

Updated values of running quark and lepton masses

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Reliable values of quark and lepton masses are important for model building at a fundamental energy scale, such as the Fermi scale $M_Z \approx 91.2$ GeV and the would-be GUT scale $\Lambda_{\text{GUT}} \sim 2 \times 10^{16}$ GeV. Using the latest data given by the Particle Data Group, we update the running quark and charged-lepton masses at a number of interesting energy scales below and above M_Z . In particular, we take into account the possible new physics scale ($\mu \sim 1$ TeV) to be explored by the CERN LHC and the typical seesaw scales ($\mu \sim 10^9$ GeV and $\mu \sim 10^{12}$ GeV) which might be relevant to the generation of neutrino masses. For illustration, the running masses of three light Majorana neutrinos are also calculated. Our up-to-date tables of running fermion masses are expected to be very useful for the study of flavor dynamics at various energy scales.

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I. INTRODUCTION

In the standard model (SM) of electroweak interactions [1], it is the Higgs mechanism that provides a self-consistent framework to generate masses for gauge bosons and charged fermions. Three neutrinos are exactly massless as a straightforward consequence of the symmetry structure of the SM. But this framework itself can neither predict the values of six quark masses and three charged-lepton masses nor interpret the observed hierarchy of their spectra [2]. On the other hand, current neutrino oscillation experiments [3–6] indicate that neutrinos are actually massive and lepton flavors are mixed. Hence one has to extend the SM in order to gain an insight into the dynamics of fermion mass generation and flavor mixing. Possible new physics beyond the SM is expected to appear far above the Fermi scale $M_Z \approx 91.2$ GeV which essentially characterizes the scale of electroweak symmetry breaking. For instance, the typical scale of grand unified theories (GUTs) for strong and electroweak interactions [7] would be $\Lambda_{\text{GUT}} \sim 2 \times 10^{16}$ GeV, while $\mu \sim 1$ TeV is the possible scale of new physics necessary to stabilize the mass of the Higgs boson and offer a natural explanation of electroweak symmetry breaking [8]. Whenever a specific mass model of leptons and/or quarks is built at a certain high energy scale, one has to make use of the technique of renormalization group equations (RGEs) [9] to bridge the gap between the model predictions (at $\mu \gg M_Z$) and the experimental data (at $\mu \lesssim M_Z$). Therefore, reliable values of running quark and lepton masses are very important for building new physics models and testing their viability.

A systematic estimate of running quark masses at various energy scales was made by Fusaoka and Koide [10] in

1998. Their numerical results have proved to be very useful for people working on both hadronic physics at relatively low energies and electroweak physics and GUTs at super-high energies. However, a nontrivial update of this work is greatly desirable today because a lot of changes in particle physics have happened since 1998. On the one hand, some new and more accurate data on fermion masses at low scales have been accumulated [11]. On the other hand, intensive interest has been paid to some new physics scales such as $\mu \sim 1$ TeV to be explored by the Large Hadron Collider (LHC) and $\mu \sim 10^{12}$ GeV which might be relevant to the seesaw mechanism of neutrino mass generation [12]. These energy scales were not considered in Ref. [10], nor were the running masses of three light neutrinos. Thus we are well motivated to do a new analysis of running fermion masses at a variety of fundamental or interesting energy scales.

Let us remark that the present paper is different from Ref. [10] and other previous works in the following aspects:

- (i) The running masses of three light Majorana neutrinos are calculated. A global analysis of current neutrino oscillation data yields strong constraints on two neutrino mass-squared differences ($\Delta m_{21}^2 \equiv m_2^2 - m_1^2 = (7.2 \dots 8.9) \times 10^{-5}$ eV² and $|\Delta m_{32}^2| \equiv |m_3^2 - m_2^2| = (1.7 \dots 3.3) \times 10^{-3}$ eV²) and three neutrino mixing angles ($30^\circ < \theta_{12} < 38^\circ$, $36^\circ < \theta_{23} < 54^\circ$, and $\theta_{13} < 10^\circ$) at the 99% confidence level [13]. In addition, the latest cosmological constraint on the absolute values of m_i is $m_1 + m_2 + m_3 < 0.61$ eV at the 95% confidence level [14]. Since the unification of leptons and quarks is only possible at a superhigh energy scale (e.g., the GUT scale or the seesaw scale), it makes sense to calculate the running masses of three light neutrinos together with those of charged fermions.

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We shall illustrate the running effects of $m_i(\mu)$ for $\mu \gg M_Z$ by using the RGEs both in the SM and in the minimal supersymmetric standard model (MSSM).

- (ii) The up-to-date values of quark masses [11] given by the Particle Data Group (PDG) at low energies are adopted, and a few new energy scales (such as $\mu \sim 1$ TeV and 10^{12} GeV) are taken into account. Any models for new physics beyond the SM may introduce new energy scales above the Fermi scale, among which the $\mu \sim 1$ TeV scale is most appealing because it will soon be probed by the LHC. Possible new physics at this energy frontier is likely to be responsible for the origin of fermion masses and flavor mixing, or it can at least shed light on these fundamental problems. On the other hand, the intriguing seesaw [12] and leptogenesis [15] mechanisms have motivated a lot of neutrino physicists to study the properties of heavy Majorana neutrinos in order to simultaneously account for the lightness of three known neutrinos and the matter-antimatter asymmetry of our universe. The particularly interesting mass scales of such heavy Majorana particles are $\mu \sim 10^9$ GeV to 10^{12} GeV, where the flavor effects on leptogenesis [16] have to be treated carefully. Hence we shall make use of the RGEs to run the lepton and quark masses to these new energy scales, just for the convenience of model builders.
- (iii) The treatment of quark masses crossing the flavor thresholds is improved. We shall use the matching conditions only at the flavor thresholds $\mu \equiv m_Q(m_Q)$ (for $Q = c, b, t$) and calculate light quark masses at any other high energy scales with the help of the RGEs. This approach has been clearly described in Ref. [17]. In comparison, the running quark masses between two neighboring flavor thresholds were just computed with the matching conditions in Ref. [10]. The running-matching-running scheme proposed in Ref. [17] and used in our calculations is no doubt more reasonable.
- (iv) The effective coupling constant α_s , which governs the strength of strong interactions, is calculated by numerically solving its RGE. In contrast, the values of α_s at different energy scales were evaluated via the analytical relation between α_s and the asymptotic scale parameter Λ in Ref. [10]. The latter method may result in some unnecessary uncertainties due to the expansion of $1/\ln(\mu^2/\Lambda^2)$, as shown in Refs. [17,18].

Our main numerical results will be tabulated, as done in Ref. [10], to serve for a useful reference in building specific models at various energy scales.

The remaining parts of this paper are organized as follows. In Sec. II, we summarize the input data which in-

clude the current masses of three light quarks at $\mu = 2$ GeV, the values of $m_c(m_c)$ and $m_b(m_b)$, the pole mass of the top quark, the strong gauge coupling α_s and the fine structure constant α at M_Z . We also collect the relevant formulas for the RGEs and matching conditions of quark masses and α_s . The strategy to deal with the flavor thresholds and evolve the charged fermion masses to the Fermi scale is outlined. Sec. III is devoted to the calculations of running fermion masses up to the GUT scale both in the SM and in the MSSM. Indeed, the true running parameters above the Fermi scale are the Yukawa couplings of leptons and quarks because these particles can only acquire their masses after the electroweak symmetry breaking (i.e., below the Fermi scale). The running masses of three light neutrinos are also illustrated for completeness. Finally, we make a brief summary of our main results in Sec. IV.

II. BELOW THE FERMİ SCALE

First of all, let us give some concise comments on the definition of fermion masses. It is important to specify the theoretical framework when discussing quark masses, since they are renormalization-scheme-dependent. There are two very common renormalization schemes. One of them is the on shell scheme, in which the position of the pole is the definition of the physical mass M . The other is to define the *running* or renormalized mass $m(\mu)$ in the dimensional regularization scheme with the modified minimal subtraction ($\overline{\text{MS}}$), where μ denotes the renormalization scale. Both definitions are suitable for charged leptons. But the situation is quite different in the quark sector due to the nonperturbative nature of the quantum chromodynamics (QCD) at low energies: light quarks are always confined in hadrons and can never be observed directly. Even for the heaviest quark, the top quark, it is impossible to completely eliminate the nonperturbative effect on the value of m_t extracted from the direct measurements. Hence the pole mass of a quark is not well defined. Note that the QCD Lagrangian has a chiral symmetry in the limit where all quark masses are vanishing. The scale of dynamical chiral symmetry breaking is $\Lambda_\chi \sim 1$ GeV [19], which can be used to distinguish between light ($m_q < \Lambda_\chi$ for $q = u, d, s$) and heavy ($m_q > \Lambda_\chi$ for $q = c, b, t$) quarks.

Another important point is the decoupling of heavy flavors. Because of the hierarchical mass spectrum of quarks, one should integrate out the heavy degrees of freedom when considering the properties of light flavors. Therefore, some consistent matching conditions should be established between the full and effective theories at the scales characterized by the masses of heavy flavors. We shall work in the framework elaborated on in Ref. [17].

A. Running quark masses

With the help of the chiral symmetry, one may extract the values of m_u/m_d and m_s/m_d from the masses of pion

and kaon in a way independent of the renormalization scale [20]. The mass of the strange quark can be determined from the spectral function sum rules or lattice QCD simulations [19,20]. The up-to-date values of m_u , m_d and m_s given by the PDG are [11]

$$\begin{aligned} m_u(2 \text{ GeV}) &= 1.5 \sim 3.0 \text{ MeV}, \\ m_d(2 \text{ GeV}) &= 3 \sim 7 \text{ MeV}, \\ m_s(2 \text{ GeV}) &= 95 \pm 25 \text{ MeV}. \end{aligned} \quad (1)$$

Note that these running masses are evaluated at $\mu = 2 \text{ GeV}$ with three active quark flavors (u, d, s). The evolution of $m_q(\mu)$ (for $q = u, d, s, c, b, t$) is governed by the following RGE:

$$\begin{aligned} \mu^2 \frac{dm_q(\mu)}{d\mu^2} &= -\gamma(\alpha_s)m_q(\mu) \\ &= -\sum_{i=0}^{\infty} \gamma_i \left(\frac{\alpha_s(\mu)}{\pi} \right)^{i+1} m_q(\mu), \end{aligned} \quad (2)$$

where $\alpha_s(\mu) \equiv g_s^2/(4\pi)$ is the effective coupling constant of strong interactions with g_s being the strong gauge coupling. The values of γ_i (for $i = 0, 1, 2, 3$) are given by [21–23]

$$\begin{aligned} \gamma_0 &= 1, \quad \gamma_1 = \frac{1}{16} \left(\frac{202}{3} - \frac{20}{9} n_q \right), \\ \gamma_2 &= \frac{1}{64} \left[1249 - \frac{(2216)}{27} + \frac{160}{3} \zeta(3) \right] n_q - \frac{140}{81} n_q^2, \\ \gamma_3 &= \frac{1}{256} \left[\frac{4 \cdot 603 \cdot 055}{162} + \frac{135 \cdot 680}{27} \zeta(3) - 8800 \zeta(5) \right. \\ &\quad - \frac{(91 \cdot 723)}{27} + \frac{34 \cdot 192}{9} - 880 \zeta(4) - \frac{18 \cdot 400}{9} \zeta(5) \left. \right] n_q \\ &\quad + \left(\frac{5242}{243} + \frac{800}{9} \zeta(3) - \frac{160}{3} \zeta(4) \right) n_q^2 - \left(\frac{332}{243} - \frac{64}{27} \zeta(3) \right) n_q^3, \end{aligned} \quad (3)$$

where n_q is the number of active quark flavors with masses $m_q < \mu$; $\zeta(3) \approx 1.202057$, $\zeta(4) = \pi^4/90 \approx 1.082323$ and $\zeta(5) \approx 1.036928$ are the Riemann zeta functions. To find the solution to Eq. (2), we write out the detailed μ dependence of $\alpha_s(\mu)$ in terms of the Callan-Symanzik beta function [11],

$$\mu^2 \frac{\partial \alpha_s(\mu)}{\partial \mu^2} = \beta(\alpha_s(\mu)) = -\sum_{i \geq 0} \beta_i \frac{\alpha_s^{i+2}(\mu)}{\pi^{i+1}}, \quad (4)$$

where

$$\begin{aligned} \beta_0 &= \frac{1}{4} (11 - \frac{2}{3} n_q), \quad \beta_1 = \frac{1}{16} (102 - \frac{38}{3} n_q), \\ \beta_2 &= \frac{1}{64} \left(\frac{2857}{2} - \frac{5033}{18} n_q + \frac{325}{54} n_q^2 \right), \\ \beta_3 &= \frac{1}{256} \left[\frac{149 \cdot 753}{6} + 3564 \zeta(3) - \left(\frac{1 \cdot 078 \cdot 361}{162} + \frac{6508}{27} \zeta(3) \right) n_q \right. \\ &\quad \left. + \left(\frac{50 \cdot 065}{162} + \frac{6472}{81} \zeta(3) \right) n_q^2 + \frac{1093}{729} n_q^3 \right]. \end{aligned} \quad (5)$$

Note that we adopt the $\overline{\text{MS}}$ scheme for the RGEs throughout this paper. The solution to Eq. (2) can be expressed as [23,24]

$$m_q(\mu) = \mathcal{R}(\alpha_s(\mu)) \hat{m}_q, \quad (6)$$

where \hat{m}_q denotes the renormalization-invariant quark mass, and

$$\begin{aligned} \mathcal{R}(\alpha_s) &= \left(\frac{\alpha_s}{\pi} \right)^{\gamma_0/\beta_0} \left[1 + \frac{\alpha_s}{\pi} C_1 + \frac{\alpha_s^2}{2\pi^2} (C_1^2 + C_2) \right. \\ &\quad \left. + \frac{\alpha_s^3}{\pi^3} \left(\frac{1}{6} C_1^3 + \frac{1}{2} C_1 C_2 + \frac{1}{3} C_3 \right) \right]. \end{aligned} \quad (7)$$

In Eq. (7), the terms of $\mathcal{O}(\alpha_s^4)$ and smaller have been omitted and the coefficients $C_{1,2,3}$ are defined as

$$\begin{aligned} C_1 &= \frac{\gamma_1}{\beta_0} - \frac{\beta_1 \gamma_0}{\beta_0^2}, \\ C_2 &= \frac{\gamma_2}{\beta_0} - \frac{\beta_1 \gamma_1}{\beta_0^2} - \frac{\beta_2 \gamma_0}{\beta_0^2} + \frac{\beta_1^2 \gamma_0}{\beta_0^3}, \\ C_3 &= \frac{\gamma_3}{\beta_0} - \frac{\beta_1 \gamma_2}{\beta_0^2} + \frac{\beta_1^2 \gamma_1}{\beta_0^3} - \frac{\beta_2 \gamma_1}{\beta_0^2} - \frac{\beta_1^3 \gamma_0}{\beta_0^4} \\ &\quad + 2 \frac{\beta_1 \beta_2 \gamma_0}{\beta_0^3} - \frac{\beta_3 \gamma_0}{\beta_0^2}. \end{aligned} \quad (8)$$

In a theory with multiple energy scales, such as the QCD with $n_l = n_Q - 1$ massless quarks and one heavy quark Q (for $m_Q \gg \mu$), one should integrate out the heavy field and construct an effective theory by requiring its consistency with the full n_Q -flavor theory at the heavy quark threshold $\mu^{(n_Q)} = \mathcal{O}(m_Q)$. In this sense, Eq. (4) is valid between two quark thresholds, which are defined by $\mu^{(n_q)} = m_q(m_q)$. Then we are in a position to address the decoupling of heavy quarks and consider the matching conditions at the flavor thresholds. In our calculations, we use the matching relation between the strong coupling constants of the neighboring flavors [17]:

$$\alpha_s^{(n_q-1)}(\mu) = \zeta_g^2 \alpha_s^{(n_q)}(\mu), \quad (9)$$

where ζ_g^2 is already known at the three-loop level, i.e.,

$$\begin{aligned} \zeta_g^2 &= 1 - \frac{\alpha_s^{(n_q)}(\mu)}{\pi} \left(\frac{1}{6} \ln \frac{\mu^2}{(\mu^{(n_q)})^2} \right) + \left(\frac{\alpha_s^{(n_q)}(\mu)}{\pi} \right)^2 \left(\frac{11}{12} - \frac{11}{24} \right. \\ &\quad \times \ln \frac{\mu^2}{(\mu^{(n_q)})^2} + \frac{1}{36} \ln^2 \frac{\mu^2}{(\mu^{(n_q)})^2} \left. \right) + \left(\frac{\alpha_s^{(n_q)}(\mu)}{\pi} \right)^3 \\ &\quad \times \left[\frac{564 \cdot 731}{124 \cdot 416} - \frac{82 \cdot 043}{27 \cdot 648} \zeta(3) - \frac{955}{576} \ln \frac{\mu^2}{(\mu^{(n_q)})^2} \right. \\ &\quad \left. + \frac{53}{576} \ln^2 \frac{\mu^2}{(\mu^{(n_q)})^2} - \frac{1}{216} \ln^3 \frac{\mu^2}{(\mu^{(n_q)})^2} - (n_q - 1) \right. \\ &\quad \left. \times \left(\frac{2633}{31 \cdot 104} - \frac{67}{576} \ln \frac{\mu^2}{(\mu^{(n_q)})^2} + \frac{1}{36} \ln^2 \frac{\mu^2}{(\mu^{(n_q)})^2} \right) \right], \end{aligned} \quad (10)$$

in the $\overline{\text{MS}}$ scheme.

Now we consider the input values of three heavy quark masses. For charm and bottom quarks, their masses ex-

tracted from the heavy quark effective theory, lattice gauge theory, and QCD sum rules are consistent with one another if they are all spelled out in the same scheme and at the same scale [11]:

$$\begin{aligned} m_c(m_c) &= 1.25 \pm 0.09 \text{ GeV}, \\ m_b(m_b) &= 4.20 \pm 0.07 \text{ GeV}. \end{aligned} \quad (11)$$

The top quark can be directly measured because its lifetime is shorter than the typical time scale of nonperturbative strong interactions $\Lambda_{\text{QCD}}^{-1}$ [25], where the magnitude of Λ_{QCD} is about several hundred MeV. The pole mass of the top quark extracted from the average of several direct measurements is [11]

$$M_t = 172.5 \pm 2.7 \text{ GeV}. \quad (12)$$

Note that the nonperturbative contribution to the top quark mass may be proportional to Λ_{QCD} , which is much smaller

than the present experimental error bar. The pole mass M_q (for $q = c, b, t$) can be translated into the running mass at $\mu = M_q$ through [24]

$$\begin{aligned} M_q &= m_q(M_q) \left[1 + \frac{4}{3} \frac{\alpha_s(M_q)}{\pi} + K_q^{(2)} \left(\frac{\alpha_s(M_q)}{\pi} \right)^2 \right. \\ &\quad \left. + K_q^{(3)} \left(\frac{\alpha_s(M_q)}{\pi} \right)^3 \right] \end{aligned} \quad (13)$$

to the accuracy of $\mathcal{O}(\alpha_s^3)$, where $\{K_c^{(2)} = 11.21, K_b^{(2)} = 10.17, K_t^{(2)} = 9.13\}$ and $\{K_c^{(3)} = 123.8, K_b^{(3)} = 101.5, K_t^{(3)} = 80.4\}$ are computed with some typical values of the pole masses of light quarks [24]. In addition to the matching condition of $\alpha_s^{(n_q)}$ in Eq. (9), we need to know the matching condition of the running quark masses at the flavor thresholds [17,26]:

$$\begin{aligned} \frac{m_q^{(n_q-1)}}{m_q^{(n_q)}} &= 1 + \frac{1}{12} \left(\frac{\alpha_s^{(n_q)}(\mu)}{\pi} \right)^2 \left(\ln^2 \frac{\mu^2}{(\mu^{(n_q)})^2} - \frac{5}{3} \ln \frac{\mu^2}{(\mu^{(n_q)})^2} + \frac{89}{36} \right) + \left(\frac{\alpha_s^{(n_q)}(\mu)}{\pi} \right)^3 \left[\frac{2951}{2916} - \frac{407}{864} \zeta(3) + \frac{5}{4} \zeta(4) - \frac{1}{36} B_4 \right. \\ &\quad - \left(\frac{311}{2592} + \frac{5}{6} \zeta(3) \right) \ln \frac{\mu^2}{(\mu^{(n_q)})^2} + \frac{175}{432} \ln^2 \frac{\mu^2}{(\mu^{(n_q)})^2} + \frac{29}{216} \ln^3 \frac{\mu^2}{(\mu^{(n_q)})^2} (n_q - 1) \left(\frac{1327}{11664} - \frac{2}{27} \zeta(3) - \frac{53}{432} \ln \frac{\mu^2}{(\mu^{(n_q)})^2} \right. \\ &\quad \left. \left. - \frac{1}{108} \ln^3 \frac{\mu^2}{(\mu^{(n_q)})^2} \right) \right], \end{aligned} \quad (14)$$

where $B_4 \simeq -1.762800$. In practical calculations, we apply Eqs. (9) and (14) to the evolution of quark masses just at the thresholds $\mu^{(n_q)}$. Between two neighboring flavor thresholds, the RGE given in Eq. (2) or Eq. (6) will be used. We remark that this treatment is more reasonable than that adopted in Ref. [10].

It is worthwhile to mention that the running and decoupling of the strong coupling and quark masses have been built into the Mathematica package *RunDec* by Chetyrkin, Kühn, and Steinhauser [17]. We use the same formulas in our calculations and find that the results are in good agreement with those calculated by using *RunDec*, if the decoupling scale is chosen as $\mu^{(n_q)} = m_q(m_q)$.

B. Running charged-lepton masses

We proceed to discuss the running masses of charged leptons [27]. The μ dependence of the fine structure constant α , including QCD corrections, is described by [28]

$$\mu^2 \frac{\partial \alpha}{\partial \mu^2} = -\frac{\alpha^2}{\pi} \left[\tilde{\beta}_0 + \tilde{\beta}_1 \left(\frac{\alpha}{\pi} \right) + \sum_{i=1}^3 \rho_i \left(\frac{\alpha_s}{\pi} \right)^i \right], \quad (15)$$

where

$$\tilde{\beta}_0 = -\frac{1}{3} \sum_f Q_f^2 N_c^f, \quad \tilde{\beta}_1 = -\frac{1}{4} \sum_f Q_f^4 N_c^f \quad (16)$$

with Q_f being the electric charge of a charged fermion (i.e., $f = e, \mu, \tau$ for charged leptons and $f = u, d, s, c, b, t$ for quarks) and N_c^f being the color factor (i.e., $N_c^l = 1$ for charged leptons and $N_c^q = 3$ for quarks), and

$$\begin{aligned} \rho_1 &= -\sum_q Q_q^2, \quad \rho_2 = \sum_q Q_q^2 \left(\frac{257}{46} - \frac{11}{72} n_q \right), \\ \rho_3 &= \sum_q Q_q^2 \left[-\frac{10487}{1728} - \frac{55}{18} \zeta(3) + \left(\frac{707}{864} + \frac{55}{54} \zeta(3) \right) n_q \right. \\ &\quad \left. + \frac{77}{3888} n_q^2 \right] - \frac{10}{3} \left(\sum_q Q_q \right)^2 \left(\frac{11}{144} - \frac{1}{6} \zeta(3) \right). \end{aligned} \quad (17)$$

The pole masses of three charged leptons can be unambiguously measured in experiments, and their values have been determined to an unprecedented degree of precision [11],

$$\begin{aligned} M_e &= 0.510998918 \pm 0.000000044 \text{ MeV}, \\ M_\mu &= 105.6583692 \pm 0.0000094 \text{ MeV}, \\ M_\tau &= 1776.99_{-0.26}^{+0.29} \text{ MeV}. \end{aligned} \quad (18)$$

One may convert the pole mass M_l into the running mass $m_l(\mu)$ by using the following equation [29]:

TABLE I. The gauge couplings $\alpha_s(\mu)$ and $\alpha(\mu)^{-1}$ at a few typical energy scales, where the first and second errors of $\alpha(\mu)^{-1}$ come from the uncertainties associated with $\alpha(M_Z)^{-1}$ and $m_q(m_q)$, respectively.

n_q	$\mu^{(n_q)} = m_q(m_q)$ (GeV)	$\alpha_s(\mu)$	$\alpha(\mu)^{-1}$
4	$m_c(m_c) = 1.25$	$0.387^{+0.027}_{-0.024}$	$134.116 \pm 0.018^{+0.109}_{-0.101}$
5	$m_b(m_b) = 4.20$	$0.223^{+0.008}_{-0.007}$	$132.406 \pm 0.018^{+0.024}_{-0.025}$
6	$m_t(m_t) = 163.6$	0.108 ± 0.002	$127.073 \pm 0.018 \pm 0.023$

 TABLE II. Running quark masses from $\mu^{(4)} = m_c(m_c)$ to $\mu^{(6)} = m_t(m_t)$, where we have taken the W -boson mass to be $M_W = 80.403$ GeV. The pole masses of three light quarks are not listed, simply because the perturbative QCD calculation is not reliable in that energy region.

μ	$m_u(\mu)$ (MeV)	$m_d(\mu)$ (MeV)	$m_s(\mu)$ (MeV)	$m_c(\mu)$ (GeV)	$m_b(\mu)$ (GeV)	$m_t(\mu)$ (GeV)
$m_c(m_c)$	$2.57^{+0.99}_{-0.84}$	$5.85^{+2.46}_{-2.38}$	111^{+31}_{-30}	1.25 ± 0.09	$5.99^{+0.28}_{-0.26}$	$384.8^{+22.8}_{-20.4}$
2 GeV	$2.2^{+0.8}_{-0.7}$	5.0 ± 2.0	95 ± 25	$1.07^{+0.12}_{-0.13}$	$5.05^{+0.16}_{-0.15}$	$318.4^{+13.3}_{-12.4}$
$m_b(m_b)$	$1.86^{+0.70}_{-0.60}$	$4.22^{+1.74}_{-1.71}$	80 ± 22	$0.901^{+0.111}_{-0.113}$	4.20 ± 0.07	$259.8^{+7.7}_{-7.4}$
M_W	$1.29^{+0.50}_{-0.43}$	$2.93^{+1.25}_{-1.21}$	56 ± 16	$0.626^{+0.084}_{-0.085}$	2.92 ± 0.09	173.8 ± 3.0
M_Z	$1.27^{+0.50}_{-0.42}$	$2.90^{+1.24}_{-1.19}$	55^{+16}_{-15}	0.619 ± 0.084	2.89 ± 0.09	171.7 ± 3.0
$m_t(m_t)$	$1.22^{+0.48}_{-0.40}$	$2.76^{+1.19}_{-1.14}$	52 ± 15	0.590 ± 0.080	2.75 ± 0.09	162.9 ± 2.8
M_q	1.77 ± 0.14	$4.91^{+0.12}_{-0.11}$	172.5 ± 2.7
$m_q(M_q)$	$1.11^{+0.11}_{-0.12}$	4.08 ± 0.08	162.2 ± 2.8

 TABLE III. Running charged-lepton masses below $m_t(m_t)$, where the uncertainties of $m_l(\mu)$ are determined by those of M_l .

μ	$m_e(\mu)$ (MeV)	$m_\mu(\mu)$ (MeV)	$m_\tau(\mu)$ (MeV)
$m_c(m_c)$	$0.495\,536\,319 \pm 0.000\,000\,043$	$104.474\,005\,6^{+0.000\,009\,3}_{-0.000\,009\,4}$	1774.90 ± 0.20
$m_b(m_b)$	$0.493\,094\,195^{+0.000\,000\,042}_{-0.000\,000\,043}$	$103.995\,189\,1 \pm 0.000\,009\,3$	1767.08 ± 0.20
M_W	$0.486\,845\,675 \pm 0.000\,000\,042$	$102.772\,088\,6 \pm 0.000\,009\,2$	$1747.12^{+0.20}_{-0.19}$
M_Z	$0.486\,570\,161 \pm 0.000\,000\,042$	$102.718\,135\,9 \pm 0.000\,009\,2$	$1746.24^{+0.20}_{-0.19}$
$m_t(m_t)$	$0.485\,289\,396 \pm 0.000\,000\,042$	$102.467\,315\,5^{+0.000\,009\,1}_{-0.000\,009\,2}$	1742.15 ± 0.20
M_l	$0.510\,998\,918 \pm 0.000\,000\,044$	$105.658\,369\,2 \pm 0.000\,009\,4$	1776.90 ± 0.20

$$m_l(\mu) = M_l \left[1 - \frac{\alpha(\mu)}{\pi} \left[1 + \frac{3}{2} \ln \left(\frac{\mu}{m_l(\mu)} \right) \right] \right], \quad (19)$$

where the subscript l runs over e , μ , and τ , and the terms of $\mathcal{O}(\alpha^2)$ and smaller have been omitted.

C. The strategy

The formulas given in the preceding subsections allow us to calculate the running masses of quarks and charged leptons up to the Fermi scale. The basic strategy of our numerical calculations is two-fold:

- (1) We shall use the values of strong and electromagnetic coupling constants given by the PDG at M_Z [11],

$$\begin{aligned} \alpha_s(M_Z) &= 0.1176 \pm 0.002, \\ \alpha(M_Z)^{-1} &= 127.918 \pm 0.018, \end{aligned} \quad (20)$$

where $M_Z = 91.1876 \pm 0.0021$ GeV is the mass of the Z boson and it lies in the range $\mu > \mu^{(5)} = m_b(m_b)$. We shall only use the central value of M_Z in

our calculations, because its error bar is negligibly small. From Eq. (4) with $n_q = 5$ and Eq. (20), we obtain the corresponding strong coupling constant at the bottom quark threshold $\alpha_s^{(5)}(\mu^{(5)}) = 0.223^{+0.008}_{-0.007}$ with $\mu^{(5)} = m_b(m_b)$. As for α^{-1} , we can directly compute its values at some typical energy scales by using Eq. (15). Our numerical results, which will be used in the subsequent calculations, are listed in Table I.

- (2) By using the RGEs with $n_q = 4$, we are able to run three light quark masses and three charged-lepton masses to the first heavy flavor threshold $\mu^{(5)}$. Then we implement the matching conditions of α_s and m_q to cross this threshold. There is no flavor threshold on the way from $\mu^{(5)}$ to M_Z , and thus the relevant equations with $n_q = 5$ can be used for numerical calculations. We have also calculated the running masses $m_Q(\mu)$ of heavy quarks Q at the scale $\mu \ll \mu^{(n_q)} = m_Q(m_Q)$, which is below the flavor threshold $\mu^{(n_q)}$. In our evaluation, the relevant equations

with n_Q active quark flavors have been used. For example, the running top quark mass at the Fermi scale $m_t(M_Z)$ is actually computed by using the RGEs with $n_q = 6$.

Our numerical results for the running masses of quarks and charged leptons below the Fermi scale are given in Tables II and III, respectively. We do not consider the running masses of three light neutrinos in this energy region, just because their interactions with other particles are too weak and their pole masses are too tiny. On the one hand, one expects that the changes of neutrino masses with respect to the energy scales are negligibly small. On the other hand, it is just the smallness of neutrino masses that hints at the possible existence of certain new physics scales (e.g., the seesaw scales $\mu \sim 10^9$ GeV to 10^{12} GeV). We shall illustrate the running effects of three neutrino masses from M_Z to $\mu \sim 10^{12}$ GeV by using the RGEs of both the SM and the MSSM in the next section.

III. ABOVE THE FERMÍ SCALE

Above the Fermi scale, the unbroken electroweak gauge symmetry forbids leptons and quarks to acquire their masses. The actual meaning of a fermion mass m_f in this energy region is a measure of the nontrivial Yukawa coupling eigenvalue y_f . One commonly defines $m_f = y_f v$ above the Fermi scale, just like the definition below the Fermi scale, where $v \approx 246$ GeV is the vacuum expectation value of the neutral Higgs field in the SM. In the conventional seesaw models [12],¹ the heavy Majorana neutrinos must be integrated out below the seesaw scale and the effective coupling matrix of three light Majorana neutrinos is given by the well-known seesaw relation $\kappa = -Y_\nu M_R^{-1} Y_\nu^T$, where Y_ν denotes the neutrino Yukawa coupling matrix and M_R is the right-handed Majorana neutrino mass matrix. The light neutrino masses are therefore given by $m_i = \kappa_i v^2$ with κ_i (for $i = 1, 2, 3$) being the eigenvalues of κ . Above the seesaw scale, the flavor threshold effects induced by the masses of heavy Majorana neutrinos have to be carefully treated and the details of Y_ν have to be model-dependently assumed [31]. Hence we shall only evaluate the running neutrino masses m_i from the Fermi scale up to the seesaw scale, in order to avoid the complications and uncertainties associated with the seesaw thresholds.

Then what we are concerned with is the evolution of four Yukawa coupling matrices Y_u, Y_d, Y_l and κ , whose eigenvalues correspond to the masses of up-type quarks (m_u, m_c, m_t), down-type quarks (m_d, m_s, m_b), charged leptons (m_e, m_μ, m_τ) and neutrinos (m_1, m_2, m_3). Their one-loop RGEs

¹Here we follow the widely accepted and well-motivated assumption that massive neutrinos are Majorana particles. The RGE running effects of Dirac neutrino masses and flavor mixing parameters have been studied in Ref. [30].

in the SM and MSSM can be found in Refs. [30,31]. The two-loop beta functions of Y_u, Y_d and Y_l have already been derived in Ref. [32]. Their lengthy expressions will not be quoted here, but they will be used in our numerical calculations. As for neutrinos, we only consider the one-loop RGEs for their running masses because our present knowledge on the absolute values of neutrino masses remain quite limited and uncertain [11]. The strategy of computing the running quark and lepton masses above the Fermi scale is outlined as follows. First, we use the quark masses and flavor mixing parameters obtained at the Fermi scale to reconstruct the Yukawa coupling matrices Y_u and Y_d . Second, the RGEs of Y_u and Y_d are solved and their eigenvalues are determined to give the running quark masses at every energy scale of our interest. We may follow a similar procedure to reconstruct Y_l and κ from current experimental data at M_Z , and then we solve their RGEs to find out the running lepton masses above M_Z . Because the RGEs of Y_u, Y_d, Y_l , and κ are more or less entangled, our numerical calculations will be done simultaneously for quarks and leptons.

Now that the running quark masses are to be evaluated in the supersymmetric framework, the matching procedure from the SM to the MSSM has to be taken into account. The RGEs in a supersymmetric theory are usually derived in the $\overline{\text{DR}}$ scheme based on the dimensional reduction and the minimal subtraction [33], while the experimental data on quark masses are extracted by using the $\overline{\text{MS}}$ scheme. Hence the transition between these two schemes needs to be treated in a consistent way [34,35]. To get around the occurrence of intermediate nonsupersymmetric effective theories, here we follow Ref. [35] to adopt the *common scale approach* with all the supersymmetric particles being roughly at a common scale \tilde{M} . In our analysis, we set the decoupling scale to be $\mu = \tilde{M} = M_Z$. Furthermore, the input values of $\alpha_s(M_Z)$ and $m_q(M_Z)$ in the $\overline{\text{MS}}$ scheme will be converted into those in the $\overline{\text{DR}}$ scheme at M_Z . First, since the two-loop RGEs will be used above the Fermi scale, we consider the transition of α_s from the $\overline{\text{MS}}$ scheme to the $\overline{\text{DR}}$ scheme at the one-loop level [36],

$$\alpha_s^{\overline{\text{MS}}} = \alpha_s^{\overline{\text{DR}}} \left(1 - \frac{\alpha_s^{\overline{\text{DR}}}}{4\pi} \right). \quad (21)$$

Given the input value of $\alpha_s(M_Z)$ in Eq. (20), $\alpha_s^{\overline{\text{DR}}}(M_Z) \approx 0.1187$ can be obtained and will be used as the input value in our numerical calculations. We take account of the matching between the SM and MSSM at M_Z ; i.e., $\alpha_s^{\text{SM}} = \zeta_s \alpha_s^{\text{MSSM}}$. In the common scale approach, the decoupling constant approximates to $\zeta_s \approx 1$ at the one-loop level, implying that the top quark and heavy sparticles are simultaneously decoupled. Second, the one-loop matching of quark masses between the $\overline{\text{MS}}$ and $\overline{\text{DR}}$ schemes is simply given by

$$m_q^{\overline{\text{DR}}} = m_q^{\overline{\text{MS}}}\left(1 - \frac{\alpha_e}{3\pi}\right), \quad (22)$$

where α_e is the evanescent coupling appearing in a non-supersymmetric theory renormalized in the $\overline{\text{DR}}$ scheme. We have $\alpha_e \approx \alpha_s^{\overline{\text{DR}}}$ up to $\mathcal{O}(\alpha_s^2)$ [35]. It is well known that the matching effects between the SM and MSSM are only significant for the masses of down-type quarks and charged leptons because of the large- $\tan\beta$ enhancement [37–40]. At the one-loop level, the threshold corrections read

$$m_i^{\text{MSSM}} = \frac{m_i^{\text{SM}}}{1 + \epsilon_i \tan\beta}, \quad (23)$$

where $i = d, s, b$ for down-type quarks or $i = e, \mu, \tau$ for charged leptons, and the definitions of ϵ_i can be found in Refs. [39,40]. In our numerical analysis with the common scale approach, ϵ_i can be as large as 1%. Hence the supersymmetric threshold corrections are particularly relevant in the case of sizable $\tan\beta$ (e.g., $\tan\beta = 50$). It is worth remarking that we have tried to avoid the details of any specific supersymmetric models within the scope of this work. If they are taken into account, however, a more careful treatment of the decoupling of supersymmetric partners will be unavoidable [37–39].

Before doing the numerical calculations, let us briefly summarize the relevant data to be input at the Fermi scale.

- (1) In the basis where the Yukawa coupling matrix Y_u is diagonal (i.e., $Y_u = \text{Diag}\{y_u, y_c, y_t\}$ with $y_q = m_q/v$ for $q = u, c, t$), the mass eigenstates of down-type quarks (d, s, b) are related to their weak eigenstates (d', s', b') by the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix V [41]. Namely, $V^\dagger Y_d Y_d^\dagger V = \text{Diag}\{y_d^2, y_s^2, y_b^2\}$ with $y_q = m_q/v$ for $q = d, s, \text{ and } b$. It is therefore possible to reconstruct Y_d from its eigenvalues and

V . Because quarks are Dirac particles, the CKM matrix V can be parametrized in terms of four independent quantities, such as the moduli of three independent elements of V and one CP -violating phase [42]. For example, one may choose the following set of parameters, which have been precisely measured, to parametrize V at M_Z [11]:

$$\begin{aligned} |V_{cb}| &= (41.6 \pm 0.6) \times 10^{-3}, \\ |V_{us}| &= 0.2257 \pm 0.0021, \\ |V_{ub}| &= (4.31 \pm 0.30) \times 10^{-3}, \\ \sin 2\beta &= 0.687 \pm 0.032, \end{aligned} \quad (24)$$

where $\beta \equiv \arg[-(V_{cd}V_{cb}^*)/(V_{td}V_{tb}^*)]$ is an inner angle of the CKM unitarity triangle. We shall only use the central values of the above parameters in our numerical calculations, because the running quark and lepton masses are actually insensitive to these inputs. On the other hand, the values of quark masses at the Fermi scale have been evaluated in Sec. II and listed in Table II.

- (2) Without loss of generality, we choose the flavor basis where the Yukawa coupling matrix Y_l is diagonal (i.e., $Y_l = \text{Diag}\{y_e, y_\mu, y_\tau\}$ with $y_l = m_l/v$ for $l = e, \mu, \tau$). In this basis, the mass eigenstates of three light Majorana neutrinos (ν_1, ν_2, ν_3) are linked to their weak eigenstates (ν_e, ν_μ, ν_τ) through the unitary Maki-Nakagawa-Sakata (MNS) matrix U [43].² Namely, $U^\dagger \kappa U^* = \text{Diag}\{\kappa_1, \kappa_2, \kappa_3\}$ with $\kappa_i = m_i/v^2$ for $i = 1, 2, \text{ and } 3$. It is then easy to reconstruct the symmetric matrix κ from m_i and U at the Fermi scale. A complete parametrization of the MNS matrix U needs three mixing angles and three CP -violating phases [45],

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_1} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_1} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_1} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_1} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_1} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (25)$$

with $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ (for $ij = 12, 13, \text{ and } 23$). A global analysis of current neutrino oscillation data yields the constraints [13]

$$30^\circ < \theta_{12} < 38^\circ, \quad 36^\circ < \theta_{23} < 54^\circ, \quad \theta_{13} < 10^\circ \quad (26)$$

at the 99% confidence level. Three CP -violating phases δ_l, ρ , and σ remain entirely unrestricted. It has been noticed that the running behaviors of neutrino masses are essentially insensitive to three neutrino mixing angles and three CP -violating phases in the SM or in the MSSM with small $\tan\beta$ [31]. Therefore, we shall simply assume $\delta_l = \rho = \sigma = 0$ and take the best-fit values $\theta_{12} \approx 33.8^\circ, \theta_{23} \approx 45^\circ$, and $\theta_{13} \approx 0^\circ$ as the typical inputs at M_Z . Note that the

absolute values of m_i remain unknown, although their upper bound is expected to be $\mathcal{O}(1)$ eV. For illustration, we only consider two possibilities in our numerical calculations: (A) the normal neutrino mass hierarchy with $m_1 = 0.001$ eV and $m_1 < m_2 \ll m_3$; and (B) the nearly degenerate neutrino mass spectrum with $m_1 = 0.2$ eV and $m_1 \lesssim m_2 \lesssim m_3$. The best-fit values $\Delta m_{21}^2 = 8.0 \times 10^{-5}$ eV² and $\Delta m_{32}^2 = 2.5 \times 10^{-3}$ eV² are input at M_Z in both cases.

²Because the seesaw scales are assumed to be extremely higher than the Fermi scale in this paper, U is expected to be unitary as an excellent approximation. See, e.g., Ref. [44] for a detailed analysis of the nonunitary neutrino mixing in the seesaw scenarios.

TABLE IV. Running quark and lepton masses above M_Z in the SM with $m_H = 140$ GeV, where the uncertainties of $m_f(\mu)$ result from those of $m_f(M_Z)$. Here we have used $\Lambda_{\text{GUT}} = 2 \times 10^{16}$ GeV. Case A and case B represent two different neutrino mass patterns with $m_1(M_Z) = 0.001$ eV and $m_1(M_Z) = 0.2$ eV, respectively.

	$\mu = M_Z$	$\mu = 1$ TeV	$\mu = 10^9$ GeV	$\mu = 10^{12}$ GeV	$\mu = \Lambda_{\text{GUT}}$
$m_u(\mu)$ (MeV)	$1.27^{+0.50}_{-0.42}$	$1.10^{+0.43}_{-0.37}$	$0.67^{+0.27}_{-0.23}$	$0.58^{+0.24}_{-0.20}$	$0.48^{+0.20}_{-0.17}$
$m_d(\mu)$ (MeV)	$2.90^{+1.24}_{-1.19}$	$2.50^{+1.08}_{-1.03}$	$1.56^{+0.69}_{-0.65}$	$1.34^{+0.60}_{-0.56}$	$1.14^{+0.51}_{-0.48}$
$m_s(\mu)$ (MeV)	55^{+16}_{-15}	47^{+14}_{-13}	30^{+9}_{-8}	26^{+8}_{-7}	22^{+7}_{-6}
$m_c(\mu)$ (GeV)	0.619 ± 0.084	$0.532^{+0.074}_{-0.073}$	$0.327^{+0.048}_{-0.047}$	$0.281^{+0.042}_{-0.041}$	$0.235^{+0.035}_{-0.034}$
$m_b(\mu)$ (GeV)	2.89 ± 0.09	2.43 ± 0.08	1.42 ± 0.06	1.21 ± 0.05	1.00 ± 0.04
$m_t(\mu)$ (GeV)	171.7 ± 3.0	150.7 ± 3.4	$99.1^{+4.0}_{-3.8}$	$86.7^{+4.0}_{-3.8}$	$74.0^{+4.0}_{-3.7}$
$m_e(\mu)$ (MeV)	$0.486\,570\,161 \pm 0.000\,000\,042$	$0.495\,901\,601 \pm 0.000\,000\,043$	$0.501\,014\,122 \pm 0.000\,000\,043$	$0.490\,856\,087^{+0.000\,000\,042}_{-0.000\,000\,043}$	$0.469\,652\,046 \pm 0.000\,000\,041$
$m_\mu(\mu)$ (MeV)	$102.718\,1359 \pm 0.000\,009\,2$	$104.688\,064\,5^{+0.000\,009\,4}_{-0.000\,009\,3}$	$105.767\,356\,2^{+0.000\,009\,5}_{-0.000\,009\,4}$	$103.622\,931\,1^{+0.000\,009\,2}_{-0.000\,009\,3}$	$99.146\,622\,6 \pm 0.000\,008\,9$
$m_\tau(\mu)$ (MeV)	$1746.24^{+0.20}_{-0.19}$	1779.74 ± 0.20	$1798.11^{+0.21}_{-0.20}$	1761.67 ± 0.20	1685.58 ± 0.19
<i>Case A</i>					
$m_1(\mu)$ (eV)	0.001	0.001	0.001	0.001	...
$\Delta m_{21}^2(\mu)$ (eV ²)	8.0×10^{-5}	9.1×10^{-5}	1.3×10^{-4}	1.5×10^{-4}	...
$\Delta m_{32}^2(\mu)$ (eV ²)	2.5×10^{-3}	2.9×10^{-3}	4.2×10^{-3}	4.6×10^{-3}	...
<i>Case B</i>					
$m_1(\mu)$ (eV)	0.20	0.21	0.26	0.27	...
$\Delta m_{21}^2(\mu)$ (eV ²)	8.0×10^{-5}	9.1×10^{-5}	1.3×10^{-4}	1.5×10^{-4}	...
$\Delta m_{32}^2(\mu)$ (eV ²)	2.5×10^{-3}	2.9×10^{-3}	4.2×10^{-3}	4.6×10^{-3}	...

TABLE V. Running quark and lepton masses above M_Z in the MSSM with $\tan\beta = 10$, where the matching effect between the MS and MSSM and the $\overline{\text{MS}}$ -to- $\overline{\text{DR}}$ transition effect on the input parameters at M_Z have been taken into account.

	$\mu = M_Z$	$\mu = 1 \text{ TeV}$	$\mu = 10^9 \text{ GeV}$	$\mu = 10^{12} \text{ GeV}$	$\mu = \Lambda_{\text{GUT}}$
$m_u(\mu)$ (MeV)	$1.27^{+0.50}_{-0.42}$	$1.15^{+0.45}_{-0.38}$	$0.75^{+0.30}_{-0.25}$	$0.62^{+0.26}_{-0.21}$	$0.49^{+0.20}_{-0.17}$
$m_d(\mu)$ (MeV)	$2.90^{+1.24}_{-1.19}$	$2.20^{+0.96}_{-0.91}$	$1.21^{+0.54}_{-0.51}$	$0.96^{+0.43}_{-0.40}$	$0.70^{+0.31}_{-0.30}$
$m_s(\mu)$ (MeV)	55^{+16}_{-15}	42 ± 12	23 ± 7	18^{+6}_{-5}	13 ± 4
$m_c(\mu)$ (GeV)	0.619 ± 0.084	$0.557^{+0.077}_{-0.076}$	$0.363^{+0.053}_{-0.052}$	$0.303^{+0.046}_{-0.045}$	$0.236^{+0.037}_{-0.036}$
$m_b(\mu)$ (GeV)	2.89 ± 0.09	2.23 ± 0.08	1.30 ± 0.05	1.05 ± 0.05	0.79 ± 0.04
$m_t(\mu)$ (GeV)	171.7 ± 3.0	$161.0^{+3.7}_{-3.6}$	$125.2^{+7.1}_{-6.5}$	$111.0^{+8.5}_{-7.4}$	$92.2^{+9.6}_{-7.8}$
$m_e(\mu)$ (MeV)	$0.486\,570\,161 \pm 0.000\,000\,042$	$0.418\,436\,115 \pm 0.000\,000\,036$	$0.358\,332\,424 \pm 0.000\,000\,031$	$0.327\,996\,884^{+0.000\,000\,028}_{-0.000\,000\,029}$	$0.283\,755\,495^{+0.000\,000\,024}_{-0.000\,000\,025}$
$m_\mu(\mu)$ (MeV)	$102.718\,1359 \pm 0.000\,009\,2$	$88.334\,701\,8 \pm 0.000\,007\,9$	$75.646\,853\,8 \pm 0.000\,006\,8$	$69.242\,937\,7 \pm 0.000\,006\,2$	$59.903\,361\,7 \pm 0.000\,005\,4$
$m_\tau(\mu)$ (MeV)	$1746.24^{+0.20}_{-0.19}$	1502.25 ± 0.17	1288.68 ± 0.15	$1180.38^{+0.13}_{-0.14}$	$1021.95^{+0.11}_{-0.12}$
<i>Case A</i>					
$m_1(\mu)$ (eV)	0.001	0.001	0.001	0.001	...
$\Delta m_{21}^2(\mu)$ (eV ²)	8.0×10^{-5}	8.7×10^{-5}	1.0×10^{-4}	1.0×10^{-4}	...
$\Delta m_{32}^2(\mu)$ (eV ²)	2.5×10^{-3}	2.7×10^{-3}	3.3×10^{-3}	3.1×10^{-3}	...
<i>Case B</i>					
$m_1(\mu)$ (eV)	0.20	0.21	0.23	0.22	...
$\Delta m_{21}^2(\mu)$ (eV ²)	8.0×10^{-5}	9.1×10^{-5}	1.5×10^{-4}	1.6×10^{-4}	...
$\Delta m_{32}^2(\mu)$ (eV ²)	2.5×10^{-3}	2.7×10^{-3}	3.3×10^{-3}	3.2×10^{-3}	...

TABLE VI. Running quark and lepton masses above M_Z in the MSSM with $\tan\beta = 50$, where the matching effect between the MS and MSSM and the $\overline{\text{MS}}$ -to- $\overline{\text{DR}}$ transition effect on the input parameters at M_Z have been taken into account.

	$\mu = M_Z$	$\mu = 1 \text{ TeV}$	$\mu = 10^9 \text{ GeV}$	$\mu = 10^{12} \text{ GeV}$	$\mu = \Lambda_{\text{GUT}}$
$m_u(\mu)$ (MeV)	$1.27^{+0.50}_{-0.42}$	$1.15^{+0.45}_{-0.38}$	$0.75^{+0.31}_{-0.26}$	$0.62^{+0.26}_{-0.22}$	$0.48^{+0.21}_{-0.17}$
$m_d(\mu)$ (MeV)	$2.90^{+1.24}_{-1.19}$	$1.51^{+0.67}_{-0.63}$	$0.86^{+0.39}_{-0.36}$	$0.69^{+0.31}_{-0.29}$	$0.51^{+0.23}_{-0.22}$
$m_s(\mu)$ (MeV)	55^{+16}_{-15}	29^{+9}_{-8}	16 ± 5	13 ± 4	10 ± 3
$m_c(\mu)$ (GeV)	0.619 ± 0.084	$0.557^{+0.077}_{-0.076}$	$0.364^{+0.054}_{-0.053}$	$0.304^{+0.046}_{-0.045}$	$0.237^{+0.037}_{-0.036}$
$m_b(\mu)$ (GeV)	2.89 ± 0.09	1.54 ± 0.06	0.96 ± 0.05	$0.79^{+0.05}_{-0.04}$	0.61 ± 0.04
$m_t(\mu)$ (GeV)	171.7 ± 3.0	161.3 ± 3.7	$127.0^{+7.4}_{-6.7}$	$113.2^{+8.9}_{-7.7}$	$94.7^{+10.3}_{-8.4}$
$m_e(\mu)$ (MeV)	$0.486\,570\,161 \pm 0.000\,000\,042$	$0.286\,562\,894 \pm 0.000\,000\,025$	$0.254\,747\,607 \pm 0.000\,000\,022$	$0.235\,672\,689^{+0.000\,000\,021}_{-0.000\,000\,020}$	$0.206\,036\,051 \pm 0.000\,000\,018$
$m_\mu(\mu)$ (MeV)	$102.718\,1359 \pm 0.000\,009\,2$	$60.496\,141\,3 \pm 0.000\,005\,4$	$53.783\,482\,3 \pm 0.000\,004\,8$	$49.757\,752\,8^{+0.000\,004\,5}_{-0.000\,004\,4}$	$43.502\,030\,5 \pm 0.000\,003\,9$
$m_\tau(\mu)$ (MeV)	$1746.24^{+0.20}_{-0.19}$	1032.61 ± 0.12	937.72 ± 0.11	875.31 ± 0.11	773.44 ± 0.10
<i>Case A</i>					
$m_1(\mu)$ (eV)	0.001	0.001	0.001	0.001	...
$\Delta m_{21}^2(\mu)$ (eV ²)	8.0×10^{-5}	8.7×10^{-5}	1.1×10^{-4}	1.0×10^{-4}	...
$\Delta m_{32}^2(\mu)$ (eV ²)	2.5×10^{-3}	2.7×10^{-3}	3.3×10^{-3}	3.2×10^{-3}	...
<i>Case B</i>					
$m_1(\mu)$ (eV)	0.20	0.21	0.23	0.23	...
$\Delta m_{21}^2(\mu)$ (eV ²)	8.0×10^{-5}	1.7×10^{-4}	7.1×10^{-4}	8.3×10^{-4}	...
$\Delta m_{32}^2(\mu)$ (eV ²)	2.5×10^{-3}	2.7×10^{-3}	3.8×10^{-3}	4.1×10^{-3}	...

This simplified treatment can easily be improved in the future, once more experimental data on m_i and U are available.

- (3) In the SM or MSSM with the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, three gauge couplings g_s , g , and g' are given by

$$\begin{aligned} g_s^2 &= 4\pi\alpha_s, & g^2 &= 4\pi\alpha/\sin^2\theta_W, \\ g' &= g \tan\theta_W, \end{aligned} \quad (27)$$

where θ_W is the weak mixing angle. In the SU(5) GUTs with or without supersymmetry, the gauge

coupling constants g_1 , g_2 , and g_3 are usually normalized as $g_3 = g_s$, $g_2 = g$, and $g_1 = \sqrt{5/3}g'$. The RGEs of g_i (for $i = 1, 2, 3$) are given in Ref. [32]. The input parameters include [11]

$$\sin^2\theta_W(M_Z) = 0.23122 \pm 0.00015, \quad (28)$$

as well as $\alpha_s(M_Z)$ and $\alpha(M_Z)$ given in Eq. (20). Besides the gauge couplings, the quartic Higgs coupling $\lambda = 2m_H^2/v^2$ in the SM is also needed in our calculations. The RGE of λ can be found in Ref. [32]. We shall typically take $m_H = 140$ GeV for the Higgs mass, just for the sake of illustration.

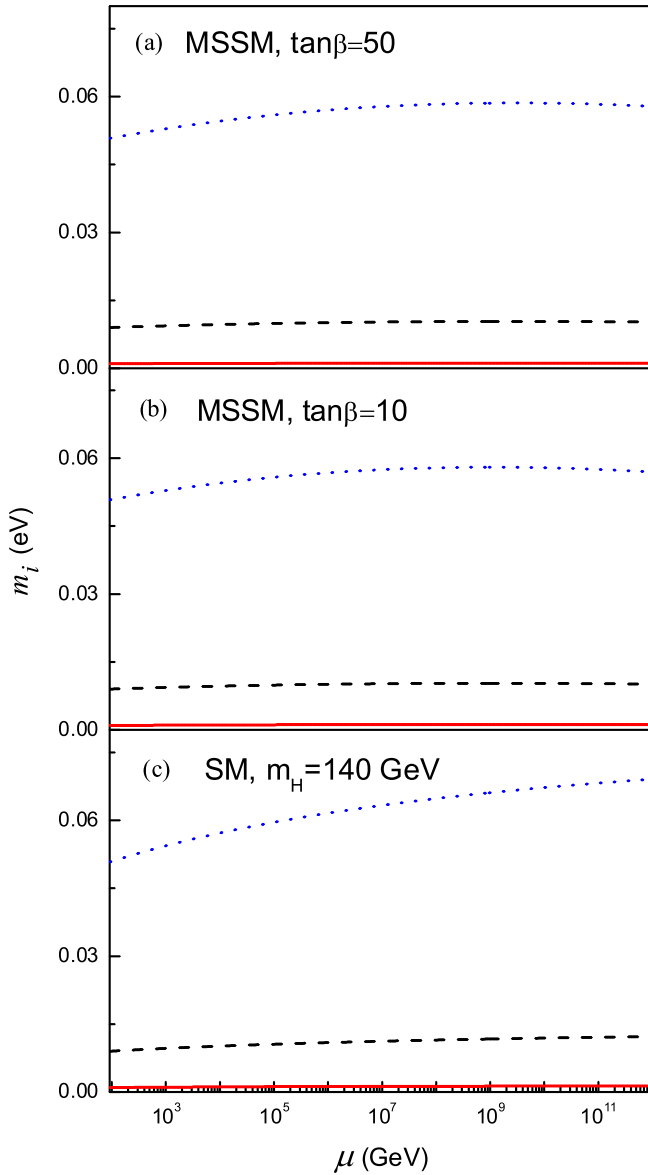


FIG. 1 (color online). The scale dependence of three neutrino masses m_1 (solid line), m_2 (dashed line), and m_3 (dotted line) in the case of a normal mass hierarchy with $m_1(M_Z) = 0.001$ eV and $m_1 < m_2 \ll m_3$.

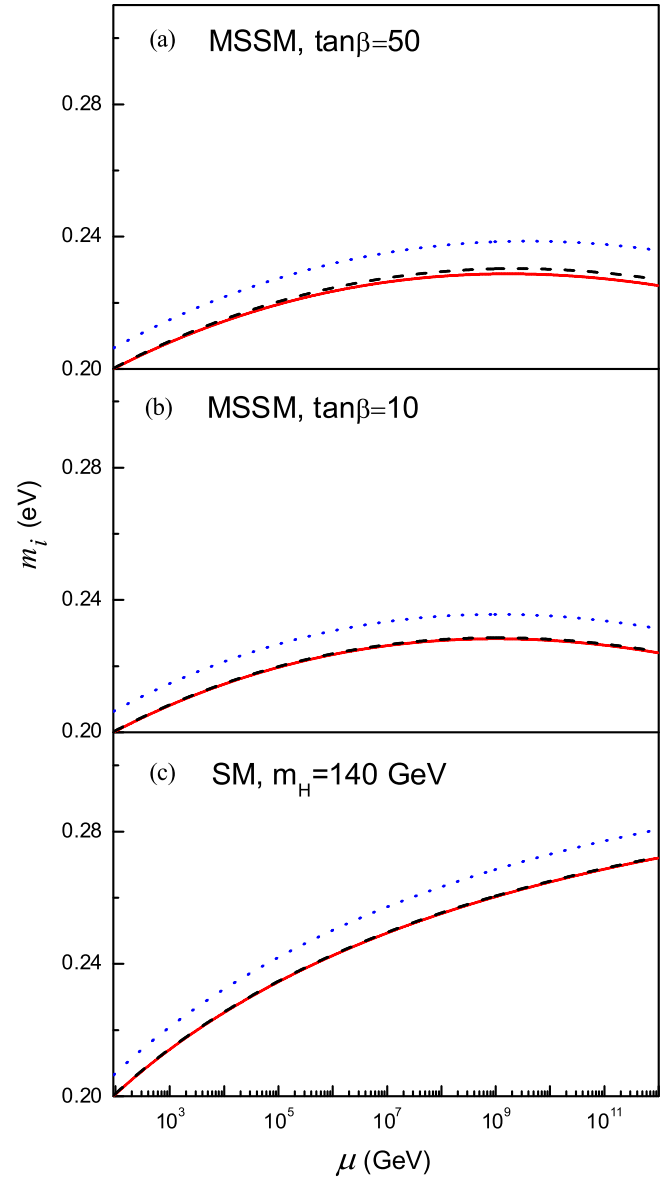


FIG. 2 (color online). The scale dependence of three neutrino masses m_1 (solid line), m_2 (dashed line), and m_3 (dotted line) in the case of a nearly degenerate mass hierarchy with $m_1(M_Z) = 0.2$ eV and $m_1 \simeq m_2 \simeq m_3$.

In the MSSM, the free parameter $\tan\beta$ is defined as $\tan\beta = v_u/v_d$ with v_d and v_u being the vacuum expectation values of the Higgs fields which couple, respectively, to the down- and up-type quarks. Note that $v^2 = v_d^2 + v_u^2 = 4M_W^2/g^2 \approx (246 \text{ GeV})^2$ has been fixed by the measurements of the W -boson mass and the electroweak gauge coupling. To be specific, we shall input $\tan\beta = 10$ and $\tan\beta = 50$ to illustrate the running fermion masses in the cases of small and large $\tan\beta$.

With the help of the above inputs, we are then able to numerically solve the RGEs and obtain the running masses of quarks and leptons at various energies above the Fermi scale.

Our numerical results at $\mu = 1 \text{ TeV}$, 10^9 GeV , 10^{12} GeV , and $\Lambda_{\text{GUT}} \sim 2 \times 10^{16} \text{ GeV}$ are summarized in Tables IV, V, and VI, where the uncertainties of the output masses mainly come from those of the input masses at M_Z . Some brief comments on these results are in order.

- (i) In both the SM and the MSSM, a common feature of the outputs is that all the quark masses decrease with increasing energy scales. Nevertheless, their running behaviors are quite model dependent.
- (ii) Tables IV and VI show that there exists a maximum for the running mass of each charged lepton in the SM. Our numerical analysis indicates that the maximal values of m_e , m_μ , and m_τ are all located around $\mu \sim 10^6 \text{ GeV}$. In the MSSM, however, three charged-lepton masses smoothly decrease as the energy scale increases.
- (iii) We have ignored large uncertainties of the input values in evaluating the running masses of three light neutrinos, just because the relevant experimental data are quite inaccurate and incomplete. Our numerical results, which rely on several assumptions made above, can only serve for illustration. We show the scale dependence of running neutrino masses in Figs. 1 and 2 for two different mass spectra. Comparing between these two fig-

ures, we observe that the RGE effects on neutrino masses are more significant in the $m_1 \lesssim m_2 \lesssim m_3$ case than in the $m_1 < m_2 \ll m_3$ case. Similar observations have been made by a number of authors before (see, e.g., Ref. [31]).

Note that the running behaviors of other parameters, such as the CKM and MNS mixing angles, CP-violating phases and gauge coupling constants, can be obtained from our program in a straightforward way. But here we only concentrate on the outputs of running quark and lepton masses.

IV. SUMMARY

In this paper, we have updated the running masses of quarks and leptons both in the SM and in the MSSM at various energy scales, including $\mu = M_Z$, 1 TeV , 10^9 GeV , 10^{12} GeV , and Λ_{GUT} . Our motivation is simple but meaningful: we want to provide a reliable and up-to-date table of the running quark and lepton masses for particle physicists. Such a table will be very useful for the analysis of hadronic physics at low energies and for the building of new physics models at superhigh energies.

The differences between our work and the previous works (in particular, Ref. [10]) have been summarized in the introductory section of this paper. For instance, the central value of the strange quark mass m_s at M_Z is about 55 MeV today, but it was about 93 MeV as given in Ref. [10] about a decade ago. Hence it makes sense to recalculate the values of m_s and other fermion masses at different energy scales. In particular, we hope that our new results for the running masses of leptons and quarks may help the model builders to get new insight into the flavor dynamics at the energy frontier set by the LHC and in the exciting era of precision neutrino physics.

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- [1] S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967); A. Salam, *Elementary Particle Theory*, edited by N. Svartholm (Almqvist and Wiksells, Stockholm, 1969), p. 367; S. L. Glashow, J. Iliopoulos, and L. Maiani, *Phys. Rev. D* **2**, 1285 (1970).
 - [2] H. Fritzsch and Z. Z. Xing, *Prog. Part. Nucl. Phys.* **45**, 1 (2000).
 - [3] Q. R. Ahmad *et al.* (SNO Collaboration), *Phys. Rev. Lett.* **89**, 011301 (2002).
 - [4] For a review, see: C. K. Jung *et al.*, *Annu. Rev. Nucl. Part. Sci.* **51**, 451 (2001).
 - [5] K. Eguchi *et al.* (KamLAND Collaboration), *Phys. Rev. Lett.* **90**, 021802 (2003).
 - [6] M. H. Ahn *et al.* (K2K Collaboration), *Phys. Rev. Lett.* **90**, 041801 (2003).
 - [7] H. Georgi and S. L. Glashow, *Phys. Rev. Lett.* **32**, 438 (1974); H. Georgi, H. R. Quinn, and S. Weinberg, *Phys. Rev. Lett.* **33**, 451 (1974); H. Fritzsch and P. Minkowski, *Ann. Phys. (N.Y.)* **93**, 193 (1975).
 - [8] G. 't Hooft, NATO advanced study institutes series Series B, *Physics* **59**, 135 (1980).
 - [9] M. Gell-Mann and F. E. Low, *Phys. Rev.* **95**, 1300 (1954);

- C. G. Callan, Phys. Rev. D **2**, 1541 (1970); K. Symanzik, Commun. Math. Phys. **18**, 227 (1970); **23**, 49 (1971).
- [10] H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998).
- [11] W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).
- [12] P. Minkowski, Phys. Lett. B **67**, 421 (1977); T. Yanagida, in *Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979); M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979); S. L. Glashow, in *Quarks and Leptons*, edited by M. Lèvy *et al.* (Plenum, New York, 1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980).
- [13] A. Strumia and F. Vissani, Nucl. Phys. **B726**, 294 (2005); arXiv:hep-ph/0606054.
- [14] E. Komatsu *et al.*, arXiv:0803.0547.
- [15] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).
- [16] R. Barbieri, P. Creminelli, A. Strumia, and N. Tetradis, Nucl. Phys. **B575**, 61 (2000); T. Endoh, T. Morozumi, and Z. H. Xiong, Prog. Theor. Phys. **111**, 123 (2004); A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada, and A. Riotto, J. Cosmol. Astropart. Phys. **04** (2006) 004; E. Nardi, Y. Nir, E. Roulet, and J. Racker, J. High Energy Phys. **01** (2006) 164; A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada, and A. Riotto, J. High Energy Phys. **09** (2006) 010; S. Antusch, S. F. King, and A. Riotto, J. Cosmol. Astropart. Phys. **11** (2006) 011; S. Pascoli, S. T. Petcov, and A. Riotto, Phys. Rev. D **75**, 083511 (2007); G. C. Branco, R. Gonzalez Felipe, and F. R. Joaquim, Phys. Lett. B **645**, 432 (2007); S. Blanchet, P. Di Bari, and G. G. Raffelt, J. Cosmol. Astropart. Phys. **03** (2007) 012; S. Pascoli, S. T. Petcov, and A. Riotto, Nucl. Phys. **B774**, 1 (2007); A. De Simone and A. Riotto, J. Cosmol. Astropart. Phys. **02** (2007) 005; Z. Z. Xing and S. Zhou, Phys. Lett. B **653**, 278 (2007); S. Antusch, Phys. Rev. D **76**, 023512 (2007).
- [17] K. G. Chetyrkin, B. A. Kniehl, and M. Steinhauser, Nucl. Phys. **B510**, 61 (1998); K. G. Chetyrkin, J. H. Kühn, and M. Steinhauser, Comput. Phys. Commun. **133**, 43 (2000).
- [18] K. G. Chetyrkin, B. A. Kniehl, and M. Steinhauser, Phys. Rev. Lett. **79**, 2184 (1997); B. A. Kniehl, A. V. Kotikov, A. I. Onishchenko, and O. L. Veretin, Phys. Rev. Lett. **97**, 042001 (2006).
- [19] A. V. Manohar and C. T. Sachrajda, review on quark masses in Ref. [11].
- [20] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).
- [21] O. V. Tarasov, Dubna Report No. JINR P2-82-900, 1982 (unpublished).
- [22] J. A. M. Vermaseren, S. A. Larin, and T. van Ritbergen, Phys. Lett. B **405**, 327 (1997).
- [23] K. G. Chetyrkin, Phys. Lett. B **404**, 161 (1997).
- [24] N. Gray, D. J. Broadhurst, W. Grafe, and K. Schilcher, Z. Phys. C **48**, 673 (1990); R. Hempfling and B. A. Kniehl, Phys. Rev. D **51**, 1386 (1995); K. G. Chetyrkin and M. Steinhauser, Phys. Rev. Lett. **83**, 4001 (1999); Nucl. Phys. **B573**, 617 (2000); K. Melnikov and T. van Ritbergen, Phys. Lett. B **482**, 99 (2000).
- [25] M. C. Smith and S. S. Willenbrock, Phys. Rev. Lett. **79**, 3825 (1997).
- [26] W. Bernreuther and W. Wetzel, Nucl. Phys. **B197**, 228 (1982); **B513**, 758(E) (1998); W. Bernreuther, Ann. Phys. (N.Y.) **151**, 127 (1983); S. A. Larin, T. van Ritbergen, and J. A. M. Vermaseren, Nucl. Phys. **B438**, 278 (1995).
- [27] P. J. Mohr and B. N. Taylor, Rev. Mod. Phys. **72**, 351 (2000).
- [28] J. Erler, Phys. Rev. D **59**, 054008 (1999).
- [29] H. Arason *et al.*, Phys. Rev. D **46**, 3945 (1992).
- [30] M. Lindner, M. Ratz, and M. A. Schmidt, J. High Energy Phys. **09** (2005) 081; C. Hagedorn and W. Rodejohann, J. High Energy Phys. **07** (2005) 034; Z. Z. Xing and H. Zhang, Commun. Theor. Phys. **48**, 525 (2007).
- [31] S. F. King and N. N. Singh, Nucl. Phys. **B591**, 3 (2000); S. Antusch, J. Kersten, M. Lindner, and M. Ratz, Phys. Lett. B **544**, 1 (2002); S. Antusch and M. Ratz, J. High Energy Phys. **11** (2002) 010; J. W. Mei and Z. Z. Xing, Phys. Rev. D **70**, 053002 (2004); Phys. Lett. B **623**, 227 (2005); S. Antusch, J. Kersten, M. Lindner, M. Ratz, and M. A. Schmidt, J. High Energy Phys. **03** (2005) 024; J. W. Mei, Phys. Rev. D **71**, 073012 (2005).
- [32] T. P. Cheng, E. Eichten, and L. F. Li, Phys. Rev. D **9**, 2259 (1974); M. Machack and M. Vaughn, Nucl. Phys. **B236**, 221 (1984); V. D. Barger, M. S. Berger, and P. Ohmann, Phys. Rev. D **47**, 1093 (1993).
- [33] W. Siegel, Phys. Lett. **84B**, 193 (1979).
- [34] H. Baer, J. Ferrandis, K. Melnikov, and X. Tata, Phys. Rev. D **66**, 074007 (2002).
- [35] R. V. Harlander, L. Mihaila, and M. Steinhauser, Phys. Rev. D **76**, 055002 (2007).
- [36] R. V. Harlander *et al.*, J. High Energy Phys. **12** (2006) 024.
- [37] H. Baer, J. Ferrandis, S. Kraml, and W. Porod, Phys. Rev. D **73**, 015010 (2006).
- [38] G. Ross and M. Serna, arXiv:0704.1248.
- [39] S. Antusch and M. Spinrath, arXiv:0804.0717.
- [40] M. Carena, D. Garcia, U. Nierste, and C. E. M. Wagner, Nucl. Phys. **B577**, 88 (2000); A. J. Buras, P. H. Chankowski, J. Rosiek, and L. Slawianowska, Nucl. Phys. **B659**, 3 (2003).
- [41] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
- [42] H. Fritzsch and Z. Z. Xing, Phys. Lett. B **353**, 114 (1995); **413**, 396 (1997); Phys. Rev. D **57**, 594 (1998).
- [43] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
- [44] See, e.g., W. Chao, S. Luo, Z. Z. Xing, and S. Zhou, Phys. Rev. D **77**, 016001 (2008); Z. Z. Xing, Phys. Lett. B **660**, 515 (2008); and references therein.
- [45] H. Fritzsch and Z. Z. Xing, Phys. Lett. B **517**, 363 (2001); Z. Z. Xing, Int. J. Mod. Phys. A **19**, 1 (2004).