Softly broken $\mu \leftrightarrow \tau$ symmetry in the minimal seesaw model

Juan Carlos Gómez-Izquierdo* and Abdel Pérez-Lorenzana+

Departamento de Física, Centro de Investigación y de Estudios Avanzados del I.P.N. Apdo. Post. 14-740, 07000, México, D.F., México

(Received 19 December 2007; published 26 June 2008)

Neutrino oscillations data indicates that neutrino mixings are consistent with an apparent $\nu_\mu - \nu_\tau$ exchange symmetry in neutrino mass matrix. We observe that in the minimally extended standard model with the seesaw mechanism, one can impose $\mu \leftrightarrow \tau$ symmetry at the tree level on all Lagrangian terms, but for the mass difference among μ and τ leptons. In the absence of any new extra physics, this mass difference becomes the only source for the breaking of such a symmetry, which induces, via quantum corrections, small but predictable values for θ_{13}, and for the deviation of $\theta_{\rm ATM}$ from maximality. In the *CP* conserving case, the predictions only depend on neutrino mass hierarchy and may provide a unique way to test for new physics with neutrino experiments.

DOI: 10.1103/PhysRevD.77.113015 PACS numbers: 14.60.Pq, 11.30.Fs, 12.15.Ff, 12.60.−i

I. INTRODUCTION

Convincing evidence that neutrinos have mass and oscillate has been provided in recent years by neutrino oscillation experiments [1]. In the standard framework, only three weak neutrino species, ν_e, ν_μ, and ν_τ, are needed to consistently describe the experimental results, with the addition of neutrino masses and mixings as new parameters to the standard model. The central idea is that neutrino mass eigenstates, $\nu_{1,2,3}$, and weak eigenstates are different, but they can be written as linear combinations of each other by using a complex unitary matrix, U, as $\nu_\ell = \sum_i U_{\ell i}\nu_i$, for $\ell = e, \mu, \tau$ and $i = 1, 2, 3$, where we refer only to left-handed states. A common parametrization for Majorana neutrinos of the U matrix is given in terms of three angles and three *CP* phases, such that $U = VK$, where $K = \mathrm{diag}\{1, e^{i\phi_1}, e^{i\phi_2}\}$, with ϕ_1, ϕ_2 the physical *CP*-odd Majorana phases, and the elements of the V mixing matrix parametrized as [2]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & z^* \\ -s_{12}c_{23} - c_{12}s_{23}z & c_{12}c_{23} - s_{12}s_{23}z & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}z & -c_{12}s_{23} - s_{12}c_{23}z & c_{23}c_{13} \end{pmatrix};$$

where c_{ij} and s_{ij} stand for $\cos\theta_{ij}$ and $\sin\theta_{ij}$ respectively and $z = s_{13}e^{i\varphi}$. The kinematical scales for the oscillation, on the other hand, are given by the two mass squared differences: the solar/KamLAND scale $\Delta m^2_\odot = \Delta m^2_{12}$; and the atmospheric scale $\Delta m^2_{\rm ATM} = |\Delta m_{23}|^2 \approx |\Delta m_{13}|^2$. Combined analysis of all data [1] indicates that at two sigma level $\Delta m^2_\odot = 7.6^{+0.5}_{-0.3} \times 10^{-5}$ eV^2; $\Delta m^2_{\rm ATM} = 2.4 \pm 0.3 \times 10^{-3}$ eV^2, whereas $\sin^2\theta_{12} = 0.32^{+0.05}_{-0.04}$, $\sin^2\theta_{\rm ATM} = \sin^2\theta_{23} = 0.5^{+0.13}_{-0.12}$, and $\sin^2\theta_{13} \leq 0.033$. Thus, data is consistent with $\theta_{13} \approx 0$, and $\theta_{\rm ATM} \approx \pi/4$, which makes the Dirac *CP* phase φ hard to be measured. Current and new experiments on neutrino physics will explore how small and how maximal, respectively, these mixing are [3], down to the level of a few times 10^{-2}.

Since the standard model was built on the assumption of zero neutrino masses, a fundamental question at this point is whether neutrino mass implies the existence of new physics, and what such physics would be. The answer, however, is not yet conclusive. It is possible to minimally extend the model by only adding three singlet right-handed neutrinos, N_i, to implement the seesaw mechanism [4], and accommodate data, without relaying in any new extra ingredient. This makes, however, the identification of any new extra physics from low energy phenomenology a difficult task. The above picture explains very well the smallness of neutrino masses, but provides no understanding for the mixings. To provide such understanding, one usually is led to invoke theoretical arguments, and many ideas exist nowadays in the literature.

It has already been observed that, in the limit with a null θ_{13} and a maximal $\theta_{\rm ATM}$, and on the basis where charged lepton masses are diagonal, the reconstructed neutrino seesaw mass matrix, $M_{\ell\ell'} = \sum_{i=1}^3 U^*_{\ell i} m_i U_{\ell' i}$, posses a $\nu_\mu - \nu_\tau$ exchange symmetry [5]. This has inspired a large number of theoretical studies [6]. Remarkably, imposing the suggested $\mu \leftrightarrow \tau$ symmetry is very possible within the minimal seesaw extension of the standard model, and it is our goal to show that the simplest realization of this idea provides a perfectly falsifiable model, with specific predictions that can easily be proved wrong by future neutrino data. Our findings, however, would show that with these minimal ingredients the prediction for both θ_{13}, and the deviation of $\theta_{\rm ATM}$ from maximality are much smaller than the forthcoming experimental sensitivities. Nevertheless, there is a positive outcome—our results establish a comparative point of reference so as to take any possible measurement of a nonzero value for those mixing parameters in near future experiments as clear indications for the existence of new physics.

It is not difficult to see that $\mu \leftrightarrow \tau$ is already a flavor symmetry in the standard model, but for the charged lepton

*jcarlos@fis.cinvestav.mx
+aplorenz@fis.cinvestav.mx

mass terms, where clearly $m_\tau \neq m_\mu$. Thus, we propose to treat $\mu \leftrightarrow \tau$ as a softly broken symmetry of the minimal seesaw model. Therefore, at tree level, all physics not directly related to $m_{\mu,\tau}$ would be described by the symmetric limit, allowing us to fix the free parameters of the model at low energy. Nevertheless, quantum corrections shall communicate the symmetry breaking to the neutrino sector [7]. In particular, one-loop corrections will already produce small deviations to $\theta_{\rm ATM}$ from $\pi/4$, and a nonzero θ_{13}. Because the model has no extra unknown ingredients, one can make definite predictions for these physical observables in terms of symmetric level results. Those are the main points we want to discuss in what follows.

II. THE MINIMAL $\mu \leftrightarrow \tau$ MODEL

The model we will explore considers, first, the minimal seesaw extension that includes three right-handed neutrinos, with all additional Lagrangian terms that are consistent with the standard model symmetries,

$$h_\ell \bar{L}_\ell H \ell_R + y_{\ell\ell'} \bar{L}_\ell \tilde{H} N_{\ell'} + (\text{H.c.}) + (M_R)_{\ell\ell'} \bar{N}^c_\ell N_{\ell'}, \quad (1)$$

where sum over indices should be understood. Here, L_ℓ stands for the standard lepton doublets and H for the Higgs field. In order to implement $\mu \leftrightarrow \tau$ symmetry in a meaningful way, we have chosen to work in the basis where the charge lepton Yukawa couplings, and so their masses, are diagonal and real. Also, we have chosen right-handed neutrinos to carry lepton number, and properly identified the index. It is worth mentioning that if $N_{i\neq\ell}$ were not subjected to $\mu \leftrightarrow \tau$ symmetry, as defined below, then, neutrino Yukawa couplings would become such that $y_{\mu i} = y_{\tau i}$, under $\mu \leftrightarrow \tau$ symmetry, regardless of the chosen basis for N_i. Following this implied degeneracy of second and third rows on the Dirac mass matrix, the left-handed massless neutrino state $\nu' = (\nu_\mu - \nu_\tau)/\sqrt{2}$ arises. Clearly, this corresponds to the third mass eigenstate in an inverted hierarchy scenario (similar results were recently found in Ref. [8]).

Next, to realize the symmetry, we require both Yukawa couplings and the Majorana mass matrix to be invariant under $\mu \leftrightarrow \tau$ exchange: $L_\mu \leftrightarrow L_\tau$; $\mu_R \leftrightarrow \tau_R$; and $N_\mu \leftrightarrow N_\tau$. One can then proceed with the diagonalization of the mass matrices. However, the analysis for the low energy phenomenology is simplified by first implementing the seesaw mechanism and observing that $\mu \leftrightarrow \tau$ symmetry also holds for the effective left-handed neutrino mass matrix, $M = -m_D M_R^{-1} m_D^T$, with m_D the Dirac neutrino mass matrix. Here, the symmetry expresses itself by two simple conditions on the matrix elements: $M_{e\mu} = M_{e\tau}$ and $M_{\mu\mu} = M_{\tau\tau}$. Thus, the most general tree-level form for M should be

$$M = M_{\mu\leftrightarrow\tau} = \begin{pmatrix} m^0_{ee} & m^0_{e\mu} & m^0_{e\mu} \\ m^0_{e\mu} & m^0_{\mu\mu} & m^0_{\mu\tau} \\ m^0_{e\mu} & m^0_{\mu\tau} & m^0_{\mu\mu} \end{pmatrix}. \quad (2)$$

Diagonalization of such a mass matrix is rather simple. We find the mass eigenvalues:

$$m_1 = m^0_{ee} - \sqrt{2}\tan\theta_{12} m^0_{e\mu};$$
$$m_2 = m^0_{ee} + \sqrt{2}\cot\theta_{12} m^0_{e\mu}; \qquad m_3 = m^0_{\mu\mu} - m^0_{\mu\tau}, \quad (3)$$

and get for the mixing angles $\theta_{\rm ATM} = \pi/4$, and $\theta_{13} = 0$, whereas the solar mixing angle is given by

$$\tan 2\theta_{12} = \frac{\sqrt{8} m^0_{e\mu}}{m^0_{\mu\mu} + m^0_{\mu\tau} - m^0_{ee}}. \quad (4)$$

Since $\sin\theta_{13} = 0$, the Dirac CP phase φ gets undefined. Thus, in this model only the two CP Majorana phases may exist at the symmetric limit. To keep our present discussion simple, we will assume them to be zero along the analysis, and so we shall take all mass parameters in Eq. (2) to be real. The analysis including CP phases will be presented elsewhere.

It is worth noticing that the seesaw mass matrix in Eq. (2) is described by only four parameters, which can be entirely fixed by the following four low energy observables: the solar mixing (θ_{12}), the mass hierarchy (m_3), solar scale $\Delta m^2_\odot = m_2^2 - m_1^2$, and the atmospheric scale that we can take as $\Delta m^2_{\rm ATM} = \frac{1}{2}|\Delta m^2_{13} + \Delta m^2_{23}|$. Of course, extra parameters exist for the whole theory [see Eq. (1)]. They belong to the high energy right-handed neutrino sector and cannot be fixed from these results. However, as we will show below, those parameters will not be required to make further predictions for the low energy physics.

III. SOFT BREAKING OF $\mu \leftrightarrow \tau$ SYMMETRY

Exact $\mu \leftrightarrow \tau$ symmetry would also imply that $h_\mu = h_\tau$, which gives the wrong result $m_\mu = m_\tau$. This is the only place where the symmetry is being explicitly broken. Henceforth, we will take $h_\mu \neq h_\tau$, a choice that of course respects all gauge symmetries, and by definition it is expected to be valid at any energy. We shall not assume any dynamical origin for such a difference on the Yukawa couplings in order to keep the model truly minimal. Notice that, as a matter of fact, all leptonic kinetic terms in the standard model, $i\bar{L}_\ell \gamma^\mu D_{\mu L} L_\ell + i\bar{\ell}_R \gamma^\mu D_{\mu R} \ell_R$ with $D^\mu_{L,R}$ the corresponding covariant derivatives, are invariant under $\mu \leftrightarrow \tau$ exchange due to the universality of gauge interactions. In contrast, out of the Lagrangian terms given in Eq. (1), the charged lepton Yukawa couplings now have the form

$$h_e \bar{L}_e H e_R + h_\mu \bar{L}_\tau H \mu_R + (h_\mu + \delta h) \bar{L}_\tau H \tau_R + \text{H.c.}, \quad (5)$$

whereas all other terms remain symmetric. Therefore, the whole tree-level Lagrangian of the model can be written as

$\mathcal{L} = \mathcal{L}_{\mu\leftrightarrow\tau} + (\delta h \bar{L}_\tau H \tau_R + \text{H.c.})$. Here, $\mathcal{L}_{\mu\leftrightarrow\tau}$ contains all symmetric terms, whereas the last term will generate the mass term $\delta m \bar{\tau}\tau$, upon standard gauge symmetry breaking. The last can be seen as a soft term whose role is to correct the mass of the tau lepton.

The breaking down of $\mu \leftrightarrow \tau$ symmetry will be communicated to all other sectors of the model via weak interactions [7]. Particularly, after including one-loop quantum corrections, muon-tau mass difference will generate a splitting in the symmetry conditions of the seesaw mass matrix, such that one should rather have $M_{e\tau} \neq M_{e\mu}$ and $M_{\tau\tau} \neq M_{\mu\mu}$, where the departure would be expected to be small due to the W mass suppressions, but calculable (see Ref. [9] for a general analysis of quantum corrections on M_ν). Notice that our calculation will be done at the low scale where observable neutrino mass parameters are being measured. No running from renormalization group equations is included, which is also known to produce very mild effects on the mixings that concern us here (for related works considering renormalization group corrections on neutrino mixings see for instance references in [10,11]). As a matter of fact, at one-loop order it is easy to see that the only diagram contributing to neutrino mass corrections which is not $\mu \leftrightarrow \tau$ invariant is the one given in Fig. 1, which explicitly involves the exchange of charge leptons through W couplings. The violation of the symmetry conditions would imply both a nonzero value for θ_{13} and the departure of θ_{ATM} from maximal, which as we will show next are completely predictable and correlated.

Therefore, after including one-loop corrections the neutrino mass matrix gets the more general form

$$M = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & m_{\mu\mu} & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & m_{\tau\tau} \end{pmatrix}, \tag{6}$$

which is now written in terms of the corrected mass parameters given by $m_{\ell\ell'} = m^0_{\ell\ell'} + I_{\ell\ell''} m^0_{\ell''\ell'} + m^0_{\ell\ell''} I_{\ell''\ell'}$, with $I_{\ell\ell'}$ the one-loop finite contributions to mass terms that come from all possible one-loop diagrams. The former matrix can be written as $M = M_{\mu\leftrightarrow\tau} + \delta M$ where the symmetric part, $M_{\mu\leftrightarrow\tau}$ has a similar parametrization as in Eq. (2), although now in terms of corrected masses. On the other hand δM encodes the only two symmetry breaking conditions, that at the lower order are, respectively, given

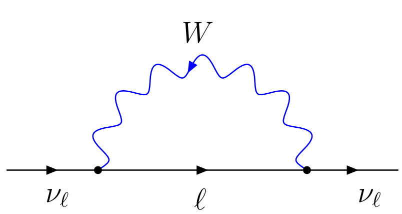

FIG. 1 (color online). 1-loop diagram that communicates the breaking of $\mu \leftrightarrow \tau$ symmetry to neutrino mass matrix.

by $\delta M_{e\tau} \equiv m_{e\tau} - m_{e\mu} \approx m^0_{e\mu}\Delta I$, and $\delta M_{\tau\tau} \equiv m_{\tau\tau} - m_{\mu\mu} \approx 2m^0_{\mu\mu}\Delta I$, where $\Delta I \equiv I_\tau - I_\mu$ with I_ℓ the one-loop contributions obtained from the diagram in Fig. 1 for the corresponding charged lepton ℓ. A quite lengthy calculation shows that

$$\Delta I \approx \frac{3g_W^2}{32\pi^2}\left[\left(\frac{m_\tau}{M_W}\right)^2 \ln\left(\frac{m_\tau}{M_W}\right) - (\tau \to \mu)\right], \tag{7}$$

which gives $\Delta I \approx -7.68 \times 10^{-6}$.

Because of the smallness of ΔI, the neutrino mass matrix in Eq. (6) can be diagonalized considering expressions up to linear order corrections in ΔI. Interestingly enough, the effect on neutrino masses and solar mixing enters as a slight modification of previous formulas that consists on the sole replacing of $m_{e\mu}$ and $m_{\mu\mu}$ by the average values $\bar{m}_{e\mu} = \frac{1}{2}(m_{e\mu} + m_{e\tau})$, and $\bar{m}_{\mu\mu} = \frac{1}{2} \times (m_{\mu\mu} + m_{\tau\tau})$, respectively, in Eqs. (3) and (4), such that we now get

$$\begin{aligned} m_1 &\approx m_{ee} - \sqrt{2}\tan\theta_{12}\bar{m}_{e\mu}; \\ m_2 &\approx m_{ee} + \sqrt{2}\cot\theta_{12}\bar{m}_{e\mu}; \\ m_3 &\approx \bar{m}_{\mu\mu} - m_{\mu\tau}, \\ \tan 2\theta_{12} &\approx \frac{\sqrt{8}\bar{m}_{e\mu}}{\bar{m}_{\mu\mu} + m_{\mu\tau} - m_{ee}}. \end{aligned} \tag{8}$$

As already observed, one can invert these equations to express the involved neutrino mass parameters in terms of neutrino observables and m_3 as the hierarchy parameter, by using $|m_1| \approx \sqrt{m_3^2 \mp \Delta m^2_{\text{ATM}} - \frac{1}{2}\Delta m^2_\odot}$, and $|m_2| \approx \sqrt{m_3^2 \mp \Delta m^2_{\text{ATM}} + \frac{1}{2}\Delta m^2_\odot}$, where the minus (plus) sign corresponds to normal (inverted) hierarchy. After some algebra one gets

$$\begin{aligned} m_{ee} &\approx m_1\cos^2\theta_{12} + m_2\sin^2\theta_{12}; \\ \bar{m}_{e\mu} &\approx \frac{1}{\sqrt{8}}\sin 2\theta_{12}(m_2 - m_1); \\ \bar{m}_{\mu\mu} &\approx \frac{1}{2}(m_1\sin^2\theta_{12} + m_2\cos^2\theta_{12} + m_3); \\ m_{\mu\tau} &\approx \frac{1}{2}(m_1\sin^2\theta_{12} + m_2\cos^2\theta_{12} - m_3). \end{aligned} \tag{9}$$

Next, we get the following predictions for other mixings, at the lower order,

$$\sin\theta_{13} \approx -\frac{\Delta I}{\sqrt{2}}\left(\frac{m_3\bar{m}_{e\mu}}{\bar{m}_{e\mu}{}^2 + m_{\mu\tau}m_*}\right) \tag{10}$$

with $m_* = m_3 - m_{ee}$, and

$$\sin\alpha = \frac{\Delta I}{2}\left(\frac{\bar{m}_{e\mu}{}^2 + \bar{m}_{\mu\mu}m_*}{\bar{m}_{e\mu}{}^2 + m_{\mu\tau}m_*}\right) \tag{11}$$

for the deviation of θ_{ATM} from maximality, where we have defined $\alpha = \theta_{\text{ATM}} - \pi/4$. In those formulas we have conveniently approximated $m^0_{e\mu} \approx \bar{m}_{e\mu}$, and $m^0_{\mu\mu} \approx \bar{m}_{\mu\mu}$, by using the tree level formulas in Eqs. (3) and (4), and comparing them with Eqs. (8). Thus, by committing an small error of order $\sim(\Delta I)^2$, this approach allows us to express $\sin\theta_{13}$ and $\sin\alpha$ in terms of all the tree level quantities obtained from neutrino low energy observables given in Eq. (9). Predicted values, however, depend not only on the hierarchy but also on the relative signs among the mass eigenvalues. This becomes clear if we study, for instance, the expressions (10) and (11) in the limit of almost degenerate neutrinos, for which the relative sign among m_1 and m_2 may enhance or suppress the contribution of $\bar{m}_{e\mu}$. These are our findings:

First, we get the approximated formulas

$$\sin\theta_{13} \approx A \cdot \frac{m_3^2 \sin 2\theta_{12}}{\Delta m^2_{\rm ATM}} \Delta I, \qquad \sin\alpha \approx \mp B \cdot \frac{2m_3^2}{\Delta m^2_{\rm ATM}} \Delta I, \tag{12}$$

where A and B coefficients are given as

(i) $A = \Delta m^2_\odot/\Delta m^2_{\rm ATM}$, and $B = 1$ for all $m_{1,2,3} > 0$;
(ii) $A = \pm 1$ and $B = c^2_{12}$ for $m_1 < 0$ and $m_{2,3} > 0$; and
(iii) $A = \mp 1$ and $B = s^2_{12}$ for $m_{1,3} > 0$ but $m_2 < 0$;

where, the upper (lower) signs corresponds to normal (inverted) hierarchy. Notice that second and third cases predict $|\sin\theta_{13}| \approx 5 \times 10^{-4}(m_3/0.4\ {\rm eV})^2$, which is larger than the case (i) prediction by a factor of about 33, whereas in all cases $|\sin\alpha| \approx B \times 10^{-3}(m_3/0.4\ {\rm eV})^2$, and so, it comes about the same order.

Finally, (iv), for $m_3 > 0$ and $m_{1,2} < 0$ one obtains

$$\sin\theta_{13} \approx -\frac{\Delta m^2_\odot \sin 2\theta_{12}}{16 m_3^2}\Delta I, \qquad \sin\alpha \approx \mp \frac{\Delta m^2_{\rm ATM}}{8m_3^2}\Delta I, \tag{13}$$

which indicates that smaller values with respect to other cases would be expected. Notice that the inverse squared m_3 mass dependence is only valid in the almost degenerate limit we are considering. For the hierarchical case former formulas become $\sin\theta_{13} \approx \sin\alpha \sim -\Delta I/2$ for normal hierarchy, whereas $\sin\alpha \approx \Delta I/2$ and $\sin\theta_{13} \sim m\Delta m^2_\odot/(\Delta m^2_{\rm ATM})^{3/2}$ for inverted hierarchy.

From the above results, it is clear that $\theta_{\rm ATM}$ should be on the first (second) octant for normal (inverted) hierarchy, whereas experimental determination of the sign of θ_{13} would discriminate cases (i) and (ii) from other ones. A measurement of $|\sin\alpha|$ would finally resolve the scenario. To get the whole picture of the parameter region that experiments should reach to test the present model, we present in Fig. 2 the two sigma regions for our predicted values of $|\sin\theta_{13}|$ and $|\sin\alpha|$ for cases (i) to (iii). Results from case (iv) are simply out of range. Plot points were obtained from a direct numerical calculation using Eqs. (10) and (11) for $m_3 < 0.4$ eV. Absolute values are

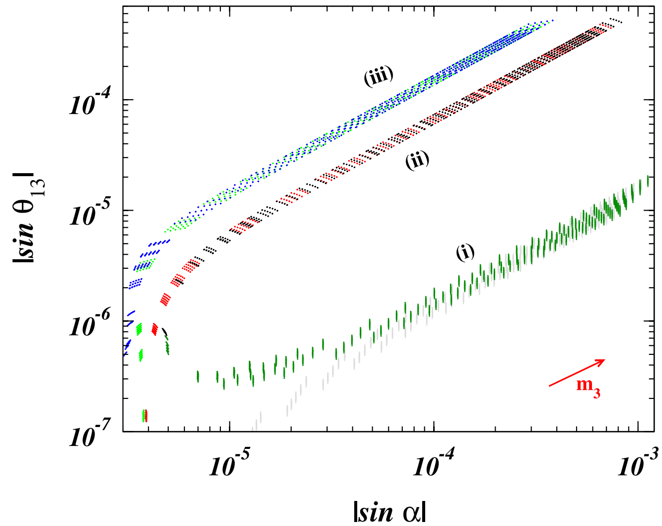

FIG. 2 (color online). Two sigma regions for $|\sin\theta_{13}|$ and $|\sin\alpha|$ predicted from the soft $\mu \leftrightarrow \tau$ model, for both normal and inverted hierarchies. Results correspond to cases (i) to (iii) as discussed in the text, for $m_3 < 0.4$ eV running from small to larger values, as indicated.

used to depict all results together. Notice that hierarchy makes a clear difference for low m_3 values in case (i) for which the upper band on the lower left corresponds to normal hierarchy. Differences also exist on other cases for small m_3, although they are less evident.

From the figure, we notice that predicted values are rather small, as we expected. Both $\sin\theta_{13}$ and $\sin\alpha$ are below 10^{-3}, which is clearly far below the expected sensitivity of the near future forthcoming experiments. Thus we would have to wait for a distant future experiment to test the depicted parameter zone to get a positive signal for the model. Nevertheless, if experiments determine values out of these regions, which could happen in the near future, that would be a clear indication that, either, (a) new physics beyond the standard model is involved in the breaking of $\mu \leftrightarrow \tau$, and the generation of the δM corrections, or (b) $\mu \leftrightarrow \tau$ is not a good symmetry to guide model building. The symmetry seems so natural that, from our point of view, it would be more likely that the first option would be the correct one in such a case.

IV. LEPTON FLAVOR VIOLATION PROCESSES

Before closing our discussion it is worth mentioning another direct implication of our model for lepton physics. Since the breaking effects of $\mu \leftrightarrow \tau$ symmetry are rather small, lepton number violation processes would be ruled in a good approximation by this symmetry. Besides, due to the lack of beyond standard model physics in our model, only W exchanged diagrams would contribute to such processes, and because they are proportional to neutrino squared masses and mixings, they are predicted to be extremely small. This provides another clear way to determine the existence of new physics if any observable effect associated to lepton flavor violating processes is detected in near future experiments.

In particular, the decay ratios for $\mu \to e\gamma$ and $\tau \to e\gamma$ at one-loop order would be [12]

$$\Gamma(\ell \to e\gamma) \approx \frac{\alpha}{4\pi^4} G_F^2 \sin^2 2\theta_\odot (\Delta m_\odot^2)^2 m_\ell, \qquad (14)$$

for $\ell = \mu$, τ and $\alpha = e^2/4\pi$. Therefore one gets the relation $\Gamma(\mu \to e\gamma)/\Gamma(\tau \to e\gamma) \approx m_\mu/m_\tau \approx 0.06$, which means $B(\mu \to e\gamma)/B(\tau \to e\gamma) \approx m_\mu\Gamma_\mu/m_\tau\Gamma_\tau \sim 8 \times 10^{-8}$ for the corresponding branching ratios.

Notice that the overall factor $G_F^2(\Delta m_\odot^2)^2$ is already too small to provide any visible effect within the reach of current and near future experimental sensitivities. Indeed, a straightforward calculation for the branching ratio, say for instance for $\mu \to e\gamma$, gives

$$B(\mu \to e\gamma) \approx \frac{48\alpha}{\pi} \sin^2 2\theta_\odot \frac{(\Delta m_\odot^2)^2}{m_\mu^4}, \qquad (15)$$

which is about 5×10^{-41}. Tau decay into muon-gamma, on the other hand, has the rate

$$\Gamma(\tau \to \mu\gamma) \approx \frac{(\Delta m_{\rm ATM}^2)^2}{\sin^2 2\theta_\odot (\Delta m_\odot^2)^2} \cdot \Gamma(\tau \to e\gamma). \qquad (16)$$

Thus, the branching ratio for $\tau \to \mu\gamma$, although enhanced by a factor of thousand respect to that for $\tau \to e\gamma$, yet remains far from reachable too.

In comparison, $\mu \to eee$ and $\tau \to eee$ decays are expected to be yet more suppressed [12]. Simple $\gamma \to ee$ insertion on previous processes will amount to an extra suppression factor of order α over above results, without altering the relation among the decay rates. Thus one would also get $\Gamma(\mu \to eee)/\Gamma(\tau \to eee) \approx m_\mu/m_\tau$.

V. CONCLUDING REMARKS

Summarizing, we have presented the minimal seesaw model that realizes $\mu \leftrightarrow \tau$ symmetry at tree level in all Lagrangian terms, but for the muon and tau mass difference, which in the absence of any extra new physics becomes the only breaking source for the symmetry. The model predicts, through quantum corrections, small values for θ_{13} and for the deviation of $\theta_{\rm ATM}$ from maximal, which, on the absence of *CP* violation, only depend on neutrino mass hierarchy. We also notice that lepton flavor violation processes are controlled by the $\mu \leftrightarrow \tau$ symmetry. However, the main contributions to such processes come out to be suppressed by a factor of $(G_F \Delta m_\odot^2)^2$, which makes them too small to be reachable by any near future experiment. We stress that even though the above results are difficult to test in any forthcoming experiment, they may have a positive outcome: since we are working in the minimal model that extends the standard model to include neutrino physics parameters, our numerical findings provide a clean point of comparison with the experiment, such as to claim that any positive experimental signal for either a nonzero θ_{13} or α mixing, or for any of the described lepton flavor violation processes, would be a clear indication for the existence of new physics beyond the present setup.

ACKNOWLEDGMENTS

This work was supported in part by CONACyT, México, Grant No. 54576.

[1] M. Maltoni, T. Schwetz, M. A. Tortola, and J. W. F. Valle, New J. Phys. **6**, 122 (2004); M. C. Gonzales-Garcia and M. Maltoni, Phys. Rep. **460**, 1 (2008).

[2] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962); B. Pontecorvo, Zh. Eksp. Teor. Fiz. **53**, 1717 (1968) [Sov. Phys. JETP **26**, 984 (1968)].

[3] K. Anderson *et al.*, arXiv:hep-ex/0402041; F. Ardellier *et al.* (Double Chooz Collaboration), arXiv:hep-ex/0606025; A. B. Balantekin *et al.* (Daya Bay Collaboration), arXiv:hep-ex/0701029; Y. Itow *et al.* (T2K Collaboration), arXiv:hep-ex/0106019; D. S. Ayres *et al.* (NOvA Collaboration), arXiv:hep-ex/0503053; D. G. Michael *et al.* (MINOS Collaboration), Phys. Rev. Lett. **97**, 191801 (2006).

[4] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by D. Freedman and P. van Nieuwenhuizen (North Holland, Amsterdam, 1979), p. 315; T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, Japan, 1979), p. 95; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); Phys. Rev. D **23**, 165 (1981); P. Minkowski, Phys. Lett. **67B**, 421 (1977); J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); **25**, 774 (1982).

[5] R. N. Mohapatra and S. Nussinov, Phys. Rev. D **60**, 013002 (1999); C. S. Lam, Phys. Lett. B **507**, 214 (2001); T. Kitabayashi and M. Yasue, Phys. Rev. D **67**, 015006 (2003); W. Grimus and L. Lavoura, Phys. Lett. B **572**, 189 (2003); Y. Koide, Phys. Rev. D **69**, 093001 (2004).

[6] For an incomplete list, see for instance: P. F. Harrison and W. G. Scott, Phys. Lett. B **547**, 219 (2002); R. N. Mohapatra, J. High Energy Phys. 10 (2004) 027; R. N. Mohapatra, S. Nasri, and H.-B. Yu, Phys. Lett. B **636**, 114 (2006); A. S. Joshipura, Eur. Phys. J. C **53**, 77 (2008); E. Ma, Phys. Rev. D **70**, 031901 (2004); K. S. Babu and R. N. Mohapatra, Phys. Lett. B **532**, 77 (2002); T. Fukuyama

and H. Nishiura, arXiv:hep-ph/9702253; K. Fuki and M. Yasue, Nucl. Phys. **B783**, 31 (2007); A. Goshal, Mod. Phys. Lett. A **19**, 2579 (2004); T. Ohlsson and G. Seidl, Nucl. Phys. **B643**, 247 (2002); Riazuddin, Eur. Phys. J. C **51**, 697 (2007); Y. Koide and E. Takasugi, Phys. Rev. D **77**, 016006 (2008).

[7] S. Weinberg, Phys. Rev. Lett. **29**, 1698 (1972); Phys. Rev. D **7**, 2887 (1973).

[8] T. Baba and M. Yasue, Phys. Rev. D **77**, 075008 (2008).

[9] M. Doi, M. Kenmoku, T. Kotani, and E. Takasugi, Phys. Rev. D **30**, 626 (1984).

[10] M. Tanimoto, Phys. Lett. B **360**, 41 (1995); K. S. Babu, C. N. Leung, and J. T. Pantaleone, Phys. Lett. B **319**, 191 (1993); P. H. Chankowski and Z. Pluciennik, Phys. Lett. B **316**, 312 (1993).

[11] A. Dighe, S. Goswami, and W. Rodejohann, Phys. Rev. D **75**, 073023 (2007); S. Luo and Z. Z. Xing, Phys. Lett. B **632**, 341 (2006); S. Antusch, J. Kersten, M. Lindner, M. Ratz, and M. A. Schmidt, J. High Energy Phys. 03 (2005) 024; J. W. Mei, Phys. Rev. D **71**, 073012 (2005); P. H. Chankowski and S. Pokorski, Int. J. Mod. Phys. A **17**, 575 (2002); S. Antusch, M. Dreeds, J. Kersten, M. Lindner, and M. Ratz, Phys. Lett. B **519**, 238 (2001); J. A. Casas, J. R. Espinosa, A. Ibarra, and I. Navarro, Nucl. Phys. **B573**, 652 (2000).

[12] S. Eliezer and D. A. Ross, Phys. Rev. D **10**, 3088 (1974); T. P. Cheng and L. F. Li, Phys. Rev. D **16**, 1565 (1977); Phys. Rev. Lett. **38**, 381 (1977).