theoretical predictions from now on. In this paper, we shall concentrate on the $B_c \rightarrow B_q^{(*)}P, B_qV$ (here P and V denote pseudoscalar and vector mesons, respectively; $q = u, d$, and s) decays in class (2) with QCD factorization approach. Now let us outline a few reasons and arguments below.

- (1) From the experimental view:
 - (i) The initial and final b -flavored mesons, i.e., the B_c and $B_q^{(*)}$, all have a long lifetime due to their decays via the weak interaction. Considered the relativistic boost kinematically due to their large momentum obtained from huge center-of-mass energy, their information would be easily recorded by the multi-purpose detectors sitting at the hadron colliders interaction regions (see [7] for details).
 - (ii) Although it is perceived that the hadron collider environment is “messy” with high backgrounds, the $B_c \rightarrow B_q^{(*)}P, B_qV$ decays are measurable due to the “clean” final states. Since the B_c meson carries charge, the final $B_q^{(*)}$ meson is tagged explicitly by the initial B_c meson. The other light meson in the final state could also be identified effectively by the conservation law of both momentum and energy, because the dedicated detectors at LHC has excellent performance on trigger, time resolution, particle identification, and so on (see [7] for details).
- (2) From the phenomenological view:
 - (i) With very high statistics, we can carefully test the various theoretical models, precisely determine the CKM elements, and meticulously search for the signals of new physics. This requires more accurate theoretical predictions. In this paper, we shall study the $B_c \rightarrow B_q^{(*)}P, B_qV$ decays with QCD factorization approach, including the contributions of both current-current and penguin operators.
 - (ii) In the rest frame of the B_c meson, the velocity $\beta_{B_q^{(*)}}$ of the $B_q^{(*)}$ meson is very small due to its large mass, not exceeding 0.18. The ratio of velocity $\beta_{P,V}/\beta_{B_q^{(*)}} \gtrsim 5.5$, which is very different from that in the two-body D meson decays where the ratio of velocities of final states is close to one. This may indicate that the final state interferences for $B_c \rightarrow B_q^{(*)}P, B_qV$ decays might not be so strong as that in D mesons. If it holds true, it will benefit us in determining the CKM elements V_{cs} and V_{cd} , the $B_c \rightarrow B_q^{(*)}$ transition form factors, etc. In this paper, we shall neglect the effects of final state interferences for the moment.

This paper is organized as follows: In Sec. II, the theoretical framework is discussed. To estimate the amplitude

of the branching ratios, the master QCD factorization (QCDF) formula are applied to the $B_c \rightarrow B_q^{(*)}P, B_qV$ decays at the leading approximation. Section III is devoted to the numerical results. Finally, we summarize in Sec. IV.

II. THEORETICAL FRAMEWORK

A. The effective Hamiltonian

Using the operator product expansion and renormalization group (RG) equation, the low energy effective Hamiltonian relevant to the $B_c \rightarrow B_q^{(*)}P, B_qV$ decays can be written as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{\substack{q_1=d,s \\ q_2=d,s}} V_{uq_1} V_{cq_2}^* [C_1(\mu)Q_1 + C_2(\mu)Q_2] + \sum_{\substack{q=u,d,s \\ i=3,\dots,10}} V_{uq} V_{cq}^* C_i(\mu)Q_i \right\} + \text{H.c.}, \quad (1)$$

where $V_{uq_1} V_{cq_2}^*$ is the CKM factor. The cases $q = d$ and $q = s$ can be treated separately and have the same Wilson coefficients $C_i(\mu)$. The expressions of the local operators are

$$\begin{aligned} Q_1 &= (\bar{u}_\alpha q_{1\alpha})_{V-A} (\bar{q}_{2\beta} c_\beta)_{V-A}, \\ Q_2 &= (\bar{u}_\alpha q_{1\beta})_{V-A} (\bar{q}_{2\beta} c_\alpha)_{V-A}, \end{aligned} \quad (2)$$

$$Q_3 = (\bar{u}_\alpha c_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \quad (3)$$

$$Q_4 = (\bar{u}_\alpha c_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A},$$

$$Q_5 = (\bar{u}_\alpha c_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \quad (4)$$

$$Q_6 = (\bar{u}_\alpha c_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A},$$

$$Q_7 = \frac{3}{2} (\bar{u}_\alpha c_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \quad (5)$$

$$Q_8 = \frac{3}{2} (\bar{u}_\alpha c_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A},$$

$$Q_9 = \frac{3}{2} (\bar{u}_\alpha c_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \quad (6)$$

$$Q_{10} = \frac{3}{2} (\bar{u}_\alpha c_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A},$$

where the summation over the repeated color indices (α and β) is understood. The Dirac current $(\bar{q}_1 q_2)_{V\pm A} = \bar{q}_1 \gamma(1 \pm \gamma_5) q_2$. q' denotes all the active quarks at scale $\mu = \mathcal{O}(m_c)$, i.e., $q' = u, d, s, c$. $e_{q'}$ denotes the electric charge of the corresponding quark q' in the unit of $|e|$,

which reflects the electroweak origin of Q_7, \dots, Q_{10} . The current-current operators (Q_1, Q_2), QCD penguin operators (Q_3, \dots, Q_6), and electroweak penguin operators (Q_7, \dots, Q_{10}) form a complete basis set under QCD and QED renormalization [38].

The effective coupling constants—Wilson coefficients $C_i(\mu)$ —are calculated in perturbative theory at a high scale $\mu \sim m_W$ and evolved down to a characteristic scale $\mu \sim m_c$ using the RG equations. The Wilson coefficient functions are given by [38]

$$\vec{C}(\mu) = U_4(\mu, \mu_b) M(\mu_b) U_5(\mu_b, \mu_W) \vec{C}(\mu_W). \quad (7)$$

Here $U_f(\mu_f, \mu_i)$ is the RG evolution matrix for f active flavors, which includes the RG-improved perturbative contribution from the initial scale μ_i down to the final scale μ_f . The $M(\mu)$ is the 10×10 quark-threshold matching matrix. The corresponding formula and expressions can be found in Ref. [38]. The Wilson coefficients $C_i(\mu)$ have been evaluated to the next-to-leading order. Their numerical values in the naive dimensional regularization scheme are listed in Table I.

B. Hadronic matrix elements within the QCDF framework

For the weak decays of hadrons, the short-distance effects are well-known and can be calculated in perturbation theory. However, the nonperturbative long-distance effects responsible for the hadronization from quarks to hadrons still remain obscure in several aspects. But to calculate the exclusive weak decays of the B_c meson, one needs to evaluate the hadronic matrix elements, i.e., the weak current operator sandwiched between the initial state of the B_c meson and the concerned final states, which is the most difficult theoretical work at present. Phenomenologically, these hadronic matrix elements are usually parametrized into the product of the decay constants and the transition form factors based on the argument of color transparency and the naive factorization scheme [39]. A few years ago, Beneke, Buchalla,

Neubert, and Sachrajda suggested a QCDF formula to compute the hadronic matrix elements in the heavy quark limit, combining the hard-scattering approach with power counting in $1/m_Q$ [40] (here m_Q is the mass of heavy quark). At leading order in the power series of heavy quark mass expansion, the hadronic matrix elements can be factorized into “nonfactorizable” corrections dominated by hard gluon exchange and a universal nonperturbative part parametrized by the form factors and meson’s light cone distribution amplitudes. This promising approach has been applied to exclusive two-body nonleptonic B_u, B_d, B_s decays [41–43]. It is found that with appropriate parameters, most of the QCDF’s predictions are in agreement with the present experimental data. In this paper, we would like to apply the QCDF approach to the $B_c \rightarrow B_q^{(*)}P$, B_qV decays.

In the heavy quark limit $m_c \gg \Lambda_{\text{QCD}}$, up to power corrections of the order of Λ_{QCD}/m_c , using the master QCDF formula, the hadronic matrix elements for the $B_c \rightarrow B_q^{(*)}M$ decays ($M = P$ or V) can be written as [40]

$$\langle B_q^{(*)}M | O_i | B_c \rangle = F^{B_c \rightarrow B_q^{(*)}} \int dz H(z) \Phi_M(z) \quad (8)$$

where $F^{B_c \rightarrow B_q^{(*)}}$ is the transition form factor and $\Phi_M(z)$ is the distribution amplitudes for the meson of M , which are assumed to be nonperturbative and dominated by the soft contributions. The hard-scattering kernels $H(z)$ can be calculated in the perturbative theory. For details about the QCDF formula Eq. (8), please refer to Ref. [40].

To estimate the branching ratios approximately and to have a sense of the order of amplitudes, we shall adopt a rough approximation, i.e., at the leading order of α_s . Within this approximation, the hard-scattering kernel functions become very simple, $H(z) = 1$. That is to say the long-distance interactions between the M and $B_c - B_q^{(*)}$ system could be neglected. So the integral of $\Phi_M(z)$ reduces to the normalization condition for the distribution amplitudes. Furthermore, according to the arguments of QCDF [40], the hard interactions with the spectator are

TABLE I. The next-to-leading order Wilson coefficients $C_i(\mu)$ in the naive dimensional regularization scheme. The input parameters are [8]: $\alpha_s(m_Z) = 0.1176$, $\alpha_{em}(m_W) = 1/128$, $m_W = 80.403$ GeV, $\Lambda_{\text{QCD}}^{(f=5)} = 220.9$ MeV, $\Lambda_{\text{QCD}}^{(f=4)} = 317.2$ MeV.

	$\mu = m_b$	$\mu = 2.0$ GeV	$\mu = 1.5$ GeV	$\mu = m_c$
C_1	1.0849	1.1497	1.1883	1.2215
C_2	-0.1902	-0.3077	-0.3717	-0.4241
C_3	0.0148	0.0238	0.0296	0.0349
C_4	-0.0362	-0.0542	-0.0652	-0.0747
C_5	0.0088	0.0105	0.0107	0.0102
C_6	-0.0422	-0.0703	-0.0896	-0.1078
C_7/α_{em}	-0.0007	-0.0164	-0.0186	-0.0181
C_8/α_{em}	0.0565	0.0964	0.1235	0.1493
C_9/α_{em}	-1.3039	-1.3966	-1.4473	-1.4901
C_{10}/α_{em}	0.2700	0.4144	0.4964	0.5656

power suppressed in the heavy quark limit. Therefore it is not surprising to reproduce the result of naive factorization.

In our paper, the annihilation amplitudes are neglected due to some reasons. (1) According to the power counting arguments of QCDF [40], compared with the leading order contributions to the hard-scattering kernel, the contributions from annihilation topologies are power suppressed. (2) The annihilation amplitudes are suppressed by the CKM elements. For the decay modes concerned, the CKM factors in the nonannihilation amplitudes are $V_{ud}V_{cs}^* \sim 1$, $V_{us}V_{cs}^* \sim \lambda$, $V_{ud}V_{cd}^* \sim \lambda$, and $V_{us}V_{cd}^* \sim \lambda^2$, while the annihilation amplitudes are proportional to the CKM factors of $V_{cb}V_{ub}^* \sim \lambda^5$.

The explicit expressions of decay amplitudes for $B_c \rightarrow B_q^{(*)}P$, B_qV decays are collected in Appendix A. In our paper, we define

$$a_i \equiv C_i + \frac{1}{N_c} C_{i+1} \quad (i = \text{odd}) \quad (9)$$

$$a_i \equiv C_i + \frac{1}{N_c} C_{i-1} \quad (i = \text{even}) \quad (10)$$

where i runs from 1 to 10, C_i are the Wilson coefficients. $N_c = 3$ is the color number.

III. NUMERICAL RESULTS AND DISCUSSIONS

Within the QCDF approach, the decay amplitudes depend on many input parameters including the CKM matrix elements, decay constants, form factors, etc. These parameters are discussed and specified below.

A. The CKM matrix elements

We will use the Wolfenstein parametrization. Phenomenologically, it is a popular approximation of the CKM matrix in which each element is expanded as a power series in the small parameter λ . Up to $\mathcal{O}(\lambda^6)$, the CKM elements can be written as [38]

$$V_{ud} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 + \mathcal{O}(\lambda^6) \quad (11)$$

$$V_{us} = \lambda + \mathcal{O}(\lambda^6) \quad (12)$$

$$V_{ub} = A\lambda^3(\rho - i\eta) \quad (13)$$

$$V_{cd} = -\lambda + A^2\lambda^5[\frac{1}{2} - (\rho + i\eta)] + \mathcal{O}(\lambda^6) \quad (14)$$

$$V_{cs} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 - \frac{1}{2}A^2\lambda^4 + \mathcal{O}(\lambda^6) \quad (15)$$

$$V_{cb} = A\lambda^2 + \mathcal{O}(\lambda^6) \quad (16)$$

$$V_{td} = A\lambda^3[1 - (\rho + i\eta)(1 - \frac{1}{2}\lambda^2)] + \mathcal{O}(\lambda^6) \quad (17)$$

$$V_{ts} = -A\lambda^2 + A\lambda^4[\frac{1}{2} - (\rho + i\eta)] + \mathcal{O}(\lambda^6) \quad (18)$$

$$V_{tb} = 1 - \frac{1}{2}A^2\lambda^4 + \mathcal{O}(\lambda^6). \quad (19)$$

The global fit for the four independent Wolfenstein parameters gives [8]

$$\begin{aligned} A &= 0.818_{-0.017}^{+0.007}, & \lambda &= 0.2272 \pm 0.0010, \\ \bar{\rho} &= 0.221_{-0.028}^{+0.064}, & \bar{\eta} &= 0.340_{-0.045}^{+0.017} \end{aligned} \quad (20)$$

where the relationship between (ρ, η) and $(\bar{\rho}, \bar{\eta})$ is [8]

$$\rho + i\eta = \frac{\sqrt{1 - A^2\lambda^4}(\bar{\rho} + i\bar{\eta})}{\sqrt{1 - \lambda^4[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]}}. \quad (21)$$

If not stated otherwise, we shall use their central values for illustration.

B. Decay constants and form factors

In principle, information about the decay constants and transition form factors of mesons can be obtained from experiments and/or theoretical estimations. Now, we specify these parameters. The decay constants f_P and f_V corresponding to the pseudoscalar and vector mesons, respectively, are defined by

$$\begin{aligned} \langle P(q) | (\bar{q}_1 q_2)_{V-A} | 0 \rangle &= -if_P q^\mu, \\ \langle V(q, \epsilon) | (\bar{q}_1 q_2)_{V-A} | 0 \rangle &= f_V m_V \epsilon^{*\mu}, \end{aligned} \quad (22)$$

where ϵ^* is the polarization vector of the vector meson V . In this paper, we assume ideal mixing between ω and ϕ mesons, i.e., $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\phi = s\bar{s}$. In fact, the $B_c \rightarrow B_u^{(*)}\phi$ decays are not possible, because the B_c meson lies below the threshold of the $B_u^{(*)}\phi$ system. As to the η and η' mesons, we take the convention in Ref. [44], adopting the Feldmann-Kroll-Stech mixing scheme. Neglecting the possible compositions of both $\eta_c = c\bar{c}$ and glueball gg , the η and η' are expressed as linear combinations of orthogonal states η_q and η_s with the flavor structure $q\bar{q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$, respectively, i.e.,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} \quad (23)$$

where $\phi = (39.3 \pm 1.0)^\circ$ [44] is the $\eta - \eta'$ mixing angle. So the decay constants related to the η and η' mesons can be defined by

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} f_q & 0 \\ 0 & f_s \end{pmatrix} \quad (24)$$

$$\langle 0 | \bar{q} \gamma_\mu \gamma_5 q | \eta^{(i)}(p) \rangle = if_{\eta^{(i)}}^q p_\mu, \quad (25)$$

$$\langle 0 | \bar{s} \gamma_\mu \gamma_5 s | \eta^{(i)}(p) \rangle = if_{\eta^{(i)}}^s p_\mu.$$

The matrix elements of the pseudoscalar densities are defined by [45]

$$\begin{aligned} \frac{\langle 0 | \bar{u} \gamma_5 u | \eta^{(\prime)} \rangle}{\langle 0 | \bar{s} \gamma_5 s | \eta^{(\prime)} \rangle} &= \frac{f_{\eta^{(\prime)}}^u}{f_{\eta^{(\prime)}}^s}, \\ \langle 0 | \bar{s} \gamma_5 s | \eta^{(\prime)} \rangle &= -i \frac{m_{\eta^{(\prime)}}^2}{2m_s} (f_{\eta^{(\prime)}}^s - f_{\eta^{(\prime)}}^u). \end{aligned} \quad (26)$$

The numerical values of the decay constants are collected in Table II. If not stated otherwise, we shall take their central values for illustration.

The transition form factors are defined as [39]

$$\begin{aligned} \langle P(k) | (\bar{q}_3 q_4)_{V-A} | B(p) \rangle &= (p+k)^\mu F_1^{B \rightarrow P}(q^2) \\ &+ \frac{m_B^2 - m_P^2}{q^2} q^\mu [F_0^{B \rightarrow P}(q^2) \\ &- F_1^{B \rightarrow P}(q^2)] \end{aligned} \quad (27)$$

$$\begin{aligned} \langle V(k, \epsilon) | (\bar{q}_3 q_4)_{V-A} | B(p) \rangle &= i \frac{\epsilon^* \cdot p}{q^2} q_\mu 2m_V A_0^{B \rightarrow V}(q^2) \\ &+ i \epsilon_\mu^* (m_B + m_V) A_1^{B \rightarrow V}(q^2) \\ &- i \frac{\epsilon^* \cdot p}{m_B + m_V} (p+k)_\mu A_2^{B \rightarrow V}(q^2) \\ &- i \frac{\epsilon^* \cdot p}{q^2} q_\mu 2m_V A_3^{B \rightarrow V}(q^2) \\ &+ \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p^\alpha k^\beta \frac{2V^{B \rightarrow V}(q^2)}{m_B + m_V} \end{aligned} \quad (28)$$

where $F_{0,1}$, V , and $A_{0,1,2,3}$ are the transition form factors, $q = p - k$. In order to cancel the poles at $q^2 = 0$, we must impose the condition

$$F_0^{B \rightarrow P}(0) = F_1^{B \rightarrow P}(0), \quad A_0^{B \rightarrow V}(0) = A_3^{B \rightarrow V}(0), \quad (29)$$

$$\begin{aligned} 2m_V A_3^{B \rightarrow V}(0) &= (m_B + m_V) A_1^{B \rightarrow V}(0) \\ &- (m_B - m_V) A_2^{B \rightarrow V}(0). \end{aligned} \quad (30)$$

In our paper, only the $B_c \rightarrow B_q^{(*)}$ transition form factors appear in the amplitudes within the ‘‘spectator’’ model where the spectator is the b quark for the concerned processes. Their numerical values are collected in Table III. From the numbers in Table III, we can see clearly that, due to the properties of nonperturbative QCD, there are large uncertainties about the form factors with different theoretical treatments. Here, we notice the fact that the velocity of the final state $B_q^{(*)}$ meson is very small in the rest frame of the initial B_c meson, as that mentioned in Sec. I. It is commonly assumed that the velocities of the b

quark in the rest frame of the b -flavored mesons should be close to zero. The $B_q^{(*)}$ meson is neither fast nor small. By intuition, the overlap between the initial and final states should be huge, close to unity, as that argued in [37]. So for illustration and simplification, we will take the same value for the transition form factors, i.e., $F_{1,0}(0) = A_0(0) = 0.8$.

C. Quark masses

In the decay amplitudes, there exist the ‘‘chirally enhanced’’ factors which are associated with the hadronic matrix elements of the scalar and pseudoscalar densities, for example, R_{c1} in Eq. (A1). These factors are formally of the order Λ_{QCD}/m_c , power suppressed in the heavy quark limit, but numerically close to unity because the mass of the c quark is not infinity in practice. The current quark masses in the denominator appear through the equations of motions and are renormalization scale dependent. Their values are [8]

$$\begin{aligned} m_u(2 \text{ GeV}) &= 3 \pm 1 \text{ MeV}, \\ m_d(2 \text{ GeV}) &= 6.0 \pm 1.5 \text{ MeV}, \\ m_s(2 \text{ GeV}) &= 103 \pm 20 \text{ MeV}, \\ m_c(m_c) &= 1.24 \pm 0.09 \text{ GeV}. \end{aligned} \quad (31)$$

Using the renormalization group equation of the running quark mass [38],

$$\begin{aligned} m(\mu) &= m(\mu_0) \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{(\gamma_m^{(0)}/2\beta_0)} \\ &\times \left\{ 1 + \left(\frac{\gamma_m^{(1)}}{2\beta_0} - \frac{\gamma_m^{(0)}\beta_1}{2\beta_0^2} \right) \frac{\alpha_s(\mu) - \alpha_s(\mu_0)}{4\pi} \right\} \end{aligned} \quad (32)$$

their corresponding values at a characteristic scale $\mu \sim m_c$ can be obtained.

D. Numerical results and discussions

The numerical results are listed in Table IV, where $\mathcal{B}r^T$ corresponds to the contributions of the current-current operators only, $\mathcal{B}r^{T+P_s}$ corresponds to the contributions of both current-current and QCD penguin operators, $\mathcal{B}r^{T+P_s+P_e}$ corresponds to the contributions of both current-current and penguin operators, i.e., Q_1, \dots, Q_{10} .

Here, we would like to point out that these numbers are just the qualitative estimations on the order of amplitudes, because many of the subtleties and details, such as final state interactions, the renormalization scale dependence, the transition form factors, the strong phases, and so on, all

TABLE II. Values of the decay constant (in the unit of MeV).

f_π	f_K	f_q	f_s	f_ρ	f_ω	f_{K^*}
131 [8]	160 [8]	$(1.07 \pm 0.02)f_\pi$ [44]	$(1.34 \pm 0.06)f_\pi$ [44]	205 ± 9 [46]	195 ± 3 [46]	217 ± 5 [46]

TABLE III. Values of transition form factors.

Ref.	$F_0^{B_c \rightarrow B_{u,d}}(0)$	$F_0^{B_c \rightarrow B_s}(0)$	$A_0^{B_c \rightarrow B_{u,d}^*}(0)$	$A_0^{B_c \rightarrow B_s^*}(0)$
[37] ^a	0.320 ~ 0.910	0.340 ~ 0.925	0.349 ~ 0.916	0.432 ~ 0.931
[47] ^b	0.3 ± 0.1	0.30 ± 0.05	0.35 ± 0.09	0.39 ± 0.05
[33] ^c	...	0.61	...	0.79
[48] ^d	...	0.403 ~ 0.617	...	0.433 ~ 0.641
[49] ^e	0.4504	0.5917	0.2691	0.4451
[50] ^f	-0.58	-0.61	0.35	0.39
[51] ^g	...	0.297	...	0.263
[21] ^h	1.27	1.3	1.29	0.94
[21] ⁱ	1.38	1.1	1.26	1.04
[17]	0.39	0.50	0.20	0.35
[52]	0.23 ± 0.03	...
[53] ^j	0.90	1.02	0.27	0.36

^aThe form factors increase with the increasing parameter $\omega = 0.4 \sim 1.0$ GeV that determines the average transverse quark momentum. The authors of [37] prefer $F_0^{B_c \rightarrow B_u}(0) = 0.831$, $F_0^{B_c \rightarrow B_s}(0) = 0.859$, $A_0^{B_c \rightarrow B_u^*}(0) = 0.869$, and $A_0^{B_c \rightarrow B_s^*}(0) = 0.842$ with the corresponding parameter $\omega = 0.8$ GeV.

^bThe definitions of the transition form factors in [47] are different from ours in Eqs. (24) and (25). The relationship is

$$F_1^{B_c \rightarrow P} = F_+, \quad A_0^{B_c \rightarrow V} = \frac{F_0^A}{2m_V} + \frac{m_{B_c}^2 - m_V^2}{2m_V} F_+^A \quad (39)$$

with the values of $F_+ = 0.3 \pm 0.1(0.30 \pm 0.05)$, $F_0^A = 4.0 \pm 1.0(4.5 \pm 0.5)$ GeV⁻¹, and $F_+^A = -0.02 \pm 0.01(-0.03 \pm 0.02)$ GeV⁻¹ for $B_c \rightarrow B_{u,d}^{(*)}(B_s^{(*)})$ transition [47].

^cUsing the relationship of Eqs. (29) and (30) with the input $A_1 = 0.52$, $A_2 = -2.79$ [33].

^dFor parameter $\omega = 0.4, 0.5$ GeV.

^eUsing the relationship of Eq. (39) with the input $F_+ = 0.4504(0.5917)$, $F_0^A = 3.383(5.506)$ GeV⁻¹, and $F_+^A = -0.0463(-0.0673)$ GeV⁻¹ for $B_c \rightarrow B_{u,d}^{(*)}(B_s^{(*)})$ transition [49].

^fUsing the relationship of Eqs. (29) and (30) with the input $A_1 = 0.27(0.33)$ and $A_2 = -0.60(-0.40)$ for $B_c \rightarrow B_{u,d}^{(*)}(B_s^{(*)})$ transition [50].

^gUsing the relationship of Eqs. (29) and (30) with the input $A_1 = 0.28$ and $A_2 = 0.49$ [51].

^hUsing the relationship of Eq. (39) with the input $F_+ = 1.27(1.3)$, $F_0^A = 9.8(8.1)$ GeV⁻¹, and $F_+^A = 0.35(0.2)$ GeV⁻¹ for $B_c \rightarrow B_{u,d}^{(*)}(B_s^{(*)})$ transition in the framework of QCD sum rules [21].

ⁱUsing the relationship of Eq. (39) with the input $F_+ = 1.38(1.1)$, $F_0^A = 9.4(8.2)$ GeV⁻¹, and $F_+^A = 0.36(0.3)$ GeV⁻¹ for $B_c \rightarrow B_{u,d}^{(*)}(B_s^{(*)})$ transition in the framework of potential model [21].

^jUsing the relationship of Eqs. (29) and (30) with the input $A_1 = 0.90(1.01)$ and $A_2 = 7.9(9.04)$ for $B_c \rightarrow B_{u,d}^{(*)}(B_s^{(*)})$ transition [53].

deserve the dedicated researches but are not considered here.

From the numbers in Table IV, we can see:

- (i) The contributions of both QCD and electroweak penguin operators are very small for $B_c \rightarrow B^{(*)}P$, BV decays, compared with those of the current-

current operators. This is very different from that of the $B_{u,d,s}$ meson decays. The reason is that the contributions of penguin operators are seriously suppressed by the CKM elements. The CKM elements corresponding to different topologies for c -quark decay in the B_c meson are listed below.

Tree topologies	Penguin topologies	Annihilation topologies
$V_{ud}V_{cs}^* \sim 1$, $V_{us}V_{cs}^* \sim +\lambda$ $V_{us}V_{cd}^* \sim \lambda^2$, $V_{ud}V_{cd}^* \sim -\lambda$	$V_{ud}V_{cd}^* + V_{us}V_{cs}^* \sim \lambda^5$	$V_{cb}V_{ub}^* \sim \lambda^5$

So, for the $B_c \rightarrow B^{(*)}P$, BV decays, the effects of new physics contributed via the penguin topologies might be tiny and not detectable even with large statistics, due to the serious suppression by the CKM elements.

- (ii) There are clear hierarchy of amplitudes of the branching ratios. According the CKM elements and the coefficients of $a_{1,2}$, these decay modes are divided into different cases listed below.

Cases	Processes	Coefficients	The CKM elements	Order of branching ratios
Case 1a	$c \rightarrow s$	a_1	$V_{ud}V_{cs}^* \sim 1$	$\sim 10^{-2}$
Case 1b	$c \rightarrow s$	a_1	$V_{us}V_{cs}^* \sim \lambda$	$\sim 10^{-3}$
	$c \rightarrow d$	a_1	$V_{ud}V_{cd}^* \sim \lambda$	$\sim 10^{-3}$
Case 1c	$c \rightarrow d$	a_1	$V_{us}V_{cd}^* \sim \lambda^2$	$\sim 10^{-4}$

Cases	Processes	Coefficients	The CKM elements	Order of branching ratios
Case 2a	$c \rightarrow u$	a_2	$V_{ud}V_{cs}^* \sim 1$	$\sim 10^{-5}$
Case 2b	$c \rightarrow u$	a_2	$V_{us}V_{cs}^* \sim \lambda$, $V_{ud}V_{cd}^* \sim \lambda$	$\sim 10^{-6}-10^{-7}$
Case 2c	$c \rightarrow u$	a_2	$V_{us}V_{cd}^* \sim \lambda^2$	$\sim 10^{-8}$

The decay modes determined by a_1 have comparatively large branching ratios, which should be detectable experimentally, especially the CKM favored decay modes, such as $B_c \rightarrow B_s^{(*)}\pi$, $B_s\rho$, might be the promising decay modes to be measured in hadron colliders. Because of the great branching ratios of the decay modes determined by a_1 , the B_c mesons can be used as a source of the B_s mesons if the B_c is produced copiously, as that stated in Ref. [10]. The decay modes determined by a_2 have comparatively small branching ratios, which are hard to detect experimentally, especially the CKM suppressed decay modes, such as $B_c \rightarrow B_u^{(*)}K^0$, B_uK^{*0} , their branching ratios are too tiny to be measured.

- (iii) Although the $B_c \rightarrow B_u^{(*)}\eta'$ decays belong to the case 2b modes, their branching ratios are abnormally small, order of 10^{-8} . This can be explained by the fact that on one hand the physical space phase available is too small, on the other hand there are

- large destructive interactions between $f_{\eta'}^u a_2$ and $f_{\eta'}^s a_2$ due to the serious cancellation between the CKM elements $V_{ud}V_{cd}^*$ and $V_{us}V_{cs}^*$.
- (iv) The relations among the $B_c \rightarrow B_q^{(*)}P$, B_qV decay mode become very simple since the effects of penguin topologies is too tiny to be considered. We can use these relations to determine and overconstrain some parameters, such as the CKM elements, the form factors, etc. In addition, in estimating and measuring these parameters, the ratios of the branching ratios can be used to cancel and/or reduce largely theoretical uncertainties and experimental errors. For example

$$\frac{\mathcal{B}r(B_c^+ \rightarrow B_d^0 \pi^+)}{\mathcal{B}r(B_c^+ \rightarrow B_d^0 K^+)} \approx \frac{V_{ud}V_{cd}^* f_\pi}{V_{us}V_{cd}^* f_K} \approx \frac{V_{ud}V_{cs}^* f_\pi}{V_{us}V_{cs}^* f_K} \approx \frac{\mathcal{B}r(B_c^+ \rightarrow B_s^0 \pi^+)}{\mathcal{B}r(B_c^+ \rightarrow B_s^0 K^+)} \quad (33)$$

TABLE IV. The branching ratios for $B_c \rightarrow B_q^{(*)}P$, B_qV . $\mathcal{B}r^T$ corresponds to the contributions of the operators Q_1 and Q_2 . $\mathcal{B}r^{T+P_s}$ corresponds to the contributions of operators $Q_1 \sim Q_6$. $\mathcal{B}r^{T+P_s+P_e}$ corresponds to the contributions of $Q_1 \sim Q_{10}$.

Modes	Case	$\mathcal{B}r^T$	$\mathcal{B}r^{T+P_s}$	$\mathcal{B}r^{T+P_s+P_e}$	$\frac{\mathcal{B}r^{T+P_s}-\mathcal{B}r^T}{\mathcal{B}r^T}$	$\frac{\mathcal{B}r^{T+P_s+P_e}-\mathcal{B}r^T}{\mathcal{B}r^T}$
$B_c^+ \rightarrow B_s^0 \pi^+$	Case 1a	5.3089×10^{-2}
$B_c^+ \rightarrow B_s^0 \rho^+$	Case 1a	6.2652×10^{-2}
$B_c^+ \rightarrow B_s^{*0} \pi^+$	Case 1a	4.5916×10^{-2}
$B_c^+ \rightarrow B_s^0 K^+$	Case 1b	3.6746×10^{-3}	3.6759×10^{-3}	3.6759×10^{-3}	3.4×10^{-4}	3.4×10^{-4}
$B_c^+ \rightarrow B_s^0 K^{*+}$	Case 1b	1.6450×10^{-3}	1.6451×10^{-3}	1.6451×10^{-3}	5.0×10^{-5}	5.0×10^{-5}
$B_c^+ \rightarrow B_s^{*0} K^+$	Case 1b	2.9772×10^{-3}	2.9766×10^{-3}	2.9766×10^{-3}	-1.9×10^{-4}	-1.9×10^{-4}
$B_c^+ \rightarrow B_d^0 \pi^+$	Case 1b	3.7283×10^{-3}	3.7272×10^{-3}	3.7272×10^{-3}	-3.0×10^{-4}	-3.0×10^{-4}
$B_c^+ \rightarrow B_d^0 \rho^+$	Case 1b	5.2745×10^{-3}	5.2742×10^{-3}	5.2742×10^{-3}	-5.0×10^{-5}	-5.0×10^{-5}
$B_c^+ \rightarrow B_d^{*0} \pi^+$	Case 1b	3.2682×10^{-3}	3.2688×10^{-3}	3.2688×10^{-3}	1.9×10^{-4}	1.9×10^{-4}
$B_c^+ \rightarrow B_d^0 K^+$	Case 1c	2.6616×10^{-4}
$B_c^+ \rightarrow B_d^0 K^{*+}$	Case 1c	2.2583×10^{-4}
$B_c^+ \rightarrow B_d^{*0} K^+$	Case 1c	2.2075×10^{-4}
$B_c^+ \rightarrow B_u^+ \bar{K}^0$	Case 2a	2.2067×10^{-5}
$B_c^+ \rightarrow B_u^+ \bar{K}^{*0}$	Case 2a	1.8434×10^{-5}
$B_c^+ \rightarrow B_u^{*+} \bar{K}^0$	Case 2a	1.8261×10^{-5}
$B_c^+ \rightarrow B_u^+ \eta$	Case 2b	1.5991×10^{-6}	1.6122×10^{-6}	1.6125×10^{-6}	8.2×10^{-3}	8.4×10^{-3}
$B_c^+ \rightarrow B_u^{*+} \eta$	Case 2b	1.3042×10^{-6}	1.2960×10^{-6}	1.2964×10^{-6}	-6.3×10^{-3}	-6.0×10^{-3}
$B_c^+ \rightarrow B_u^+ \pi^0$	Case 2b	4.5968×10^{-7}	4.5161×10^{-7}	4.5134×10^{-7}	-1.8×10^{-2}	-1.8×10^{-2}
$B_c^+ \rightarrow B_u^+ \rho^0$	Case 2b	6.5030×10^{-7}	6.4823×10^{-7}	6.4776×10^{-7}	-3.2×10^{-3}	-3.9×10^{-3}
$B_c^+ \rightarrow B_u^+ \omega$	Case 2b	5.7921×10^{-7}	5.8199×10^{-7}	5.8212×10^{-7}	4.8×10^{-3}	5.0×10^{-3}
$B_c^+ \rightarrow B_u^{*+} \pi^0$	Case 2b	4.0262×10^{-7}	4.0722×10^{-7}	4.0685×10^{-7}	1.1×10^{-2}	1.0×10^{-2}
$B_c^+ \rightarrow B_u^+ \eta'$	Case 2d	8.8676×10^{-8}	8.7700×10^{-8}	8.7738×10^{-8}	-1.1×10^{-2}	-1.1×10^{-2}
$B_c^+ \rightarrow B_u^{*+} \eta'$	Case 2d	1.7401×10^{-8}	1.7728×10^{-8}	1.7731×10^{-8}	1.9×10^{-2}	1.9×10^{-2}
$B_c^+ \rightarrow B_u^+ K^0$	Case 2c	6.5428×10^{-8}
$B_c^+ \rightarrow B_u^+ K^{*0}$	Case 2c	5.4658×10^{-8}
$B_c^+ \rightarrow B_u^{*+} K^0$	Case 2c	5.4143×10^{-8}

$$\begin{aligned}\frac{\mathcal{B}r(B_c^+ \rightarrow B_s^0 \pi^+)}{\mathcal{B}r(B_c^+ \rightarrow B_d^0 \pi^+)} &\approx \frac{V_{cd}^* F_0^{B_c \rightarrow B_d} (m_{B_c}^2 - m_{B_d}^2)}{V_{cs}^* F_0^{B_c \rightarrow B_s} (m_{B_c}^2 - m_{B_s}^2)} \\ &\approx \frac{\mathcal{B}r(B_c^+ \rightarrow B_s^0 K^+)}{\mathcal{B}r(B_c^+ \rightarrow B_d^0 K^+)}\end{aligned}\quad (34)$$

$$\begin{aligned}\frac{\mathcal{B}r(B_c^+ \rightarrow B_d^0 \pi^+)}{\mathcal{B}r(B_c^+ \rightarrow B_d^0 \rho^+)} &\approx \frac{f_\pi F_0^{B_c \rightarrow B_d}}{f_\rho F_1^{B_c \rightarrow B_d}} \frac{\mathcal{B}r(B_c^+ \rightarrow B_d^0 K^+)}{\mathcal{B}r(B_c^+ \rightarrow B_d^0 K^{*+})} \\ &\approx \frac{f_K F_0^{B_c \rightarrow B_d}}{f_{K^*} F_1^{B_c \rightarrow B_d}}\end{aligned}\quad (35)$$

$$\begin{aligned}\frac{\mathcal{B}r(B_c^+ \rightarrow B_d^0 \pi^+)}{\mathcal{B}r(B_c^+ \rightarrow B_d^{*0} \pi^+)} &\approx \frac{F_0^{B_c \rightarrow B_d}}{A_0^{B_c \rightarrow B_d^*}} \\ &\approx \frac{\mathcal{B}r(B_c^+ \rightarrow B_d^0 K^+)}{\mathcal{B}r(B_c^+ \rightarrow B_d^{*0} K^+)}\end{aligned}\quad (36)$$

$$\begin{aligned}\frac{\mathcal{B}r(B_c^+ \rightarrow B_s^0 \pi^+)}{\mathcal{B}r(B_c^+ \rightarrow B_s^0 \rho^+)} &\approx \frac{f_\pi F_0^{B_c \rightarrow B_s}}{f_\rho F_1^{B_c \rightarrow B_s}} \frac{\mathcal{B}r(B_c^+ \rightarrow B_s^0 K^+)}{\mathcal{B}r(B_c^+ \rightarrow B_s^0 K^{*+})} \\ &\approx \frac{f_K F_0^{B_c \rightarrow B_s}}{f_{K^*} F_1^{B_c \rightarrow B_s}}\end{aligned}\quad (37)$$

$$\begin{aligned}\frac{\mathcal{B}r(B_c^+ \rightarrow B_s^0 \pi^+)}{\mathcal{B}r(B_c^+ \rightarrow B_s^{*0} \pi^+)} &\approx \frac{F_0^{B_c \rightarrow B_d}}{A_0^{B_c \rightarrow B_d^*}} \\ &\approx \frac{\mathcal{B}r(B_c^+ \rightarrow B_s^0 K^+)}{\mathcal{B}r(B_c^+ \rightarrow B_s^{*0} K^+)}.\end{aligned}\quad (38)$$

IV. SUMMARY AND CONCLUSION

In prospects of the huge statistics of the B_c mesons at the hadron colliders, accurate and thorough studies of the B_c physics will be accessible very soon. In this paper, we study the two-body nonleptonic c -quark decays in the B_c mesons, i.e., $B_c \rightarrow B_q^{(*)} P$, $B_q V$ decays within the QCDF approach for the leading approximation, and estimate their branching ratios. We find that the contributions of the penguin operators are very small to the decay amplitudes due to the serious suppression by the CKM elements. The decay modes determined by a_1 have comparatively large branching ratios. The most promising decay modes are $B_c \rightarrow B_s^{(*)} \pi$, $B_s \rho$, which might be easily detected at the hadron colliders.

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APPENDIX: AMPLITUDES FOR $B_c \rightarrow B_q^{(*)} P$, $B_q V$ DECAYS

(1) $c \rightarrow d$ processes:

$$\begin{aligned}\mathcal{A}(B_c^+ \rightarrow B_d^0 \pi^+) &= -i \frac{G_F}{\sqrt{2}} f_\pi F_0^{B_c \rightarrow B_d} (m_{B_c}^2 - m_{B_d}^2) \\ &\times \left\{ V_{ud} V_{cd}^* a_1 + (V_{ud} V_{cd}^* \right. \\ &\quad \left. + V_{us} V_{cs}^*) \left[a_4 - \frac{1}{2} a_{10} \right] \right. \\ &\quad \left. + R_{c1} \left(a_6 - \frac{1}{2} a_8 \right) \right\}\end{aligned}\quad (A1)$$

where $R_{c1} = \frac{2m_{\pi^+}^2}{(m_d + m_u)(m_c - m_d)}$.

$$\begin{aligned}\mathcal{A}(B_c^+ \rightarrow B_d^0 K^+) &= -i \frac{G_F}{\sqrt{2}} f_K F_0^{B_c \rightarrow B_d} (m_{B_c}^2 - m_{B_d}^2) \\ &\times V_{us} V_{cd}^* a_1\end{aligned}\quad (A2)$$

$$\begin{aligned}\mathcal{A}(B_c^+ \rightarrow B_d^0 \rho^+) &= \sqrt{2} G_F f_\rho F_1^{B_c \rightarrow B_d} m_{\rho^+} (\varepsilon \cdot p_{B_c}) \\ &\times \{ V_{ud} V_{cd}^* a_1 + (V_{ud} V_{cd}^* \\ &\quad + V_{us} V_{cs}^*) [a_4 - \frac{1}{2} a_{10}] \}\end{aligned}\quad (A3)$$

$$\begin{aligned}\mathcal{A}(B_c^+ \rightarrow B_d^0 K^{*+}) &= \sqrt{2} G_F f_{K^*} F_1^{B_c \rightarrow B_d} m_{K^{*+}} (\varepsilon \cdot p_{B_c}) \\ &\times V_{us} V_{cd}^* a_1\end{aligned}\quad (A4)$$

$$\begin{aligned}\mathcal{A}(B_c^+ \rightarrow B_d^{*0} \pi^+) &= \sqrt{2} G_F f_\pi A_0^{B_c \rightarrow B_d^*} m_{B_d^*} (\varepsilon \cdot p_{B_c}) \\ &\times \{ V_{ud} V_{cd}^* a_1 + (V_{ud} V_{cd}^* \\ &\quad + V_{us} V_{cs}^*) [a_4 - \frac{1}{2} a_{10}] \\ &\quad + Q_{c1} (a_6 - \frac{1}{2} a_8) \}\end{aligned}\quad (A5)$$

where $Q_{c1} = \frac{-2m_{\pi^+}^2}{(m_d + m_u)(m_c + m_d)}$.

$$\begin{aligned}\mathcal{A}(B_c^+ \rightarrow B_d^{*0} K^+) &= \sqrt{2} G_F f_K \\ &\times A_0^{B_c \rightarrow B_d^*} m_{B_d^*} (\varepsilon \cdot p_{B_c}) \\ &\times V_{us} V_{cd}^* a_1.\end{aligned}\quad (A6)$$

(2) $c \rightarrow s$ processes:

$$\begin{aligned}\mathcal{A}(B_c^+ \rightarrow B_s^0 \pi^+) &= -i \frac{G_F}{\sqrt{2}} f_\pi F_0^{B_c \rightarrow B_s} (m_{B_c}^2 - m_{B_s}^2) \\ &\times V_{ud} V_{cs}^* a_1\end{aligned}\quad (A7)$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_s^0 K^+) &= -i \frac{G_F}{\sqrt{2}} f_K F_0^{B_c \rightarrow B_s^0} (m_{B_c}^2 - m_{B_s}^2) \\ &\times \left\{ V_{us} V_{cs}^* a_1 + (V_{ud} V_{cd}^* \right. \\ &+ V_{us} V_{cs}^*) \left[a_4 - \frac{1}{2} a_{10} \right. \\ &\left. \left. + R_{c2} \left(a_6 - \frac{1}{2} a_8 \right) \right] \right\} \quad (\text{A8}) \end{aligned}$$

$$\text{where } R_{c2} = \frac{2m_{K^+}^2}{(m_s + m_u)(m_c - m_s)}$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_s^0 \rho^+) &= \sqrt{2} G_F f_\rho F_1^{B_c \rightarrow B_s^0} m_{\rho^+} (\varepsilon \cdot p_{B_c}) \\ &\times V_{ud} V_{cs}^* a_1 \quad (\text{A9}) \end{aligned}$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_s^0 K^{*+}) &= \sqrt{2} G_F f_{K^*} F_1^{B_c \rightarrow B_s^0} m_{K^{*+}} (\varepsilon \cdot p_{B_c}) \\ &\times \{ V_{us} V_{cs}^* a_1 + (V_{ud} V_{cd}^* \\ &+ V_{us} V_{cs}^*) [a_4 - \frac{1}{2} a_{10}] \} \quad (\text{A10}) \end{aligned}$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_s^{*0} \pi^+) &= \sqrt{2} G_F f_\pi A_0^{B_c \rightarrow B_s^{*0}} m_{B_s^*} (\varepsilon \cdot p_{B_c}) \\ &\times V_{ud} V_{cs}^* a_1 \quad (\text{A11}) \end{aligned}$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_s^{*0} K^+) &= \sqrt{2} G_F f_K A_0^{B_c \rightarrow B_s^{*0}} m_{B_s^*} (\varepsilon \cdot p_{B_c}) \\ &\times \{ V_{us} V_{cs}^* a_1 + (V_{ud} V_{cd}^* \\ &+ V_{us} V_{cs}^*) [a_4 - \frac{1}{2} a_{10} \\ &+ Q_{c2} (a_6 - \frac{1}{2} a_8)] \} \quad (\text{A12}) \end{aligned}$$

$$\text{where } Q_{c2} = \frac{-2m_{K^+}^2}{(m_s + m_u)(m_c + m_s)}.$$

(3) $c \rightarrow u$ processes:

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_u^+ \pi^0) &= -i \frac{G_F}{2} f_\pi F_0^{B_c \rightarrow B_u^+} (m_{B_c}^2 - m_{B_u}^2) \\ &\times \left\{ -V_{ud} V_{cd}^* a_2 + (V_{ud} V_{cd}^* \right. \\ &+ V_{us} V_{cs}^*) \left[a_4 + a_{10} \right. \\ &\left. \left. - \frac{3}{2} (a_7 - a_9) + R_{c3} (a_6 + a_8) \right] \right\} \quad (\text{A13}) \end{aligned}$$

$$\text{where } R_{c3} = \frac{2m_{\pi^0}^2}{(m_d + m_u)(m_c - m_u)}$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_u^+ \bar{K}^0) &= -i \frac{G_F}{\sqrt{2}} f_K F_0^{B_c \rightarrow B_u^+} (m_{B_c}^2 - m_{B_u}^2) \\ &\times V_{ud} V_{cs}^* a_2 \quad (\text{A14}) \end{aligned}$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_u^+ K^0) &= -i \frac{G_F}{\sqrt{2}} f_K F_0^{B_c \rightarrow B_u^+} (m_{B_c}^2 - m_{B_u}^2) \\ &\times V_{us} V_{cd}^* a_2 \quad (\text{A15}) \end{aligned}$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_u^+ \eta^{(\prime)}) &= -i \frac{G_F}{\sqrt{2}} f_{\eta^{(\prime)}}^u F_0^{B_c \rightarrow B_u^+} (m_{B_c}^2 - m_{B_u}^2) \\ &\times \left\{ V_{ud} V_{cd}^* a_2 + \frac{f_{\eta^{(\prime)}}^s}{f_{\eta^{(\prime)}}^u} V_{us} V_{cs}^* a_2 \right. \\ &+ (V_{ud} V_{cd}^* + V_{us} V_{cs}^*) \\ &\times \left[\frac{f_{\eta^{(\prime)}}^s}{f_{\eta^{(\prime)}}^u} \left\{ a_3 - a_5 + \frac{1}{2} (a_7 - a_9) \right\} \right. \\ &+ 2(a_3 - a_5) + a_4 + a_{10} \\ &\left. - \frac{1}{2} (a_7 - a_9) + \left(1 - \frac{f_{\eta^{(\prime)}}^u}{f_{\eta^{(\prime)}}^s} \right) \right. \\ &\left. \left. \times R_{c4}^{(\prime)} (a_6 + a_8) \right] \right\} \quad (\text{A16}) \end{aligned}$$

$$\text{where } R_{c4}^{(\prime)} = \frac{2m_{\eta^{(\prime)}}^2}{(m_s + m_s)(m_c - m_u)}$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_u^+ \rho^0) &= G_F f_\rho F_1^{B_c \rightarrow B_u^+} m_{\rho^0} (\varepsilon \cdot p_{B_c}) \\ &\times \{ -V_{ud} V_{cd}^* a_2 + (V_{ud} V_{cd}^* \\ &+ V_{us} V_{cs}^*) [a_4 + a_{10} \\ &+ \frac{3}{2} (a_7 + a_9)] \} \quad (\text{A17}) \end{aligned}$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_u^+ \omega) &= G_F f_\omega F_1^{B_c \rightarrow B_u^+} m_\omega (\varepsilon \cdot p_{B_c}) \\ &\times \{ V_{ud} V_{cd}^* a_2 + (V_{ud} V_{cd}^* \\ &+ V_{us} V_{cs}^*) [2(a_3 + a_5) \\ &+ a_4 + a_{10} + \frac{1}{2} (a_7 + a_9)] \} \quad (\text{A18}) \end{aligned}$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_u^+ \bar{K}^{*0}) &= \sqrt{2} G_F f_{\bar{K}^*} F_1^{B_c \rightarrow B_u^+} m_{\bar{K}^{*0}} (\varepsilon \cdot p_{B_c}) \\ &\times V_{ud} V_{cs}^* a_2 \quad (\text{A19}) \end{aligned}$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_u^+ K^{*0}) &= \sqrt{2} G_F f_{K^{*0}} F_1^{B_c \rightarrow B_u^+} m_{K^{*0}} (\varepsilon \cdot p_{B_c}) \\ &\times V_{us} V_{cd}^* a_2 \quad (\text{A20}) \end{aligned}$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_u^{*+} \pi^0) &= G_F f_\pi A_0^{B_c \rightarrow B_u^{*+}} m_{B_u^*} (\varepsilon \cdot p_{B_c}) \\ &\times \{ -V_{ud} V_{cd}^* a_2 + (V_{ud} V_{cd}^* \\ &+ V_{us} V_{cs}^*) [a_4 + a_{10} \\ &- \frac{3}{2} (a_7 - a_9) + Q_{c3} (a_6 + a_8)] \} \quad (\text{A21}) \end{aligned}$$

$$\text{where } Q_{c3} = \frac{-2m_{\pi^0}^2}{(m_d + m_u)(m_c + m_u)}$$

$$\mathcal{A}(B_c^+ \rightarrow B_u^{*+} \bar{K}^0) = \sqrt{2} G_F f_K A_0^{B_c \rightarrow B_u^{*+}} m_{B_u^{*+}} (\varepsilon \cdot p_{B_c}) V_{ud} V_{cs}^* a_2 \quad (\text{A22})$$

$$\mathcal{A}(B_c^+ \rightarrow B_u^{*+} K^0) = \sqrt{2} G_F f_K A_0^{B_c \rightarrow B_u^{*+}} m_{B_u^{*+}} (\varepsilon \cdot p_{B_c}) V_{us} V_{cd}^* a_2 \quad (\text{A23})$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_u^{*+} \eta^{(\prime)}) &= \sqrt{2} G_F f_{\eta^{(\prime)}}^u A_0^{B_c \rightarrow B_u^{*+}} m_{B_u^{*+}} (\varepsilon \cdot p_{B_c}) \left\{ V_{ud} V_{cd}^* a_2 + \frac{f_{\eta^{(\prime)}}^s}{f_{\eta^{(\prime)}}^u} V_{us} V_{cs}^* a_2 + (V_{ud} V_{cd}^* + V_{us} V_{cs}^*) \right. \\ &\quad \times \left[\frac{f_{\eta^{(\prime)}}^s}{f_{\eta^{(\prime)}}^u} \left\{ a_3 - a_5 + \frac{1}{2}(a_7 - a_9) \right\} + 2(a_3 - a_5) + a_4 + a_{10} - \frac{1}{2}(a_7 - a_9) \right. \\ &\quad \left. \left. + \left(1 - \frac{f_{\eta^{(\prime)}}^u}{f_{\eta^{(\prime)}}^s} \right) Q_{c4}^{(\prime)} (a_6 + a_8) \right] \right\} \quad (\text{A24}) \end{aligned}$$

$$\text{where } Q_{c4}^{(\prime)} = \frac{-2m_{\eta^{(\prime)}}^2}{(m_s + m_s)(m_c + m_u)}.$$

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